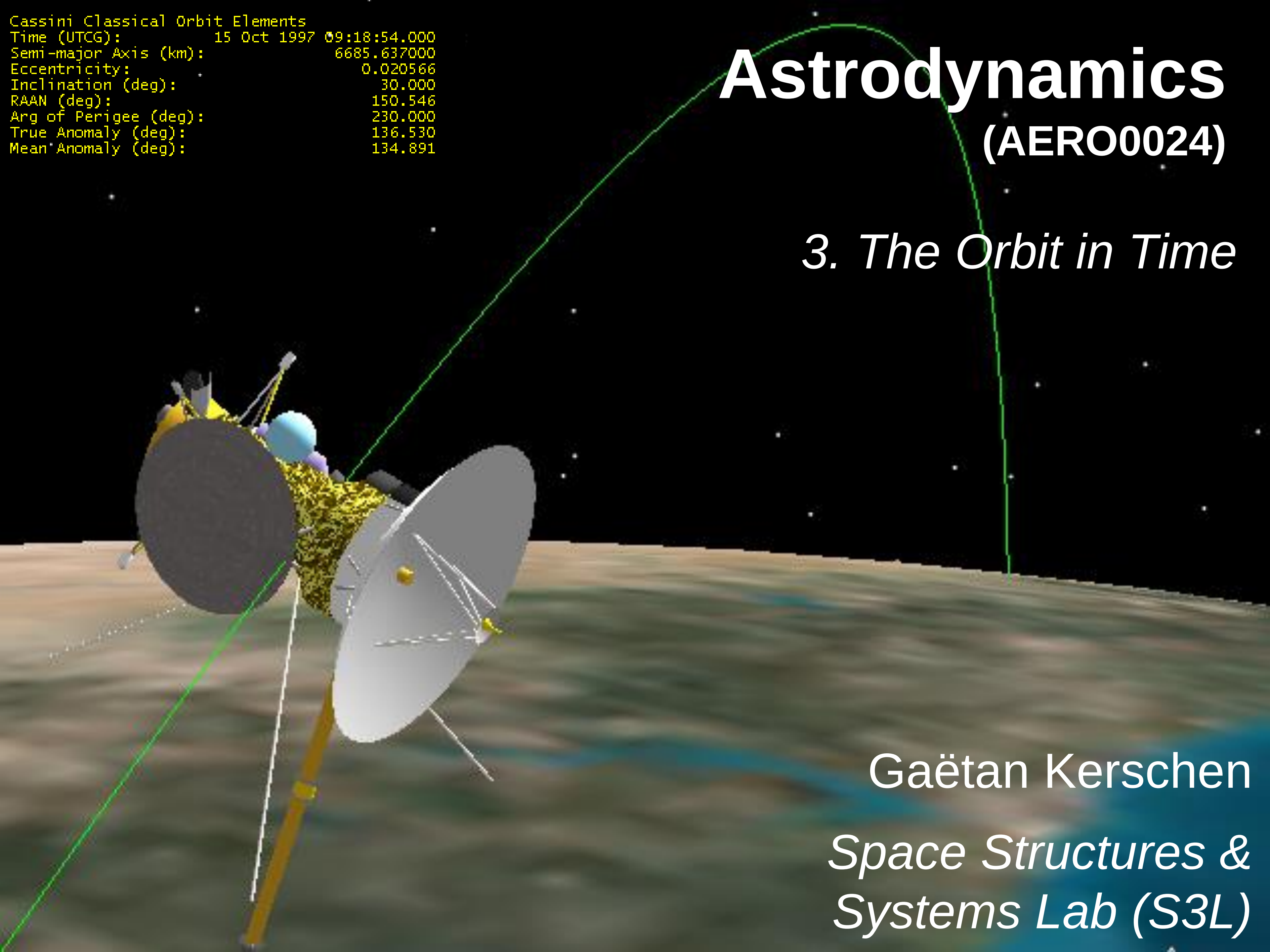

Astrodynamics

(AERO0024)

3. The Orbit in Time

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Space Structures & Systems Lab (S3L)

Newton's laws

$$\mathbf{F} = m\mathbf{a}$$

$$\mathbf{F}_g = G\frac{m_1 m_2}{r^2}\hat{\mathbf{u}}_r$$

Relative motion

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

Energy conserv.

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

Angular mom.

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}}$$

Kepler's 1st law

$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta}$$

The orbit equation

Azim. velocity

$$v_\perp = h / r$$

Kepler's 2nd law

$$dA/dt = h/2$$

Kepler's 3rd law

$$T = 2\pi a^{1.5} / \mu^{0.5}$$

We have lost track of the time variable ! We must relate true anomaly to time

Importance/Complexity of Time Measurement

Applications such as GPS rely on an extremely precise time measurement system:

