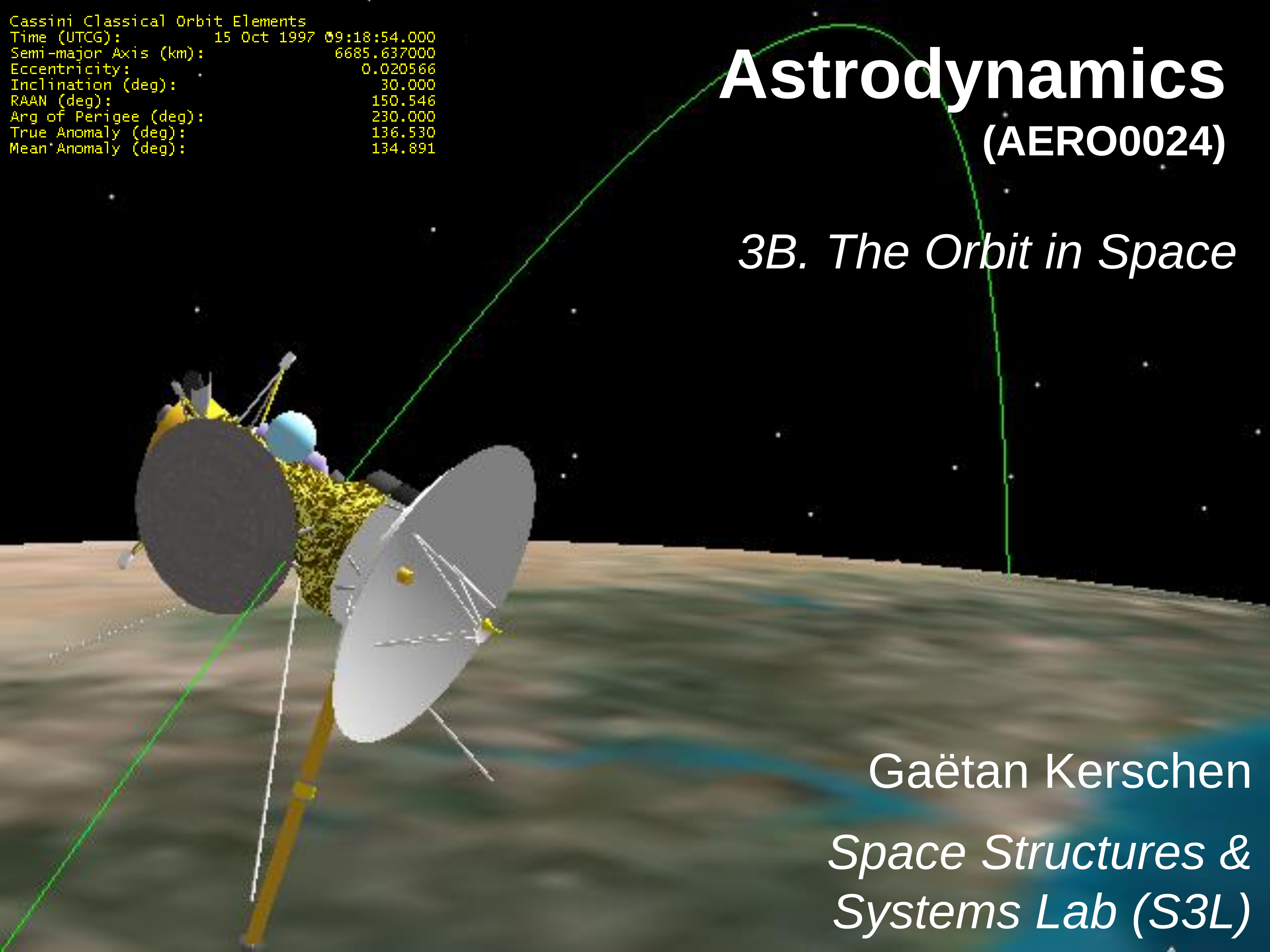


Astrodynamics
(AERO0024)

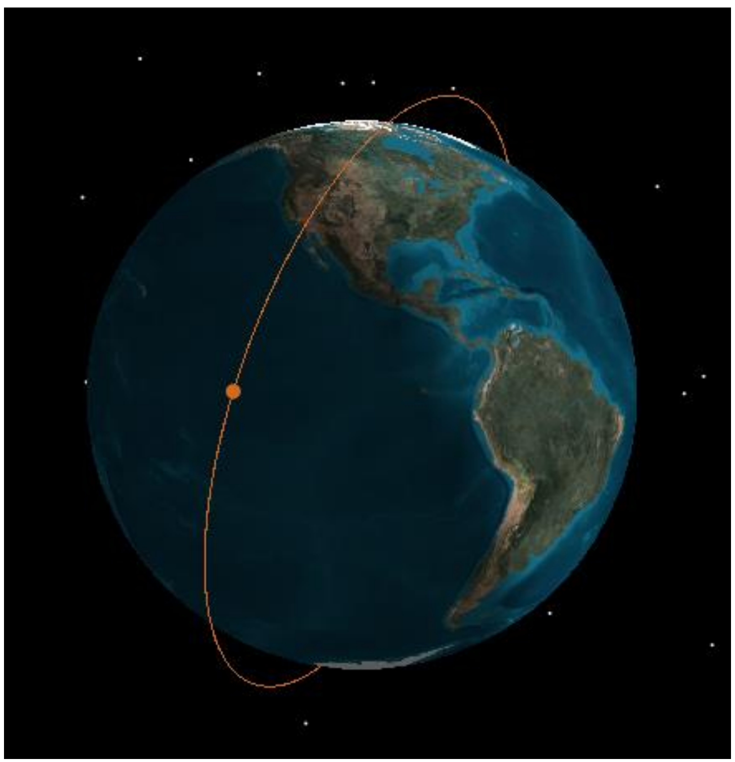
3B. The Orbit in Space

Gaëtan Kerschen

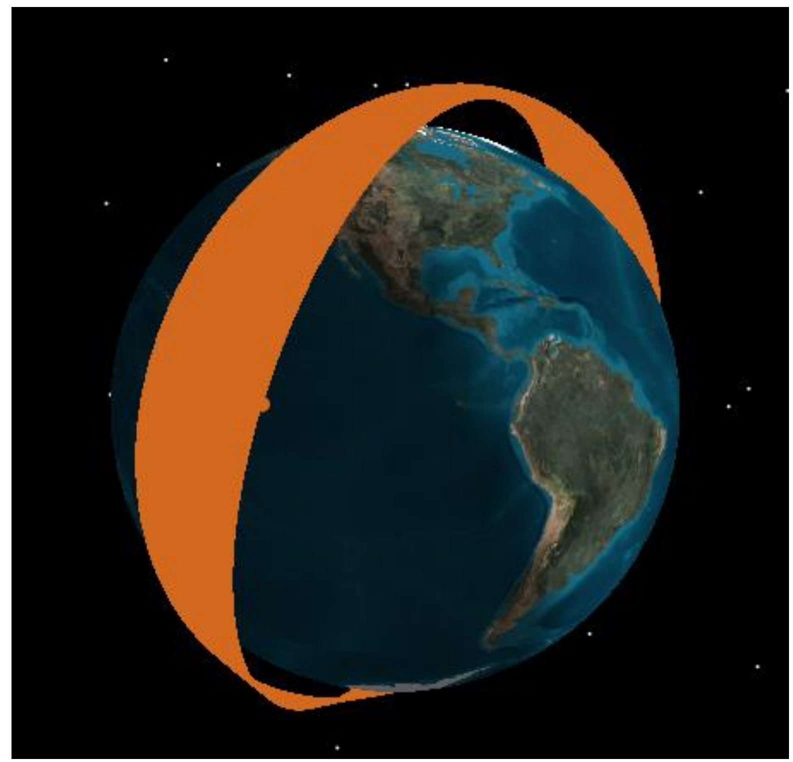
Space Structures & Systems Lab (S3L)

Motivation: Space

We need means of describing orbits in three-dimensional space.

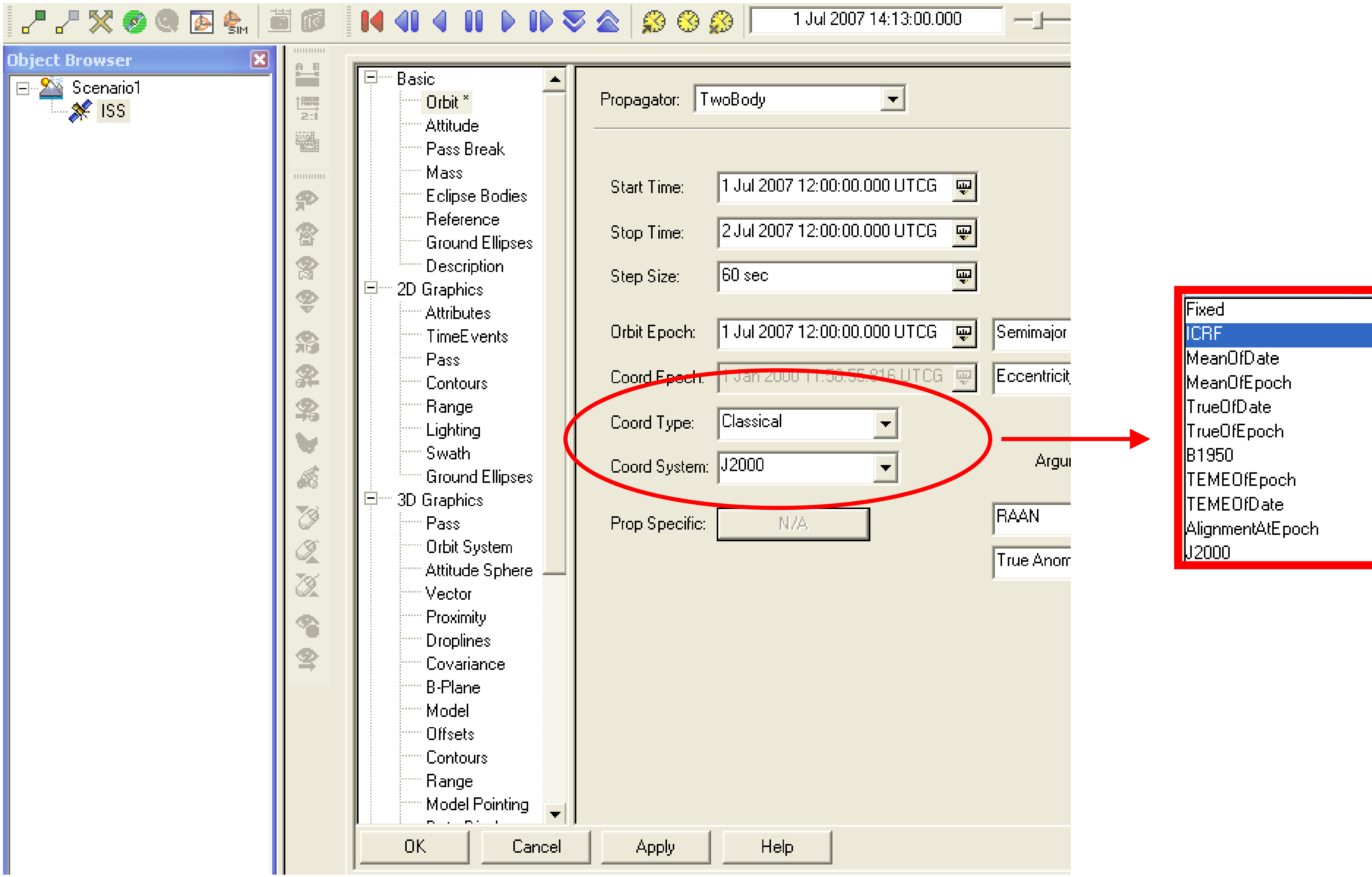


Two-body propagator

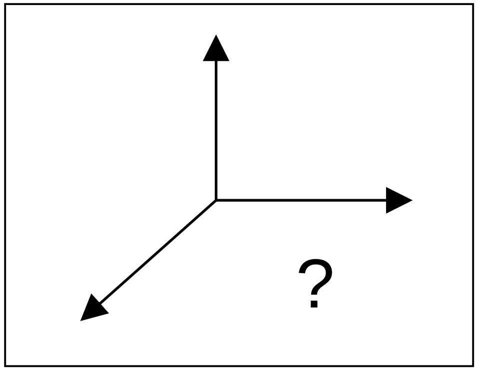


J2 propagator

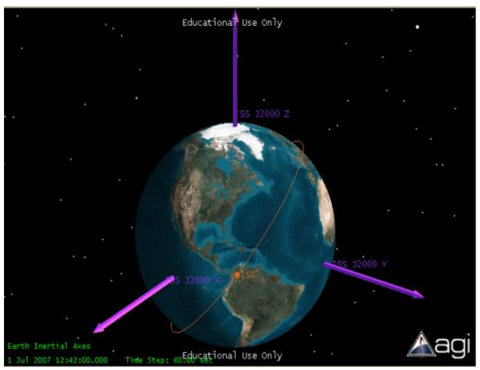
Complexity of Coordinate Systems: STK



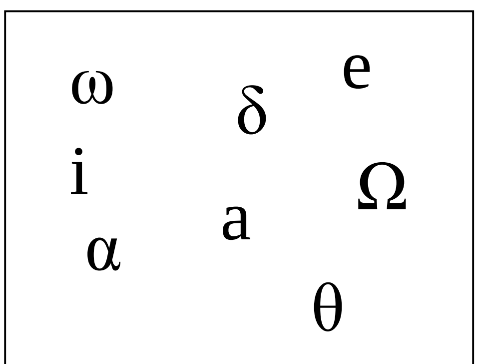
The Orbit in Space



Inertial frames



Coordinate systems



Coordinate types

Importance of Inertial Frames

An inertial reference frame is defined as a system that is neither rotating nor accelerating relative to a certain reference point.

Suitable inertial frames are required for orbit description (remember that Newton’s second law is to be expressed in an inertial frame).

An inertial frame is also an appropriate coordinate system for expressing positions and motions of celestial objects.

Reference System and Reference Frame

Distinction between reference system and a reference frame:

1. A reference system is the complete specification of how a celestial coordinate system is to be formed. For instance, it defines the origin and fundamental planes (or axes) of the coordinate system.

2. A reference frame consists of a set of identifiable points on the sky along with their coordinates, which serves as the practical realization of a reference system.

International Celestial Reference System (ICRS)

The ICRS is the reference system of the International Astronomical Union (IAU) for high-precision astronomy.

Its origin is located at the barycenter of the solar system.

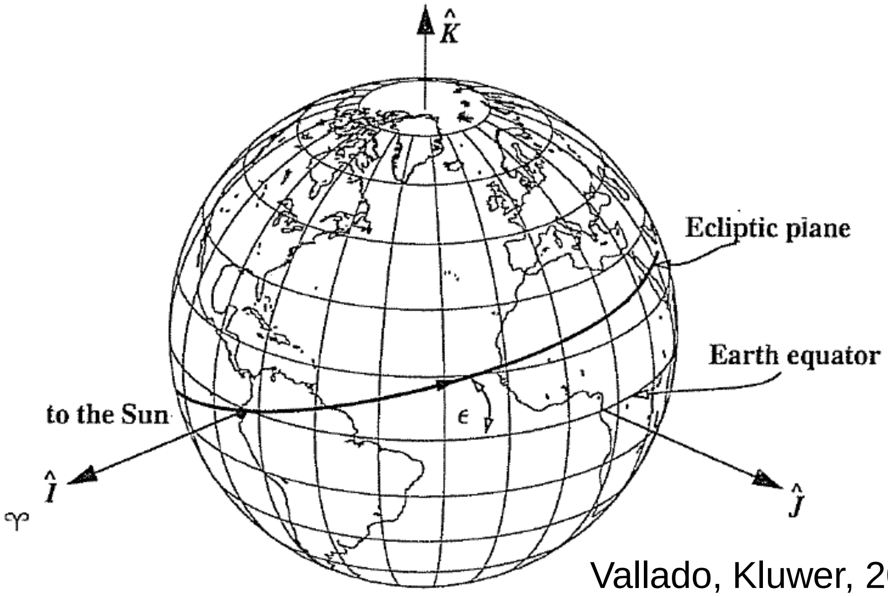
Definition of non-rotating axes:

1. The celestial pole is the Earth’s north pole (or the fundamental plane is the Earth's equatorial plane).

2. The reference direction is the vernal equinox (point at which the Sun crosses the equatorial plane moving from south to north).

3. Right-handed system.

Vernal Equinox ?



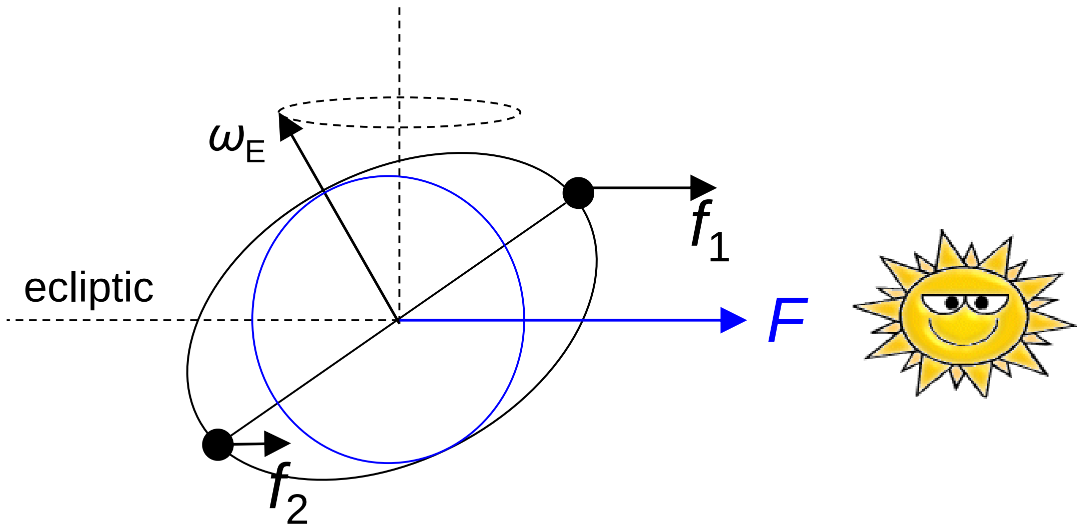
Vallado, Kluwer, 2001.

