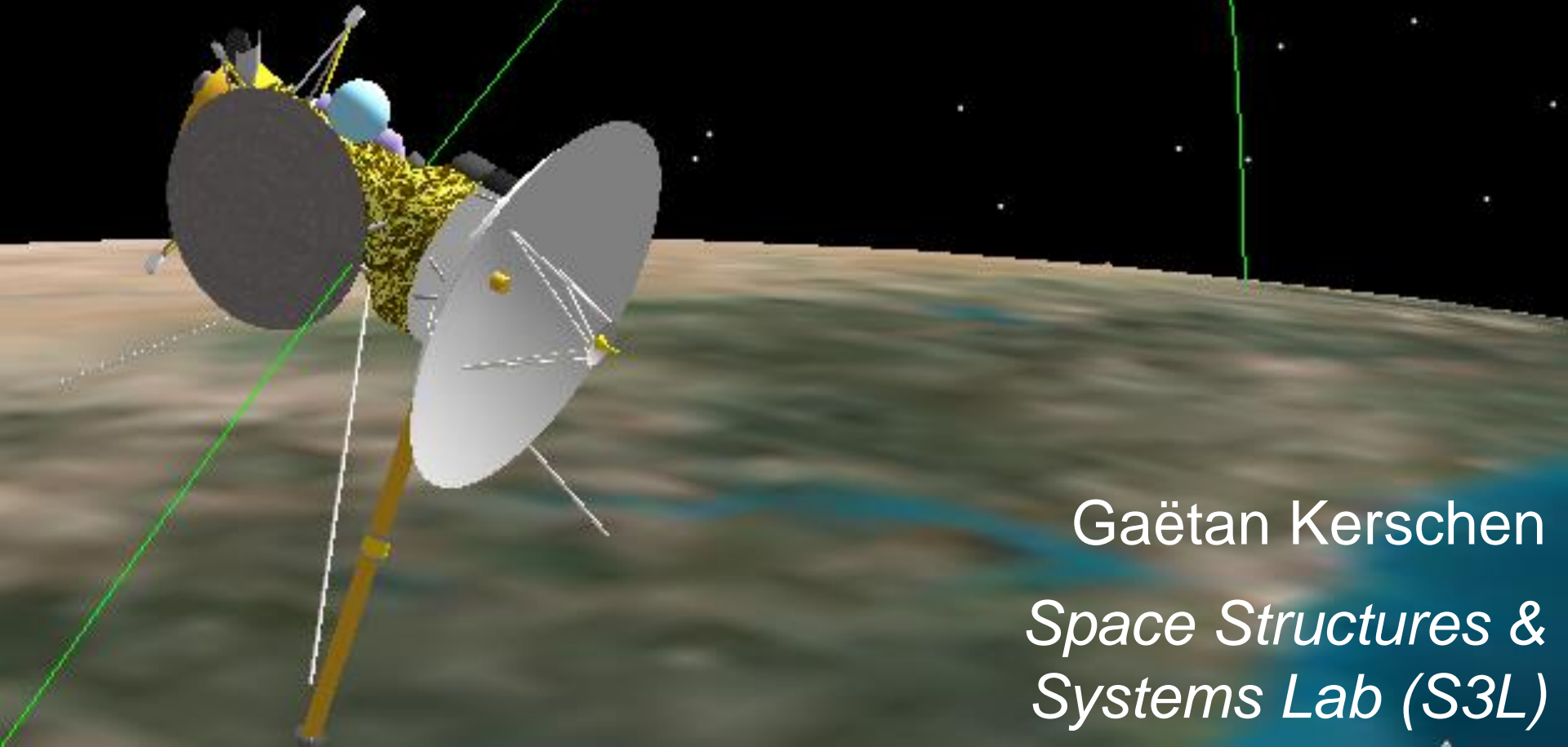


Cassini Classical Orbit Elements
Time (UTCG): 15 Oct 1997 09:18:54.000
Semi-major Axis (km): 6685.637000
Eccentricity: 0.020566
Inclination (deg): 30.000
RAAN (deg): 150.546
Arg of Perigee (deg): 230.000
True Anomaly (deg): 136.530
Mean Anomaly (deg): 134.891

Aerodynamics (AERO0024)

5. Dominant Perturbations



Gaëtan Kerschen
*Space Structures &
Systems Lab (S3L)*

Motivation



Assumption of a two-body system in which the central body acts gravitationally as a point mass.

In many practical situations, a satellite experiences significant perturbations (accelerations).

These perturbations are sufficient to cause predictions of the position of the satellite based on a Keplerian approach to be in significant error in a brief time.

The Effect of Earth Oblateness

Keplerian Parameters	
Semi-major axis [m]	6778e3
Eccentricity	0.0
Inclination [deg]	51
Argument of perigee [deg]	0.0
RAAN [deg]	20
True anomaly [deg]	0.0

Control	Date
<input type="radio"/> None	Year <input type="text" value="2010"/>
<input type="radio"/> Cross-Section	Month <input type="text" value="10"/>
<input type="radio"/> Attitude	Day <input type="text" value="23"/>
	Hours <input type="text" value="19"/>
	Minutes <input type="text" value="40"/>
	Seconds <input type="text" value="00"/>
	Simulation time [s] <input type="text" value="5*24 * 3600"/>

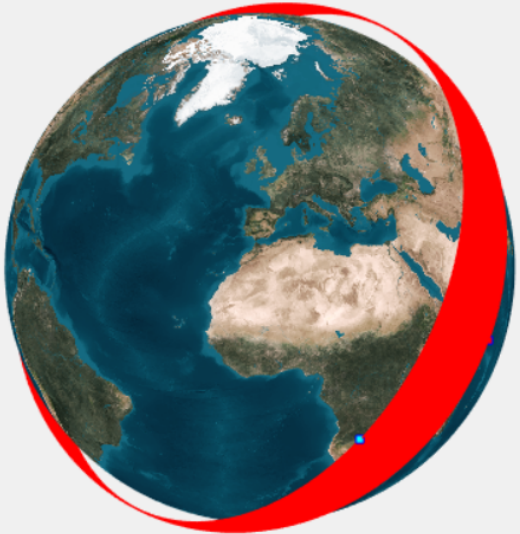
Force Model
<input checked="" type="checkbox"/> Non-spherical
<input type="checkbox"/> Drag
<input type="checkbox"/> SRP
<input type="checkbox"/> Third-body Sun
<input type="checkbox"/> Third-body Moon

ECI to ECEF	Integration Parameters
<input type="checkbox"/> Precession	Relative tolerance <input type="text" value="1e-13"/>
<input type="checkbox"/> Nutation	Absolute tolerance <input type="text" value="1e-13"/>
<input type="checkbox"/> Polar Wandering	Output time step [s] <input type="text" value="60"/>
<input type="button" value="Simplified"/>	

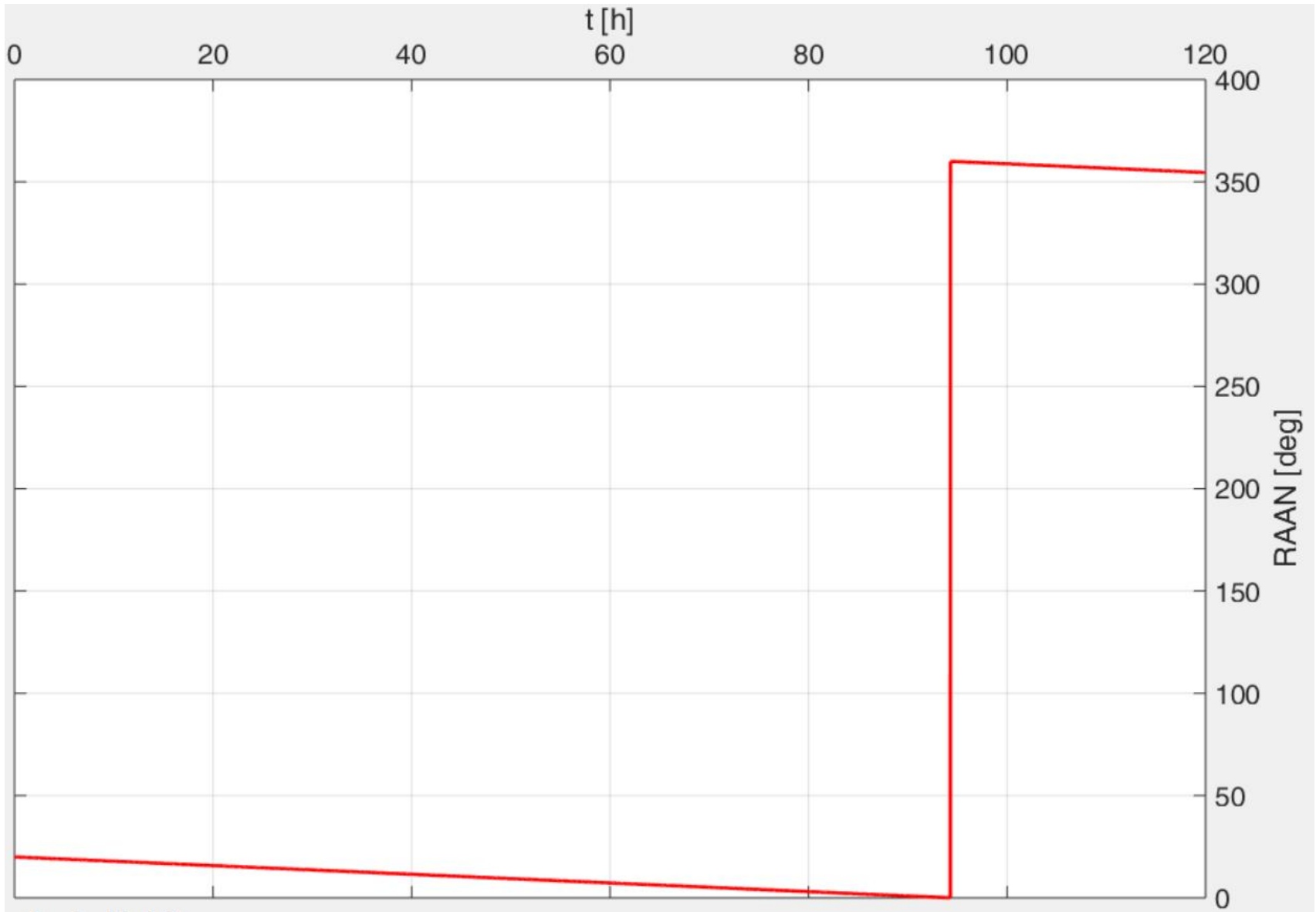
Spacecraft Properties	
Mass [kg]	<input type="text" value="4"/>
Sizes [m, m, m]	<input type="text" value="[0.3, 0.1, 0.1]"/>
Cross-section to TAS [m^2]	<input type="text" value="0.03"/>
Cross-section to Sun [m^2]	<input type="text" value="0.03"/>
Drag Coefficient	<input type="text" value="4"/>
Reflectivity Coefficient	<input type="text" value="[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]"/>

Density Model	Density Parameters	Gravity Model
<input type="radio"/> Harris-Priester	Harris-Priester coeff. <input type="text" value="0"/>	Maximum Degree <input type="text" value="2"/>
<input type="radio"/> Jacchia 71	DailyF10.7 <input type="text" value="155"/>	Maximum Order <input type="text" value="0"/>
<input type="radio"/> Jacchia-Roberts	Averaged F10.7 <input type="text" value="155"/>	
<input type="checkbox"/> Measured data	Geomagnetic activity <input type="text" value="3"/>	
		<input type="checkbox"/> Download Data

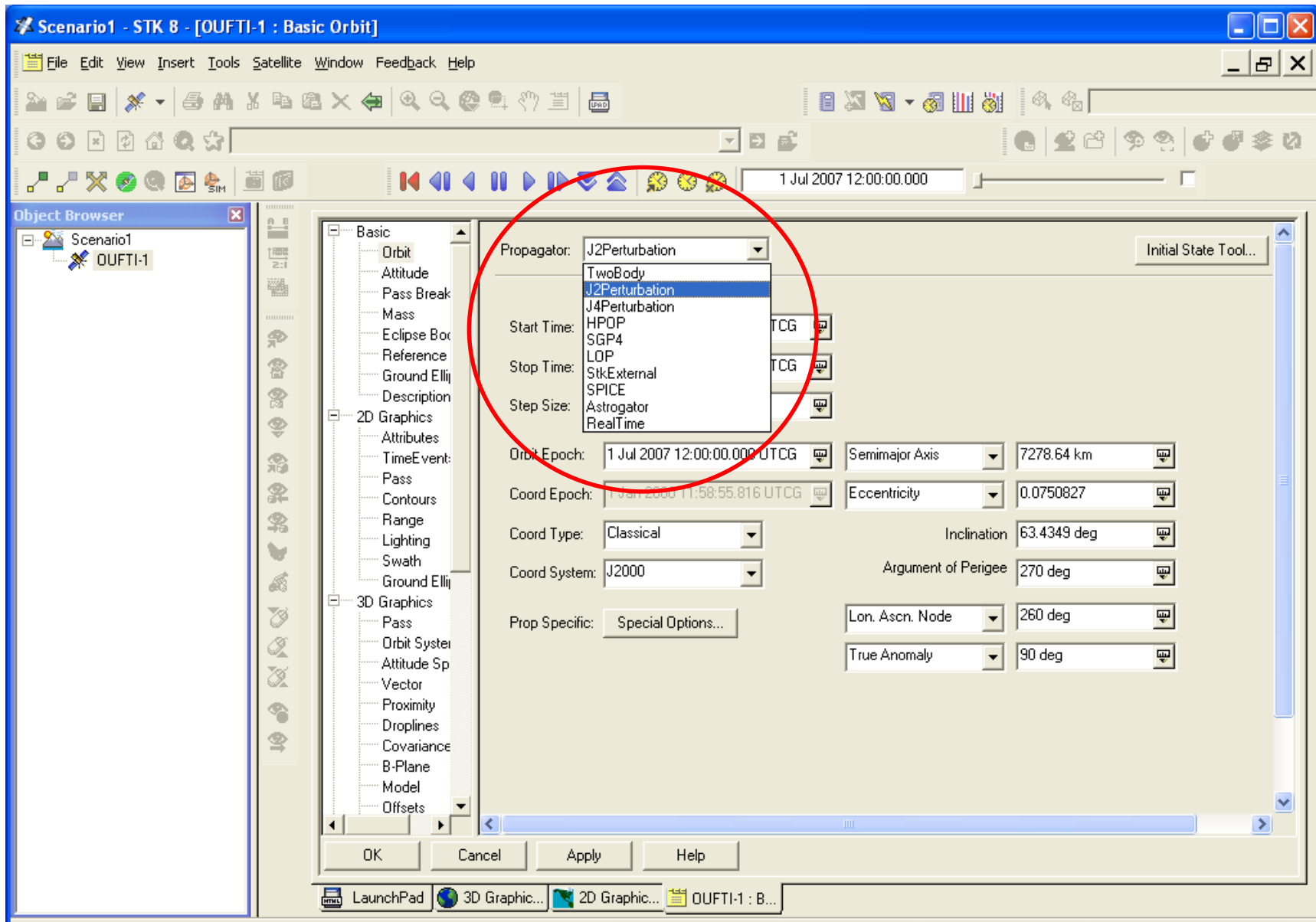
Orbit 3D
<input type="button" value="RUN!"/>