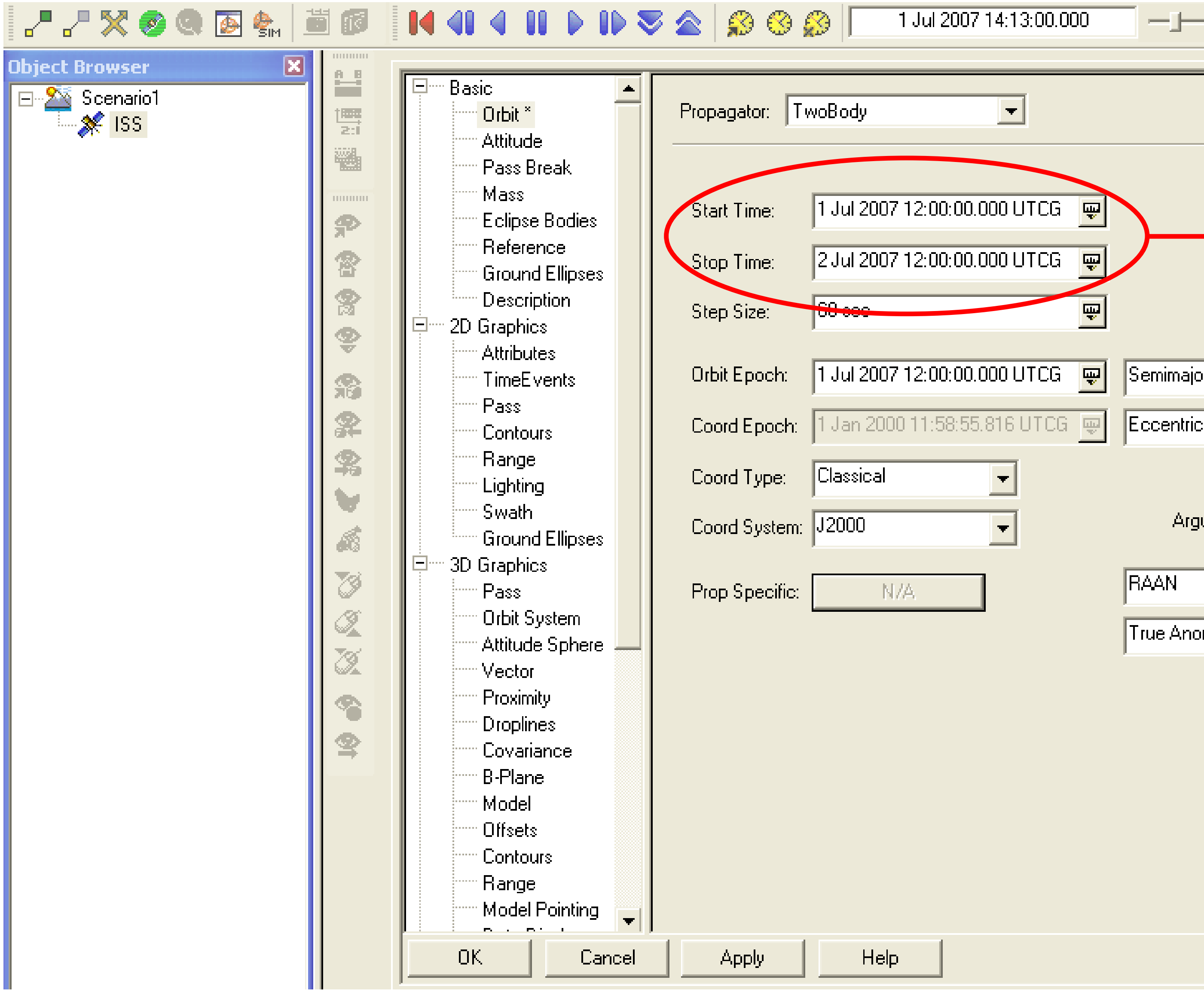
an error of 1 nanosecond translates into an error of 30cm in the distance.

What time is it? Well, no one knows for sure

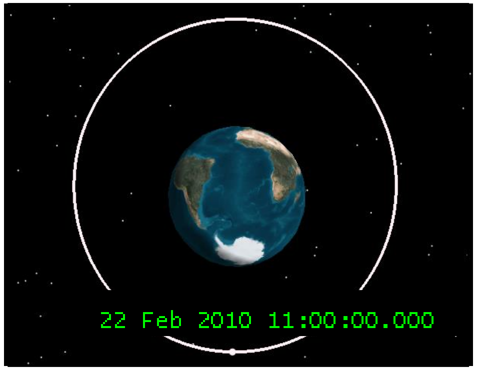
guardian.co.uk

As the Earth spins slower, methods of telling time diverge. Experts warn this could end in disaster

Complexity: STK



The Orbit in Time



Orbital position as a function of time

Time systems

Time Since Periapsis

What is the time required to fly between any two true anomaly ?

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{h}{2} = \text{constant}$$

Kepler's second law

$$r = \frac{h^2}{\mu} \frac{1}{1 + e\cos\theta}$$

Orbit equation

$$\frac{\mu^2}{h^3} dt = \frac{d\theta}{(1 + e\cos\theta)^2}$$

$$\frac{\mu^2}{h^3}(t - t_p) = \int_0^\theta \frac{d\theta}{(1 + e\cos\theta)^2}$$

Sixth missing constant (t_p=0)

$$\frac{\mu^2}{h^3} t = \int_0^\theta \frac{d\theta}{(1 + e\cos\theta)^2}$$

Circular Orbits

$$\frac{\mu^2}{h^3} t = \int_0^\theta \frac{d\theta}{(1+e\cos\theta)^2}$$

$$\Downarrow$$

$$\frac{\mu^2}{h^3} t = \int_0^\theta d\theta$$

$$\Downarrow$$

$$t = \frac{h^3}{\mu^2}\theta = \frac{r^{3/2}}{\sqrt{\mu}}\theta = \frac{T}{2\pi}\theta$$

$$\boxed{\rightarrow \theta = \frac{2\pi}{T} t}$$

Obvious, because the angular velocity is constant.

Elliptic Orbits

Because the angular velocity of a spacecraft along an eccentric orbit is continuously varying, the expression of the angular position versus time is no longer trivial.

$$\frac{\mu^2}{h^3} t = \int_0^\theta \frac{d\theta}{(1+e\cos\theta)^2}$$

$$\overset{0<e<1}{=} \frac{1}{(1-e^2)^{3/2}} \left[\overbrace{2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right) - \frac{e\sqrt{1-e^2}\sin\theta}{1+e\cos\theta}}^{\text{Mean anomaly, } M} \right]$$

$$T_{ellip} = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$h = \sqrt{\mu a(1-e^2)}$$