The vernal equinox is the intersection of the ecliptic and equator planes, where the sun passes from the southern to the northern hemisphere (First day of spring in the northern hemisphere).

Today, the vernal equinox points in the direction of the constellation Pisces, whereas it pointed in the direction of the constellation Ram during Christ's lifetime. Why ?

Rotation Axis: Lunisolar Precession

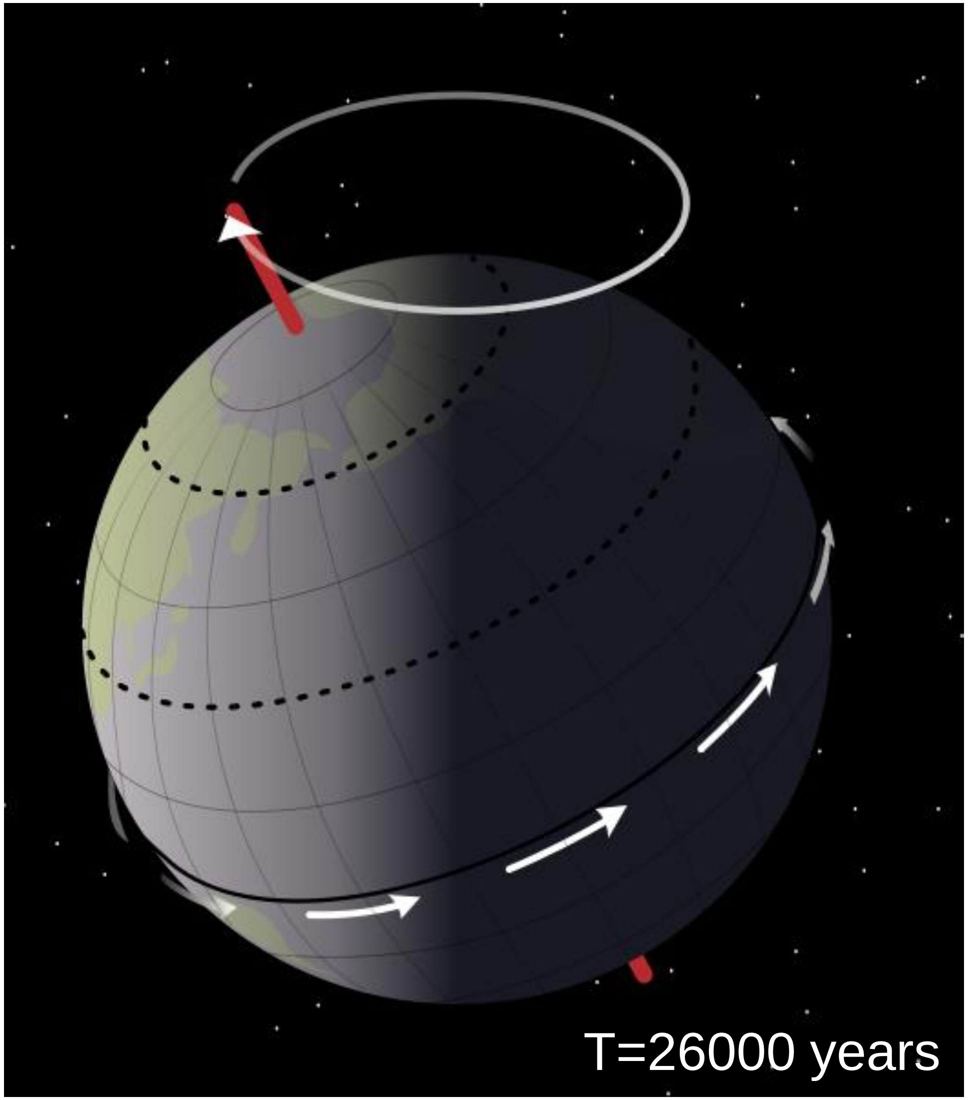
Because of the gravitational tidal forces of the Moon and Sun, the Earth’s spin axis precesses westward around the normal to the ecliptic at a rate of 1.4°/century. The Earth's axis sweeps out a cone of 23.3 degrees in 26000 years.



F: dominant force on the spherical mass.

f_1, f_2: forces due to the bulging sides; $f_1 > f_2$, which implies a net clockwise moment.

Rotation Axis: Lunisolar Precession



Competition between two effects:

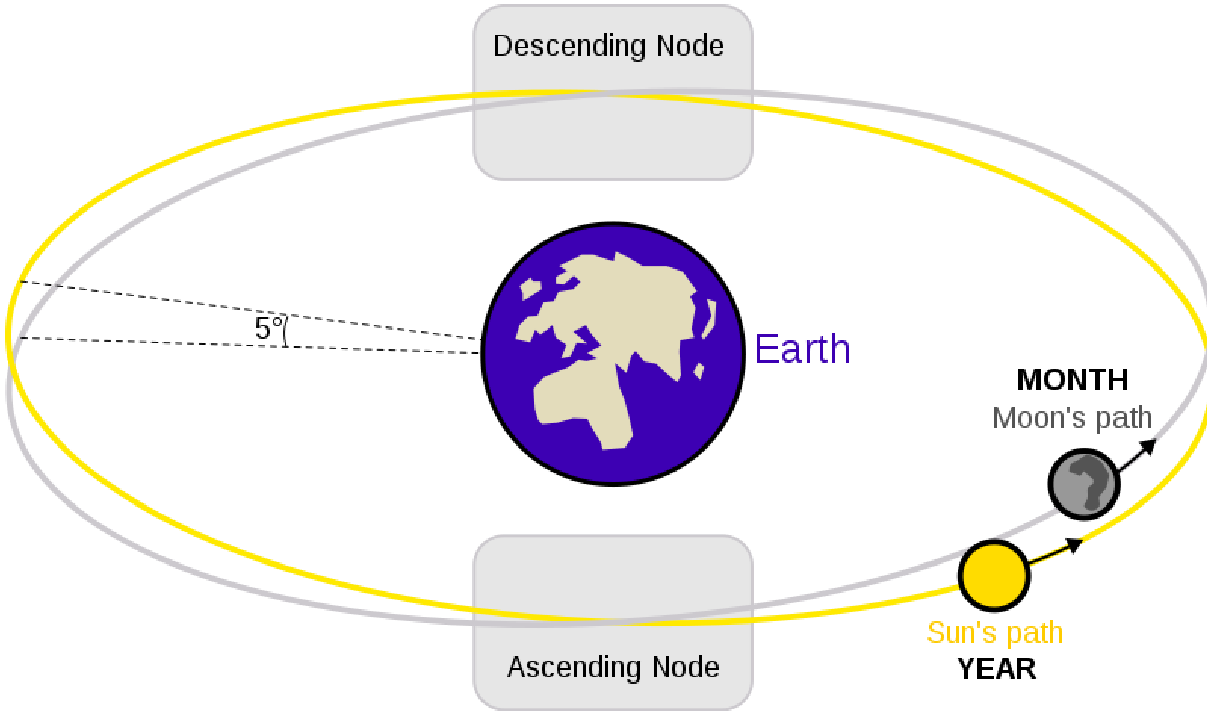
1. Gyroscopic stiffness of the spinning Earth (maintain orientation in inertial space).

2. Gravity gradient torque (pull the equatorial bulge into the plane of the ecliptic).

Rotation Axis: Nutation

The obliquity of the Earth varies with a maximum amplitude of 0.00025° over a period of 18.6 years.

This nutation is caused by the precession of the Moon's orbital nodes. They complete a revolution in 18.6 years.



Yet Another Disturbance: Polar Motion

Movement of Earth’s rotation axis across its surface.

Difference between the instantaneous rotational axis and the conventional international origin (CIO — a conventionally defined reference axis of the pole's average location over the year 1900).

The drift, about 20 m since 1900, is partly due to motions in the Earth's core and mantle, and partly to the redistribution of water mass as the Greenland ice sheet melts.

Yet Another Disturbance: Polar Motion

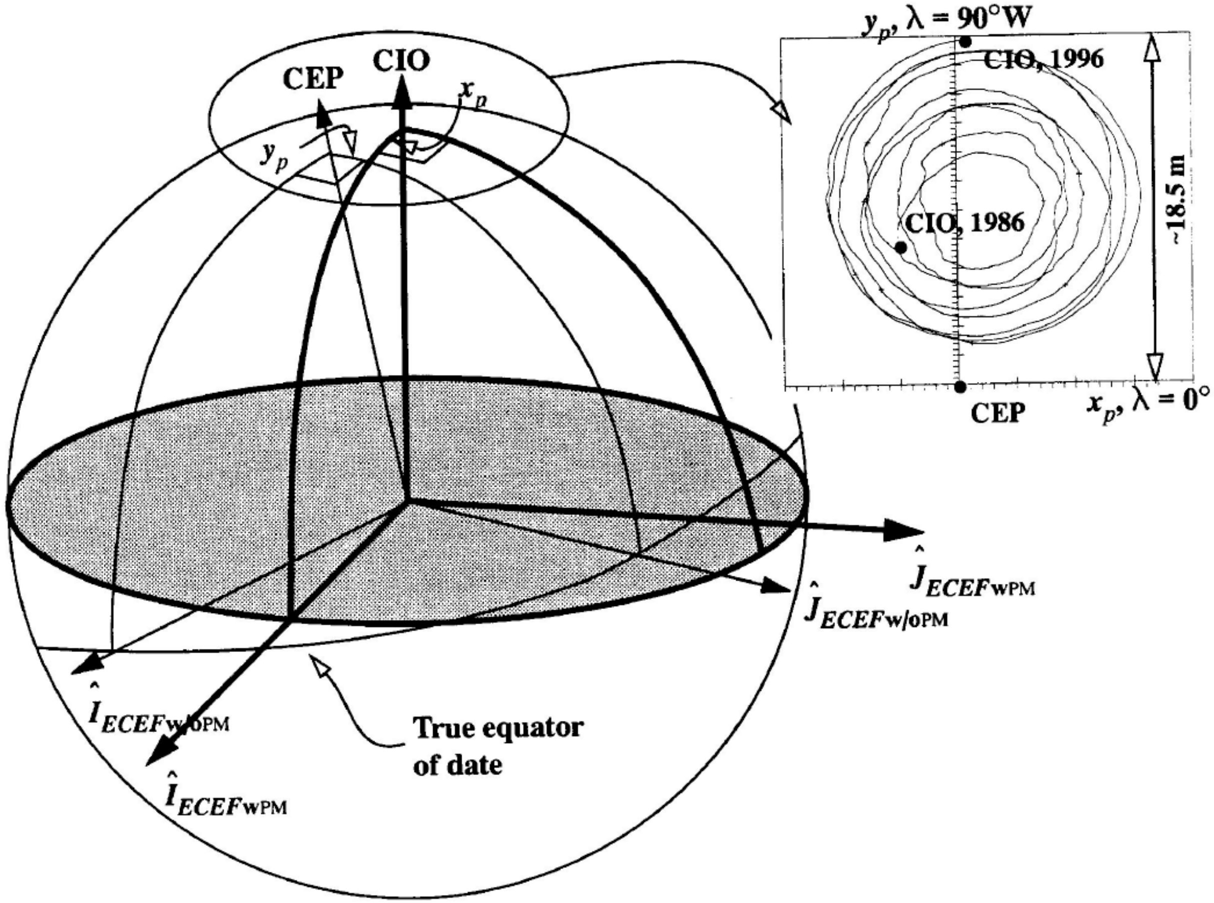
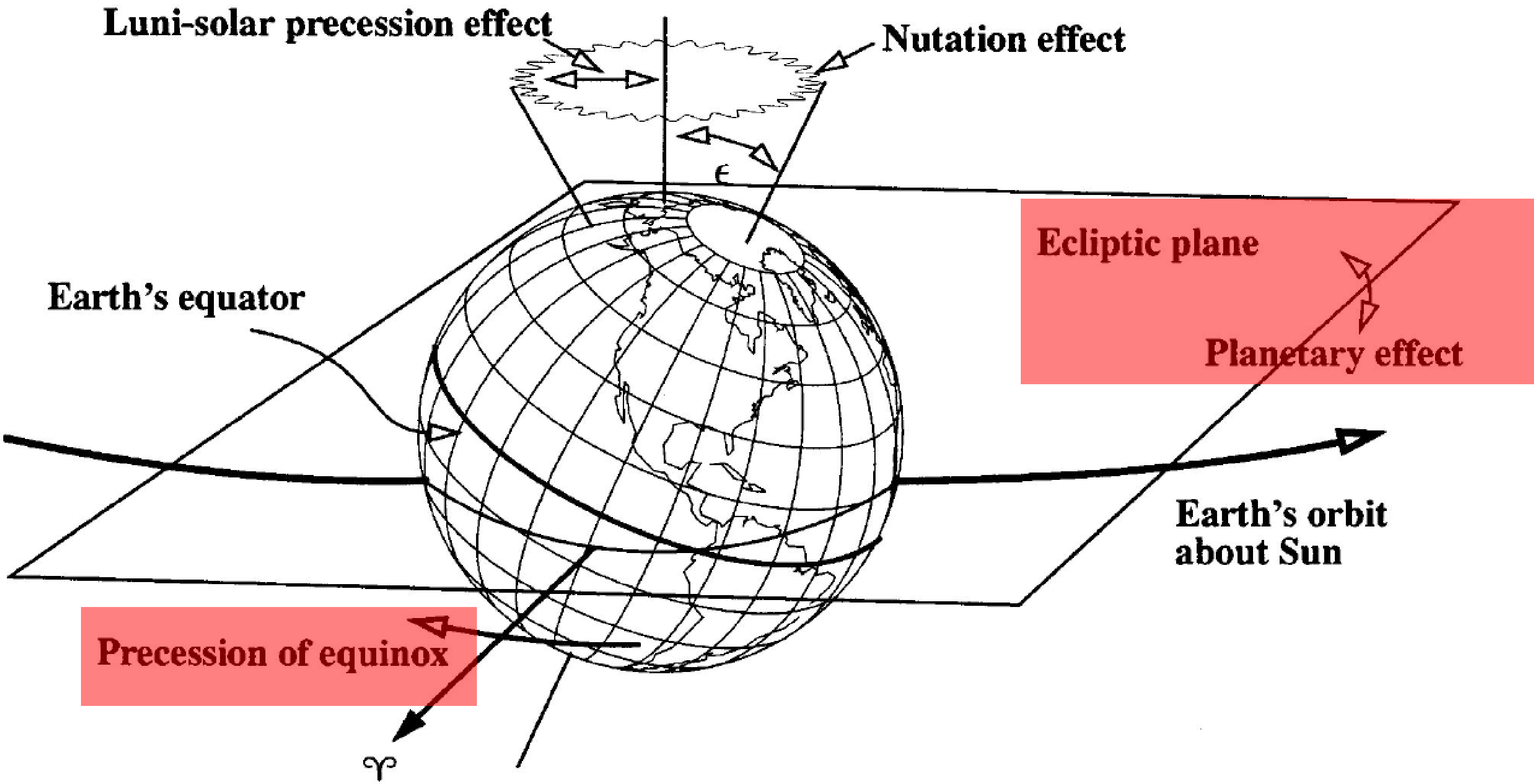


Figure 1-35. Transformation Geometry Due to Polar Motion. Accounting for polar motion takes into account the actual location of the Celestial Ephemeris Pole (CEP) over time. It moves from an *ECEF* system without polar motion through the CEP, to an *ECEF* system with polar motion using the Conventional International Origin (CIO). This correction changes the values very little, but highly accurate studies should include it. The inset plot shows the motion for the CIO from May 1986 to May 1996.

CIO: fixed with respect to the surface of the Earth

CEP: periodic motion (celestial ephemeris pole)

Complicated Motion of the Earth



Need To Specify a Date

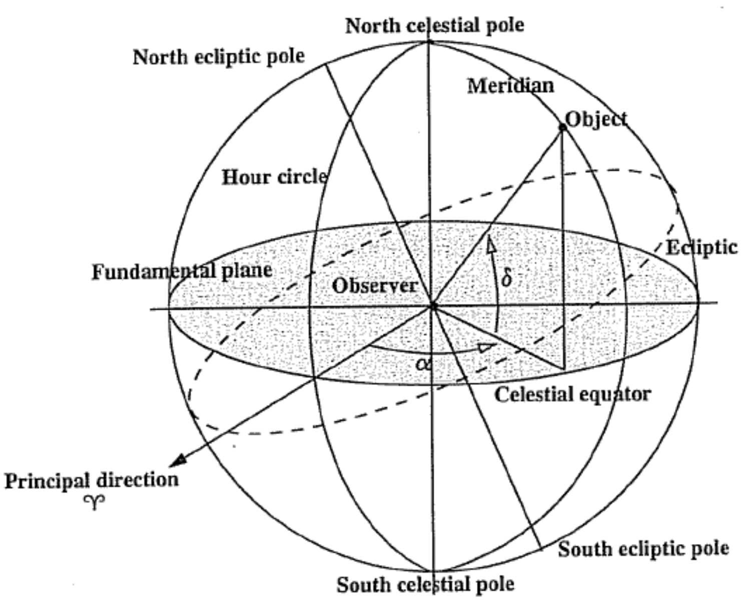
Because the ecliptic and equatorial planes are moving, the coordinate system must have a corresponding date:

"the pole/equator and equinox of [some date]".