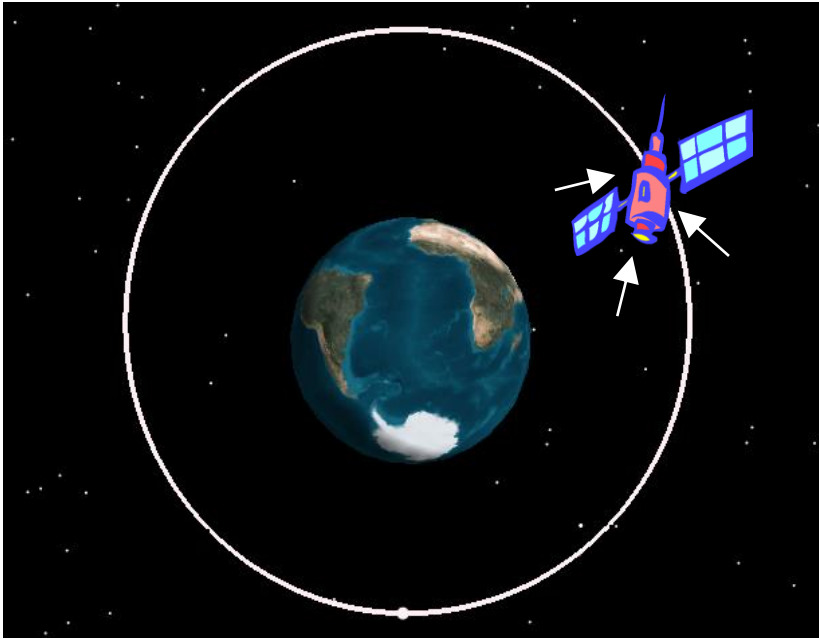
The Effect of Earth Oblateness



STK: Different Propagators



Non-Keplerian Motion



Dominant perturbations

Earth's gravity field

Atmospheric drag

Third-body perturbations

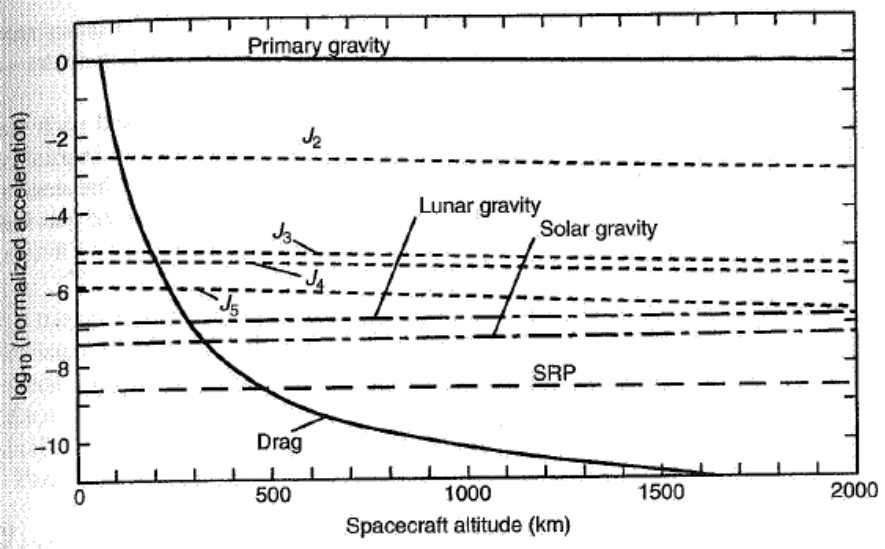
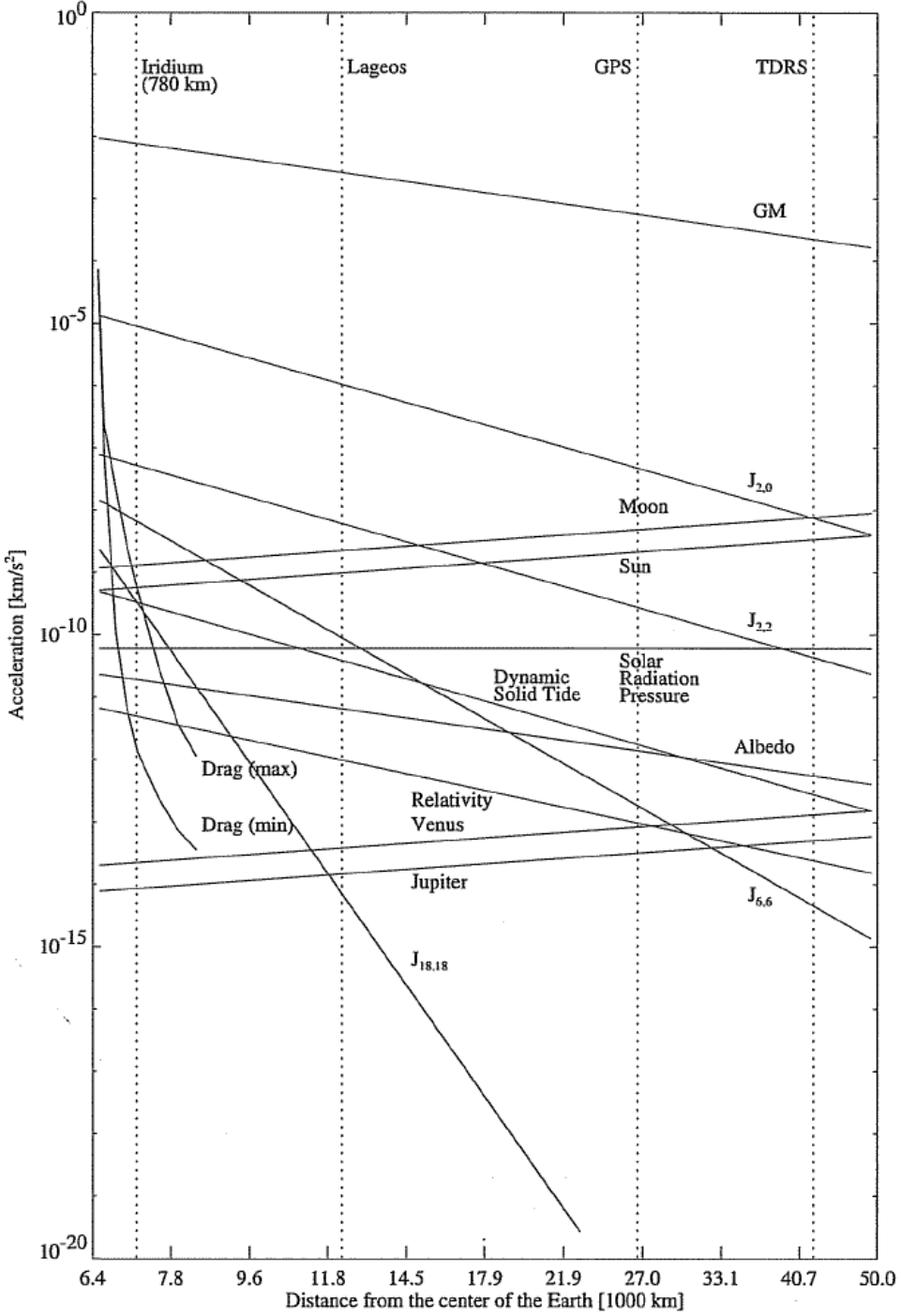
Solar radiation pressure

Different Perturbations and Importance ?

In low-earth orbit (LEO) ?

In geostationary orbit (GEO) ?

Satellite dependent !



Montenbruck and Gill, *Satellite orbits*, Springer, 2000

Fortescue et al., *Spacecraft systems engineering*, 2003

Orders of Magnitude

400 kms

1000 kms

36000 kms

Oblateness

Drag

Oblateness

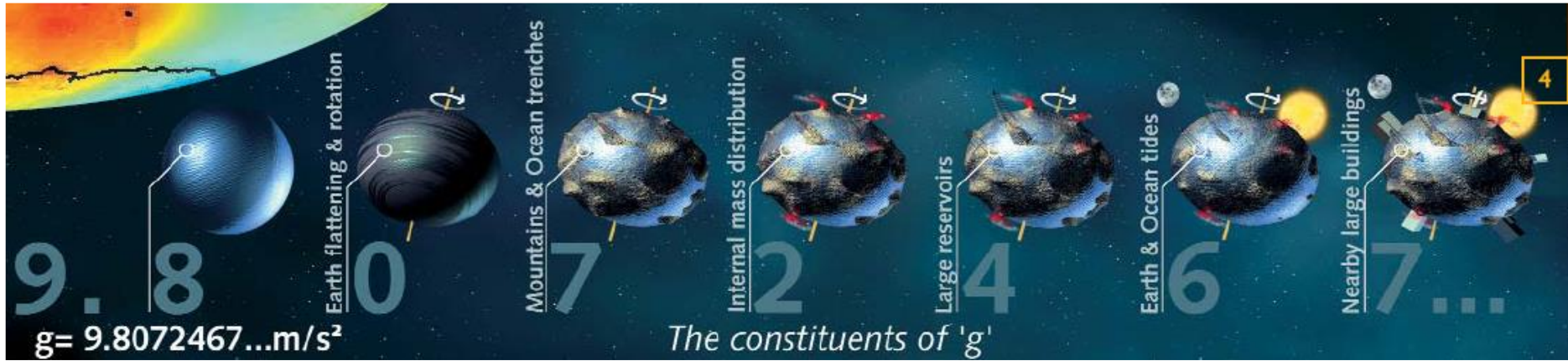
Sun and moon

Oblateness

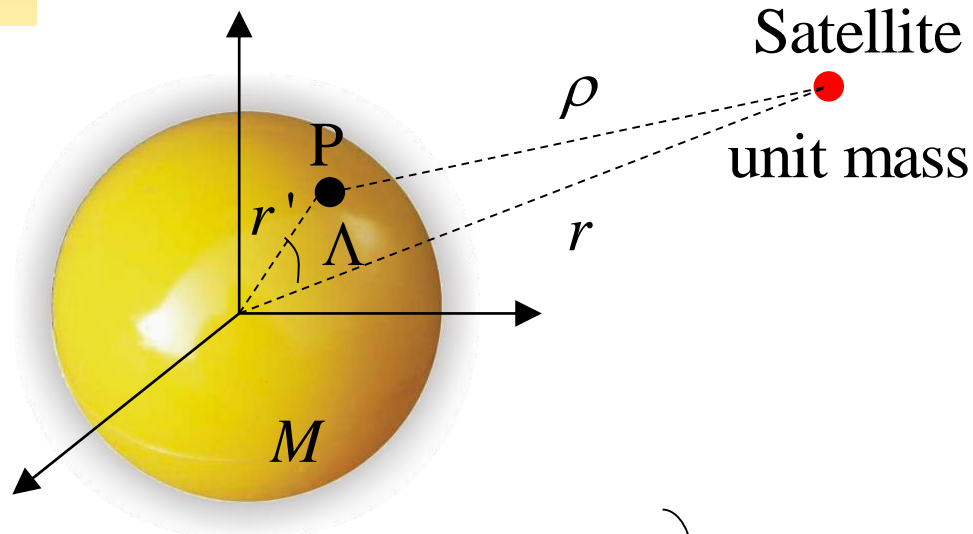
Sun and moon

SRP

The Earth is not a Sphere...



Mathematical Modeling



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r' = \sqrt{\xi^2 + \eta^2 + \zeta^2}$$

$U = -V$, potential, $\ddot{\mathbf{r}} = \nabla U$

V , potential energy

$$U = G \int_{body} \frac{dm}{\rho}$$

$$\rho = \sqrt{r^2 + r'^2 - 2r'r \cos \Lambda}$$

$$\cos \Lambda = \frac{\mathbf{r} \cdot \mathbf{r}'}{r \cdot r'} \quad \alpha = \frac{r'}{r} < 1$$

$$U = G \int_{body} \frac{dm}{r \sqrt{1 - 2\alpha \cos \Lambda + \alpha^2}}$$

Legendre Polynomials

First introduced in 1782 by Legendre

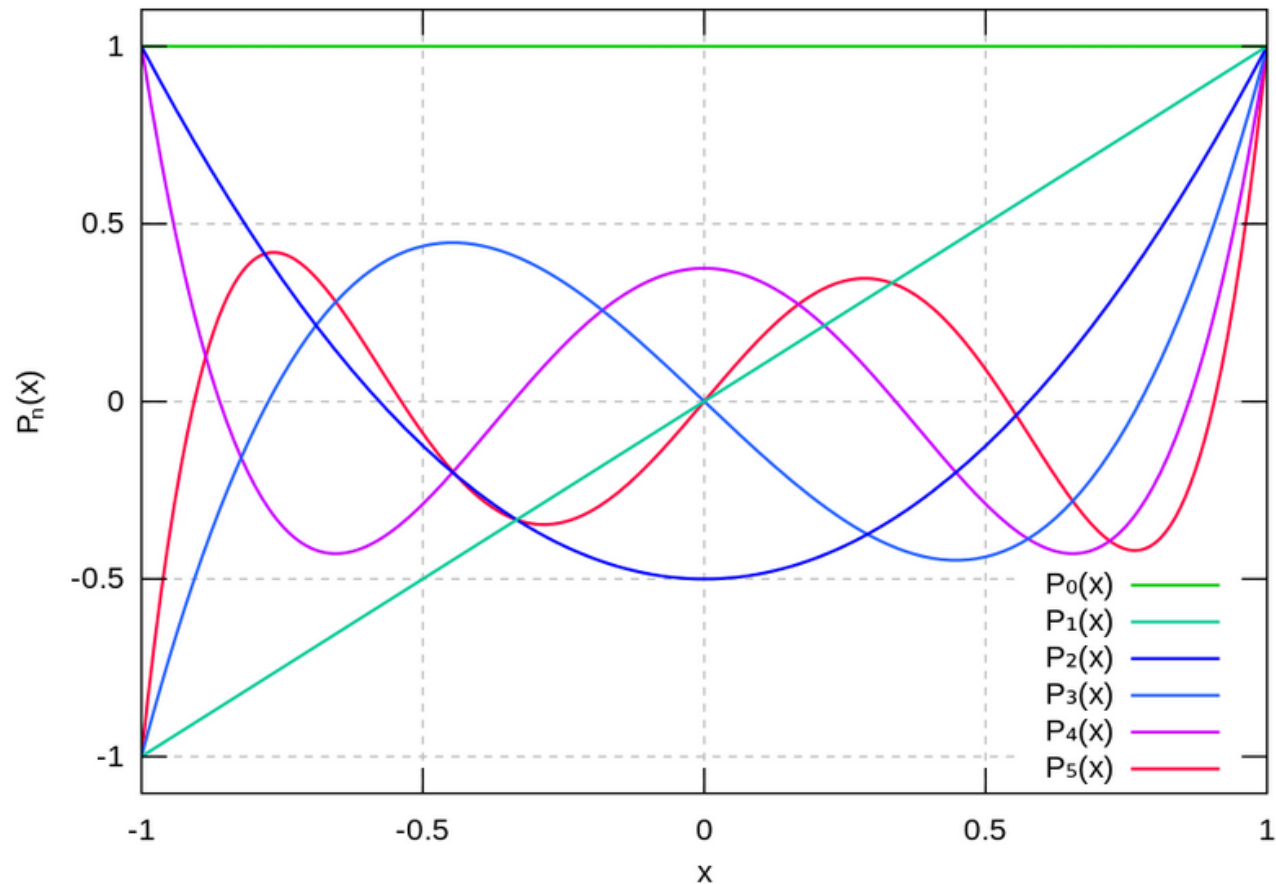
$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

n	$P_n(x)$
0	1
1	x
2	$\frac{1}{2} (3x^2 - 1)$
3	$\frac{1}{2} (5x^3 - 3x)$
4	$\frac{1}{8} (35x^4 - 30x^2 + 3)$