$$\Rightarrow$$

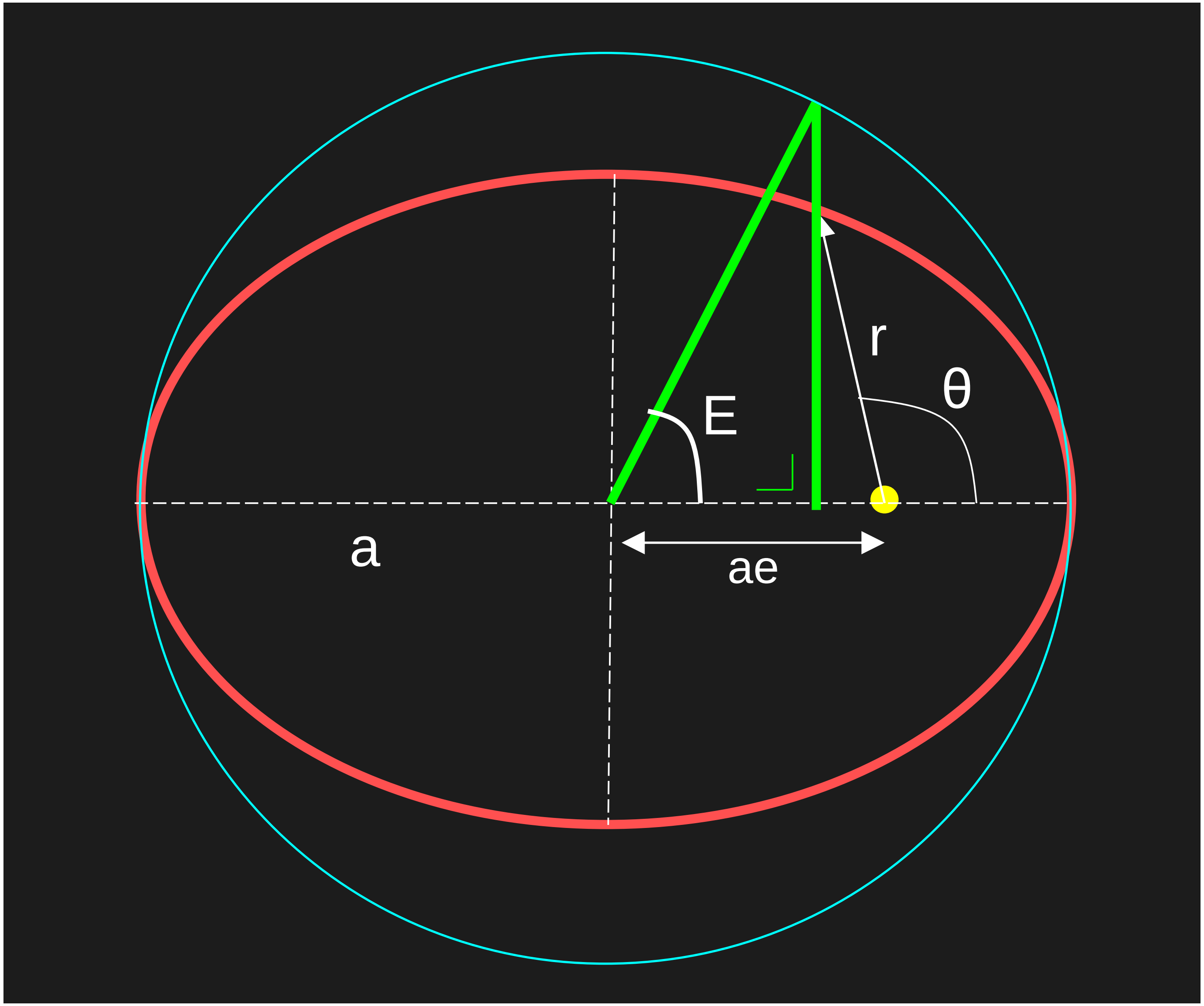
$$\boxed{M = \frac{\mu^2}{h^3}(1-e^2)^{3/2} t = \frac{2\pi}{T} t = nt}$$

Mean Anomaly Is Related to Time

For circular orbits, the mean M and true anomalies θ are identical.

For elliptic orbits, the mean anomaly represents the angular displacement of a fictitious body moving around the ellipse at the constant angular speed n.

Eccentric Anomaly Is Related to Position



Eccentric Anomaly: Relation with Mean Anomaly ?

$$\cos^2 E + \sin^2 E = 1$$

$a\cos E = ae + r\cos\theta$ Graph

$$r = a\frac{1-e^2}{1+e\cos\theta}$$

L2

$$\cos E = \frac{e+\cos\theta}{1+e\cos\theta} \Rightarrow \boxed{\sin E = \frac{\sqrt{1-e^2}\sin\theta}{1+e\cos\theta}}$$

trig. id.

$$\cos E = \frac{e+\cos\theta}{1+e\cos\theta}$$

$$\sin E = \frac{\sqrt{1-e^2}\sin\theta}{1+e\cos\theta}$$

$$\Rightarrow \tan^2\frac{E}{2} = \frac{1-\cos E}{1+\cos E} = \frac{1-e}{1+e}\frac{1-\cos\theta}{1+\cos\theta} = \frac{1-e}{1+e}\tan^2\frac{\theta}{2}$$

$$\boxed{E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right)}$$

Kepler's Equation

$$M = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right) - \frac{e\sqrt{1-e^2}\sin\theta}{1+e\cos\theta} \qquad E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right)$$

$$\Downarrow$$

$$\boxed{M = nt = E - e\sin E}$$

It relates time, in terms of $M=nt$, to position, in terms of E, $r=a(1-e.\cos E)$.

Usefulness of Kepler's Equation

θ and orbit are given

$\Downarrow$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right)$$

$\Downarrow$

$$M = E - e\sin E$$

$\Downarrow$

$$t = \frac{M}{2\pi}T$$

Practical application:

Determine the time at which a satellite passes from sunlight into the Earth's shadow (the location of this point is known from the geometry).

Example

A geocentric elliptic orbit has a perigee radius of 9600 km and an apogee radius of 21000 km. Calculate the time to fly from perigee to a true anomaly of 120°.

$$e \overset{L2}{=} \frac{r_a - r_p}{r_a + r_p} = \frac{21000 - 9600}{21000 + 9600} = 0.37255$$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right) = 1.7281\,\mathrm{rad}$$

$$M = E - e\sin E = 1.3601\,\mathrm{rad}$$

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} = 2\pi\sqrt{\left(\frac{r_p + r_a}{2}\right)^3 / \mu} = 18834\,\mathrm{s}$$

$$t = \frac{M}{2\pi}T = \frac{1.3601}{2\pi}18834 = 4077\,\mathrm{s} = 1\mathrm{h}07\mathrm{m}57\mathrm{s}$$

Usefulness of Kepler's Equation

t is given

$$\Downarrow$$

$$M = \frac{2\pi t}{T}$$

$$\Downarrow$$

$$M = E - e \sin E$$

$$\Downarrow$$

Transcendental equation !!!
(with a unique solution)

Practical application:

Perform a rendez-vous with the ISS (ATV, STS, Soyuz, Progress).