For ICRS, the equator and equinox are considered at the epoch J2000.0 (January 1, 2000 at 11h58m56s UTC).

ICRS in Summary

Quasi-equatorial coordinates at the solar system barycenter !



An object is located in the ICRS using right ascension and declination

But how to realize ICRS practically ?

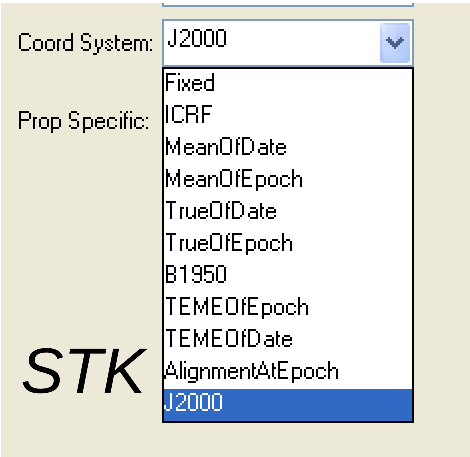
Previous Realizations: B1950 and J2000

B1950 and J2000 were considered the best realized inertial axes until the development of ICRF.

They exploit star catalogs (FK4 and FK5, respectively) which provide mean positions and proper motions for classical fundamental stars (optical measurements):

FK4 was published in 1963 and contained 1535 stars in various equinoxes from 1950 to 1975.

FK5 was an update of FK4 in 1988 with new positions for the 1535 stars.



1	0	8	23.265	+1.039	+29	5	25.58	-16.33	0	5	47.877	+1.036	+28	48	51.96	-16.33	43.31	0.7	2.0	33.00	1.3	3.1	2.06	A0p	+0.024	-11.7	358	BD+28	4	127
2	0	9	10.695	+6.827	+59	8	59.18	-18.09	0	6	29.765	+6.774	+58	52	26.54	-18.06	54.34	1.4	5.0	42.22	1.6	4.2	2.27	F5	+0.072	+11.8	432	BD+58	3	147
3	0	9	24.659	+1.186	-45	44	50.79	-18.11	0	6	52.788	+1.192	-46	1	23.36	-18.11	58.56	2.5	9.3	45.98	3.6	11.1	3.88	K0	+0.059	-9.2	496	CD-46	18	158
4	0	10	19.257	+0.074	+46	4	20.21	+0.03	0	7	42.779	+0.074	+45	47	38.71	+0.03	56.07	1.1	4.4	47.67	1.8	6.0	5.03	F0		-5.4	571	BD+45	17	169
5	0	11	34.437	+0.079	-27	47	59.12	+1.65	0	9	2.265	+0.079	-28	4	41.19	+1.65	56.47	1.6	6.5	46.63	2.9	9.7	5.42	K0		-5.7	720	CD-28	26	197
6	0	11	44.014	+1.412	-35	7	59.17	+11.86	0	9	11.739	+1.417	-35	24	46.32	+11.86	57.75	2.2	8.1	45.78	3.2	10.3	5.25	F5	+0.027	-1.7	739	CD-35	42	202
7	0	13	14.154	+0.019	+15	11	0.80	-1.20	0	10	39.483	+0.019	+14	54	20.50	-1.20	43.14	0.6	1.9	31.35	1.3	3.4	2.83	B2		+4.1	886	BD+14	14	238
9	0	19	25.674	-0.093	-08	49	26.14	-3.61	0	16	52.829	-0.093	-09	6	3.45	-3.61	43.93	0.8	2.5	32.95	1.6	4.8	3.56	K0	+0.010	+19.6	1522	BD-09	48	388
10	0	20	4.251	+26.778	-64	52	29.25	+116.39	0	17	28.799	+27.076	-65	10	6.35	+116.41	57.53	4.0	15.8	44.61	3.2	10.3	4.23	F8	+0.134	+8.7	1581	CP-65	13	401
11	0	25	45.056	+66.919	-77	15	15.40	+32.37	0	23	9.318	+68.429	-77	32	8.15	+32.37	43.89	6.4	17.2	28.53	2.6	6.8	2.80	G0	+0.153	+22.8	2151	CP-77	16	503
12	0	26	17.030	+1.833	-42	18	21.81	-39.57	0	23	49.051	+1.844	-42	34	38.31	-39.57	58.62	2.1	7.5	49.96	3.0	9.1	2.39	K0	+0.035	+74.6	2261	CD-42	116	519
13	0	30	2.362	+0.074	-03	57	26.39	-1.23	0	27	29.198	+0.074	-04	14	0.15	-1.23	42.48	0.7	2.4	30.56	1.5	4.5	5.72	K5		+4.7	2637	BD-04	54	584
14	0	30	22.661	-0.177	-23	47	15.72	+1.27	0	27	52.782	-0.177	-24	3	50.53	+1.27	53.36	1.4	5.1	46.67	2.6	9.5	5.19	A3	+0.012	+1.0	2696	CD-24	179	590
15	0	31	24.988	+1.449	-48	48	12.67	+1.75	0	29	0.619	+1.457	-49	4	47.11	+1.76	61.25	2.6	12.0	50.27	3.2	11.6	4.77	A2	+0.019	-5.0	2834	CD-49	115	619
16	0	32	59.982	+0.044	+62	55	54.40	-0.33	0	30	8.387	+0.044	+62	39	21.80	-0.33	51.76	1.7	6.0	41.93	1.7	4.9	4.16	B0		-2.3	2905	BD+62	102	645
17	0	36	58.291	+0.219	+53	53	48.92	-0.91	0	34	10.364	+0.218	+53	37	19.16	-0.91	55.12	1.2	4.6	42.60	1.7	5.0	3.66	B3		+2.1	3360	BD+53	105	727
18	0	36	52.858	+0.124	+33	43	9.63	-0.40	0	34	12.218	+0.124	+33	26	39.60	-0.40	53.90	0.9	3.4	45.10	1.8	5.6	4.36	B3		+8.7	3369	BD+32	101	729
19	0	38	33.350	-1.739	+29	18	42.30	-25.41	0	35	54.458	-1.732	+29	2	25.94	-25.42	52.82	0.8	3.1	45.56	1.5	5.1	4.37	G5	+0.031	-83.6	3546	BD+28	103	759
20	0	39	19.697	+1.060	+30	51	39.43	-9.15	0	36	38.890	+1.058	+30	35	15.48	-9.14	49.10	1.0	3.2	39.99	1.8	5.4	3.27	K2	+0.024	-7.3	3627	BD+30	91	774
21	0	40	30.450	+0.636	+56	32	14.46	-3.19	0	37	39.341	+0.632	+56	15	48.33	-3.18	50.70	1.2	3.8	32.85	1.5	3.4	2.23	K0	+0.009	-3.8	3712	BD+55	139	792
22	0	43	35.372	+1.637	-17	59	11.82	+3.25	0	41	4.844	+1.639	-18	15	38.66	+3.27	36.70	0.8	2.4	25.46	1.6	4.5	2.04	K0	+0.057	+13.1	4128	BD-18	115	965