$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n$$

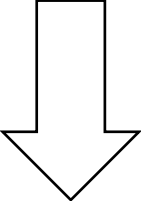
Legendre Polynomials Are Orthogonal

$$\int_{-1}^1 P_m(x)P_n(x) dx = 0 \quad \text{if } n \neq m.$$



Let's Use Them

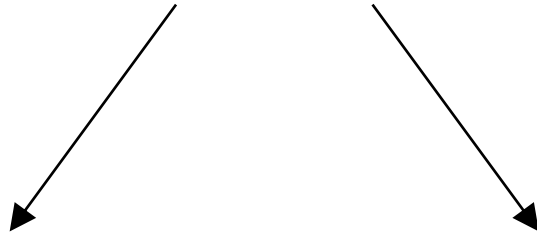
$$U = G \int_{body} \frac{dm}{r \sqrt{1 - 2\alpha \cos \Lambda + \alpha^2}}$$


$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n$$

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l[\cos(\Lambda)] dm$$

Summing Up...

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l[\cos(\Lambda)] dm$$



Geometric method (intuitive feel for gravity and inertia)

$$U = U_0 + U_1 + U_2 + \dots$$

Theory

Spherical-harmonic expansion

Experiments

Geometric Method: First Term

$$U_0 = \frac{G}{r} \int dm = \frac{\mu}{r}$$

Two-body
potential

Geometric Method: Second Term

$$\begin{aligned} U_1 &= \frac{G}{r} \int \cos(\Lambda) \alpha \, dm = \frac{G}{r} \int \frac{x\xi + y\eta + z\zeta}{r^2} \, dm \\ &= \frac{G}{r^3} \left(x \int \xi \, dm + y \int \eta \, dm + z \int \zeta \, dm \right) = 0 \end{aligned}$$

Center of mass at the origin of
the coordinate frame

Geometric Method: Third Term

$$\begin{aligned}U_2 &= \frac{G}{r} \int \frac{\alpha^2}{2} (3 \cos^2 \Lambda - 1) dm \\&= \frac{G}{2r^3} \int 2r'^2 dm - \frac{G}{2r^3} \int 3r'^2 \sin^2 \Lambda dm \\&= \frac{G}{2r^3} (A + B + C - 3I)\end{aligned}$$

$$\begin{aligned}\int 2r'^2 dm &= \int (\eta^2 + \zeta^2) dm + \int (\xi^2 + \zeta^2) dm + \int (\eta^2 + \xi^2) dm \\&= A + B + C \quad \text{Moments of inertia}\end{aligned}$$

$$\int r'^2 \sin^2 \Lambda dm = I \quad \text{Polar moment of inertia}$$

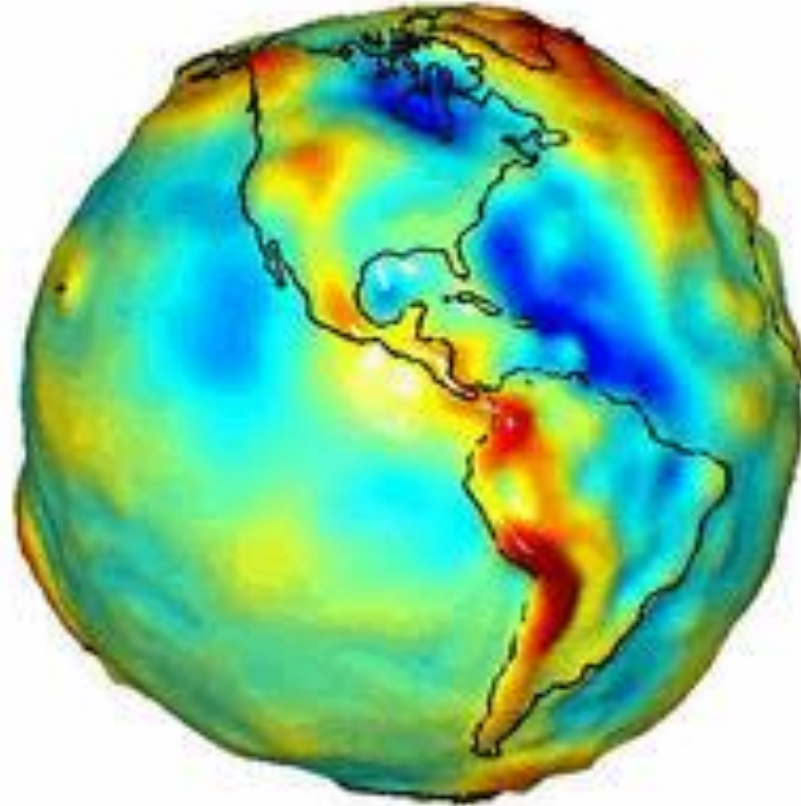
Geometric Method: MacCullagh's Formula

$$U = \frac{Gm_{\oplus}}{r} + \frac{G}{2r^3} (A + B + C - 3I) + \dots$$

Some of the simplest assumptions are

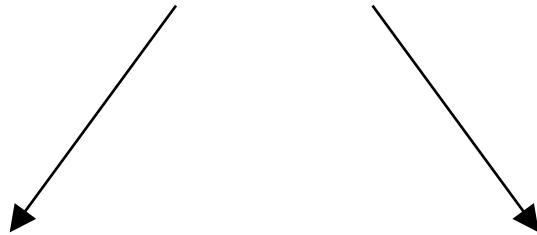
- the ellipsoidal Earth (oblate spheroid) with uniform density ($a=b>c$).
- triaxial ellipsoid ($a>b>c$).

Geometric Method: Difficult to Go Further...



Summing up...

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l[\cos(\Lambda)] dm$$



Geometric method (intuitive feel for gravity and inertia)

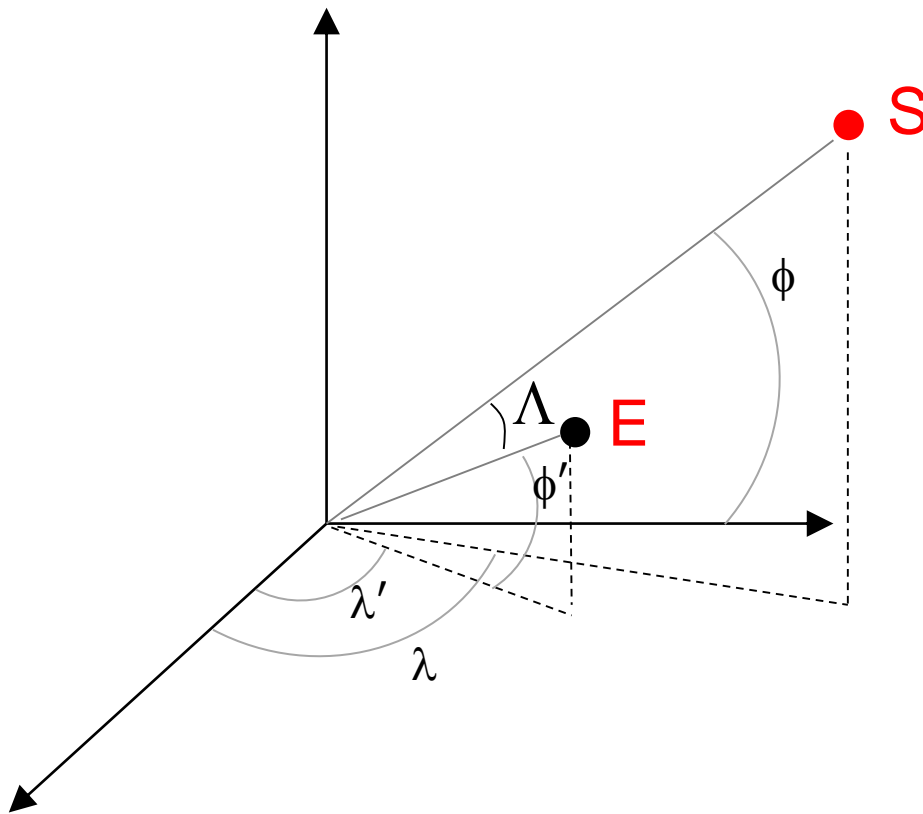
$$U = U_0 + U_1 + U_2 + \dots$$

Theory

Spherical-harmonic expansion

Experiments

Spherical Trigonometry



$\phi \rightarrow$ latitude sat
 $\lambda \rightarrow$ longitude sat
 $\phi' \rightarrow$ latitude Earth
 $\lambda' \rightarrow$ longitude Earth

$$\cos \Lambda = \cos(90 - \phi') \cos(90 - \phi) + \sin(90 - \phi') \sin(90 - \phi) \cos(\lambda - \lambda')$$

Addition Theorem for Spherical Harmonics

If $\cos \Lambda = \cos(90 - \phi') \cos(90 - \phi) + \sin(90 - \phi') \sin(90 - \phi) \cos(\lambda - \lambda')$

Then,

$$P_l(\cos \Lambda) = \sum_{m=0}^l (2 - \delta_{0m}) \frac{(l-m)!}{(l+m)!} P_{lm}(\sin \phi) P_{lm}(\sin \phi') \cos(m(\lambda - \lambda'))$$

where $P_{lm}(u) = (1 - u^2)^{m/2} \frac{d}{du^m} P_n(u)$

Associated Legendre polynomial of degree l and order m

$$U = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \alpha^l P_l(\cos \Lambda) dm = \frac{G}{r} \int_{body} \sum_{l=0}^{\infty} \left(\frac{r'}{r}\right)^l P_l(\cos \Lambda) dm$$

$$P_l(\cos \Lambda) = \sum_{m=0}^l (2 - \delta_{0m}) \frac{(l-m)!}{(l+m)!} P_{lm}(\sin \phi) P_{lm}(\sin \phi') \cos(m(\lambda - \lambda'))$$

\swarrow
 $(\cos m\lambda \cos m\lambda' + \sin m\lambda \sin m\lambda')$



Depends only on the satellite (r, ϕ, λ)

$$U = \frac{GM_{\oplus}}{r} \left\{ \sum_{l=0}^{\infty} \sum_{m=0}^l \left(\frac{R_{\oplus}}{r}\right)^l P_{lm}(\sin \phi) (C_{lm} \cos m\lambda + S_{lm} \sin m\lambda) \right\}$$

Depends only on the Earth (ϕ', λ') : spherical harmonics

$$C_{lm} = \frac{(2 - \delta_{0m}) (l-m)!}{M_{\oplus} (l+m)!} \int \left(\frac{r'}{R_{\oplus}}\right)^l P_{lm}(\sin \phi') \cos m\lambda' dm$$

$$S_{lm} = \frac{(2 - \delta_{0m}) (l-m)!}{M_{\oplus} (l+m)!} \int \left(\frac{r'}{R_{\oplus}}\right)^l P_{lm}(\sin \phi') \sin m\lambda' dm$$

Normalization: End Result

$$U = \frac{GM_{\oplus}}{r} \left\{ \sum_{l=0}^{\infty} \sum_{m=0}^l \left(\frac{R_{\oplus}}{r} \right)^l \bar{P}_{lm}(\sin \phi) (\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda) \right\}$$

$$\begin{Bmatrix} \bar{C}_{lm} \\ \bar{S}_{lm} \end{Bmatrix} = \sqrt{\frac{(l+m)!}{(2-\delta_{0m})(2n+1)(l-m)!}} \begin{Bmatrix} C_{lm} \\ S_{lm} \end{Bmatrix}$$

$$\bar{P}_{lm} = \sqrt{\frac{(2-\delta_{0m})(2n+1)(l-m)!}{(l+m)!}} P_{lm}$$

Very Important Remark

Many different expressions exist in the literature:

$$\Rightarrow V = \pm V$$

$$\Rightarrow P_l^m = (-1)^m P_{lm}$$

\Rightarrow Normalized or non-normalized coefficients

\Rightarrow Latitude or colatitude ($\sin \phi$ or $\cos \phi$)

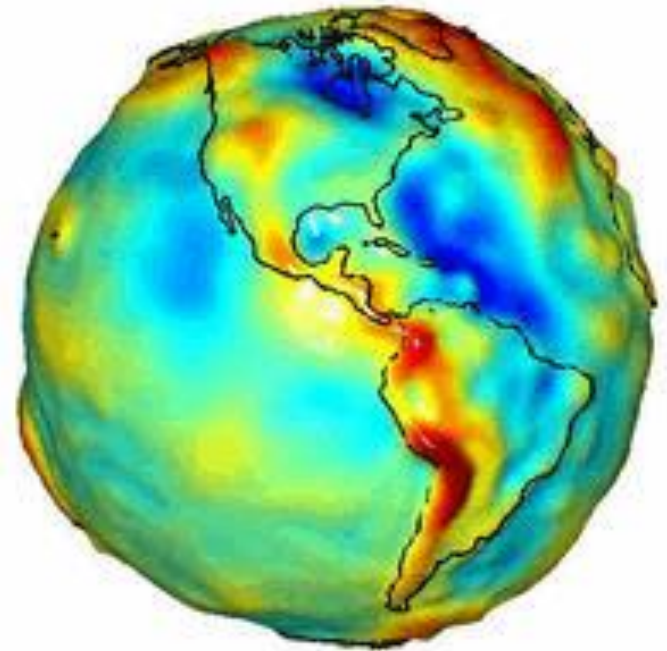
$\Rightarrow \dots$

Be always aware of the conventions/definitions used !

Spherical Harmonics

A set of functions used to represent functions on the surface of the sphere. They are a higher-dimensional analogy of Fourier series.