Numerical Solution: Newton-Raphson

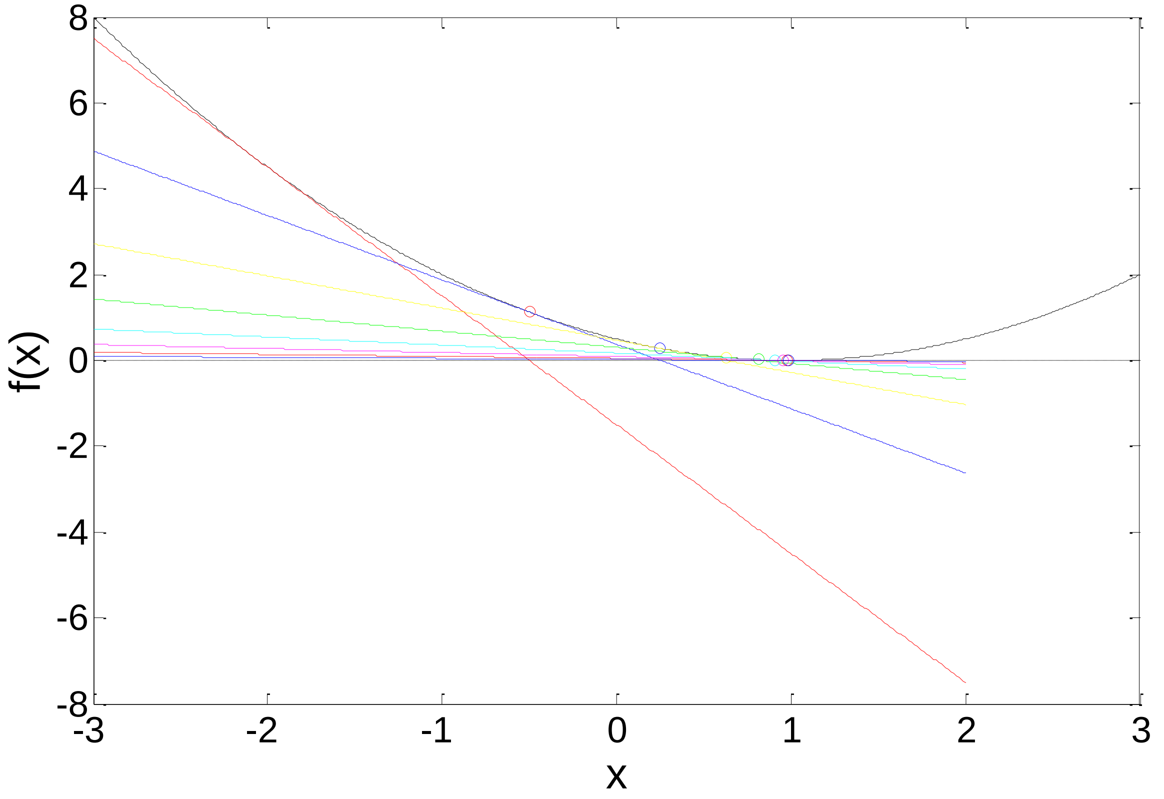
Algorithm for finding approximations to the zeros of a nonlinear function.

Recursive application of Taylor series truncated after the first derivative.

The initial guess should be close enough to the actual solution.

Numerical Solution: Newton-Raphson

Example: find the zero of $f(x)=0.5(x-1)^2$



Analytic Solution: Lagrange

$$E = M + \sum_{n=1}^{\infty} a_n e^n$$

$$a_n = \frac{1}{2^{n-1}} \sum_{k=0}^{floor(n/2)} (-1)^k \frac{1}{(n-k)!k!} (n-2k)^{n-1} \sin\left[(n-2k)M\right]$$

Convergence if e<0.663.

For small values of the eccentricity a good agreement with the exact solution is obtained using a few terms (e.g., 3).

Analytic Solution: Bessel Functions

$$E = M + \sum_{n=1}^{\infty} \frac{2}{n} J_n(ne) \sin nM$$

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(n+k)!\,k!} \left(\frac{x}{2}\right)^{n+2k}$$

Convergence for all values of the eccentricity less than 1.

Parabolic and Hyperbolic Orbits

$$\frac{\mu^2}{h^3} t = \int_0^{\theta} \frac{d\theta}{\left(1+\cos\theta\right)^2} = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2}$$

$$M = \frac{\mu^2}{h^3} t = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2}$$

$$F = \ln\left[\frac{\sqrt{e+1} - \sqrt{e-1}\tan(\theta/2)}{\sqrt{e+1} - \sqrt{e-1}\tan(\theta/2)}\right]$$

$$M = e\sinh F - F$$

Prediction of the Position and Velocity

If the position and velocity $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ of an orbiting body are known at a given instant t_0, how can we compute the position and velocity $\boldsymbol{r}$ and $\boldsymbol{v}$ at any later time t ?

Concept of *f* and *g* function and series:

$$\mathbf{r}(t) = f(t, t_0, \mathbf{r}_0, \mathbf{v}_0)\,\mathbf{r}_0 + g(t, t_0, \mathbf{r}_0, \mathbf{v}_0)\,\mathbf{v}_0$$

Prussing and Conway

$$f = 1 - \frac{a}{r_0}\left[1 - \cos(E - E_0)\right]$$

J.E. Prussing, B.A. Conway, *Orbital Mechanics*, Oxford University Press

$$g = (t - t_0) - \sqrt{\frac{a^3}{\mu}}\left[(E - E_0) - \sin(E - E_0)\right]$$

Prediction of the Position and Velocity

Some form of Kepler's equation must still be solved by iteration. However, Gauss developed a series expansion in the elapsed time parameter t-t_0, and there is no longer the need to solve Kepler's equation:

$$\mathbf{r}(t) = f(t, t_0, \mathbf{r}_0, \mathbf{v}_0)\,\mathbf{r}_0 + g(t, t_0, \mathbf{r}_0, \mathbf{v}_0)\,\mathbf{v}_0$$

$$f = \left[1 - \frac{\mu}{2r_0^3}(t - t_0)^2 + \frac{\mu}{2}\frac{\mathbf{r}_0.\mathbf{v}_0}{r_0^5}(t - t_0)^3 + \ldots\right]$$

$$g = \left[(t - t_0) - \frac{\mu}{6r_0^3}(t - t_0)^3 + \frac{\mu}{4}\frac{\mathbf{r}_0.\mathbf{v}_0}{r_0^5}(t - t_0)^4 + \ldots\right]$$

The Orbit in Time

Time systems

- Clocks
- Universal time
- Earth's rotation
- Atomic time
- Coordinated universal time
- Julian date

"Everyday" Time Systems

Are conventional local time systems adequate for orbital mechanics ?