Fifth Fundamental Catalog (FK5), available on the web site

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Byte-by-byte description of the file: catalog
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  1-  4     I4      ---       FK5       *[1/1670]+ FK5 number
  6-  7     I2      h         RAh        Right ascension, hours,
                                           Equinox=J2000, Epoch=J2000
  9- 10     I2      min       RAm        Right ascension minutes (J2000.0)
 12- 17     F6.3    s         RAs       *Right ascension seconds (J2000.0)
 19- 25     F7.3    s/ha      pmRA       Proper motion in RA (J2000.0)
     27     A1      ---       DE-        Sign of declination (Dec) (J2000.0)
 28- 29     I2      deg       DEd        Declination degrees (J2000.0)
 31- 32     I2      arcmin    DEm        Declination arcminutes (J2000.0)
 34- 38     F5.2    arcsec    DEs       *Declination arcseconds (J2000.0)
 40- 46     F7.2  arcsec/ha   pmDE       Proper motion in DE (J2000.0)
 48- 49     I2      h         RA1950h    Right ascension, hours
                                           Equinox=B1950, Epoch=B1950
 51- 52     I2      min       RA1950m    Right ascension minutes (B1950.0)
 54- 59     F6.3    s         RA1950s   *Right ascension seconds (B1950.0)
 61- 67     F7.3    s/ha      pmRA1950   Proper motion in RA (B1950.0)
     69     A1      ---       DE1950-    Sign of declination (B1950.0)
 70- 71     I2      deg       DE1950d    Declination degrees (B1950.0)
 73- 74     I2      arcmin    DE1950m    Declination arcminutes (B1950.0)
 76- 80     F5.2    arcsec    DE1950s   *Declination arcseconds (B1950.0)
 82- 88     F7.2  arcsec/ha   pmDE1950   Proper motion in DE (B1950.0)
 90- 94     F5.2    a        EpRA-1900  *Mean Epoch of observed RA
 96- 99     F4.1    ms        e_RAs     *Mean error in RA
101-105     F5.1    ms/ha     e_pmRA     Mean error in pmRA
107-111     F5.2    a        EpDE-1900  *Mean Epoch of observed DE
113-116     F4.1    carcsec   e_DEs     *Mean error in Declination
118-122     F5.1  carcsec/ha e_pmDE      Mean error in pmDE
124-128     F5.2    mag       Vmag      *V magnitude
    129     A1      ---       n_Vmag    *[VvD] Magnitude flag
131-137     A7      ---       SpType    *Spectral type(s)
139-144     F6.3    arcsec    plx       *?Parallax
147-152     F6.1    km/s      RV        *?Radial velocity
155-159     A5      ---       AGK3R      AGK3R number (Catalog <I/72>)
```

Star Catalogs: Limitations and Improvement

1. The uncertainties in the star positions of the FK5 are about 30-40 milliarcseconds over most of the sky.

2. A stellar reference frame is time-dependent because stars exhibit detectable motions.

1. Uncertainties of radio source positions are now typically less than one milliarcsecond, and often a factor of ten better.

2. Radio sources are not expected to show measurable intrinsic motion.

1	0	8	23.265	+1.039	+29	5	25.58	-16.33	0	5	47.877	+1.036	+28	48	51.96	-16.33	43.31	0.7	2.0	33.00	1.3	3.1	2.06	A0p	+0.024	-11.7	358	BD+29	4	127
2	0	9	10.695	+6.827	+59	8	59.18	-18.09	0	6	29.765	+6.774	+58	52	26.54	-18.06	54.34	1.4	5.0	42.22	1.6	4.2	2.27	F5	+0.072	+11.8	432	BD+58	3	147
3	0	9	24.659	+1.186	-45	44	50.79	-18.11	0	6	52.788	+1.192	-46	1	23.36	-18.11	58.56	2.5	9.3	45.98	3.6	11.1	3.88	K0	+0.059	-9.2	496	CD-46	18	158
4	0	10	19.257	+0.074	+46	4	20.21	+0.03	0	7	42.779	+0.074	+45	47	38.71	+0.03	56.07	1.1	4.4	47.67	1.8	6.0	5.03	F0		-5.4	571	BD+45	17	169
5	0	11	34.437	+0.079	-27	47	59.12	+1.65	0	9	2.265	+0.079	-28	4	41.19	+1.65	56.47	1.6	6.5	46.63	2.9	9.7	5.42	K0		-5.7	720	CD-28	26	197
6	0	11	44.014	+1.412	-35	7	59.17	+11.86	0	9	11.739	+1.417	-35	24	46.32	+11.86	57.75	2.2	8.1	45.79	3.2	10.3	5.25	F5	+0.027	-1.7	739	CD-35	42	202
7	0	13	14.154	+0.019	+15	11	0.80	-1.20	0	10	39.483	+0.019	+14	54	20.50	-1.20	43.14	0.6	1.9	31.35	1.3	3.4	2.83	B2		+4.1	886	BD+14	14	238
9	0	19	25.674	-0.093	-08	49	26.14	-3.61	0	16	52.829	-0.093	-09	6	3.45	-3.61	43.93	0.8	2.5	32.95	1.6	4.8	3.56	K0	+0.010	+18.6	1522	BD-09	48	388
10	0	20	4.251	+26.778	-64	52	29.25	+116.39	0	17	28.799	+27.076	-65	10	6.35	+116.41	57.53	4.0	15.8	44.61	3.2	10.3	4.23	F8	+0.134	+8.7	1581	CP-65	13	401
11	0	25	45.056	+66.919	-77	15	15.40	+32.37	0	23	9.318	+68.429	-77	32	8.15	+32.37	43.89	6.4	17.2	28.53	2.6	6.8	2.80	G0	+0.153	+22.8	2151	CP-77	16	503
12	0	26	17.030	+1.833	-42	18	21.81	-39.57	0	23	49.051	+1.844	-42	34	38.31	-39.57	58.62	2.1	7.5	49.96	3.0	9.1	2.39	K0	+0.035	+74.6	2261	CD-42	116	519
13	0	30	2.362	+0.074	-03	57	26.39	-1.23	0	27	29.198	+0.074	-04	14	0.15	-1.23	42.48	0.7	2.4	30.56	1.5	4.5	5.72	K5		+4.7	2637	BD-04	54	584
14	0	30	22.661	-0.177	-23	47	15.72	+1.27	0	27	52.782	-0.177	-24	3	50.53	+1.27	53.36	1.4	5.1	46.67	2.6	8.5	5.19	A3	+0.012	+1.0	2696	CD-24	179	590
15	0	31	24.988	+1.449	-48	48	12.67	+1.75	0	29	0.619	+1.457	-49	4	47.11	+1.76	61.25	2.6	12.0	50.27	3.2	11.6	4.77	A2	+0.019	-5.0	2834	CD-49	115	619
16	0	32	59.982	+0.044	+62	55	54.40	-0.33	0	30	8.387	+0.044	+62	39	21.80	-0.33	51.76	1.7	6.0	41.93	1.7	4.9	4.16	B0		-2.3	2905	BD+62	102	645
17	0	36	58.291	+0.219	+53	53	48.92	-0.91	0	34	10.364	+0.218	+53	37	19.16	-0.91	55.12	1.2	4.6	42.60	1.7	5.0	3.66	B3		+2.1	3360	BD+53	105	727
18	0	36	52.858	+0.124	+33	43	9.63	-0.40	0	34	12.218	+0.124	+33	26	39.60	-0.40	53.90	0.9	3.4	45.10	1.8	5.6	4.36	B3		+8.7	3369	BD+32	101	729
19	0	38	33.350	-1.739	+29	18	42.30	-25.41	0	35	54.458	-1.732	+29	2	25.94	-25.42	52.82	0.8	3.1	45.56	1.5	5.1	4.37	G5	+0.031	-83.6	3546	BD+28	103	759
20	0	39	19.697	+1.060	+30	51	39.43	-9.15	0	36	38.890	+1.058	+30	35	15.48	-9.14	49.10	1.0	3.2	39.99	1.8	5.4	3.27	K2	+0.024	-7.3	3627	BD+30	91	774
21	0	40	30.450	+0.636	+56	32	14.46	-3.19	0	37	39.341	+0.632	+56	15	48.33	-3.18	50.70	1.2	3.8	32.85	1.5	3.4	2.23	K0	+0.009	-3.8	3712	BD+55	139	792
22	0	43	35.372	+1.637	-17	59	11.82	+3.25	0	41	4.844	+1.639	-18	15	38.66	+3.27	36.70	0.8	2.4	25.46	1.6	4.5	2.04	K0	+0.057	+13.1	4128	BD-18	115	965

Fifth Fundamental Catalog (FK5), available on the web site

Byte-by-byte description of the file: catalog

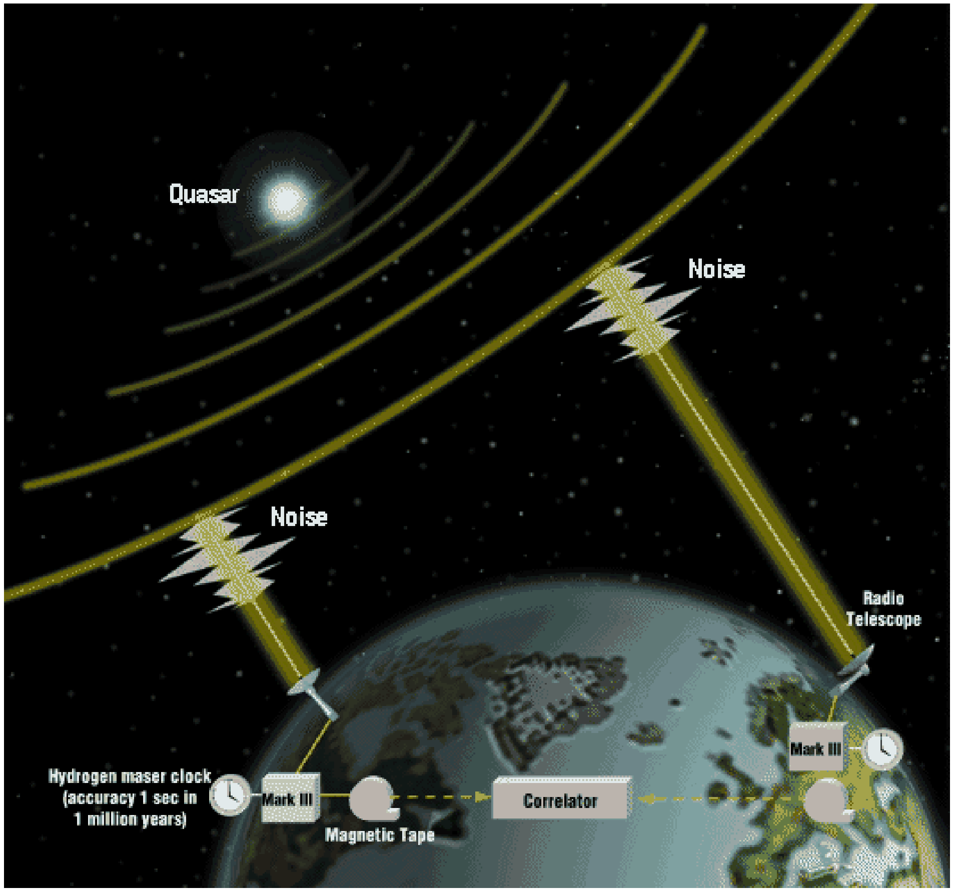
Bytes	Format	Units	Labels	Explanations
1- 4	I4	---	FK5	*[1/1670]+ FK5 number
6- 7	I2	h	RAh	Right ascension, hours, Equinox=J2000, Epoch=J2000
9- 10	I2	min	RAm	Right ascension minutes (J2000.0)
12- 17	F6.3	s	RAs	*Right ascension seconds (J2000.0)
19- 25	F7.3	s/ha	pmRA	Proper motion in RA (J2000.0)
27	A1	---	DE-	Sign of declination (Dec) (J2000.0)
28- 29	I2	deg	DEd	Declination degrees (J2000.0)
31- 32	I2	arcmin	DEm	Declination arcminutes (J2000.0)
34- 38	F5.2	arcsec	DEs	*Declination arcseconds (J2000.0)
40- 46	F7.2	arcsec/ha	pmDE	Proper motion in DE (J2000.0)
48- 49	I2	h	RA1950h	Right ascension, hours Equinox=B1950, Epoch=B1950
51- 52	I2	min	RA1950m	Right ascension minutes (B1950.0)
54- 59	F6.3	s	RA1950s	*Right ascension seconds (B1950.0)
61- 67	F7.3	s/ha	pmRA1950	Proper motion in RA (B1950.0)
69	A1	---	DE1950-	Sign of declination (B1950.0)
70- 71	I2	deg	DE1950d	Declination degrees (B1950.0)
73- 74	I2	arcmin	DE1950m	Declination arcminutes (B1950.0)
76- 80	F5.2	arcsec	DE1950s	*Declination arcseconds (B1950.0)
82- 88	F7.2	arcsec/ha	pmDE1950	Proper motion in DE (B1950.0)
90- 94	F5.2	a	EpRA-1900	*Mean Epoch of observed RA
96- 99	F4.1	ms	e_RAs	*Mean error in RA
101-105	F5.1	ms/ha	e_pmRA	Mean error in pmRA
107-111	F5.2	a	EpDE-1900	*Mean Epoch of observed DE
113-116	F4.1	carcsec	e_DEs	*Mean error in Declination
118-122	F5.1	carcsec/ha	e_pmDE	Mean error in pmDE
124-128	F5.2	mag	Vmag	*V magnitude
129	A1	---	n_Vmag	*[VvD] Magnitude flag
131-137	A7	---	SpType	*Spectral type(s)
139-144	F6.3	arcsec	plx	*?Parallax
147-152	F6.1	km/s	RV	*?Radial velocity
155-159	A5	---	AGK3R	AGK3R number (Catalog <I/72>)

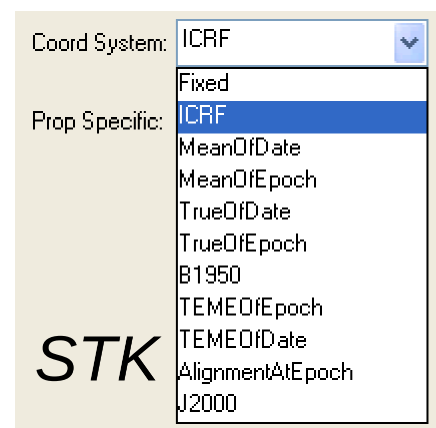
ICRF is the Current Realization of ICRS

Since 1998, IAU adopted the International Celestial Reference Frame (ICRF) as the standard reference frame: quasi-inertial reference frame with barely no time dependency.

It represents an improvement upon the theory behind the J2000 frame, and it is the best realization of an inertial frame constructed to date.

Very Long Baseline Interferometry





Further Reading on the Web Site

THE ASTRONOMICAL JOURNAL, 116:516–546, 1998 July

THE INTERNATIONAL CELESTIAL REFERENCE FRAME AS REALIZED BY VERY LONG BASELINE INTERFEROMETRY

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TABLE 3

COORDINATES OF THE 212 DEFINING SOURCES IN THE ICRF

		Note[c]								Epoch of Observation[d]				
Designation[a]	Source[b]	X	S	H	α (J2000.0)	δ (J2000.0)	σ_α (s)	σ_δ (arcsec)	$C_{\alpha\text{-}\delta}$	Mean	First	Last	N_{exp}[e]	N_{obs}[f]