So any object that looks « kind-of-spherical » can be decomposed into an infinite sum of basic functions, as long as you multiply each basic function by the right coefficient



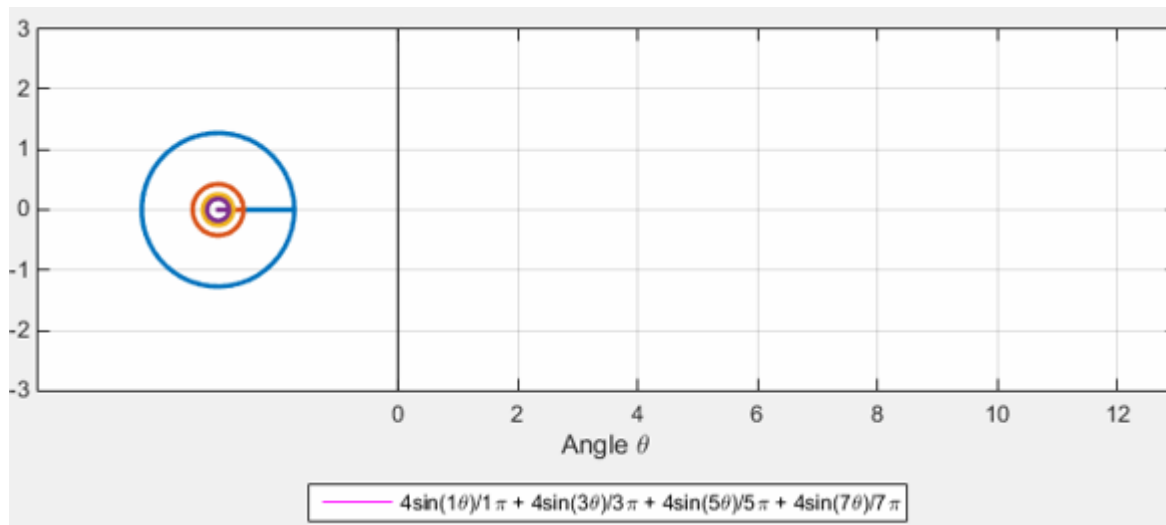
Our objective !

Fourier Series

A Fourier series is an expansion of a periodic function in terms of an infinite sum of sines and cosines.

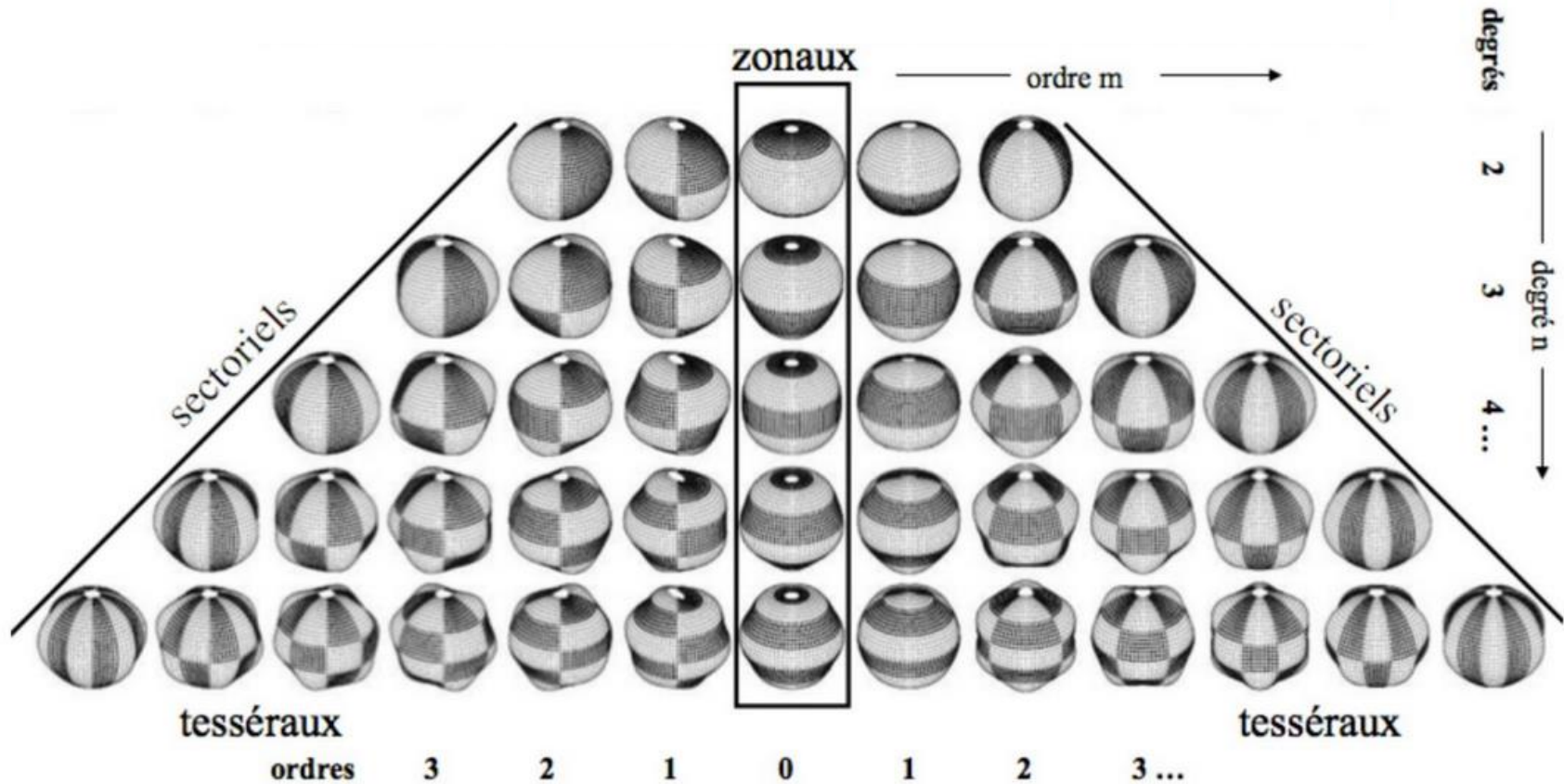
Fourier series make use of the orthogonality of sine and cosine functions.

<https://www.youtube.com/watch?v=LznpjC4Lo7IE>



A square wave defined using 4 Fourier terms

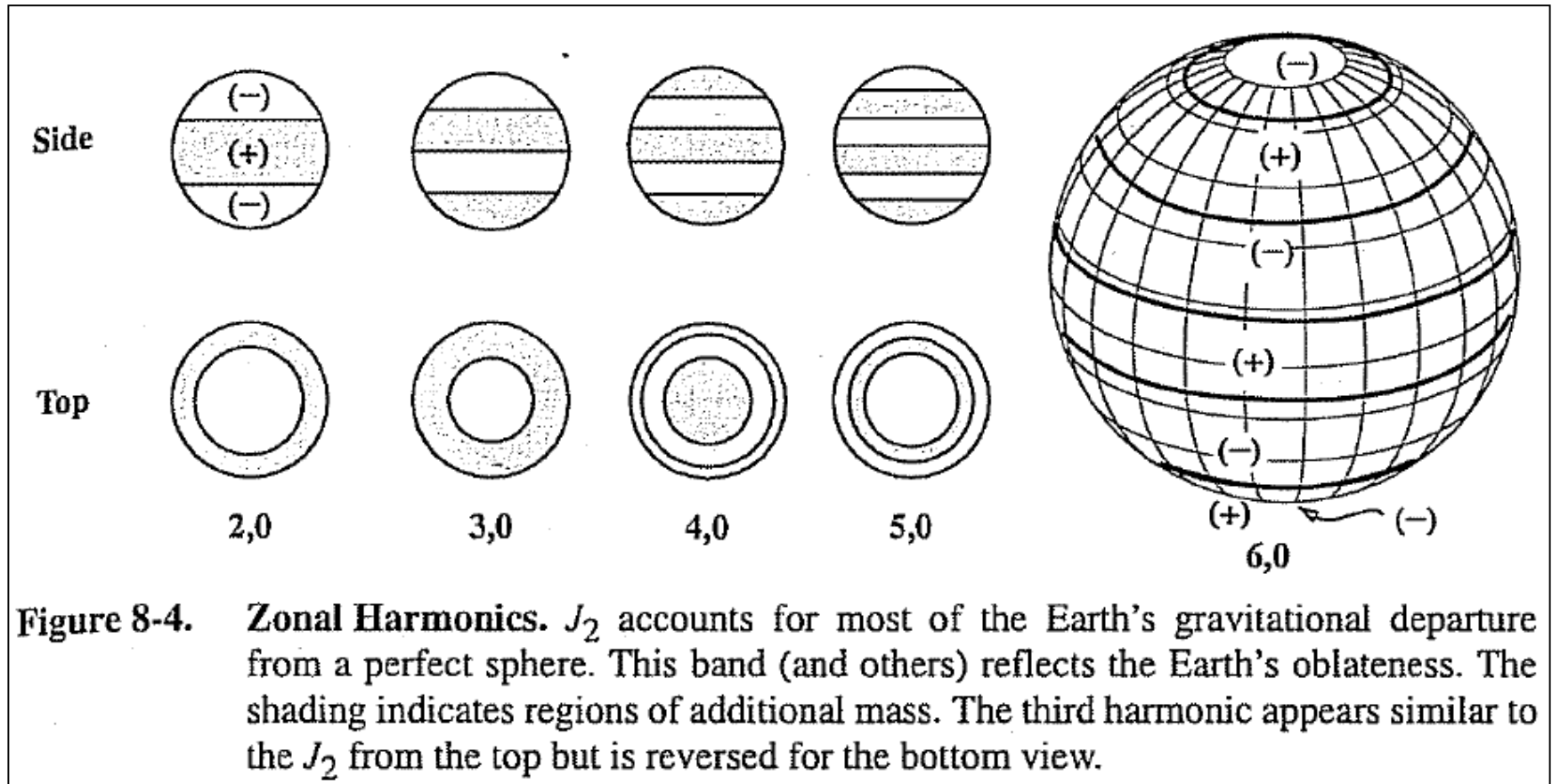
Spherical Harmonics



The degree « n » is the total number of waves. The order « m » is the number of waves in longitude. The number of waves in latitude is thus « $n - m$ ».

Zonal Harmonics ($m=0$)

Each boundary is a root of the Legendre polynomial.



Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

Zonal Harmonics (m=0)

The zonal coefficients are independent of longitude (symmetry with respect to the rotation axis).

$$U = \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{R_{\oplus}}{r} \right)^l \bar{P}_{lm} [\sin \phi_{sat}] \left[\bar{C}_{l,m} \cos(m\lambda_{sat}) + \bar{S}_{l,m} \sin(m\lambda_{sat}) \right] \right\}$$

$$\begin{array}{l} \Downarrow \\ J_l = -C_{l,0} \\ S_{l,0} = 0 \text{ (definition)} \end{array}$$

$$U = \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \left(\frac{R_{\oplus}}{r} \right)^l \left[-J_l P_l [\sin \phi_{sat}] + \sum_{m=1}^l \bar{P}_{lm} [\sin \phi_{sat}] \left[\bar{C}_{l,m} \cos(m\lambda_{sat}) + \bar{S}_{l,m} \sin(m\lambda_{sat}) \right] \right] \right\}$$

EGM96

0,1 ?

Degree and Order		Normalized Gravitational Coefficients	
n	m	\bar{C}_{nm}	\bar{S}_{nm}
2	0	-.484165371736E -03	
2	1	-.186987635955E -09	.119528012031E-08
2	2	.243914352398E -05	-.140016683654E-05
3	0	.957254173792E -06	
3	1	.202998882184E -05	.248513158716E-06
3	2	.904627768605E -06	-.619025944205E-06
3	3	.721072657057E -06	.141435626958E-05
4	0	.539873863789E -06	
4	1	-.536321616971E -06	-.473440265853E-06
4	2	.350694105785E -06	.662671572540E-06
4	3	.990771803829E -06	-.200928369177E-06
4	4	-.188560802735E -06	.308853169333E-06
5	0	.685323475630E -07	
5	1	-.621012128528E -07	-.944226127525E-07
5	2	.652438297612E -06	-.323349612668E-06
5	3	-.451955406071E -06	-.214847190624E-06
5	4	-.295301647654E -06	.496658876769E-07
5	5	.174971983203E -06	-.669384278219E-06

First Zonal Harmonic: J_{2,0} or J₂

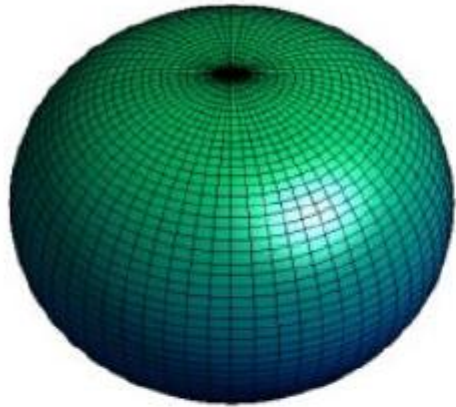
It represents the Earth's equatorial bulge and quantifies the major effects of oblateness on orbits.

It is almost a thousand times as large as any of the other coefficients.