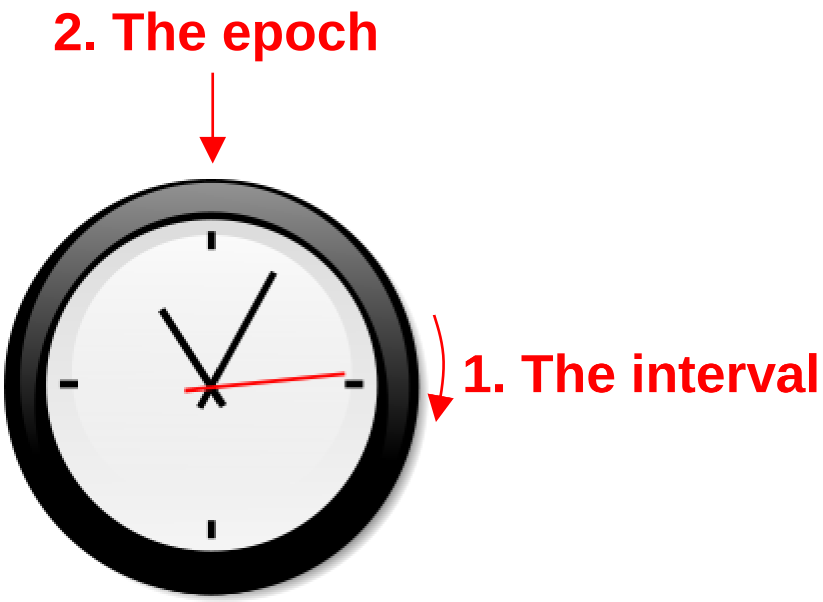
$\Rightarrow$ They depend on the user's position on Earth.

$\Rightarrow$ They are in a format (Y/M/D/H/M/S) that does not lend itself to use in a computer-implemented algorithm. For instance, what is the difference between any two dates ?

Objective of this section:

What would be a meaningful time system ?

What are the Ingredients of a Time System ?



1. The interval (a time reckoner): a repeatable phenomenon whose motion or change of state is observable and obeys a definite law.

2. The epoch (a time reference) from which to count intervals

Historical Perspective

From remote antiquity, the celestial bodies have been the fundamental reckoners of time (e.g. rising and setting of the Sun).

Sundials were among the first instruments used to measure the time of the day. The Egyptians divided the day and night into 12h each, which varied with the seasons (unequal seasonal hour)

Historical Perspective

It was not until the 14th century that an hour of uniform length became customary due to the invention of mechanical clocks.

Quartz-crystal clocks were developed in the 1920s.

The first atomic clock was constructed in 1948, and the first caesium atomic clock in 1955.

Quantum Clocks Soon ?

ULTRA-PRECISE QUANTUM-LOGIC CLOCK TRUMPS OLD ATOMIC CLOCK

Scientists have built a clock which is 37 times more precise than the existing international standard.

The quantum-logic clock, which detects the energy state of a single aluminum ion, keeps time to within a second every 3.7 billion years. The new timekeeper could one day improve GPS or detect the slowing of time predicted by Einstein's theory of general relativity.

https://www.wired.com/2010/02/quantum-logic-atomic-clock/

Tow Important Time Scales

1. *Universal time*: the time scale based on the rotation of the Earth on its axis.

2. *Atomic time*: the time scale based on the quantum mechanics of the atom.

Can We Use the Real Sun ?

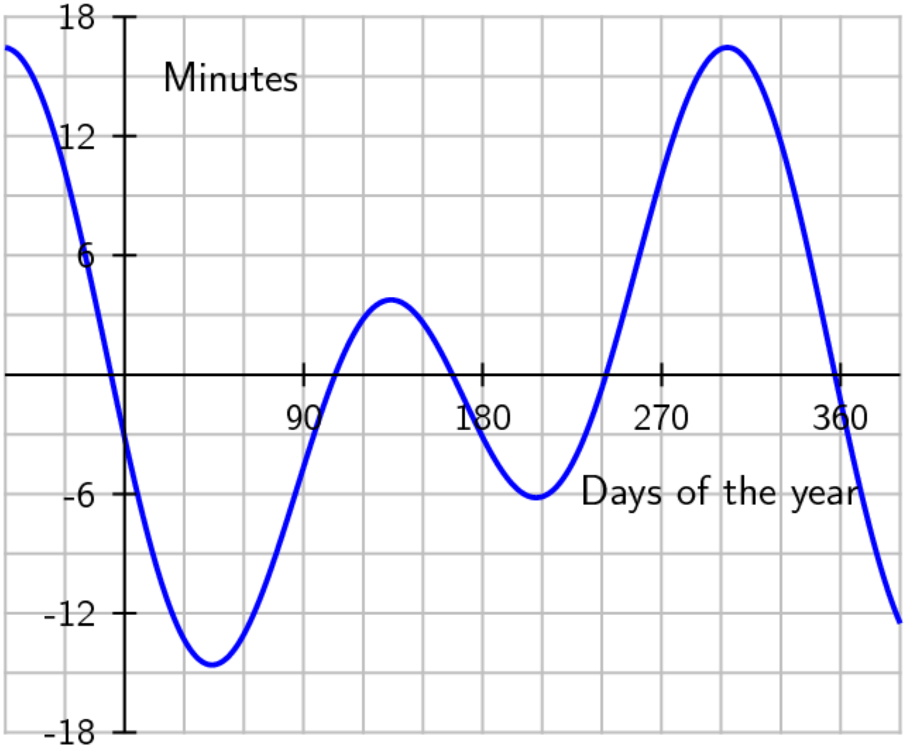
Apparent solar time, as read directly by a sundial, is the local time defined by the actual diurnal motion of the Sun.

Apparent solar day is the time required for the sun to lie on the same meridian.

Due to the eccentricity of Earth's orbit, the length of the apparent solar day varies throughout the year.

$\Rightarrow$ The real sun is not well suited for time reckoning purposes.

Apparent and Mean Solar Days



Approximation where E is in minutes, sin and cos in degrees, and N is the day number:

$$E = 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B$$

$$B = \frac{360^\circ (N - 81)}{365}$$

Equation of time

Can We Use a Fictitious Sun ?

At noon the fictitious sun lies on the Greenwich meridian.

A mean solar day comprises 24 hours. It is the time interval between successive transits of a fictitious mean sun over a given meridian. A constant velocity in the motion about the sun is therefore assumed.

The mean solar second can be defined as 1/86400 of a **mean solar day**.

Universal Time

Universal time is today's realization of a **mean solar time** (introduced in 1920s).

It is the same everywhere on Earth.

It is referred to the meridian of Greenwich and reckoned from midnight.