ICRF J000557.1+382015......	0003+380	...	...		00 05 57.175409	38 20 15.14857	0.000041	0.00051	−0.041	49,087.0	48,720.9	49,554.8	2	41
ICRF J001031.0+105829......	0007+106	...	...		00 10 31.005888	10 58 29.50412	0.000032	0.00068	0.540	47,938.9	47,288.7	49,690.0	10	74
ICRF J001033.9+172418......	0007+171	...	...		00 10 33.990619	17 24 18.76135	0.000021	0.00035	−0.402	48,730.8	47,931.6	49,662.8	19	57
ICRF J001331.1+405137......	0010+405	2	1		00 13 31.130213	40 51 37.14407	0.000026	0.00034	−0.038	49,549.6	48,434.7	49,820.5	7	219
ICRF J001708.4+813508......	0014+813	...	...		00 17 08.474953	81 35 08.13633	0.000121	0.00026	0.012	49,505.2	47,023.7	49,924.8	78	1453
ICRF J004204.5+232001......	0039+230	...	...		00 42 04.545183	23 20 01.06129	0.000036	0.00060	0.090	48,898.1	48,328.5	49,533.8	3	44
ICRF J004959.4−573827......	0047−579	...	...		00 49 59.473091	−57 38 27.33992	0.000047	0.00053	0.298	48,697.0	47,626.5	49,407.6	13	46
ICRF J011205.8+224438......	0109+224	...	...	Y	01 12 05.824718	22 44 38.78619	0.000027	0.00049	0.082	48,733.1	48,434.7	49,736.9	7	97
ICRF J012642.7+255901......	0123+257	...	...		01 26 42.792631	25 59 01.30079	0.000030	0.00054	0.167	48,856.4	48,328.5	49,659.8	4	71
ICRF J013305.7−520003......	0131−522	...	...		01 33 05.762585	−52 00 03.94693	0.000049	0.00081	0.399	49,039.1	48,162.4	49,895.6	6	30
ICRF J013658.5+475129......	0133+476	2	2		01 36 58.594810	47 51 29.10006	0.000026	0.00027	0.021	48,629.0	45,138.8	49,750.8	190	2196
ICRF J013738.3−243053......	0135−247	...	...		01 37 38.346378	−24 30 53.88526	0.000055	0.00042	−0.188	48,321.8	47,640.2	49,790.7	3	29
ICRF J014125.8−092843......	0138−097	2	1		01 41 25.832025	−09 28 43.67381	0.000081	0.00088	0.063	47,138.1	46,875.8	49,498.8	2	20
ICRF J015127.1+274441......	0148+274	...	...		01 51 27.146149	27 44 41.79365	0.000031	0.00043	−0.064	48,963.9	48,328.5	49,659.8	5	112
ICRF J015218.0+220707......	0149+218	...	...		01 52 18.059047	22 07 07.70004	0.000020	0.00029	−0.437	48,294.0	46,977.9	49,848.8	50	243
ICRF J015734.9+744243......	0153+744	4	3	Y	01 57 34.964908	74 42 43.22998	0.000091	0.00031	0.059	49,495.7	47,019.9	49,820.5	11	400
ICRF J020333.3+723253......	0159+723	...	...		02 03 33.385004	72 32 53.66741	0.000072	0.00031	0.033	48,800.7	47,011.4	49,667.9	17	108
ICRF J020504.9+321230......	0202+319	...	...		02 05 04.925371	32 12 30.09560	0.000022	0.00030	−0.441	48,017.7	45,466.3	49,736.9	35	214
ICRF J021748.9+014449......	0215+015	1	1		02 17 48.954740	01 44 49.69909	0.000022	0.00039	−0.215	49,302.1	48,328.5	49,547.8	5	133
ICRF J022239.6+430207......	0219+428	...	...		02 22 39.611500	43 02 07.79884	0.000034	0.00043	−0.098	49,103.6	48,650.8	49,554.8	7	64
ICRF J022256.4−344128......	0220−349	...	...		02 22 56.401625	−34 41 28.73011	0.000050	0.00044	−0.209	48,679.5	47,640.2	49,790.7	4	35
ICRF J022850.0+672103......	0224+671	...	...		02 28 50.051459	67 21 03.02926	0.000052	0.00031	−0.080	45,097.6	44,090.5	49,600.3	42	801
ICRF J022934.9−784745......	0230−790	...	...		02 29 34.946647	−78 47 45.60129	0.000149	0.00049	0.028	48,828.1	47,626.5	49,895.6	11	52
ICRF J023838.9+163659......	0235+164	1	1		02 38 38.930108	16 36 59.27471	0.000018	0.00027	0.090	47,475.7	44,447.0	49,909.6	194	2595
ICRF J024229.1+110100......	0239+108	2	2		02 42 29.170847	11 01 00.72823	0.000018	0.00030	−0.483	48,582.3	47,511.1	49,662.8	43	153
ICRF J025134.5+431515......	0248+430	...	...		02 51 34.536779	43 15 15.82858	0.000027	0.00033	−0.074	49,109.4	47,931.6	49,690.0	10	169
ICRF J025927.0+074739......	0256+075	...	...		02 59 27.076633	07 47 39.64323	0.000021	0.00035	−0.607	48,247.0	47,011.4	49,445.6	44	190
ICRF J030350.6−621125......	0302−623	...	...		03 03 50.631333	−62 11 25.54983	0.000047	0.00033	0.129	49,059.2	48,162.4	49,650.8	15	97
ICRF J030903.6+102916......	0306+102	...	...		03 09 03.623523	10 29 16.34082	0.000023	0.00042	−0.804	48,974.1	47,394.1	49,667.9	18	76
ICRF J030956.0−605839......	0308−611	...	...		03 09 56.099167	−60 58 39.05628	0.000038	0.00029	0.037	49,029.5	47,626.5	49,895.6	79	738
ICRF J031301.9+412001......	0309+411	...	...	Y	03 13 01.962129	41 20 01.18353	0.000026	0.00031	−0.321	48,371.0	47,165.8	49,848.8	29	127
ICRF J034506.4+145349......	0342+147	...	...		03 45 06.416546	14 53 49.55818	0.000021	0.00032	−0.622	48,809.6	47,394.1	49,445.6	23	177
ICRF J040305.5+260001......	0400+258	3	2	Y	04 03 05.586048	26 00 01.50274	0.000020	0.00030	−0.127	48,990.5	47,005.8	49,820.5	37	397
ICRF J040922.0+121739......	0406+121	2	1		04 09 22.008740	12 17 39.84750	0.000021	0.00033	−0.704	48,399.2	46,977.9	49,565.9	28	149
ICRF J041636.5−185108......	0414−189	...	...		04 16 36.544466	−18 51 08.34012	0.000051	0.00048	−0.078	47,814.6	46,840.8	49,790.7	3	31
ICRF J042442.2−375620......	0422−380	...	...		04 24 42.243727	−37 56 20.78423	0.000033	0.00119	0.251	49,081.7	48,162.4	49,750.8	11	60
ICRF J042446.8+003606......	0422+004	2	1		04 24 46.842052	00 36 06.32983	0.000020	0.00063	0.038	48,938.2	45,997.8	49,820.5	11	245
ICRF J042636.6+051819......	0423+051	...	...		04 26 36.604102	05 18 19.87204	0.000031	0.00087	0.101	48,977.3	48,194.7	49,667.9	9	64
ICRF J042840.4−375619......	0426−380	...	...		04 28 40.424306	−37 56 19.58031	0.000036	0.00047	0.011	48,125.7	47,640.2	49,692.6	5	39
ICRF J043900.8−452222......	0437−454	...	...		04 39 00.854714	−45 22 22.56260	0.000057	0.00078	−0.123	49,443.5	48,766.9	49,895.6	7	32
ICRF J044238.6−001743......	0440−003	1	1		04 42 38.660762	−00 17 43.41910	0.000025	0.00064	0.262	47,735.2	47,011.4	49,576.9	15	111
ICRF J044907.6+112128......	0446+112	...	...		04 49 07.671119	11 21 28.59662	0.000024	0.00051	−0.143	49,312.0	47,394.1	49,854.8	5	32
ICRF J045005.4−810102......	0454−810	...	...		04 50 05.440195	−81 01 02.23146	0.000137	0.00032	−0.005	48,784.2	47,626.5	49,895.6	18	148
ICRF J045952.0+022931......	0457+024	...	...		04 59 52.050664	02 29 31.17631	0.000019	0.00032	0.062	48,993.4	47,005.8	49,750.8	36	394
ICRF J050145.2+135607......	0458+138	2	2		05 01 45.270840	13 56 07.22063	0.000037	0.00064	−0.770	48,830.7	47,394.1	49,848.8	13	20
ICRF J050523.1+045942......	0502+049	...	...		05 05 23.184723	04 59 42.72448	0.000037	0.00060	−0.584	48,897.7	47,394.1	49,667.9	6	28
ICRF J050643.9−610940......	0506−612	...	...		05 06 43.988739	−61 09 40.99328	0.000047	0.00035	0.145	48,760.5	48,110.9	49,594.7	16	69
ICRF J050842.3+843204......	0454+844	...	...		05 08 42.363503	84 32 04.54402	0.000194	0.00028	−0.046	48,674.7	46,977.9	49,611.9	42	250
ICRF J051002.3+180041......	0507+179	2	2		05 10 02.369122	18 00 41.58171	0.000020	0.00030	−0.396	49,401.9	47,605.1	49,820.5	24	339
ICRF J051644.9−620705......	0516−621	...	...		05 16 44.926178	−62 07 05.38930	0.000048	0.00042	0.202	49,455.4	48,749.6	49,895.6	9	56

Formal Definition of ICRS

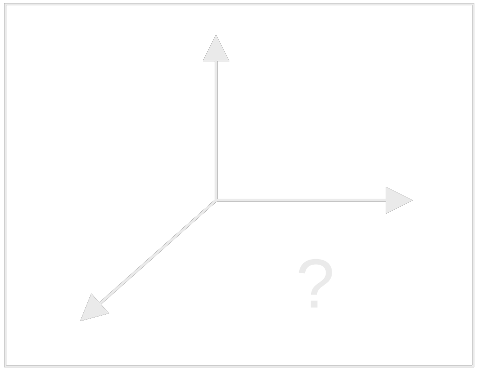
It is defined by the measured positions of 212 extragalactic sources (mainly quasars).

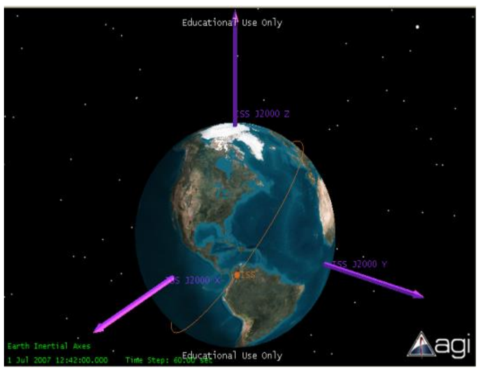
1. Its **origin** is located at the barycenter of the solar system through appropriate modeling of VLBI observations in the framework of general relativity.

2. Its **pole** is in the direction defined by the conventional IAU models for precession (Lieske et al. 1977) and nutation (Seidelmann 1982).

3. Its **origin of right ascensions** was implicitly defined by fixing the right ascension of the radio source 3C273B to FK5 J2000 value.

3. The Orbit in Space





Coordinate systems

Coordinate Systems

Now that we have defined an inertial reference frame, other reference frames can be defined according to the needs of the considered application.

Coordinate transformations between two reference frames involve rotation and translation.

What are the possibilities for a satellite in Earth orbit ?

Geocentric — Inertial (ECI)

A geocentric-equatorial system is clearly convenient.

The geocentric celestial reference frame (GCRF) is the geocentric counterpart of the ICRF and is the standard inertial coordinate system for the Earth.

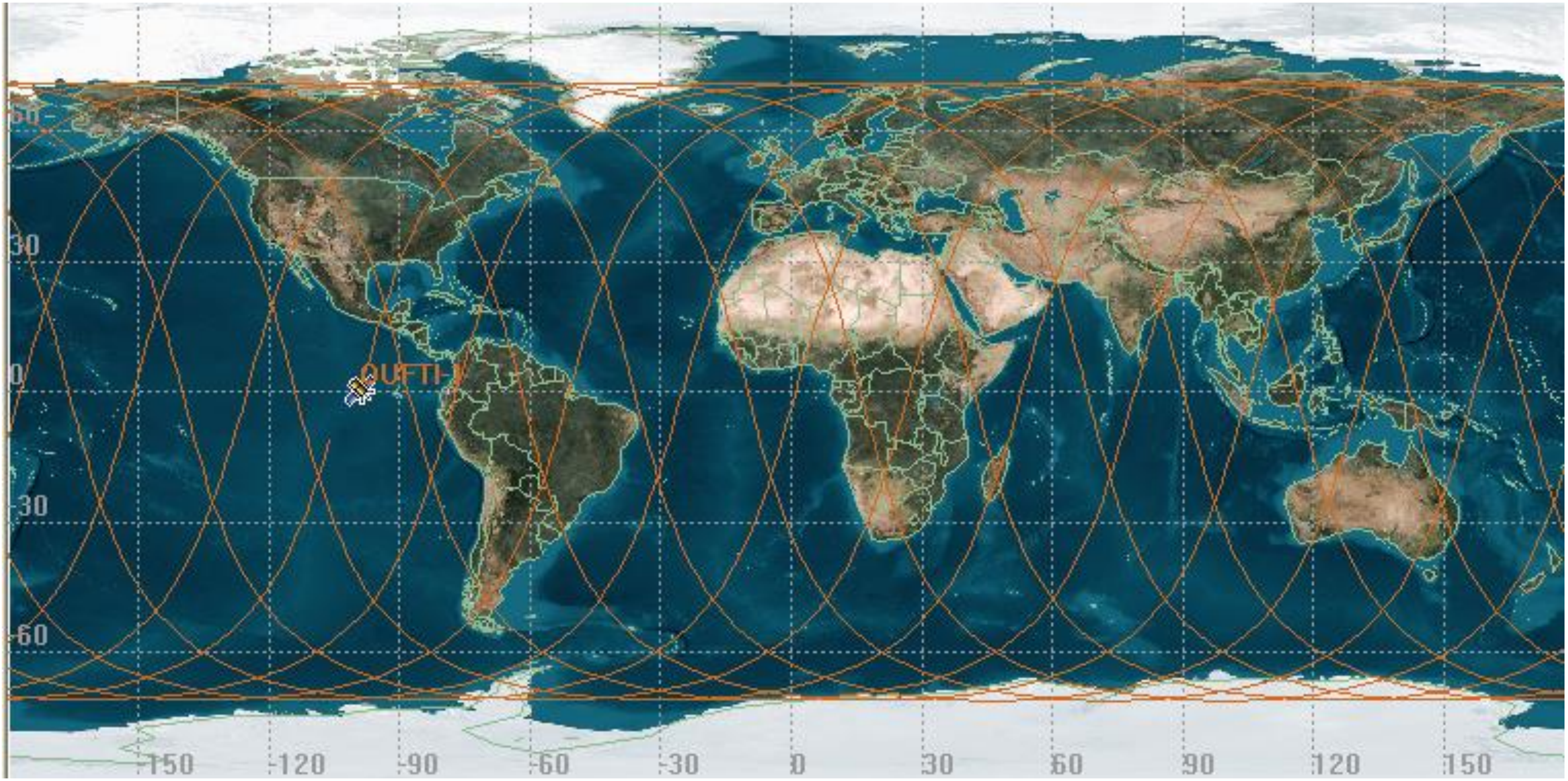
Geocentric — Fixed (ECEF)

Origin at the Earth's center.

- ⇒ z-axis is parallel to Earth's rotation vector.
- ⇒ x-axis passes through the Greenwich meridian.
- ⇒ y-axis: right-handed set.

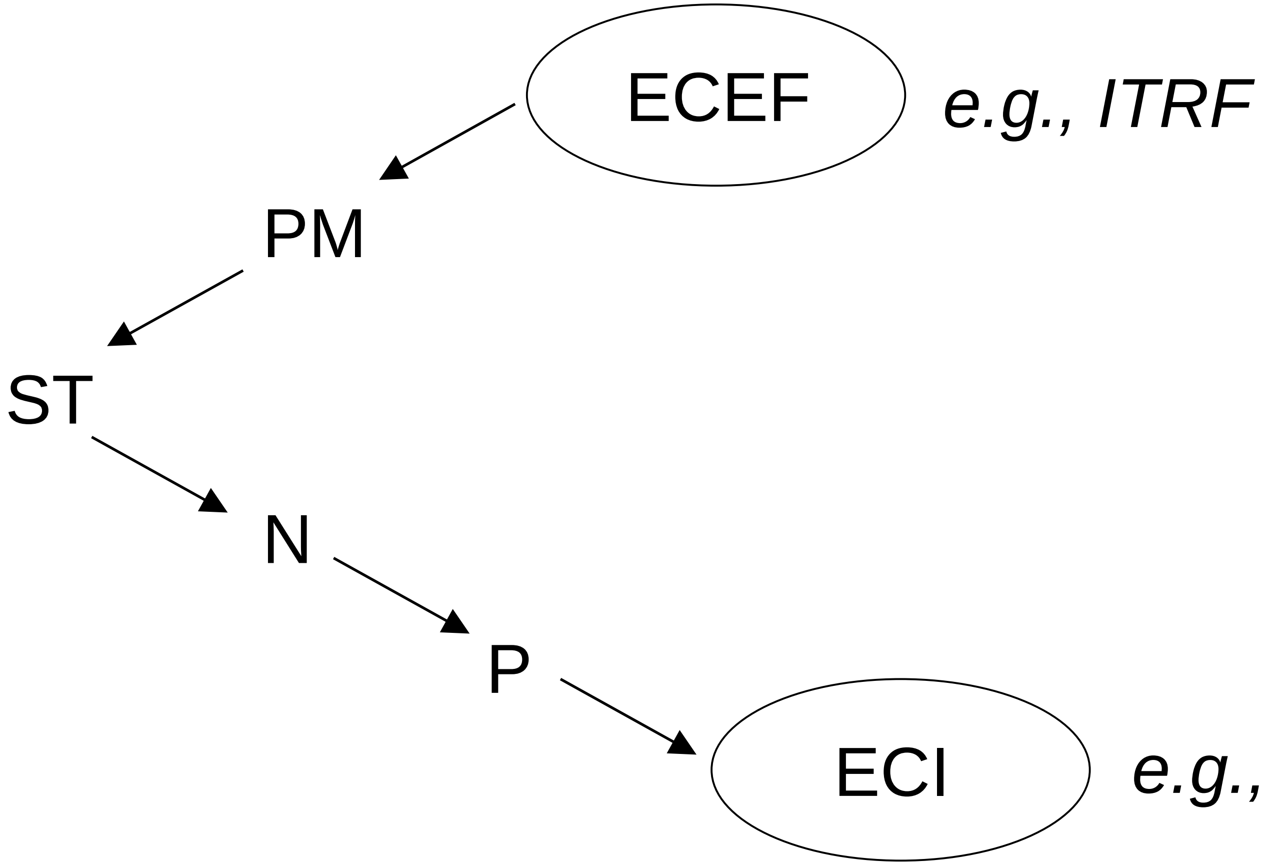
For ground tracks and force computation.

ECEF



ECEF-ECI Transformation

It includes precession, nutation, and rotation effects, as well as pole wander and frame corrections.



ECEF-ECI Transformation

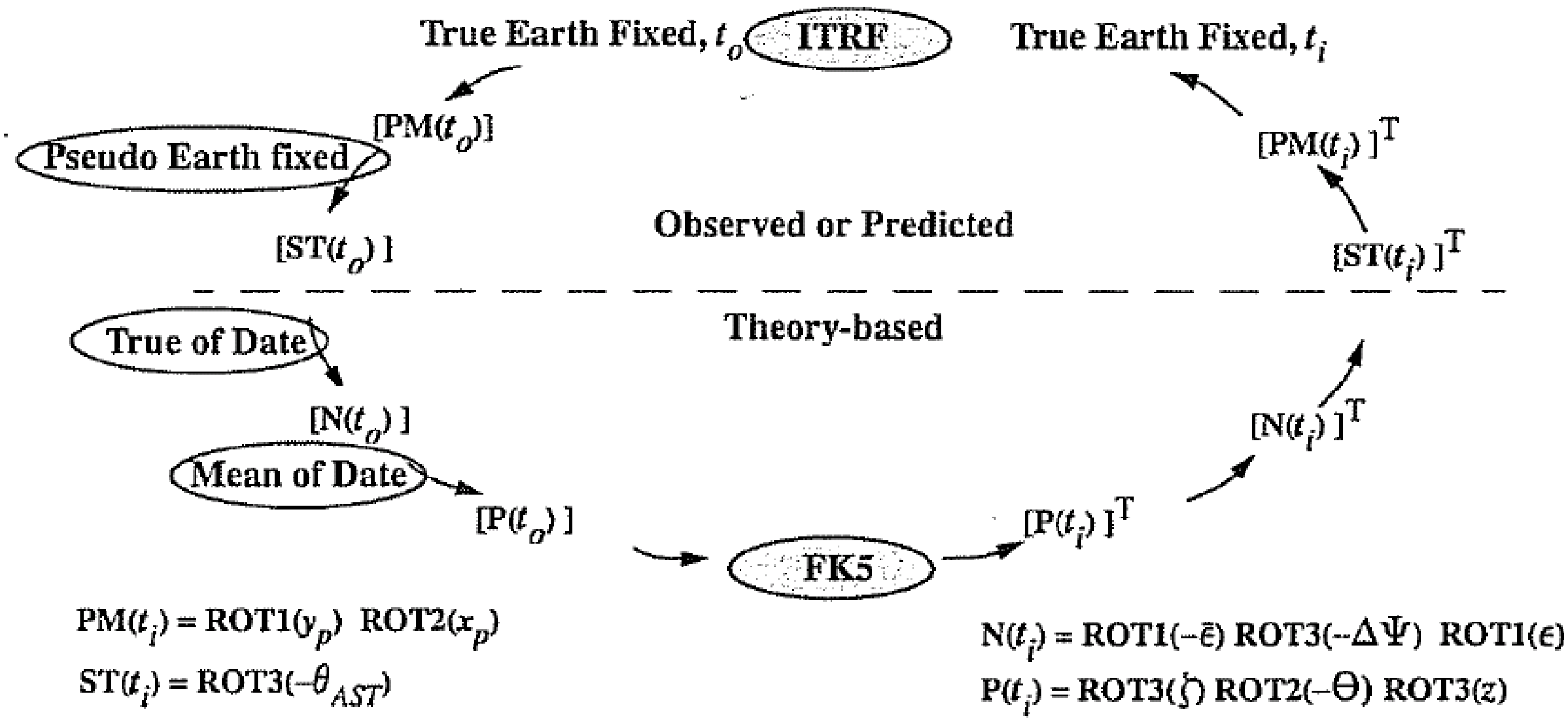


Figure 3-29. Classical Transformation. This figure depicts the transformation of a state vector in the body fixed (ITRF) frame to the inertial (FK5) frame. This two-way conversion is necessary for many orbit determination problems. The clear ellipses show the intermediate frames.

Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

ECEF-ECI Transformation

Simplified transformation

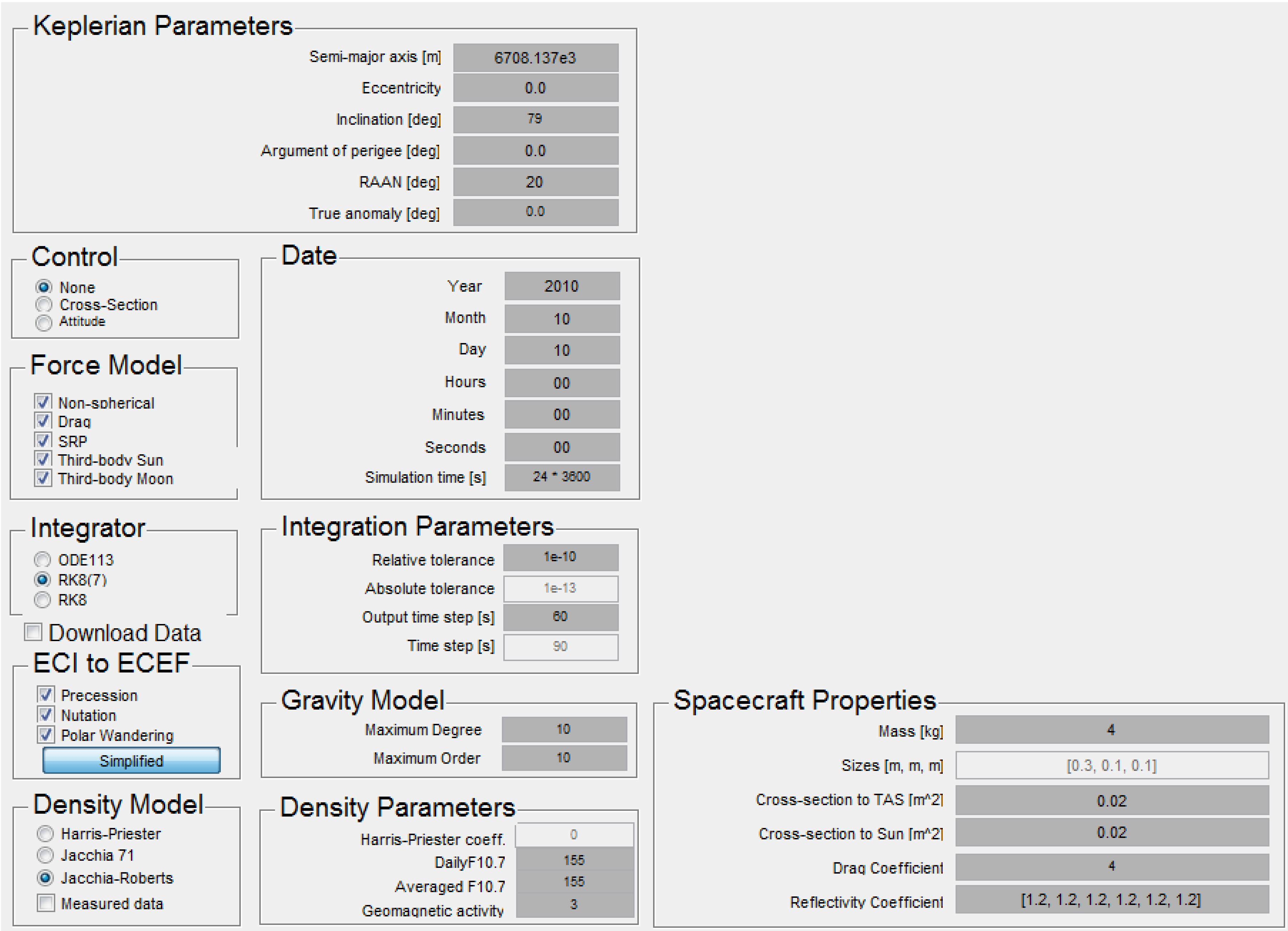
$$\omega_{\oplus} = 0.000{,}072{,}921{,}158{,}553{,}0 \text{ rad/s}$$

$$\theta_{\text{GMST,2000}} = 1.7447671633061 \text{ rad}$$

$$\theta_{\text{GMST}} = \theta_{\text{GMST,2000}} + \omega_{\oplus} \times 86400 \times (t + 0.5) \text{ rad}$$

$$\begin{pmatrix} \text{x} \\ \text{y} \\ \text{z} \end{pmatrix}_{ECI} = \begin{bmatrix} \cos(\theta_{\text{GMST}}) & -\sin(\theta_{\text{GMST}}) & 0 \\ \sin(\theta_{\text{GMST}}) & \cos(\theta_{\text{GMST}}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{ECEF}$$

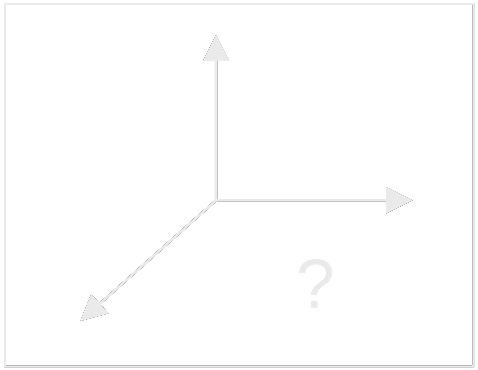
Precession, nutation, polar motion ignored

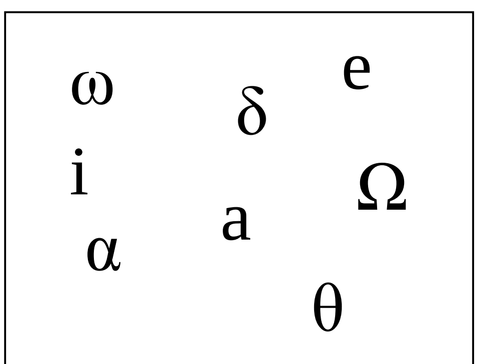

Keplerian Parameters
Semi-major axis [m]
6708.137e3
Eccentricity
0.0
Inclination [deg]
79
Argument of perigee [deg]
0.0
RAAN [deg]
20
True anomaly [deg]
0.0
Control
None
Cross-Section
Attitude
Date
Year
2010
Month
10
Day
10
Hours
00
Minutes
00
Seconds
00
Simulation time [s]
24 * 3600
Force Model
Non-spherical
Drag
SRP
Third-body Sun
Third-body Moon
Integrator
ODE113
RK8(7)
RK8
Integration Parameters
Relative tolerance
1e-10
Absolute tolerance
1e-13
Output time step [s]
60
Time step [s]
90
Download Data
ECI to ECEF
Precession
Nutation
Polar Wandering
Simplified
Gravity Model
Maximum Degree
10
Maximum Order
10
Spacecraft Properties
Mass [kg]
4
Sizes [m, m, m]
[0.3, 0.1, 0.1]
Cross-section to TAS [m^2]
0.02
Cross-section to Sun [m^2]
0.02
Drag Coefficient
4
Reflectivity Coefficient
[1.2, 1.2, 1.2, 1.2, 1.2, 1.2]
Density Model
Harris-Priester
Jacchia 71
Jacchia-Roberts
Measured data
Density Parameters
Harris-Priester coeff.
0
DailyF10.7
155
Averaged F10.7
155
Geomagnetic activity
3

Yet More Coordinate Systems !

Satellite coordinate system	**For ADCS**
Perifocal coordinate system	**Natural frame for an orbit (z is zero)**
Heliocentric coordinate system	**For interplanetary missions**
Non-singular elements	**For particular orbits**

3. The Orbit in Space



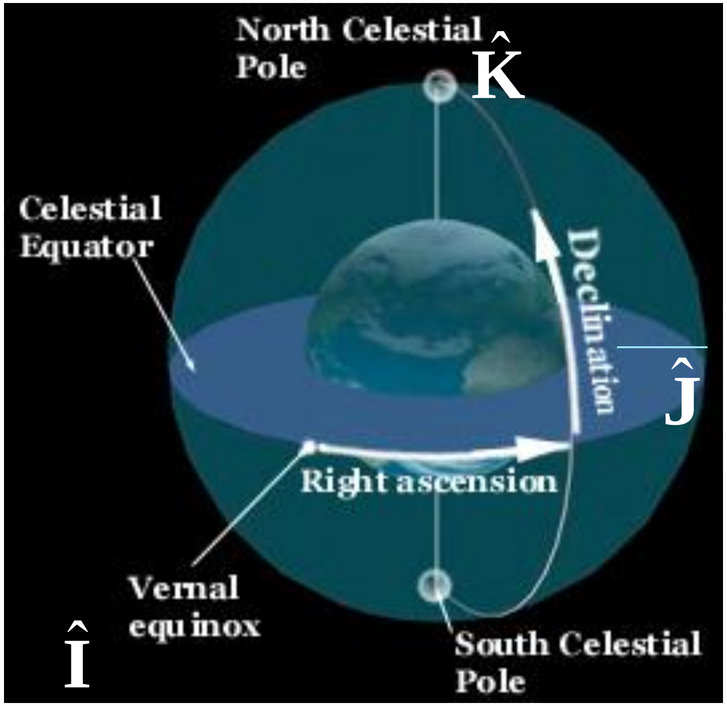


Coordinate types

Cartesian and Spherical

1. Cartesian: **for computations**

2. Spherical: azimuth and elevation (**for ground station**) — right ascension and declination (**for astronomers**)



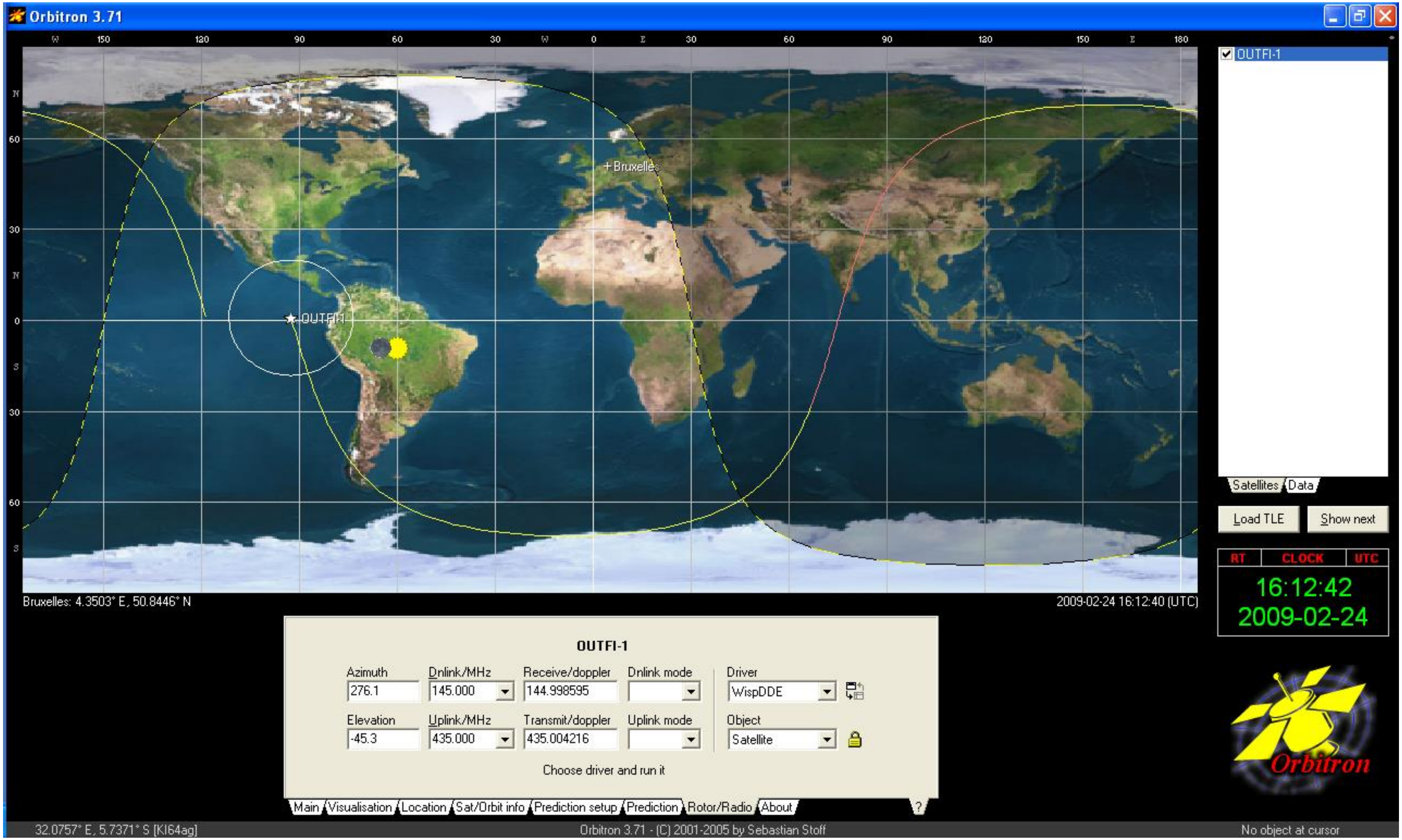
Cartesian ↔ Spherical

$$\mathbf{r} = X\hat{\mathbf{I}} + Y\hat{\mathbf{J}} + Z\hat{\mathbf{K}} = r\hat{\mathbf{u}}_r$$

$$\Updownarrow$$

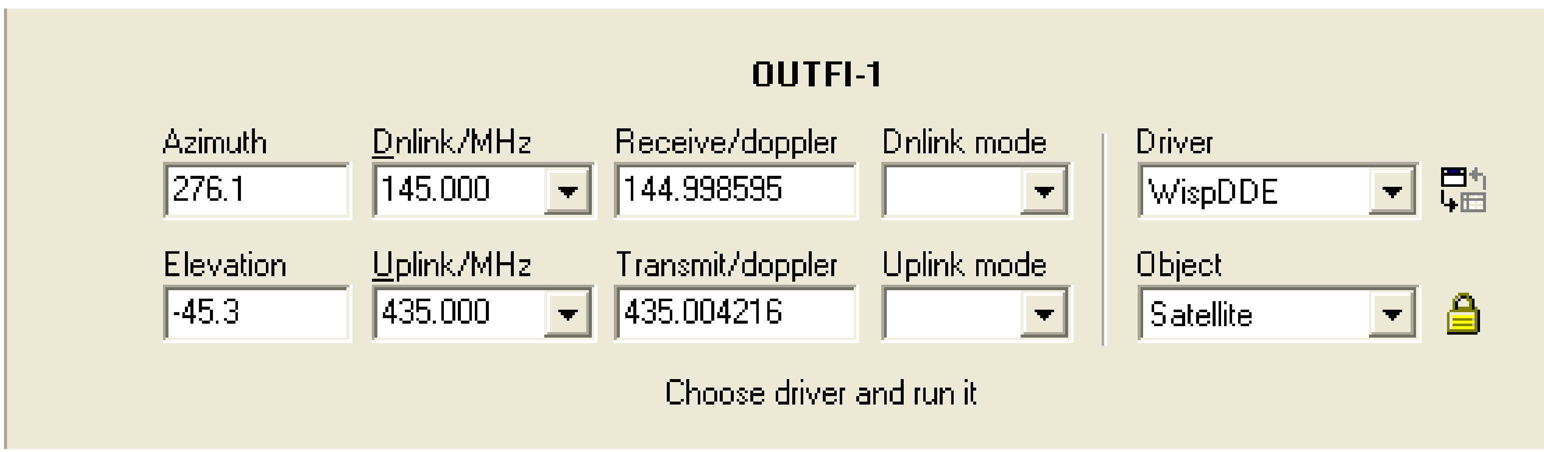
$$\hat{\mathbf{u}}_r = \cos\delta\cos\alpha\hat{\mathbf{I}} + \cos\delta\sin\alpha\hat{\mathbf{J}} + \sin\delta\hat{\mathbf{K}}$$

Orbitron



http://www.stoff.pl/

Orbitron: Close-Up



Orbital (Keplerian) Elements

For interpretation

r and **v** do not directly yield much information about the orbit. We cannot even infer from them what type of conic the orbit represents !

Another set of six variables, which is much more descriptive of the orbit, is needed.

6 Orbital (Keplerian) Elements

1. *e*: shape of the orbit

2. *a*: size of the orbit

definition of the ellipse

3. *i*: orients the orbital plane with respect to the ecliptic plane

4. Ω: longitude of the intersection of the orbital and ecliptic planes

definition of the orbital plane

5. ω: orients the semi-major axis with respect to the ascending node

orientation of the ellipse within the orbital plane

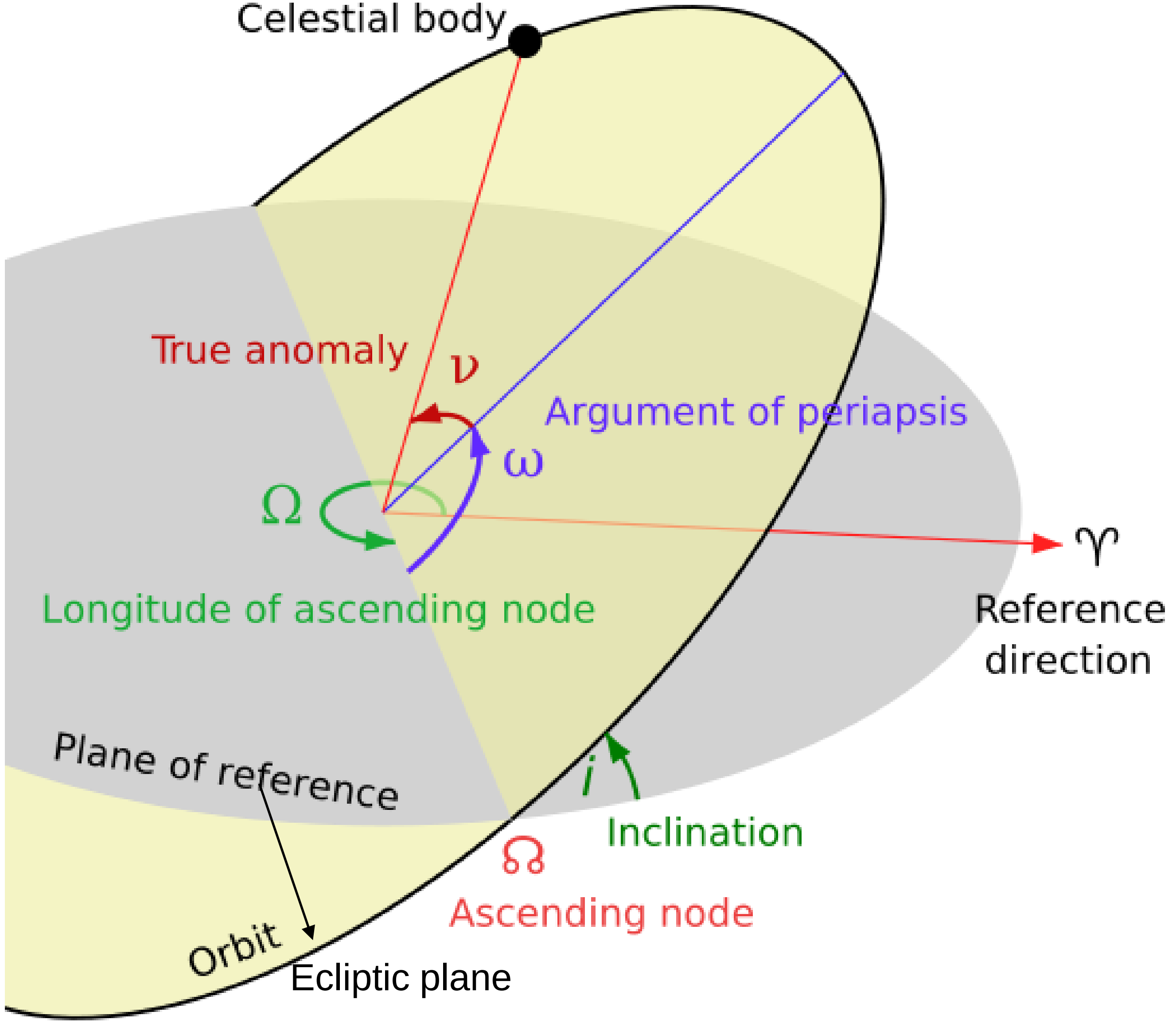
6. ν: orients the celestial body in space

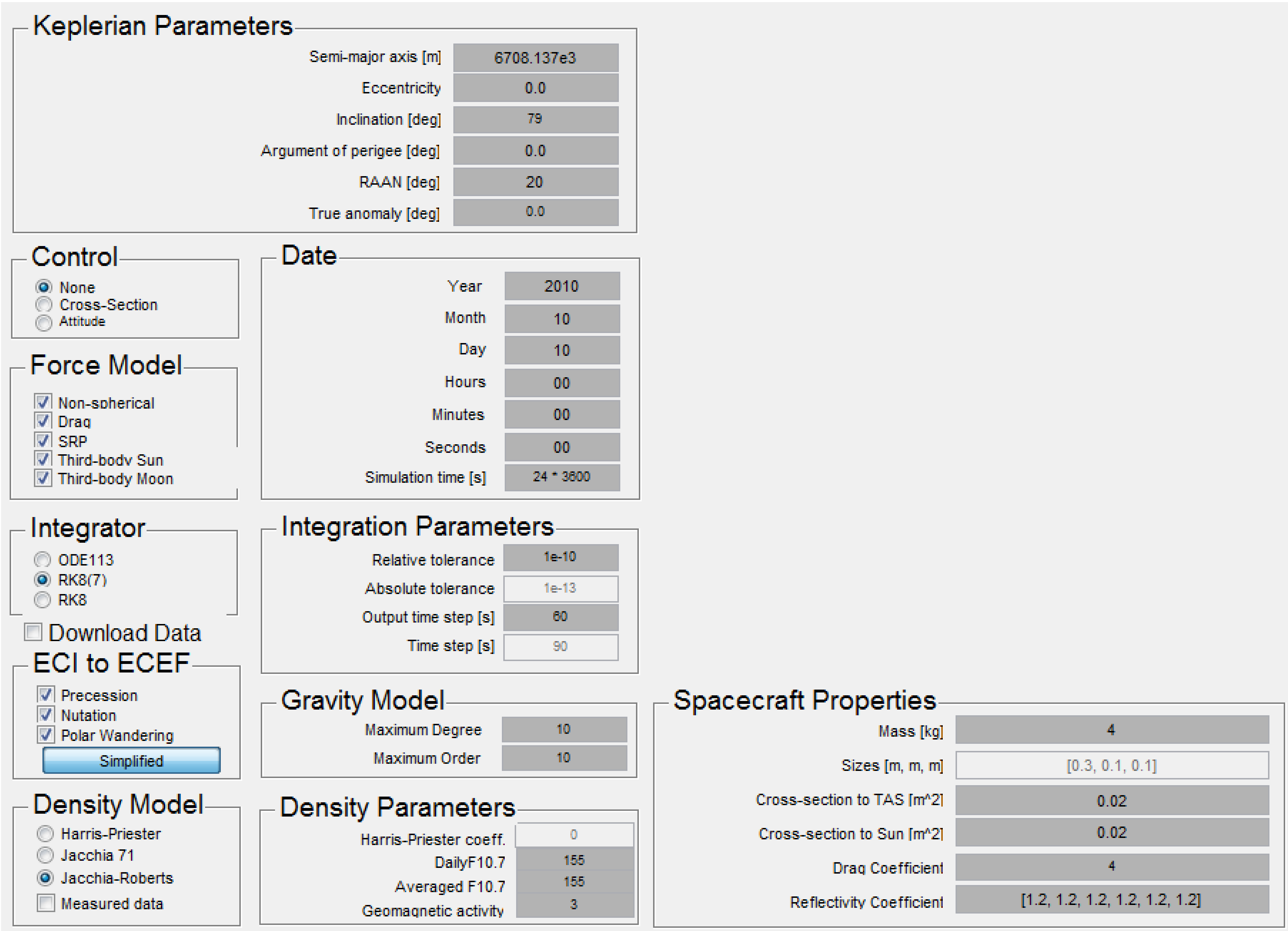
position of the satellite on the ellipse

Orbital plane

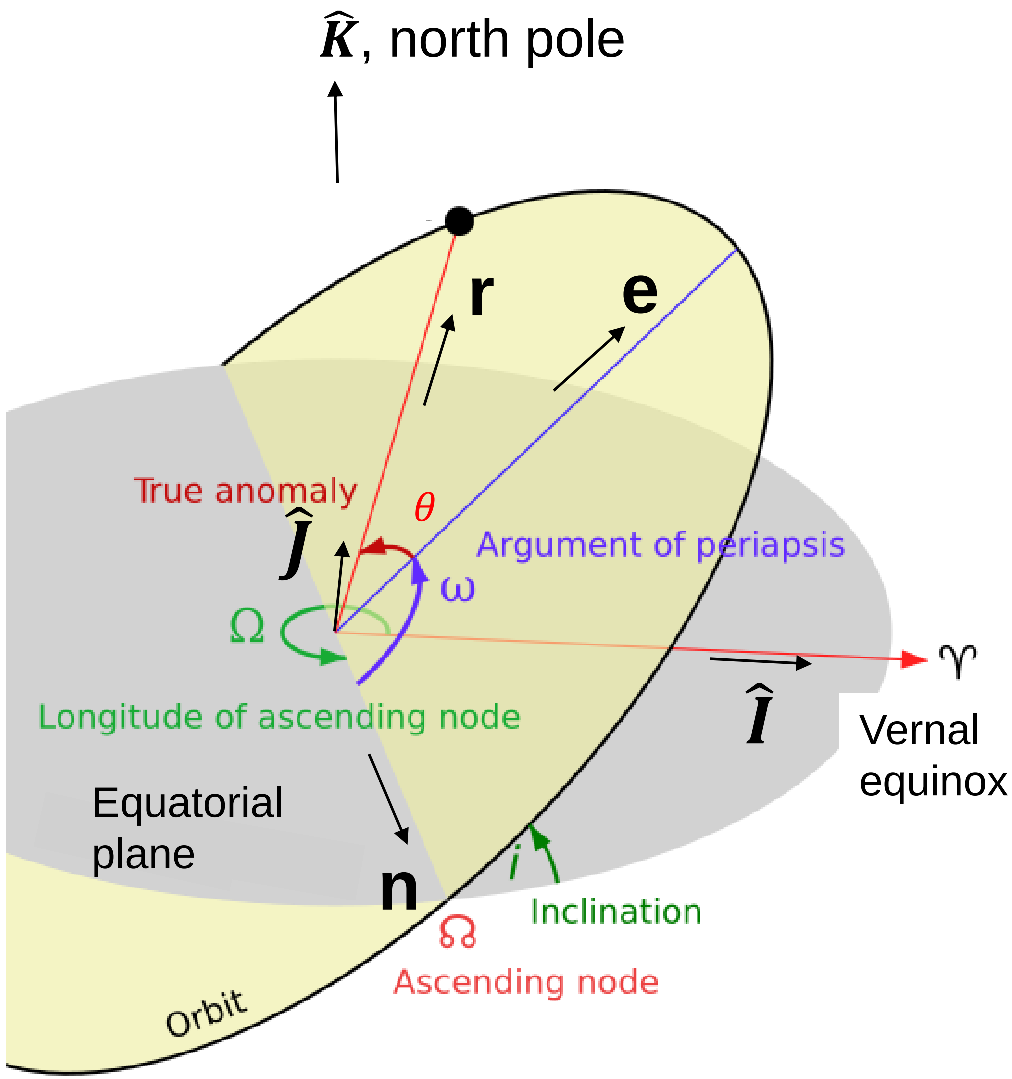
orientation of the ellipse

position of the satellite




Keplerian Parameters
Semi-major axis [m] 6708.137e3
Eccentricity 0.0
Inclination [deg] 79
Argument of perigee [deg] 0.0
RAAN [deg] 20
True anomaly [deg] 0.0
Control
None
Cross-Section
Attitude
Date
Year 2010
Month 10
Day 10
Hours 00
Minutes 00
Seconds 00
Simulation time [s] 24 * 3600
Force Model
Non-spherical
Drag
SRP
Third-body Sun
Third-body Moon
Integrator
ODE113
RK8(7)
RK8
Integration Parameters
Relative tolerance 1e-10
Absolute tolerance 1e-13
Output time step [s] 60
Time step [s] 90
Download Data
ECI to ECEF
Precession
Nutation
Polar Wandering
Simplified
Gravity Model
Maximum Degree 10
Maximum Order 10
Spacecraft Properties
Mass [kg] 4
Sizes [m, m, m] [0.3, 0.1, 0.1]
Cross-section to TAS [m^2] 0.02
Cross-section to Sun [m^2] 0.02
Drag Coefficient 4
Reflectivity Coefficient [1.2, 1.2, 1.2, 1.2, 1.2, 1.2]
Density Model
Harris-Priester
Jacchia 71
Jacchia-Roberts
Measured data
Density Parameters
Harris-Priester coeff. 0
DailyF10.7 155
Averaged F10.7 155
Geomagnetic activity 3

Orbital Elements a,e,i,Ω,ω,θ from r,v ?



e and a from the 2-body Problem

$$\mu \boldsymbol{e} = \boldsymbol{v} \times \boldsymbol{h} - \mu \frac{\boldsymbol{r}}{r}$$

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

⇩

$$e = \left\| \frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r} \right\|$$

$$a = \frac{r}{2 - \frac{r v^2}{\mu}}$$

$$r = \|\boldsymbol{r}\|, v = \|\boldsymbol{v}\|$$

Inclination

Angle between the orbital and equatorial planes:

Normal to the orbit plane

Normal to the equatorial plane

$$\cos i = \frac{\boldsymbol{h}.\widehat{\boldsymbol{K}}}{\|\boldsymbol{h}\|}$$

$$\boldsymbol{h} = \boldsymbol{r} \times \boldsymbol{v}$$

$$i = \cos^{-1}\left(\frac{(\boldsymbol{r} \times \boldsymbol{v}).\widehat{\boldsymbol{K}}}{\|\boldsymbol{r} \times \boldsymbol{v}\|}\right)$$

Longitude Ω

Angle between the nodal vector **n** and the vernal equinox:

$$\cos\Omega = \frac{\boldsymbol{n}.\hat{\boldsymbol{I}}}{\|\boldsymbol{n}\|}$$

The nodal vector **n** is in the orbital and equatorial planes:

$$\boldsymbol{n} = \hat{\boldsymbol{K}} \times \frac{\boldsymbol{h}}{h}$$

$$\Omega = cos^{-1}\frac{\boldsymbol{n}.\hat{\boldsymbol{I}}}{\|\boldsymbol{n}\|} = cos^{-1}\left(\frac{\left(\hat{\boldsymbol{K}} \times \frac{\boldsymbol{r}\times\boldsymbol{v}}{\|\boldsymbol{r}\times\boldsymbol{v}\|}\right).\hat{\boldsymbol{I}}}{\left\|\hat{\boldsymbol{K}} \times \frac{\boldsymbol{r}\times\boldsymbol{v}}{\|\boldsymbol{r}\times\boldsymbol{v}\|}\right\|}\right) \qquad \boldsymbol{n}.\hat{\boldsymbol{J}} \geq \boldsymbol{0}$$

$$\Omega = 360° - \Omega \qquad \boldsymbol{n}.\hat{\boldsymbol{J}} < \boldsymbol{0}$$

Argument of Perigee

Angle between the nodal and eccentricity vectors:

$$\cos\omega = \frac{\boldsymbol{e}.\boldsymbol{n}}{\|\boldsymbol{e}\|\|\boldsymbol{n}\|}$$

$$\boldsymbol{n} = \widehat{\boldsymbol{K}} \times \frac{\boldsymbol{h}}{h}, \boldsymbol{e} = \frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}$$

$$\omega = cos^{-1}\left(\frac{\left(\widehat{\boldsymbol{K}} \times \frac{\boldsymbol{r} \times \boldsymbol{v}}{\|\boldsymbol{r} \times \boldsymbol{v}\|}\right).\left(\frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}\right)}{\left\|\widehat{\boldsymbol{K}} \times \frac{\boldsymbol{r} \times \boldsymbol{v}}{\|\boldsymbol{r} \times \boldsymbol{v}\|}\right\|\left\|\frac{\boldsymbol{v} \times (\boldsymbol{r} \times \boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}\right\|}\right) \qquad \boldsymbol{e}.\widehat{\boldsymbol{K}} \geq \boldsymbol{0}$$

$$\omega = 360° - \omega \qquad \boldsymbol{e}.\widehat{\boldsymbol{K}} < \boldsymbol{0}$$

True Anomaly

Angle between the position and eccentricity vectors

$$\cos\theta = \frac{\boldsymbol{r}.\boldsymbol{e}}{r\|\boldsymbol{e}\|}$$

$$\theta = cos^{-1}\left(\frac{\boldsymbol{r}.\left(\frac{\boldsymbol{v}\times(\boldsymbol{r}\times\boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}\right)}{r\left\|\frac{\boldsymbol{v}\times(\boldsymbol{r}\times\boldsymbol{v})}{\mu} - \frac{\boldsymbol{r}}{r}\right\|}\right) \qquad \boldsymbol{r}.\boldsymbol{v} \geq \boldsymbol{0}$$

$$\theta = 360° - \theta \qquad \boldsymbol{r}.\boldsymbol{v} < \boldsymbol{0}$$

r,v from a,e,i,Ω,ω,θ ? From Vallado

2.6 Application: *r* and *v* from Orbital Elements

We've seen how to find the orbital elements from the position and velocity vectors, but we often need the reverse process to complete certain astrodynamic studies. We'll call the process ***RANDV*** to indicate that we're determining the position and velocity vectors. The overall idea is to determine the position and velocity vectors in the perifocal coordinate system, PQW, and then rotate to the geocentric equatorial system. Although the orbit may not be elliptical, and therefore the PQW system would actually be undefined, we can elegantly work around this limitation. We can also make the method completely generic through several short, simple substitutions.