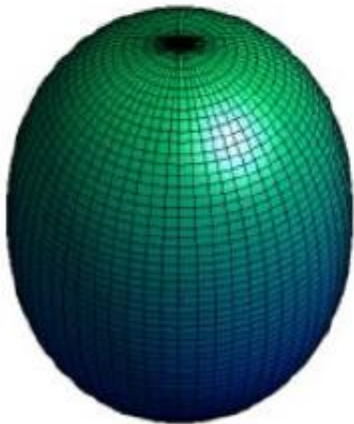
$$J_2 = -C_{2,0} = \sqrt{\frac{2.1 \cdot (2.2 + 1)}{2}} 0.4841 \times 10^{-3} = 0.001082$$

Degree and Order		Normalized Gravitat
n	m	\bar{C}_{nm}
2	0	-.484165371736E -03
2	1	-.186987635955E -09
2	2	2.12014252208E -05

First Zonal Harmonic: $J_{2,0}$ or J_2



Oblate planet: $J_2 > 0$



Prolate planet: $J_2 < 0$

Calculation of the Rotational Flattening

Equilibrium of a rotating self gravitating fluidlike body
(uniform density)

<http://farside.ph.utexas.edu/teaching/336k/Newton/node109.html>

$$\frac{R_e - R_p}{R} = \frac{5\Omega^2 R^3}{4GM} \quad R \text{ is the mean radius}$$

$$\frac{R_e - R_p}{R} = \frac{5(7.27 \times 10^{-5})^2 (6.37 \times 10^6)^3}{4 \cdot 6.67 \times 10^{-11} \cdot 5.97 \times 10^{24}} = 0.0043$$

$$\frac{R_e - R_p}{R} = 0.0043 \times 6.37 \times 10^6 = 27\text{km vs } 21\text{km}$$

First Zonal Harmonic of Other Planets

Planet	J_2
Mercury	60e-6
Venus	4.46e-6
Earth	1.08e-3
Moon	2.03e-4
Jupiter	1.47e-2
Saturn	1.63e-2

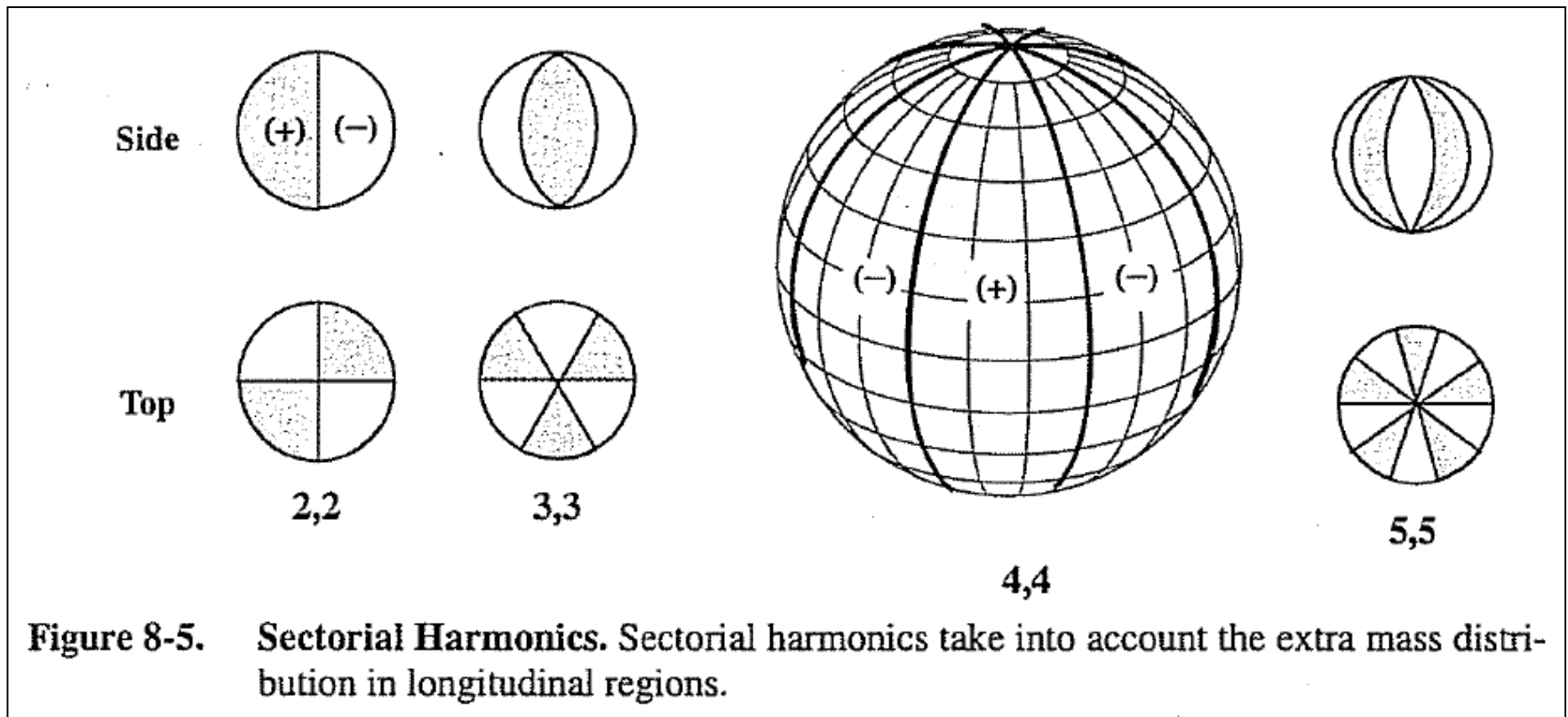
$$\frac{R_e - R_p}{R} = \frac{5\Omega^2 R^3}{4GM}$$

Celestial objects	Rotation period
Sun	25.379995 days (Carrington rotation) 35 days (high latitude)
Mercury	58.6462 days ^[7]
Venus	-243.0187 days ^{[7][8]}
Earth	0.99726968 days ^{[7][9]}
Moon	27.321661 days ^[10] (synchronous toward Earth)
Mars	1.02595675 days ^[7]
Ceres	0.37809 days ^[11]
Jupiter	0.4135344 days (deep interior) ^[12] 0.41007 days (equatorial) 0.41369942 days (high latitude)
Saturn	0.44403 days (deep interior) ^[12] 0.426 days (equatorial) 0.443 days (high latitude)
Uranus	-0.71833 days ^{[7][8][12]}
Neptune	0.67125 days ^{[7][12]}
Pluto	-6.38718 days ^{[7][8]} (synchronous with Charon)

Sectorial Harmonics ($l=m$)

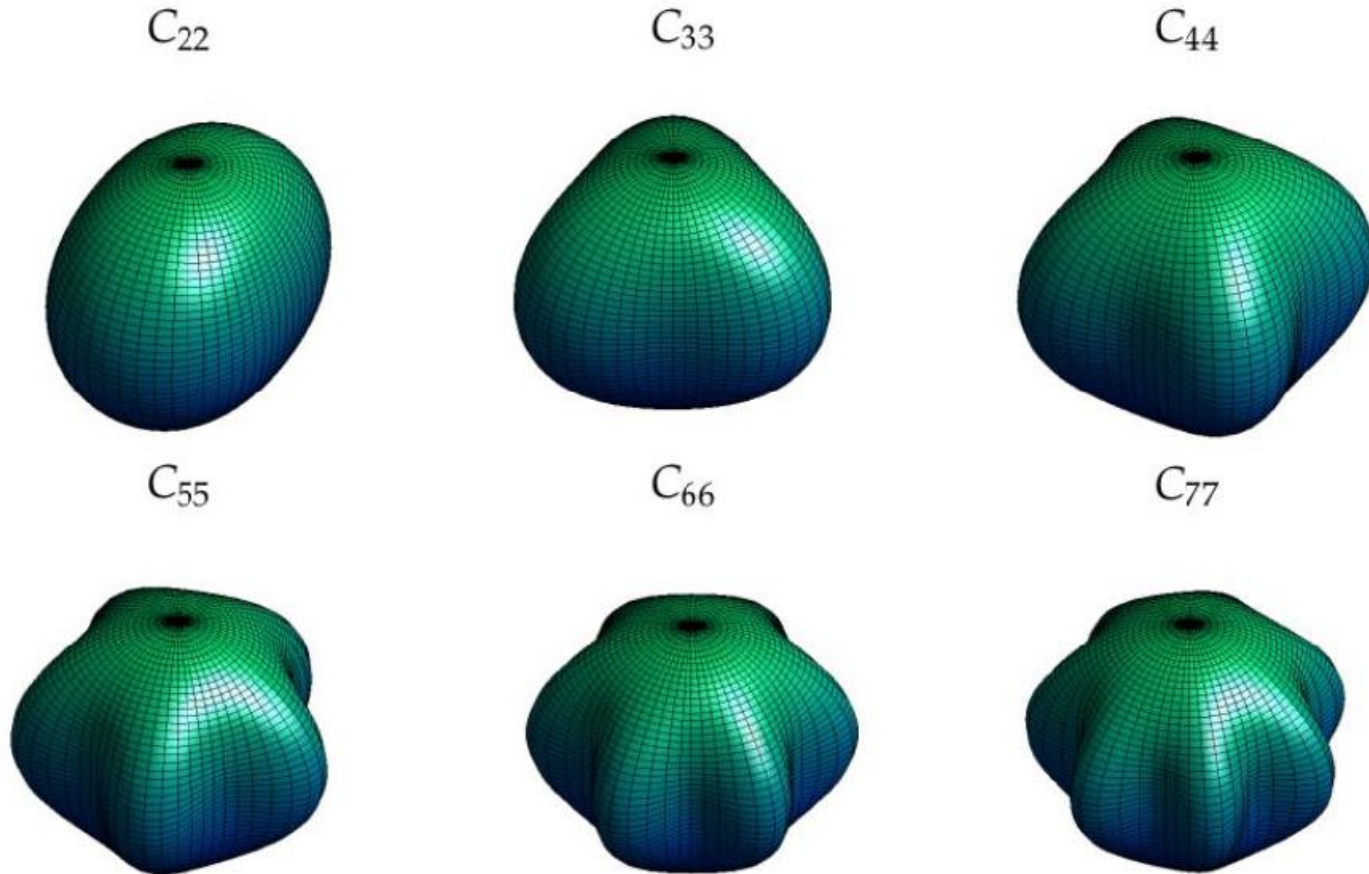
The sectorial coefficients represent bands of longitude.

The polynomials $P_{l,l}$ are zero only at the poles.



Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

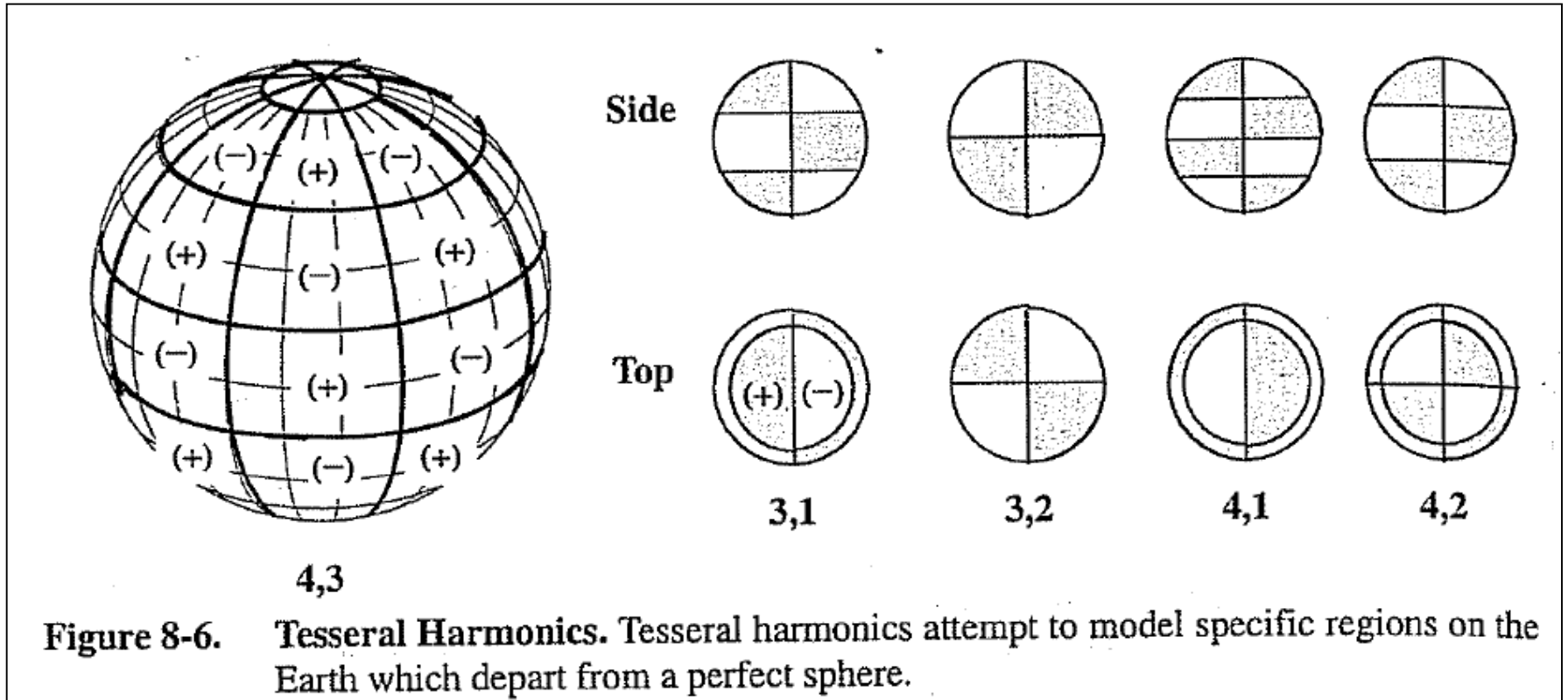
Sectorial Harmonics ($l=m$)



Sectorial harmonics preserve symmetry with respect to the equatorial plane

Polynomials $P_{l,l}$ are zero only at the poles

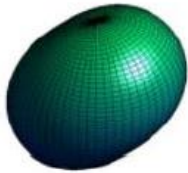
Tesseral Harmonics ($l \neq m \neq 0$)



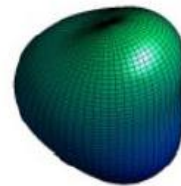
Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

Tesseral Harmonics ($l \neq m \neq 0$)

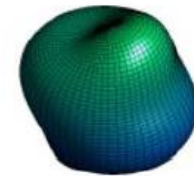
C_{21}



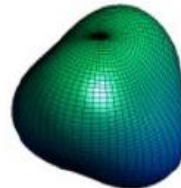
C_{31}



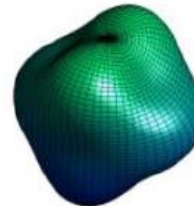
C_{41}



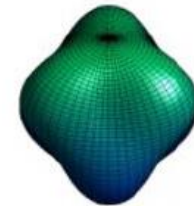
C_{32}



C_{42}



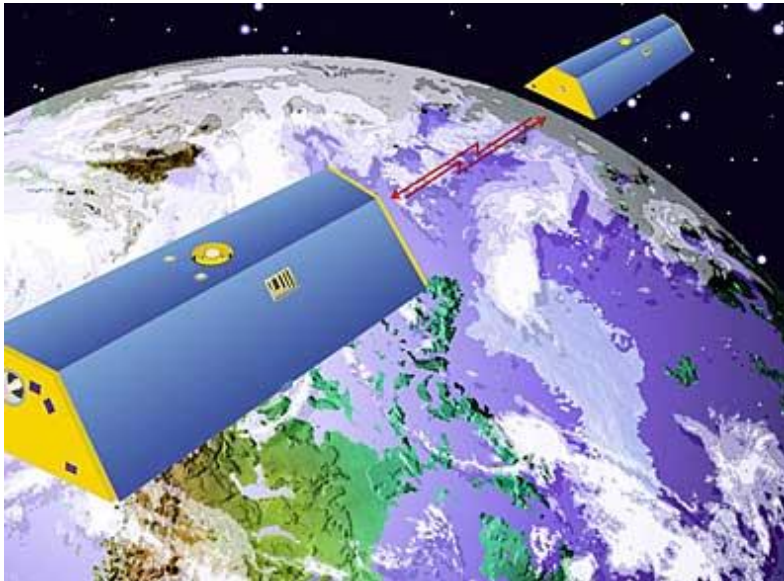
C_{43}



Determination of Gravitational Coefficients

Because the internal distribution of the Earth is not known, the coefficients cannot be calculated from their definition.

They are determined experimentally; e.g, using satellite tracking.



Satellite-to-satellite tracking: GRACE employs microwave ranging system to measure changes in the distance between two identical satellites as they circle Earth. The ranging system detects changes as small as 10 microns over a distance of 220 km.