Universal Time UT1

UT1 is the observed rotation of the Earth with respect to the mean sun.

It is based on the measurement of the Earth rotation angle with respect to an inertial reference frame (sidereal day).

A conversion from mean sidereal day to mean solar day is therefore necessary.

Earth rotation angle (sidereal day) → UT1 (mean solar day)

IAU2000 Definition of Universal Time UT1

Earth rotation angle
(sidereal day)

UT1
(mean solar day)

$$ERA = 2\pi(0.7790572732640 + 1.00273781191135448\, T_u) \text{ radians}$$

IAU2000 Definition of Universal Time UT1

Earth rotation angle
at J2000.0 UT1

Explanation 1

$$ERA = 2\pi(0.7790572732640 + 1.00273781191135448\ Tu)\ \text{radians}$$

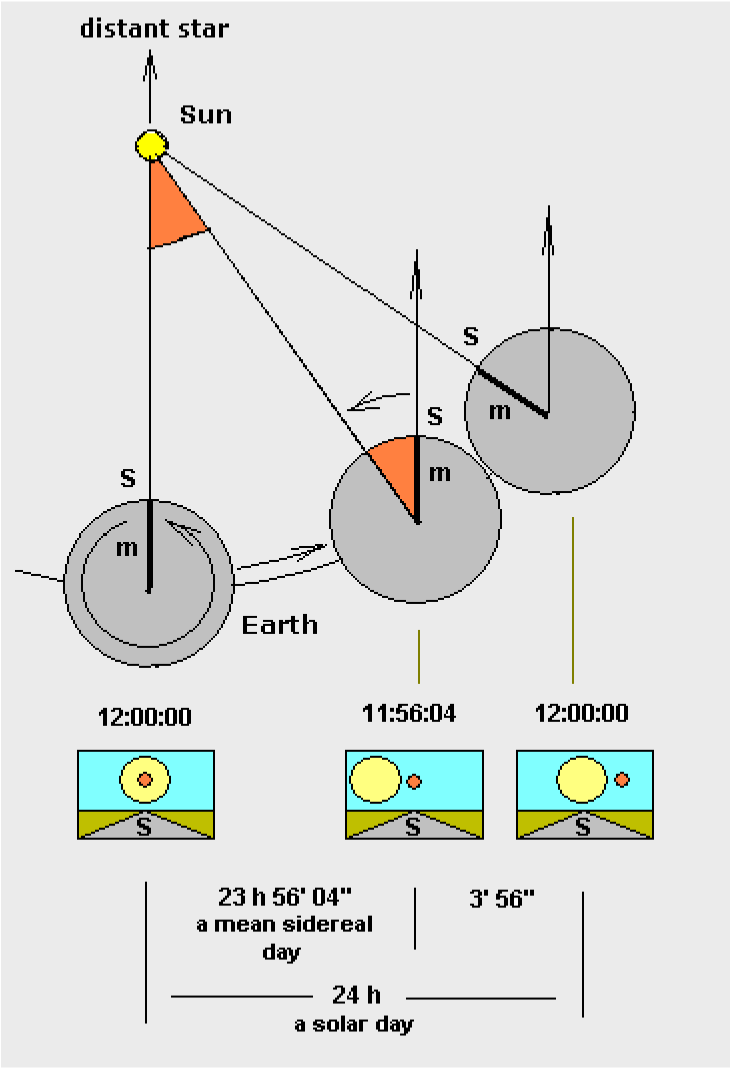
Earth rotation angle w.r.t. ICRF.
Its time derivative is the Earth's angular velocity.

Explanation 3

Tu is directly related to UT1:
Julian UT1 date - 2451545.0

Explanation 2

Explanation 1: Mean Solar Sidereal Days



1 solar day= 1.0027378119113548 sidereal day

Explanation 2: Julian Date

The Julian day number is the number of days since noon January 1, 4713 BC $\rightarrow$ Continuous time scale and no negative dates.

For historical reasons, the Julian day begins at noon, and not midnight, so that astronomers observing the heavens at night do not have to deal with a change of date.

The number of days between two events is found by subtracting the Julian day of one from that of the other.

Forward Computation of the Julian Date

Conversion from conventional time (YYYY, MM, DD, hh:mm:ss.ss) to Julian Date:

$$JD = 367.YYYY - \text{floor}\left\{\frac{7}{4}\left[YYYY + \text{floor}\left(\frac{MM+9}{12}\right)\right]\right\} + \text{floor}\left(\frac{275.M}{9}\right) + DD + 1721013.5 + \frac{1}{24}\left[\frac{1}{60}\left(\frac{ss}{60} + mm\right) + hh\right]$$

Valid for the period 1st March 1900 and 28th February 2100

Backward Computation of the Julian Date

$$a = \text{floor}\{MJD\} + 2{,}400{,}001 \qquad MJD = JD - 2{,}400{,}000.5$$

$$q = MJD - \text{floor}\{MJD\}$$

$$b = \begin{cases} 0 & \text{if } a < 2299161 \\ \text{floor}\{(a - 1867216.25)/36524.25\} & \text{otherwise} \end{cases}$$

$$c = \begin{cases} a + 1524 & \text{if } a < 2299161 \\ a + b - \text{floor}\{b/4\} + 1525 & \text{otherwise} \end{cases}$$

$$d = \text{floor}\{(c - 121.1)/365.25\}$$

$$e = \text{floor}\{365.25d\}$$

$$f = \text{floor}\{(c - e)/30.6001f\} + q$$

$$D = c - e - \text{floor}\{30.6001f\} + q$$

$$[D = \text{floor}\{D\}]$$

$$M = f - 1 - 12 \times \text{floor}\{f/14\}$$

$$Y = d - 4715 - \text{floor}\{(7 + M)/10\}$$

$$hr = \text{floor}\{q \times 24\}$$

$$min = \text{floor}\{q \times 24 \times 60\} - (hr \times 60)$$

$$sec = (q \times 24 \times 60 \times 60) - (60 \times min) - (60 \times 60 \times hr)$$

Computation of Elapsed Time

Find the elapsed time between 4 October 1957 at 19:26:24 UTC and 12 May 2004 at 14:45:30 UTC