First, we must use the semiparameter instead of the semimajor axis. As previously mentioned, the semimajor axis is infinite for the parabola, whereas the semiparameter is defined for all orbits. The second requirement concerns how we treat the auxiliary classical orbital elements for the special cases of circular and equatorial orbits.

Let's begin by finding the position and velocity vectors in the perifocal coordinate system. We've developed and presented these equations previously but show them here coupled with the trajectory equation. Notice the use of the semiparameter to replace dependence on the semimajor axis.

$$\vec{r}_{PQW} = \begin{bmatrix} \dfrac{p\,\text{COS}(\nu)}{1 + e\,\text{COS}(\nu)} \\ \dfrac{p\,\text{SIN}(\nu)}{1 + e\,\text{COS}(\nu)} \\ 0 \end{bmatrix} \qquad (2\text{-}100)$$

An immediate difficulty arises when attempting to define the true anomaly for circular orbits. It turns out that the orbital elements may be *temporarily* replaced with the alternate elements to provide the necessary values for the calculations. Although you can design a change like this so it's transparent to users, make sure any changes or alternate codings use temporary variables and don't alter the original elements. It's possible to substitute values:

$$\begin{gathered} \text{IF Circular Equatorial} \\ \text{let } \omega = 0.0, \Omega = 0.0, \text{ and } \nu = \lambda_{true} \\ \text{IF Circular Inclined} \\ \text{let } \omega = 0.0 \text{ and } \nu = u \end{gathered} \qquad (2\text{-}101)$$

The rationale for assigning ω and Ω to zero will be clear shortly; however, we haven't violated any assumptions because ω and Ω are undefined for circular orbits. Be careful not to return any changed variables in computer applications.

Find the velocity vector by differentiating the position vector:

$$\vec{v}_{PQW} = \begin{bmatrix} \dot{r}\ \text{COS}(\nu) - r\dot{\nu}\,\text{SIN}(\nu) \\ \dot{r}\ \text{SIN}(\nu) + r\dot{\nu}\,\text{COS}(\nu) \\ 0 \end{bmatrix}$$

Remembering the geometry from Fig. 1-13, solve Eq. (1-18) as

$$r\dot{\nu} = \frac{h}{r}$$

Now, substitute the definitions of position and angular momentum:

$$r\dot{\nu} = \frac{\sqrt{\mu p}(1 + e\,\mathrm{COS}(\nu))}{p} = \sqrt{\frac{\mu}{p}}(1 + e\,\mathrm{COS}(\nu))$$

Using Eq. (1-25) and the equation above, write

$$\dot{r} = \sqrt{\frac{\mu}{p}}(e\,\mathrm{SIN}(\nu))$$

Substituting these results into the differentiated vector gives us the final solution:

$$\vec{v}_{PQW} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}}\mathrm{SIN}(\nu) \\ \sqrt{\frac{\mu}{p}}(e + \mathrm{COS}(\nu)) \\ 0 \end{bmatrix} \quad (2\text{-}102)$$

The next step is to rotate the position and velocity vectors to the geocentric equatorial frame. Although this is relatively easy for standard, elliptical, inclined orbits, we'll need to take certain precautions in order to account for special cases, as described with the true anomaly above. We've discussed two of these special cases; the third is the elliptical equatorial case:

$$\begin{aligned} &\text{IF Elliptical Equatorial} \\ &\text{set } \Omega = 0.0 \text{ and } \omega = \tilde{\omega}_{true} \end{aligned} \quad (2\text{-}103)$$

The assumptions remain intact because Ω is undefined for elliptical equatorial orbits.

We can now do the coordinate transformations using Eq. (3-28). We may want to multiply out these operations to reduce trigonometric operations. The rationale for setting certain variables to zero should now be apparent. For the special cases, a zero rotation causes the vector to remain unchanged, whereas a desired angular value causes a change.

Implementing RANDV

Computational efficiency results from assigning the trigonometric terms [SIN(ν), COS(ν)] and (μ/p) to temporary variables. This saves *many* transcendental operations and requires very little extra work. There are also some savings in treating special-case orbits if we reuse the same rotation matrices, but there may be some redundancy in special cases.

As with the ***ELORB*** algorithm, we may run many test cases to verify the routine. Because ***RANDV*** is simply designed to be a mirror calculation of the ***ELORB*** routine, we can use the same set of test reference data. But we must test several limiting cases. Algorithm 10 summarizes the process.

ALGORITHM 10: RANDV ($p, e, i, \Omega, \omega, \nu(u, \lambda_{true}, \tilde{\omega}_{true}) \Rightarrow \vec{r}_{IJK}\ \vec{v}_{IJK}$)

IF Circular Equatorial

SET $(\omega, \Omega) = 0.0$ and $\nu = \lambda_{true}$

IF Circular Inclined

SET $\omega = 0.0$ and $\nu = u$

IF Elliptical Equatorial

SET $\Omega = 0.0$ and $\omega = \tilde{\omega}_{true}$

$$\vec{r}_{PQW} = \begin{bmatrix} \dfrac{p\cos(\nu)}{1 + e\cos(\nu)} \\ \dfrac{p\sin(\nu)}{1 + e\cos(\nu)} \\ 0 \end{bmatrix} \qquad \vec{v}_{PQW} = \begin{bmatrix} -\sqrt{\dfrac{\mu}{p}}\sin(\nu) \\ \sqrt{\dfrac{\mu}{p}}(e + \cos(\nu)) \\ 0 \end{bmatrix}$$

$$\vec{r}_{IJK} = [\text{ROT3}(-\Omega)][\text{ROT1}(-i)][\text{ROT3}(-\omega)]\vec{r}_{PQW} = [\tfrac{IJK}{PQW}]\vec{r}_{PQW}$$

$$\vec{v}_{IJK} = [\text{ROT3}(-\Omega)][\text{ROT1}(-i)][\text{ROT3}(-\omega)]\vec{v}_{PQW} = [\tfrac{IJK}{PQW}]\vec{v}_{PQW}$$

$$\left[\frac{IJK}{PQW}\right] = \begin{bmatrix} \cos(\Omega)\cos(\omega) - \sin(\Omega)\sin(\omega)\cos(i) & -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(\omega)\cos(i) & \sin(\Omega)\sin(i) \\ \sin(\Omega)\cos(\omega) + \cos(\Omega)\sin(\omega)\cos(i) & -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(\omega)\cos(i) & -\cos(\Omega)\sin(i) \\ \sin(\omega)\sin(i) & \cos(\omega)\sin(i) & \cos(i) \end{bmatrix}$$

An example demonstrates the technique.

▼ **Example 2-6. Finding Position and Velocity Vectors (RANDV Test Case).**

GIVEN: $p = 11{,}067.790$ km $= 1.735\ 27$ ER, $e = 0.832\ 85$, $i = 87.87°$, $\Omega = 227.89°$, $\omega = 53.38°$, $\nu = 92.335°$

FIND: $\vec{r}_{IJK}\ \vec{v}_{IJK}$

We have to change the rotation angles if we're using special orbits (equatorial or circular), but this orbit doesn't have special cases. From the given information, form the PQW position and velocity vectors:

$$\vec{r}_{PQW} = \begin{bmatrix} \dfrac{p\cos(\nu)}{1 + e\cos(\nu)} \\ \dfrac{p\sin(\nu)}{1 + e\cos(\nu)} \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{1.735\ 27\cos(92.336)°}{1 + 0.832\ 84\cos(92.336)°} \\ \dfrac{1.735\ 27\sin(92.336)}{1 + 0.832\ 84\cos(92.336)°} \\ 0 \end{bmatrix} = \begin{bmatrix} -0.073\ 186\ 7 \\ 1.794\ 733\ 9 \\ 0 \end{bmatrix} \text{ER}$$

$$\vec{v}_{PQW} = \begin{bmatrix} -\sqrt{\frac{\mu}{p}}\,\text{SIN}(\nu) \\ \sqrt{\frac{\mu}{p}}(e + \text{COS}(\nu)) \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{\frac{1}{1.735\,27}}\,\text{SIN}(92.336) \\ \sqrt{\frac{1}{1.735\,27}}(0.832\,84 + \text{COS}(92.336)) \\ 0 \end{bmatrix} = \begin{bmatrix} -0.758\,499\,8 \\ 0.601\,313\,6 \\ 0 \end{bmatrix} \frac{\text{ER}}{\text{TU}}$$

Rotate these vectors to the geocentric equatorial system using the following rotation matrices:

$$\vec{r}_{IJK} = [\text{ROT3}(-\Omega)][\text{ROT1}(-i)][\text{ROT3}(-\omega)]\vec{r}_{PQW}$$

$$\vec{v}_{IJK} = [\text{ROT3}(-\Omega)][\text{ROT1}(-i)][\text{ROT3}(-\omega)]\vec{v}_{PQW}$$

Or, use the expanded matrix with a computer to do the many trigonometric operations, which result in the transformation matrix

$$\left[\frac{IJK}{PQW}\right] = \begin{bmatrix} -0.377\,736\,47 & 0.554\,597\,39 & -0.741\,442\,44 \\ -0.462\,538\,21 & 0.580\,670\,14 & 0.669\,985\,52 \\ 0.802\,105\,71 & 0.596\,023\,42 & 0.037\,182\,20 \end{bmatrix}$$

Finally, multiply each vector to apply the transformation:

$$\vec{r}_{IJK} = \left[\frac{IJK}{PQW}\right]\vec{r}_{PQW} = \begin{bmatrix} -0.377\,736\,47 & 0.554\,597\,39 & -0.741\,442\,44 \\ -0.462\,538\,21 & 0.580\,670\,14 & 0.669\,985\,52 \\ 0.802\,105\,71 & 0.596\,023\,42 & 0.037\,182\,20 \end{bmatrix} \begin{bmatrix} -0.073\,186\,7 \\ 1.794\,733\,9 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.023 \\ 1.076 \\ 1.011 \end{bmatrix} \text{ER} = \begin{bmatrix} 6524.834 \\ 6862.875 \\ 6448.296 \end{bmatrix} \text{km}$$

$$\vec{v}_{IJK} = \left[\frac{IJK}{PQW}\right]\vec{v}_{PQW} = \begin{bmatrix} -0.377\,736\,47 & 0.554\,597\,39 & -0.741\,442\,44 \\ -0.462\,538\,21 & 0.580\,670\,14 & 0.669\,985\,52 \\ 0.802\,105\,71 & 0.596\,023\,42 & 0.037\,182\,20 \end{bmatrix} \begin{bmatrix} -0.758\,499\,8 \\ 0.601\,313\,6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.62 \\ 0.70 \\ 0.75 \end{bmatrix} \text{ER/TU} = \begin{bmatrix} 4.901\,320 \\ 5.533\,756 \\ -1.976\,341 \end{bmatrix} \text{km/s}$$

Two-Line Elements (TLE)