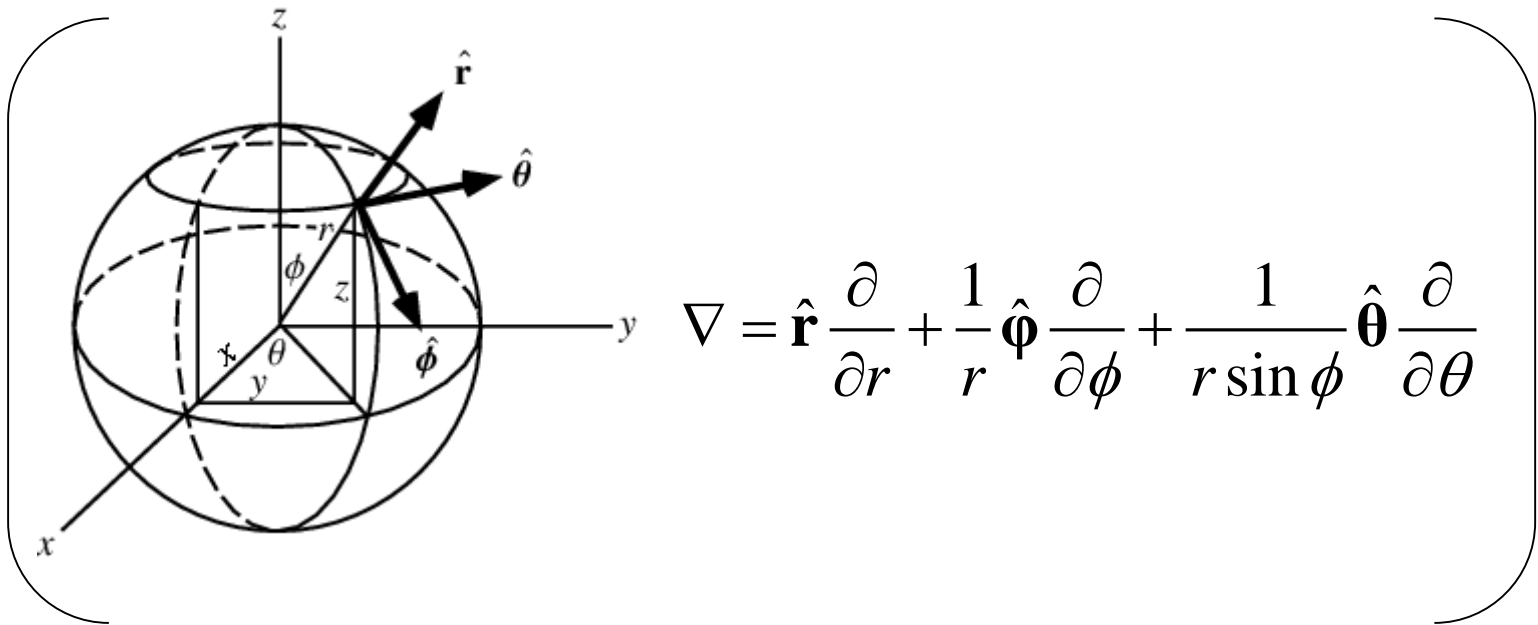
Gravitational Coefficients: GRACE

EGM-2008 has been publicly released:

- ⇒ Extensive use of GRACE twin satellites.
- ⇒ 4.6 million terms in the spherical expansion (130317 in EGM-96)
- ⇒ Geoid with a resolution approaching 10 km (5'x5').

Resulting Force

$$\mathbf{F} = \nabla U \quad \text{with} \quad \nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \cos \phi} \hat{\boldsymbol{\lambda}} \frac{\partial}{\partial \lambda}$$



$$\nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \sin \phi} \hat{\boldsymbol{\theta}} \frac{\partial}{\partial \theta}$$

Spherical Earth

Gravitational force acts through the Earth's center.

$$U = \frac{\mu}{r}$$

$$\nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \cos \phi} \hat{\boldsymbol{\lambda}} \frac{\partial}{\partial \lambda}$$

Oblate Earth: J2

$$\left\{ \begin{aligned} U &= \frac{\mu}{r} \left\{ 1 + \sum_{l=2}^{\infty} \left(\frac{R_{\oplus}}{r} \right)^l \left[-J_l P_l [\sin \phi_{sat}] + \sum_{m=1}^l \bar{P}_{lm} [\sin \phi_{sat}] \left[\bar{C}_{l,m} \cos(m\lambda_{sat}) + \bar{S}_{l,m} \sin(m\lambda_{sat}) \right] \right] \right\} \\ P_2[\gamma] &= \frac{1}{2} (3\gamma^2 - 1) \end{aligned} \right.$$

$$U = \frac{\mu}{r} \left\{ 1 - J_2 \left(\frac{R_{\oplus}}{r} \right)^2 \frac{3 \sin^2 \phi_{sat} - 1}{2} \right\}$$

$$\nabla = \hat{\mathbf{r}} \frac{\partial}{\partial r} + \frac{1}{r} \hat{\boldsymbol{\phi}} \frac{\partial}{\partial \phi} + \frac{1}{r \cos \phi} \hat{\boldsymbol{\lambda}} \frac{\partial}{\partial \lambda}$$

Perturbation of the radial acceleration

Longitudinal acceleration that can be decomposed into an azimuth and normal accelerations

STK: Gravity Models (HPOP)

The screenshot shows the 'Force Model for ISS_HPOP' dialog box in STK. The 'Gravity' field is circled in red. A 'Select a Gravity Field' dialog box is open over it, showing a file browser with the following files:

- GDALData
- maps
- nad
- Ptxmaps
- Rf
- Saa
- EGM96
- EGM2008
- GEMT1
- GGM01C
- GGM02C
- JGM2
- JGM3
- WGS72_ZonalsToJ4
- WGS84
- WGS84_EGM96
- WGS84_old

The 'File name' field is empty, and the 'Files of type' is set to '*.grv'. The 'Open as read-only' checkbox is unchecked.

The 'Force Model for ISS_HPOP' dialog box has the following settings:

- Central Body Gravity: Use
- Gravity: WGS84_EGM96.grv
- Maximum Degree: 21
- Maximum Order: 21
- Solid Tides: Permanent tide only
- Use Ocean Tides:
- Solar Radiation Pressure: Use
- Model: Spherical
- Cr: 1.000000
- Area/Mass Ratio: 0.005 m²/kg
- Shadow Model: Dual Cone
- Use Boundary Mitigation:
- Third Body Gravity:

Name	Use	Source	
Sun	<input checked="" type="checkbox"/>	Cb file	1.32712200
Moon	<input checked="" type="checkbox"/>	Cb file	4.902801076000e+003 km ³ /sec ²
Jupiter	<input type="checkbox"/>	Cb file	1.267127648383e+008 km ³ /sec ²
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km ³ /sec ²
Saturn	<input type="checkbox"/>	Cb file	3.794058536168e+007 km ³ /sec ²

Atmospheric Drag

Atmospheric forces represent the largest nonconservative perturbations acting on low-altitude satellites.

The drag is directly opposite to the velocity of the satellite, hence decelerating the satellite.

The lift force can be neglected in most cases.

STK – Atmospheric Models (HPOP)

Force Model for ISS

Central Body Gravity

Gravity: WGS84_EGM96.grv

Maximum Degree: 21

Maximum Order: 21

Solid Tides: Permanent tide only

Use Ocean Tides

Solar Radiation Pressure

Use

Cr: 1.000000

Area/Mass Ratio: 0.02 m²/kg

Shadow Model: Dual Cone

Use Boundary Mitigation

Drag

Use

Cd: 2.200000

Area/Mass Ratio: 0.02 m²/kg

Atm. Density Model: Jacchia-Roberts

SolarFlux/GeoMag

Daily F10.7:

Average F10.7:

Geomagnetic Index (Kp):

1976 Standard
Harris-Priester
Jacchia 1970
Jacchia 1971
NRLMSISE 2000
MSISE 1990
MSIS 1986
Jacchia 1960
Jacchia-Roberts
CIRA 1972

Third Body Gravity

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km ³ /sec ²
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km ³ /sec ²
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km ³ /sec ²
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km ³ /sec ²
Saturn	<input type="checkbox"/>	Cb file	3.794062606114e+007 km ³ /sec ²

More Options...

OK Cancel Help

STK – Solar Activity (HPOP)

The screenshot displays the 'Force Model for ISS_HPOP' dialog box in the STK software. The left sidebar shows a tree view with categories like 'Basic', '2D Graphics', and '3D Graphics'. The main dialog area is organized into several sections:

- Central Body Gravity:** Gravity is set to 'WGS84_EGM96.grv'. Maximum Degree and Maximum Order are both set to 21. Solid Tides is set to 'Permanent tide only'. There is an unchecked checkbox for 'Use Ocean Tides'.
- Drag:** The 'Use' checkbox is checked. Cd is 2.070000. Area/Mass Ratio is 0.005 m²/kg. Atm. Density Model is set to 'Harris-Priester'.
- Solar Radiation Pressure:** The 'Use' checkbox is checked. The Model is set to 'Spherical'. Cr is 1.000000. Area/Mass Ratio is 0.005 m²/kg. Shadow Model is set to 'Dual Cone'. There is an unchecked checkbox for 'Use Boundary Mitigation'.
- SolarFlux/GeoMag:** This section is circled in red. It features a dropdown menu set to 'Enter Manually'. Below it are three input fields: 'Daily F10.7' (150.00000000), 'Average F10.7' (150.00000000), and 'Geomagnetic Index (Kp)' (3.00000000).

At the bottom right of the dialog, there is a button labeled 'Eclipsing Bodies...'.

Mathematical Modeling

Knowledge of attitude Velocity with respect to the atmosphere

$$\ddot{\mathbf{r}}_{\oplus sat} = -\frac{1}{2} C_D \frac{A}{m} \rho v_r^2 \frac{\mathbf{v}_r}{v_r}$$

[1.5-3] Atmospheric density

The diagram shows the drag force equation $\ddot{\mathbf{r}}_{\oplus sat} = -\frac{1}{2} C_D \frac{A}{m} \rho v_r^2 \frac{\mathbf{v}_r}{v_r}$ in a grey box. Three arrows point from the box to labels: one from C_D to 'Knowledge of attitude', one from $\frac{\mathbf{v}_r}{v_r}$ to 'Velocity with respect to the atmosphere', and one from ρ to '[1.5-3] Atmospheric density'.

The atmosphere co-rotates with the Earth.

$$\mathbf{v}_r = \mathbf{v} - \boldsymbol{\omega}_{\oplus} \times \mathbf{r}$$

Inertial velocity Earth's angular velocity

The diagram shows the equation $\mathbf{v}_r = \mathbf{v} - \boldsymbol{\omega}_{\oplus} \times \mathbf{r}$. Two arrows point from the terms \mathbf{v} and $\boldsymbol{\omega}_{\oplus}$ to the labels 'Inertial velocity' and 'Earth's angular velocity' respectively.

All these parameters are difficult to estimate !

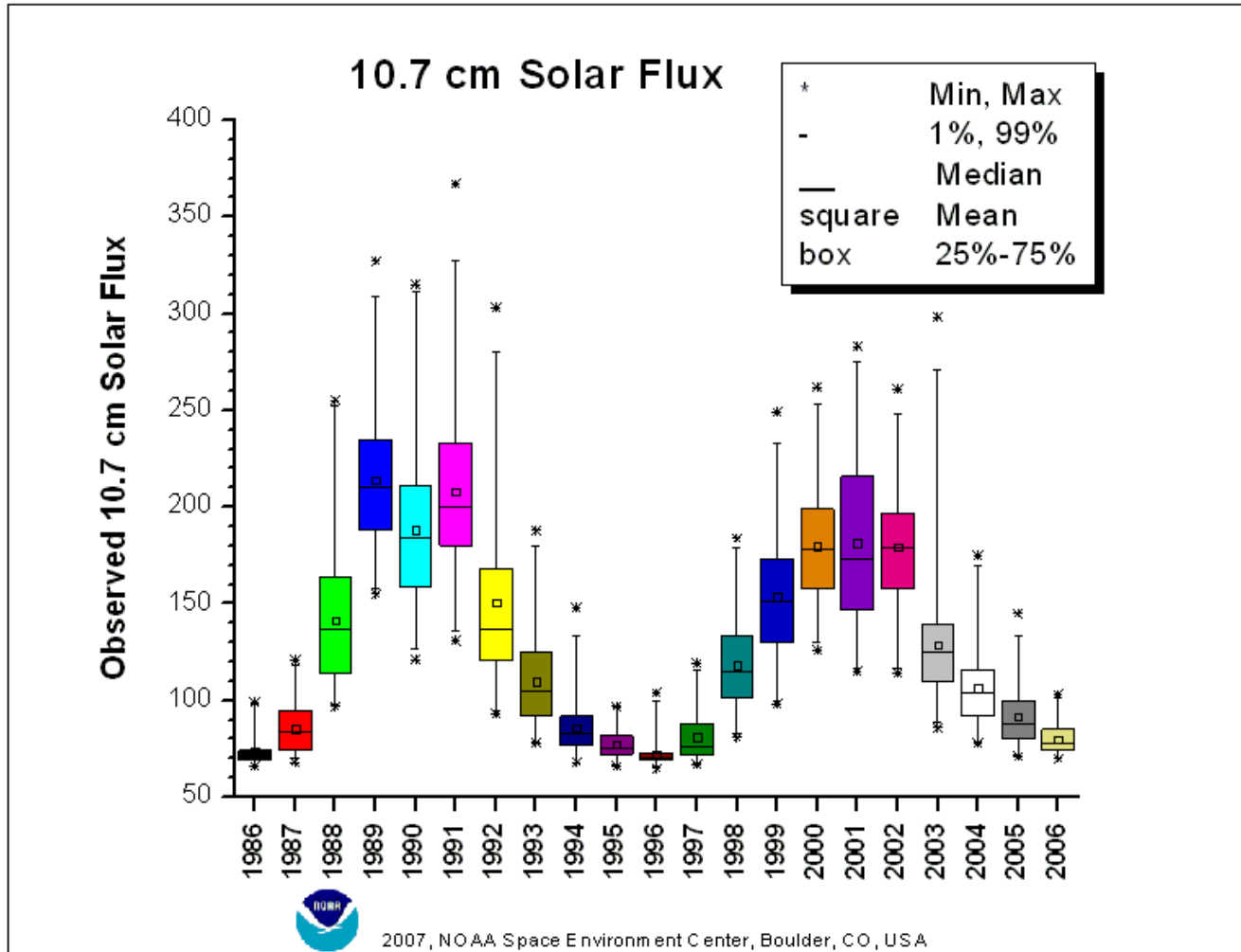
Atmospheric Density

The gross behavior of the atmospheric density is well established, but it is still this factor which makes the determination of satellite lifetimes so uncertain.