4 October 1957 at 19:26:24 UTC: 2436116.3100 days

12 May 2004 at 14:45:30 UTC: 2453138.11493056 days

$\rightarrow$ The elapsed time is 17021.805 days

Standard Epoch Used Today: J2000

To lessen the magnitude of the Julian date, a constant offset can be introduced. A different reference epoch 1st January 2000 at noon is used:

$$J2000 = JD - 2451545$$

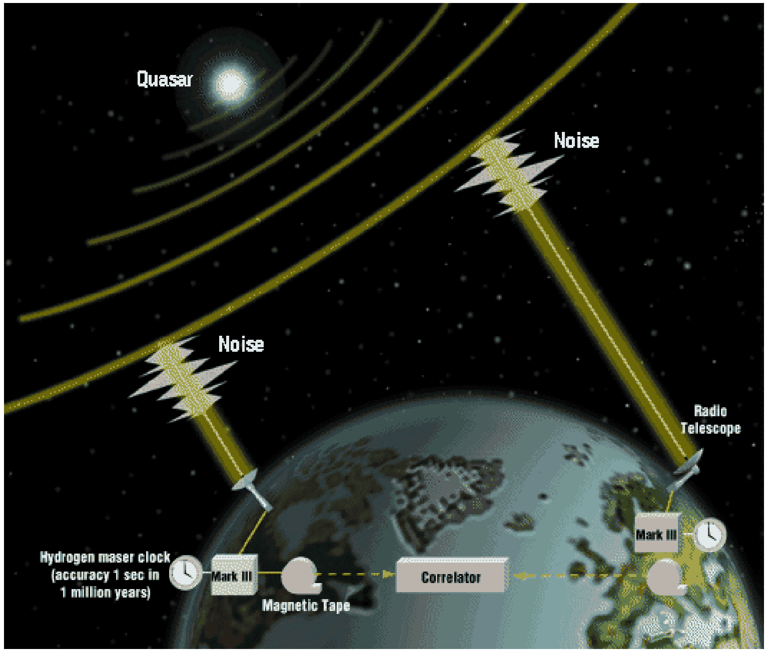
Explanation 3: Accurate Determination of ERA

The most remote objects in the universe are quasars in a distance of about 3-15 billion light years. Because quasars are at such great distances that their motions across the sky are undetectable, they form a quasi-inertial reference frame, called the international celestial reference frame.

Quasars can be detected with very sensitive radiotelescopes.

By observing the diurnal motion of distant quasars (more precise than sun-based observations), it is possible to relate the position, orientation and rotation of the Earth to the inertial reference frame realized by these quasars.

Very Long Baseline Interferometry



A radio telescope with a cryogenic dual band S/X-band receiver (TIGO, Concepcion, Chili)

Can We Trust the Earth's Rotation ?

?

$$ERA = 2\pi(0.7790572732640 + 1.00273781191135448\,Tu) \text{ radians}$$

Can We Trust the Earth's Rotation ?

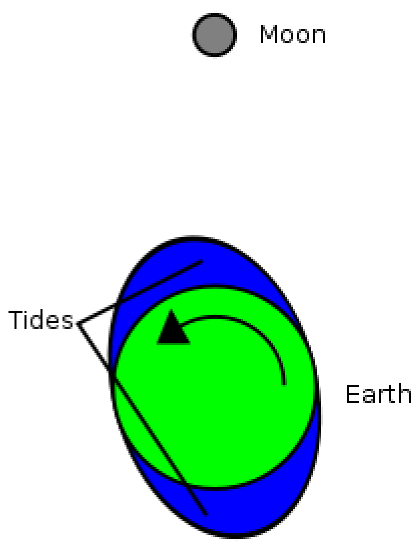
No !

$\Rightarrow$ The Earth's rotation rate is not uniform. It exhibits changes on the order of 2 milliseconds per day. Corals dating from 370 millions years ago indicate that the number of days was between 385 and 410.

$\Rightarrow$ There also exists random and seasonal variations.

In addition, the axis of rotation is not fixed in space.

Rotation Rate: Steady Deceleration (Cause 1)



Energy transfer from the Earth to the Moon

The Moon is at the origin of tides: the water of the oceans bulges out along both ends of an axis passing through the centers of the Earth and Moon.

The tidal bulge closely follows the Moon in its orbit, and the Earth rotates under this bulge in a day. Due to friction, the rotation drags the position of the tidal bulge ahead of the position directly under the Moon.

A substantial amount of mass in the bulge is offset from the line through the centers of the Earth and Moon. Because of this offset, there exists a torque which boosts the Moon in its orbit, and decelerates the rotation of the Earth.

Rotation Rate: Steady Deceleration (Cause 2)

In addition to this tidal acceleration of the Moon, the Earth is also slowing down due to tidal friction.

Tides stretch the oceans, and to a small extent, the solid mass of a planet or satellite. In one complete rotation, the planet material keeps deforming and relaxing. This takes energy away from the rotation, transforming it into heat.