```
ISS (ZARYA)
1 25544U 98067A   08264.51782528 -.00002182  00000-0 -11606-4 0  2927
2 25544  51.6416 247.4627 0006703 130.5360 325.0288 15.72125391563537
```

For monitoring by Norad *

The meaning of this data is as follows:

LINE 1:

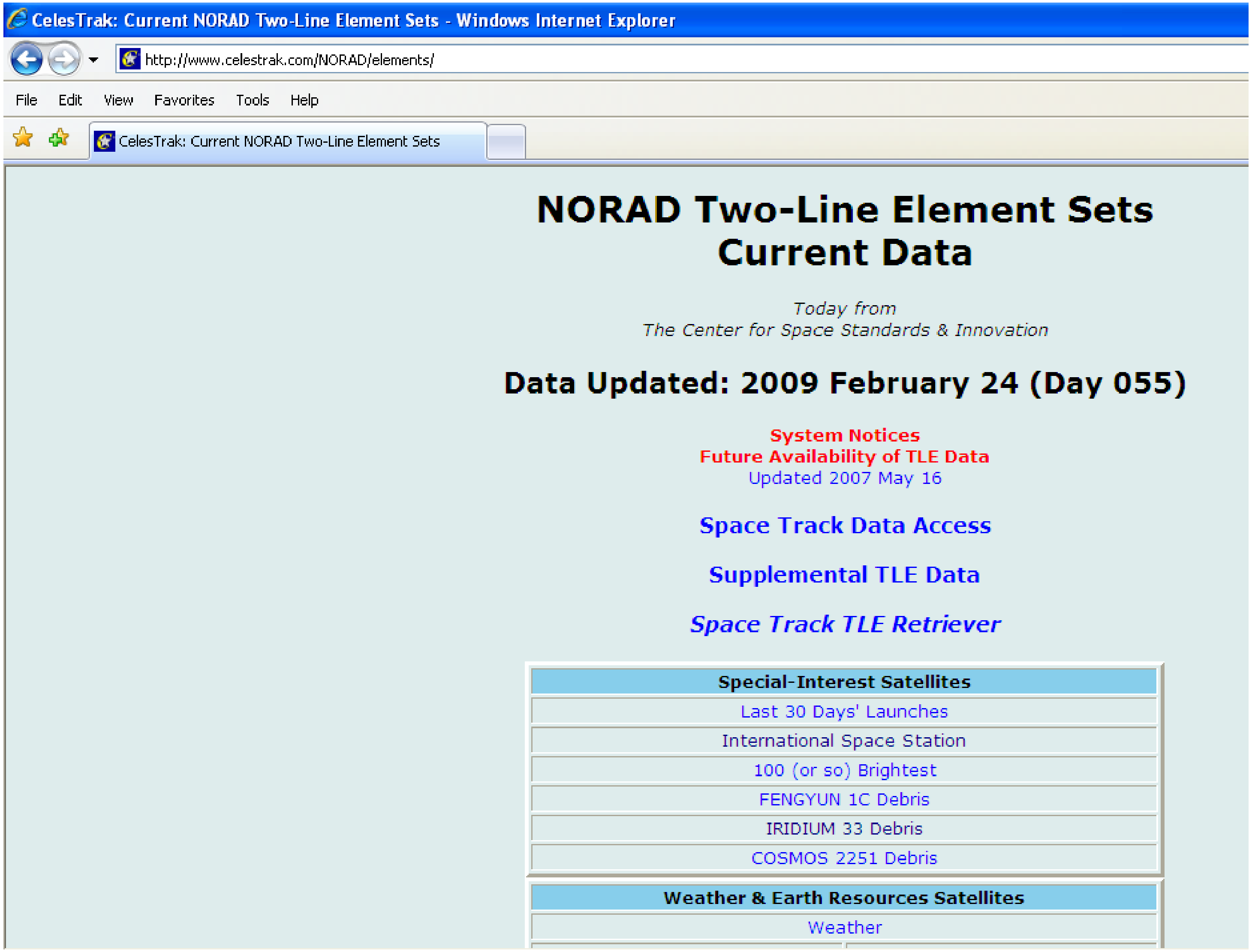
FIELD	COLS	CONTENT	EXAMPLE
1	01-01	Line number	1
2	03-07	Satellite number	25544
3	08-08	Classification (U=Unclassified)	U
4	10-11	International Designator (Last two digits of launch year)	98
5	12-14	International Designator (Launch number of the year)	067
6	15-17	International Designator (Piece of the launch)	A
7	19-20	Epoch Year (Last two digits of year)	08
8	21-32	Epoch (Day of the year and fractional portion of the day)	264.51782528
9	34-43	First Time Derivative of the Mean Motion	-.00002182
10	45-52	Second Time Derivative of Mean Motion (decimal point assumed)	00000-0
11	54-61	BSTAR drag term (decimal point assumed)	-11606-4
12	63-63	The number 0 (Originally this should have been "Ephemeris type")	0
13	65-68	Element number	292
14	69-69	Checksum (Modulo 10)	7

LINE 2:

FIELD	COLS	CONTENT	EXAMPLE
1	01-01	Line number	2
2	03-07	Satellite number	25544
3	09-16	Inclination [Degrees]	51.6416
4	18-25	Right Ascension of the Ascending Node [Degrees]	247.4627
5	27-33	Eccentricity (decimal point assumed)	0006703
6	35-42	Argument of Perigee [Degrees]	130.5360
7	44-51	Mean Anomaly [Degrees]	325.0288
8	53-63	Mean Motion [Revs per day]	15.72125391
9	64-68	Revolution number at epoch [Revs]	56353
10	69-69	Checksum (Modulo 10)	7

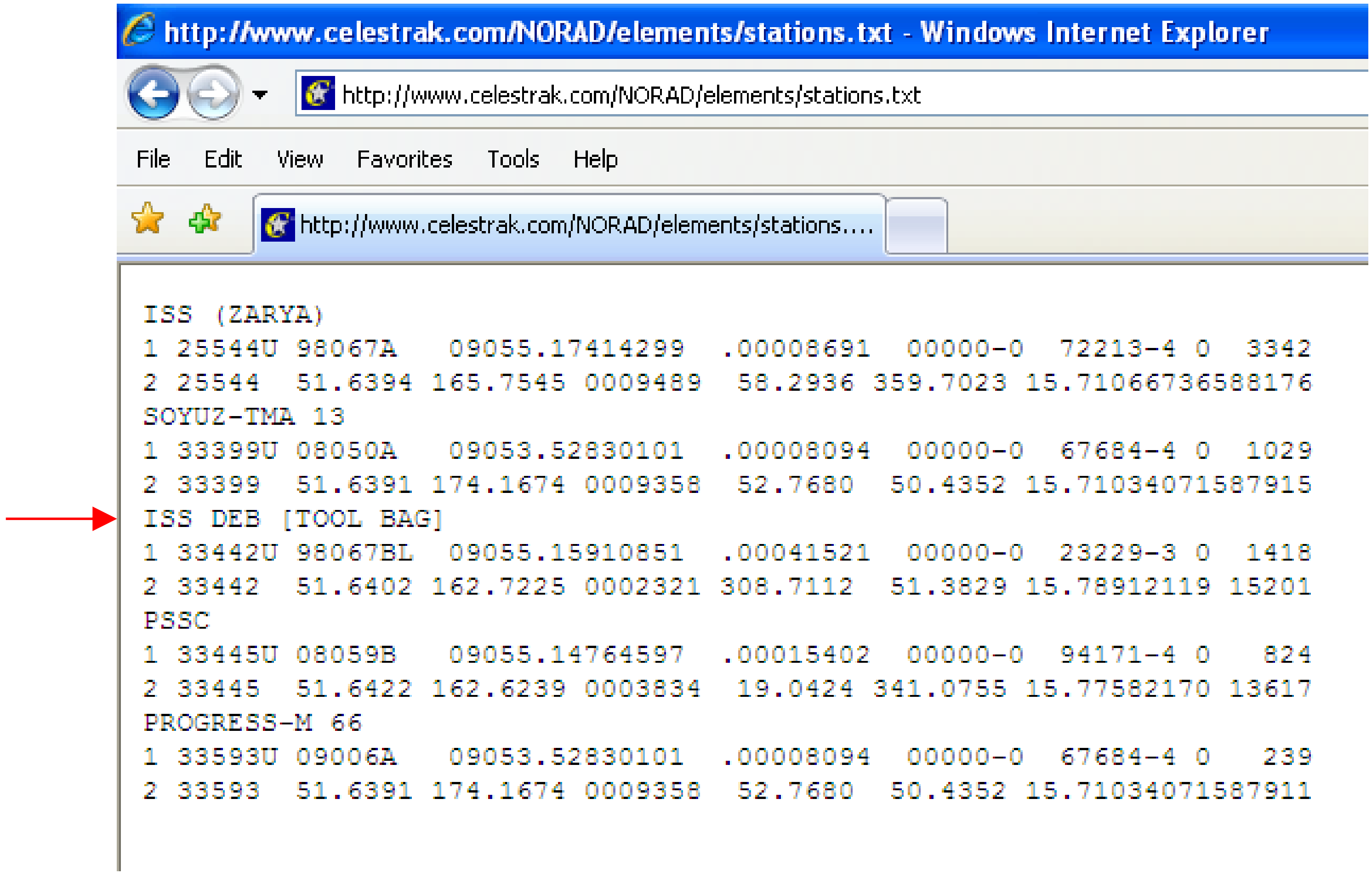
* North American Aerospace Defense Command

Celestrak: Update TLE



http://www.celestrak.com/NORAD/elements/

Celestrak: ISS, February 24, 2009



https://www.youtube.com/watch?v=1vXdRUIZ_EM

Lost ISS Toolbag

Heidemarie Stefanyshyn-Piper

From Wikipedia, the free encyclopedia

Heidemarie Martha Stefanyshyn-Piper (born on February 7, 1963
experienced salvage officer. Her major salvage projects include de-s
Peruvian submarine Pacocha.

Stefanyshyn-Piper has received numerous honors and awards, such
115 and STS-126, during which she completed five spacewalks tota

Contents [hide]

Celestrak: IRIDIUM 33, February 24, 2009

http://www.celestrak.com/NORAD/elements/iridium-33-debris.txt - Windows Internet Explorer

http://www.celestrak.com/NORAD/elements/iridium-33-debris.txt

File Edit View Favorites Tools Help

http://www.celestrak.com/NORAD/elements/iridium-3...

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IRIDIUM 33
1 24946U 97051C   09054.26512449  .00000041  00000-0  80365-5 0  5064
2 24946  86.3883 116.0443 0010244  51.3346 308.8760 14.32467868599267
IRIDIUM 33 DEB
1 33771U 97051J   09054.24783998 -.00001731  00000-0 -62285-3 0    48
2 33771  86.4104 116.1345 0003197 215.5839 144.5272 14.34418186  1781
IRIDIUM 33 DEB
1 33772U 97051K   09054.43464044  .00002688  00000-0  12018-2 0    46
2 33772  86.4348 116.2882 0062040  44.9699 315.6532 14.21284224  1813
IRIDIUM 33 DEB
1 33773U 97051L   09053.68724689 -.00000074  00000-0 -33189-4 0    40
2 33773  86.4034 116.3390 0013869 128.8329 231.4104 14.34743465  1684
IRIDIUM 33 DEB
1 33774U 97051M   09054.23963679  .00001124  00000-0  44869-3 0    48
2 33774  86.4214 116.2430 0031089  41.9851 318.3830 14.27407906  1734
IRIDIUM 33 DEB
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2 33775  86.3685 116.0217 0016761  40.0111 320.2326 14.30888357  1746
IRIDIUM 33 DEB
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IRIDIUM 33 DEB
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IRIDIUM 33 DEB
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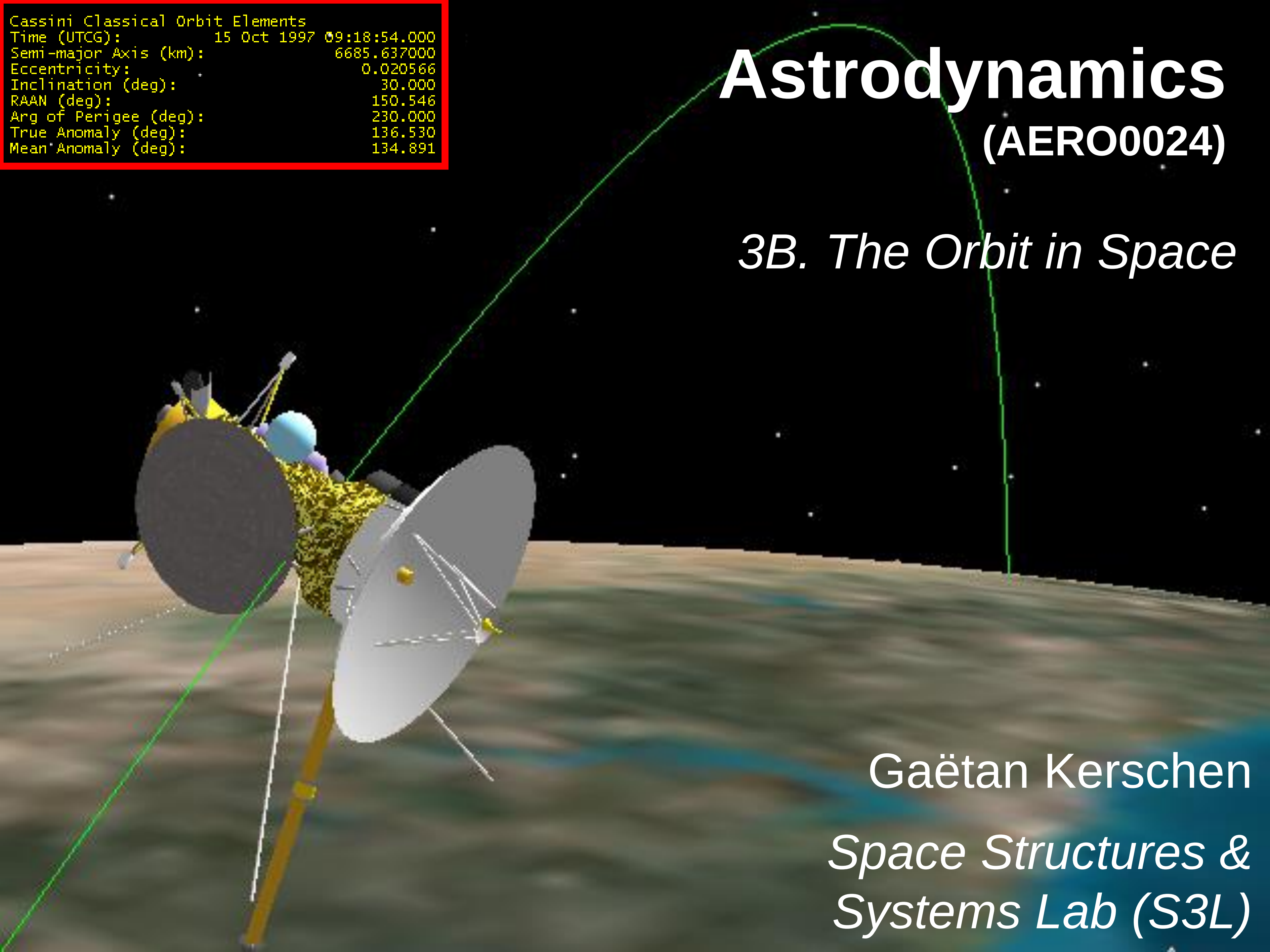
Astrodynamics
(AERO0024)

3B. The Orbit in Space



Gaëtan Kerschen

Space Structures & Systems Lab (S3L)