There exist several models (e.g., Jacchia-Roberts, Harris-Priester).

Dependence on temperature, molecular weight, altitude, solar activity, etc.

Solar Activity



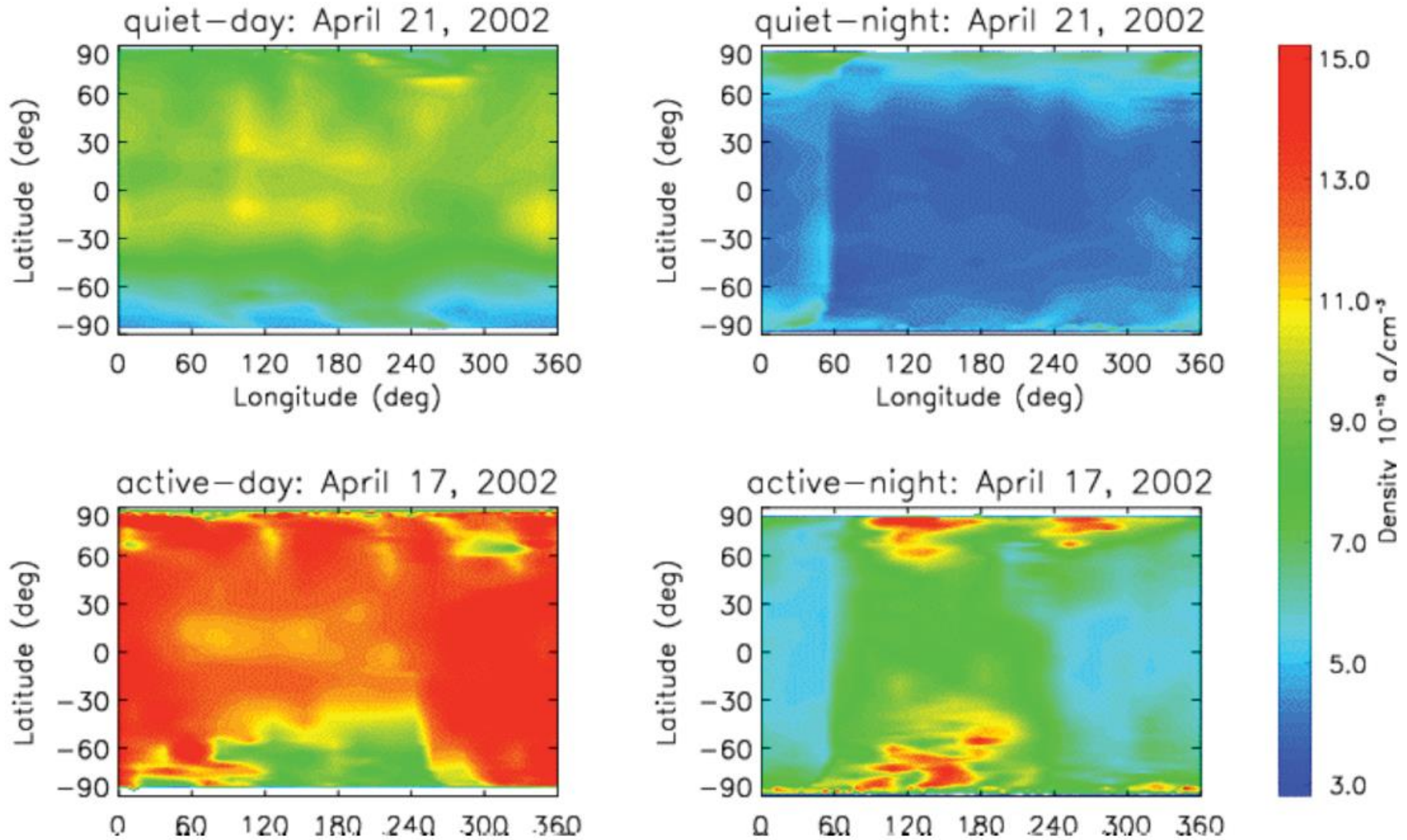
Atmospheric Density using CHAMP



An accelerometer measures the non-gravitational accelerations in three components, of which the along-track component mainly represents the atmospheric drag.

By subtracting modeled accelerations for SRP and Earth Albedo, the drag acceleration is isolated and is proportional to the atmospheric density.

CHAMP Density at 410 kms



Further Reading



Available online at www.sciencedirect.com



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**Planetary
and
Space Science**

www.elsevier.com/locate/pss

Atmospheric densities derived from CHAMP/STAR accelerometer observations

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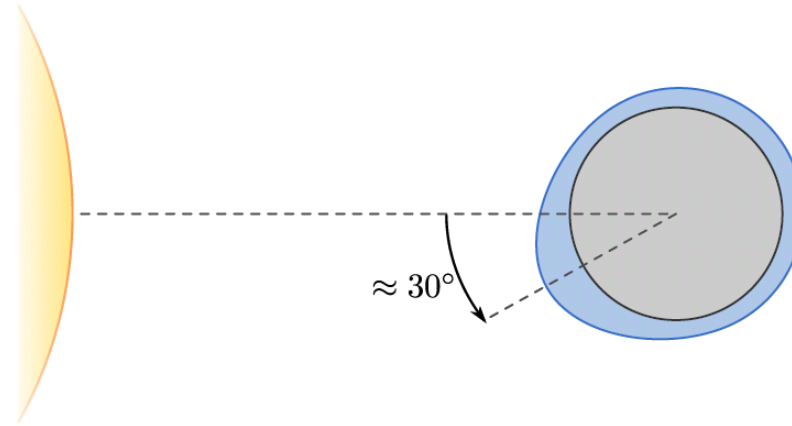
Harris-Priester (120-2000km)

Static model (e.g., no variation with the 27-day solar rotation).

Interpolation determines the density at a particular time.

Simple, computationally efficient and fairly accurate.

Atmospheric Bulge



The high atmosphere bulges toward a point in the sky some 15° to 30° east of the sun (density peak at 2pm local solar time).

The observed accelerations of Vanguard satellite (1958) indicated that the air density at 665 km is about 10 times as great when perigee passage occur one hour after noon as when it occurs during the night !

Harris-Priester (120-2000km)

The modified Harris-Priester (HP) model may be considered as a middle ground between the two extremes (Harris and Priester, 1962; Long et al., 1989; Hatten and Russell, 2016). Like the Standard Atmosphere, HP relies on exponential interpolation of density between values tabulated at discrete altitudes. However, HP also uses functional dependencies to model latitudinal and diurnal effects. Further, HP may be revised to take into account varying levels of solar activity. This effect has been achieved by including a set of 10 tables, each of which corresponds to a different value of the 81-day centered average 10.7 cm solar flux index $\bar{F}_{10.7}$. Given a value of $\bar{F}_{10.7}$, an interpolation scheme, such as nearest-neighbor (Dowd and Tapley, 1979) or linear (Tolman et al., 2004), is used to calculate density values.

Thus, HP may produce significantly more accurate density values than a simple exponential atmospheric model while executing in a fraction of the time of more complex models (Montenbruck and Gill, 2001). Such balance makes HP a suitable candidate for use in preliminary studies in which a combination of high speed and reasonable accuracy is paramount. However, even in this context, the HP model is not without its deficiencies. This work addresses

Harris-Priester (120-2000km)

Account for diurnal density bulge due to solar radiation

$$\rho(h) = \rho_m(h) + [\rho_M(h) - \rho_m(h)] \cos^n \left(\frac{\psi}{2} \right)$$

Height above the Earth's
reference ellipsoid

Angle between
satellite position
vector and the apex
of the diurnal bulge

Interpolation Between Altitudes

$$\rho_m(h) = \rho_m(h_i) \exp\left(\frac{h_i - h}{H_{m_i}}\right), \quad h_i \leq h \leq h_{i+1}$$

$$\rho_M(h) = \rho_M(h_i) \exp\left(\frac{h_i - h}{H_{M_i}}\right), \quad h_i \leq h \leq h_{i+1},$$

$$H_{m_i} = \frac{h_i - h_{i+1}}{\ln\left(\frac{\rho_m(h_{i+1})}{\rho_m(h_i)}\right)}$$

$$H_{M_i} = \frac{h_i - h_{i+1}}{\ln\left(\frac{\rho_M(h_{i+1})}{\rho_M(h_i)}\right)}$$

h [km]	ρ_m [g/km ³]	ρ_M [g/km ³]	h [km]	ρ_m [g/km ³]	ρ_M [g/km ³]
100	497400.0	497400.0	420	1.558	5.684
120	24900.0	24900.0	440	1.091	4.355
130	8377.0	8710.0	460	0.7701	3.362
140	3899.0	4059.0	480	0.5474	2.612
150	2122.0	2215.0	500	0.3916	2.042
160	1263.0	1344.0	520	0.2819	1.605
170	800.8	875.8	540	0.2042	1.267
180	528.3	601.0	560	0.1488	1.005
190	361.7	429.7	580	0.1092	0.7997
200	255.7	316.2	600	0.08070	0.6390
210	183.9	239.6	620	0.06012	0.5123
220	134.1	185.3	640	0.04519	0.4121
230	99.49	145.5	660	0.03430	0.3325
240	74.88	115.7	680	0.02632	0.2691
250	57.09	93.08	700	0.02043	0.2185
260	44.03	75.55	720	0.01607	0.1779
270	34.30	61.82	740	0.01281	0.1452
280	26.97	50.95	760	0.01036	0.1190
290	21.39	42.26	780	0.008496	0.09776
300	17.08	35.26	800	0.007069	0.08059
320	10.99	25.11	840	0.004680	0.05741
340	7.214	18.19	880	0.003200	0.04210
360	4.824	13.37	920	0.002210	0.03130
380	3.274	9.955	960	0.001560	0.02360
400	2.249	7.492	1000	0.001150	0.01810

Mean solar activity

Atmospheric Bulge Position

Spacecraft position (ECI) \swarrow

Unit vector toward the apex of the diurnal bulge in ECI coordinates \nearrow

$$\cos^n \left(\frac{\psi}{2} \right) = \left(\frac{1}{2} + \frac{\mathbf{r}^T \mathbf{u}_b}{2r} \right)^{\frac{n}{2}}$$

$$\mathbf{u}_b = \begin{pmatrix} \cos(\delta_s) \cos(\alpha_s + \lambda_{lag}) \\ \cos(\delta_s) \sin(\alpha_s + \lambda_{lag}) \\ \sin(\delta_s) \end{pmatrix}$$

Lag \rightarrow

Sun declination \swarrow

Sun right ascension \searrow

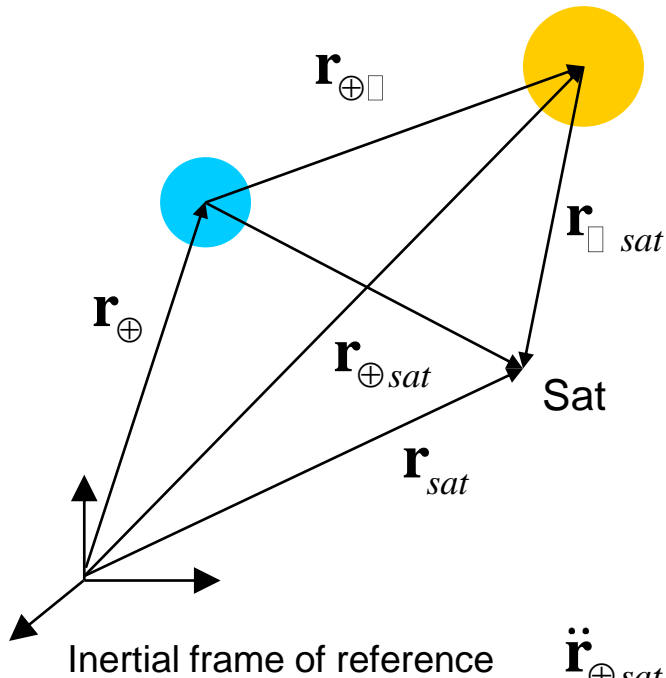
Third-Body Perturbations



For an Earth-orbiting satellite, the Sun and the Moon should be modeled for accurate predictions.

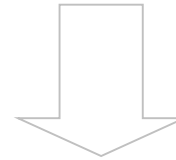
Their effects become noticeable when the effects of drag begin to diminish.

Mathematical Modeling (Sun Example)

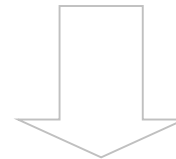


$$m_{sat} \ddot{\mathbf{r}}_{sat} = - \frac{Gm_{\oplus} m_{sat} \mathbf{r}_{\oplus sat}}{r_{\oplus sat}^3} - \frac{Gm_{\square} m_{sat} \mathbf{r}_{\square sat}}{r_{\square sat}^3}$$

Relative motion
is of interest



$$\ddot{\mathbf{r}}_{\oplus sat} = - \frac{Gm_{\oplus} \mathbf{r}_{\oplus sat}}{r_{\oplus sat}^3} - \frac{Gm_{\square} \mathbf{r}_{\square sat}}{r_{\square sat}^3} - \frac{Gm_{sat} \mathbf{r}_{\oplus sat}}{r_{\oplus sat}^3} - \frac{Gm_{\square} \mathbf{r}_{\oplus \square}}{r_{\oplus \square}^3}$$



$$\ddot{\mathbf{r}}_{\oplus sat} = - \frac{G(m_{\oplus} + m_{sat}) \mathbf{r}_{\oplus sat}}{r_{\oplus sat}^3} + Gm_{\square} \left(\frac{\mathbf{r}_{sat \square}}{r_{sat \square}^3} - \frac{\mathbf{r}_{\oplus \square}}{r_{\oplus \square}^3} \right)$$

direct indirect

STK: Third-Body Gravity (HPOP)

The screenshot shows the STK software interface with the 'Force Model for OUFTI-1' dialog box open. The dialog is configured for HPOP propagation. The 'Central Body Gravity' section is set to 'WGS84_EGM96.grv'. The 'Drag' section is checked and configured with a Cd of 2.200000 and an Area/Mass Ratio of 0.02 m²/kg. The 'Solar Radiation Pressure' section is also checked. The 'Third Body Gravity' section is highlighted with a red circle and contains the following table:

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km ³ /sec ²
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km ³ /sec ²
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km ³ /sec ²
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km ³ /sec ²
Saturn	<input type="checkbox"/>	Cb file	3.794062606114e+007 km ³ /sec ²

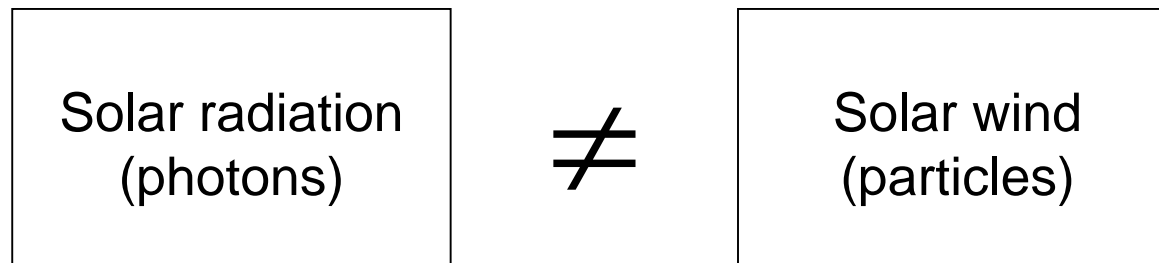
The dialog also shows the 'Propagator' set to HPOP, 'Start Time' as 1 Jul 2007 12:00:00.000 UTCG, 'Stop Time' as 2 Jul 2007 12:00:00.000 UTCG, and 'Step Size' as 60 sec. The 'Coord System' is set to J2000. The 'Object Browser' on the left shows the 'OUFTI-1' object selected under the 'Scenario1'.

Solar Radiation Pressure

It produces a nonconservative perturbation on the spacecraft, which depends upon the distance from the sun.

It is usually very difficult to determine precisely.

It is NOT related to solar wind, which is a continuous stream of particles emanating from the sun.



800km is regarded as a transition altitude between drag and SRP.

Mathematical Modeling

$$\mathbf{F}_{SR} = -p_{SR} c_R A \mathbf{e}_{sc / sun}$$

$$p_{SR} = \frac{1350 \text{ W/m}^2}{3e8 \text{ m/s}} = 4.51 \times 10^{-6} \text{ N/m}^2$$

The reflectivity c_R is a value between 0 and 2:

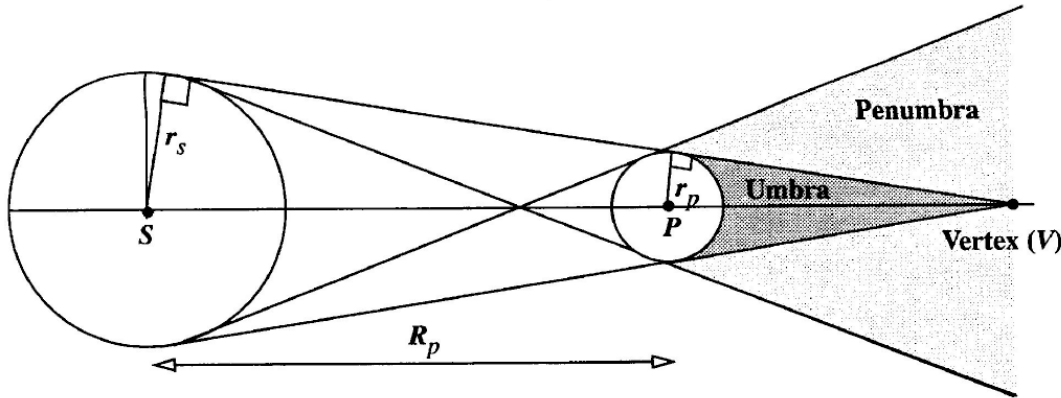
- 0: translucent to incoming radiation.
- 1: all radiation is absorbed (black body).
- 2: all radiation is reflected.

The incident area exposed to the sun must be known. The normals to the surfaces are assumed to point in the direction of the sun (e.g., solar arrays).

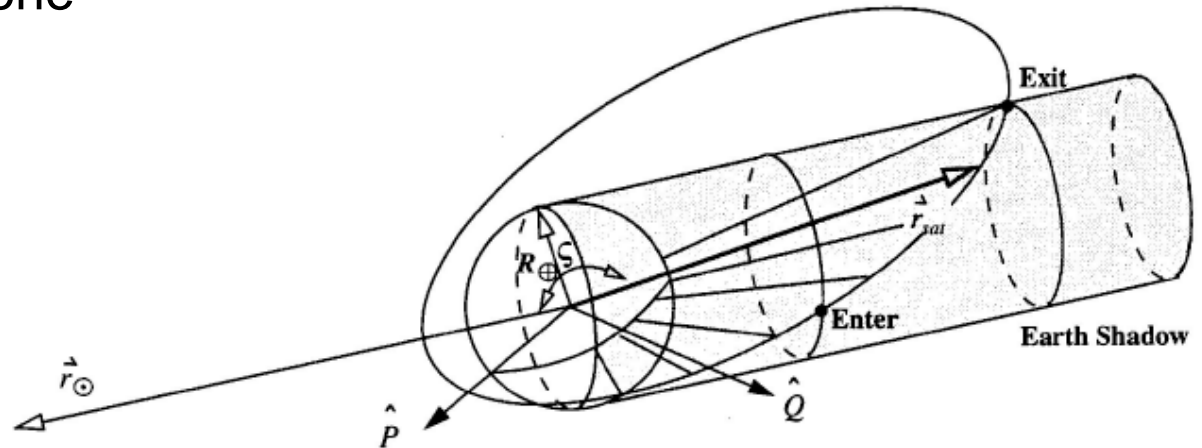
Mathematical Modeling: Eclipses

Use of shadow functions:

$$\mathbf{F}_{SR} = 0$$



Dual cone



Cylindrical

STK: Solar Radiation Pressure (HPOP)

The screenshot shows the STK software interface with the 'Force Model for OUFTI-1' dialog box open. The 'Solar Radiation Pressure' section is highlighted with a red circle. The dialog includes settings for Central Body Gravity, Drag, and Third Body Gravity.

Central Body Gravity

- Gravity: WGS84_EGM96.grv
- Maximum Degree: 21
- Maximum Order: 21
- Solid Tides: Permanent tide only
- Use Ocean Tides

Drag

- Use
- Cd: 2.200000
- Area/Mass Ratio: 0.02 m²/kg
- Atm. Density Model: Jacchia-Roberts

Solar Radiation Pressure

- Use
- Cr: 1.000000
- Area/Mass Ratio: 0.02 m²/kg
- Shadow Model: Dual Cone
- Use Boundary Mitigation

Third Body Gravity

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km ³ /sec ²
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km ³ /sec ²
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km ³ /sec ²
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km ³ /sec ²
Saturn	<input type="checkbox"/>	Cb file	3.794062606114e+007 km ³ /sec ²

More Options...

OK Cancel Help

STK: Shadow Models (HPOP)

Force Model for Satellite1

Central Body Gravity

Gravity: WGS84_EGM96.grv ...

Maximum Degree: 21

Maximum Order: 21

Solid Tides: Permanent tide only

Use Ocean Tides

Solar Radiation Pressure

Use

Cr: 1.000000

Area/Mass Ratio: 0.02 m²/kg

Shadow Model: Dual Cone

Use Boundary Model

Drag

Use

Cd: 2.200000

Area/Mass Ratio: 0.02 m²/kg

Atm. Density Model: Jacchia-Roberts

SolarFlux/GeoMag

Enter Manually

Daily F10.7: 150.00000000

Average F10.7: 150.00000000

Geomagnetic Index (Kp): 3.00000000

Third Body Gravity

Name	Use	Source	Gravity Value
Sun	<input checked="" type="checkbox"/>	Cb file	1.327122000000e+011 km ³ /sec ²
Moon	<input checked="" type="checkbox"/>	Cb file	4.902802953597e+003 km ³ /sec ²
Jupiter	<input type="checkbox"/>	Cb file	1.267127678578e+008 km ³ /sec ²
Venus	<input type="checkbox"/>	Cb file	3.248585920790e+005 km ³ /sec ²
Saturn	<input type="checkbox"/>	Cb file	3.794062608114e+007 km ³ /sec ²

More Options...

OK Cancel Help

STK: Central Body Pressure (HPOP)

The screenshot displays the STK software interface with two dialog boxes open. The 'Force Model for Satellite1' dialog is in the background, and the 'Force Model Options - Satellite1' dialog is in the foreground. Both dialog boxes have red circles highlighting the 'More Options...' button in the background and the 'Central Body Radiation Pressure' section in the foreground.

Force Model for Satellite1

Central Body Gravity

Gravity: WGS84_EGM96.grv

Maximum Degree: 21

Maximum Order: 21

Solid Tides: Permanent tide only

Use Ocean Tides

Solar Radiation Pressure

Use

Cr: 1.000000

Area/Mass Ratio: 0.02 m²/kg

Shadow Model: Dual Cone

Use Boundary Mitigation

Third Body Gravity

Name	Use	Source	Gravity Value
Pluto	<input type="checkbox"/>	Cb file	1.0090760000000e+003 km ³ /sec ²
Charon	<input type="checkbox"/>	Cb file	1.0810260000000e+002 km ³ /sec ²
Phobos	<input type="checkbox"/>	Cb file	7.0933990000000e-004 km ³ /sec ²
Deimos	<input type="checkbox"/>	Cb file	1.5881740000000e-004 km ³ /sec ²

More Options...

OK

Force Model Options - Satellite1

Drag

Use Approximate Altitude

Use Apparent Sun Position

Satellite Mass: 1000 kg

Include Relativistic Accelerations

Solar Radiation Pressure

Method to Compute Sun Position: Apparent To True CB

Atmospheric Altitude for the shape of the central body for Eclipse: 0 km

Solid Tides

Include Time Dependent Solid Tides

Minimum Amplitude: 0 m

Ocean Tides

Maximum Degree: 4

Maximum Order: 4

Minimum Amplitude: 0 m

Propagator Plugin

Use Plugin

Plugin Name:

Central Body Radiation Pressure

Include Albedo Ck: 1.000000

Include Thermal Area to Mass Ratio: 0.02 m²/k

Ground Reflection Model File: SimpleReflectionModel.txt

OK Cancel Help

S3L Propagator

Keplerian Parameters

Semi-major axis [m]	6778e3
Eccentricity	0.0
Inclination [deg]	51
Argument of perigee [deg]	0.0
RAAN [deg]	20
True anomaly [deg]	0.0

Control

None
 Cross-Section
 Attitude

Date

Year	2010
Month	10
Day	23
Hours	19
Minutes	40
Seconds	00
Simulation time [s]	5*24 * 3600

Force Model

Non-spherical
 Drag
 SRP
 Third-body Sun
 Third-body Sun

ECI to ECEF

Precession
 Nutation
 Polar Wandering

Simplified

Integration Parameters

Relative tolerance	1e-13
Absolute tolerance	1e-13
Output time step [s]	60

Spacecraft Properties

Mass [kg]	4
Sizes [m, m, m]	[0.3, 0.1, 0.1]
Cross-section to TAS [m^2]	0.03
Cross-section to Sun [m^2]	0.03
Drag Coefficient	4
Reflectivity Coefficient	[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]

Density Model

Harris-Priester
 Jacchia 71
 Jacchia-Roberts
 Measured data

Density Parameters

Harris-Priester coeff.	0
DailyF10.7	155
Averaged F10.7	155
Geomagnetic activity	3

Gravity Model

Maximum Degree	2
Maximum Order	0

Download Data



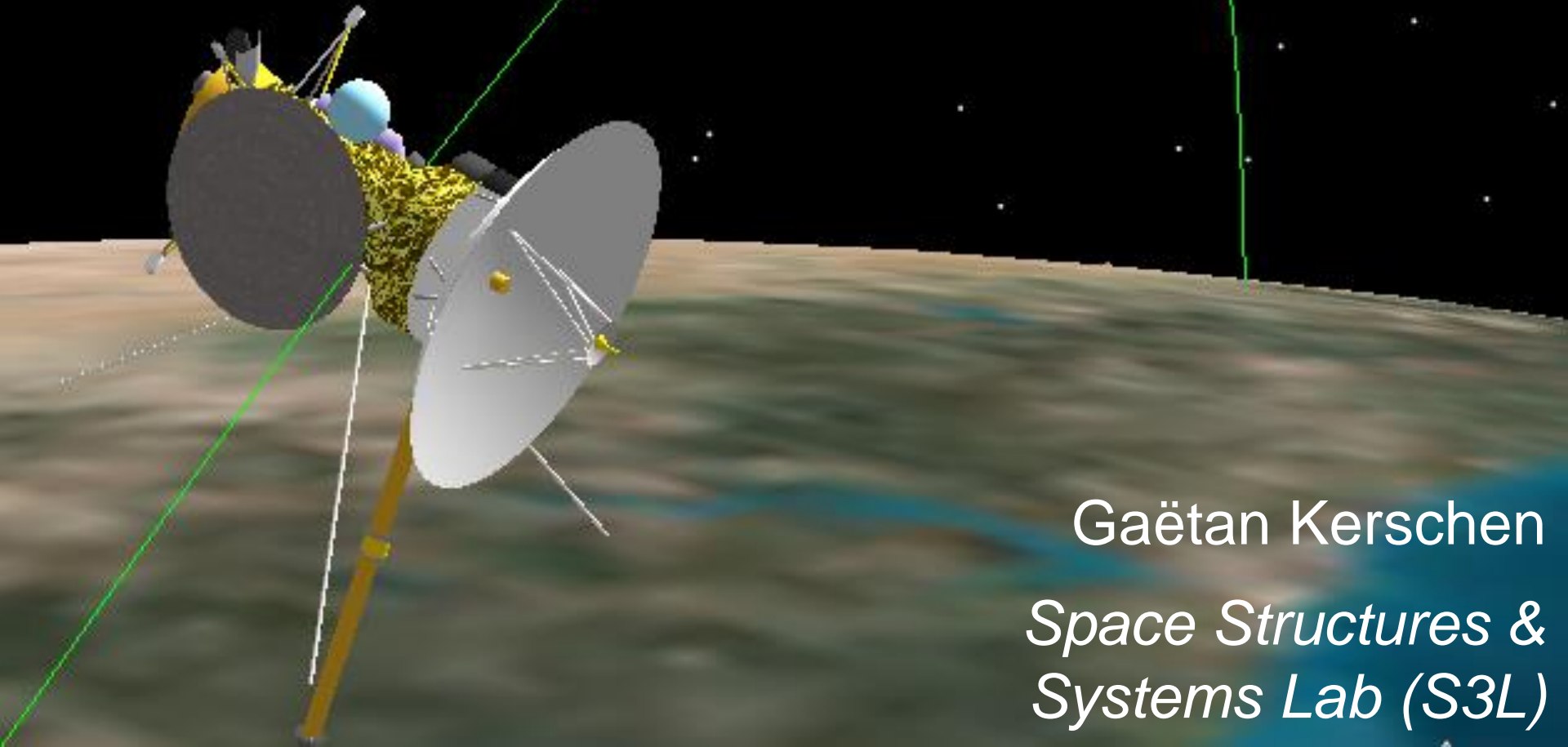
Orbit 3D

RUN!

Cassini Classical Orbit Elements
Time (UTCG): 15 Oct 1997 09:18:54.000
Semi-major Axis (km): 6685.637000
Eccentricity: 0.020566
Inclination (deg): 30.000
RAAN (deg): 150.546
Arg of Perigee (deg): 230.000
True Anomaly (deg): 136.530
Mean Anomaly (deg): 134.891

Aerodynamics (AERO0024)

5. Dominant Perturbations



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Systems Lab (S3L)*