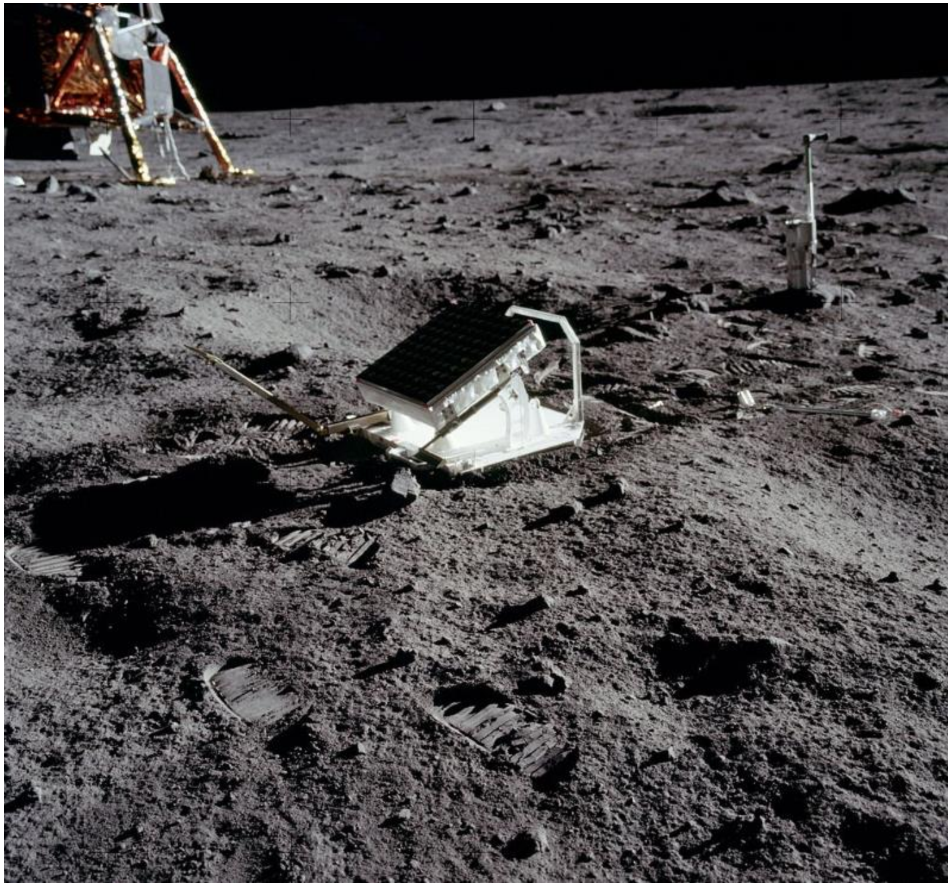
The Moon Is Moving Away from the Earth

The secular acceleration of the Moon is small but it has a cumulative effect on the Moon's position when extrapolated over many centuries.

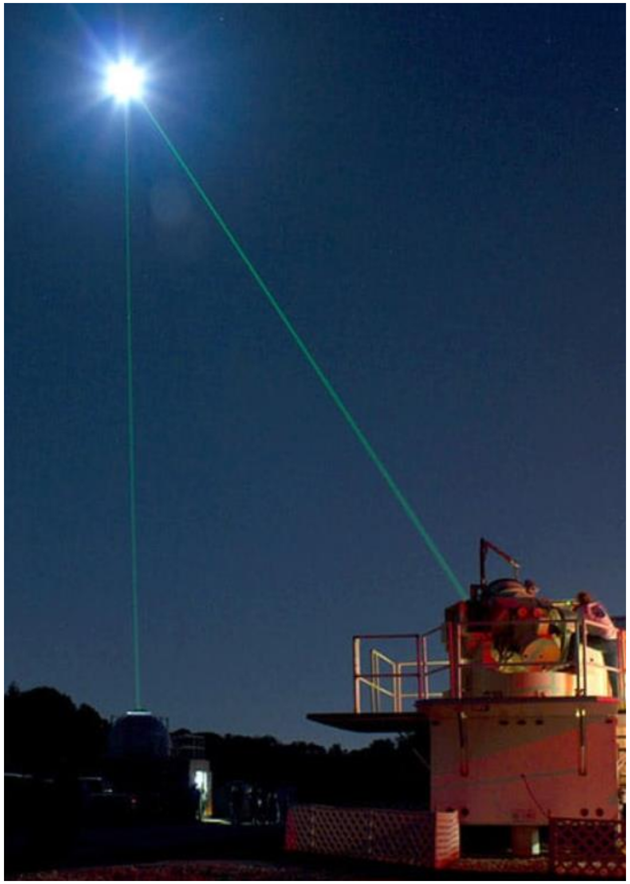
Direct measurements of the acceleration have been possible since 1969 using the Apollo retro-reflectors left on the Moon.

The results from Lunar Laser Ranging show that the Moon's mean distance from Earth is increasing by 3.8 cm per year.

Lunar Laser Ranging Experiment (Apollo 11)



Lunar Laser Ranging Experiment from the Apollo 11 mission



NASA Goddard
(Lunar Reconnaissance Orbiter)

Le séisme au Chili pourrait avoir raccourci les jours

Rédaction en ligne

mardi 02 mars 2010, 19:48

Le violent séisme qui a frappé le Chili a peut-être fait bouger l'axe de la Terre et, par conséquent, diminué la longueur du jour terrestre. La masse de la Terre étant désormais répartie autrement, notre planète tourne plus vite.

©AFP

Le séisme de magnitude 8,8 samedi au Chili pourrait avoir eu des effets sur l'axe de rotation de la Terre, entraînant un infime raccourcissement de la durée du jour, selon des scientifiques du Jet Propulsion Laboratory (JPL) de la NASA, l'agence spatiale américaine.

D'après des calculs préliminaires, issus d'une simulation informatique, le séisme du 27 février pourrait avoir entraîné un décalage de huit centimètres de l'axe de rotation terrestre, a expliqué mardi Richard Gross, du JPL, à Pasadena (Californie). Cela devrait provoquer un raccourcissement des jours de 1,26 microseconde, soit 1,26 millionième de seconde, ajoute-t-il. D'après Richard Gross, l'analyse des données du séisme permettra d'affiner les calculs.

Le phénomène n'est pas inédit. Comme dans tous les séismes majeurs, la Terre peut changer sa vitesse de rotation. Sous l'effet du séisme, la circonférence de la terre rétrécit très légèrement.

Le phénomène se retrouve lorsqu'une patineuse sur glace accélère sa rotation en fermant les bras et les rapprochant du corps. Très légère, cette accélération peut cependant être mesurée par satellite.

A titre d'exemple, le plus grand séisme du XXe siècle, d'une magnitude de 9,6 en 1960 au Chili, a fait diminuer la longueur du jour de huit microsecondes, selon une estimation des chercheurs.

Reste que l'atmosphère (frottements, jeu des masses d'air) et les marées océaniques ont une influence bien plus importante sur la durée du jour.

Quant à l'axe de la Terre, il varie naturellement tout le temps, décrivant en gros, à l'échelle d'une année, un cercle d'une dizaine de mètres. Le petit déplacement subi par l'axe de la Terre à cause du séisme chilien, estimé à huit centimètres, est donc moins élevé que le mouvement naturel de la Terre.

A ce mouvement mécanique s'ajoutent les mouvements des océans, des marées, l'influence de l'atmosphère, les éruptions volcaniques.

Pour provoquer un cataclysme et modifier réellement l'orbite terrestre, soulignent les sismologues, il faudrait une cause extérieure comme la collision avec un astéroïde.

What is Your Conclusion ?

We cannot “trust” the Earth’s rotation $\Rightarrow$ the length of one second of UT1 is not constant !

Its offset from atomic time is continually changing in a not completely predictable way.

International Atomic Time (TAI)

Since the advent of atomic time in 1955 there has been a steady transition from reliance on the Earth's rotation to the use of atomic time as the standard for the SI unit of duration (second).

The second is the duration of 9.192.631.770 cycles of the radiation corresponding to the transition between two hyperfine levels of the ground state of ^{133}Cs.

Weighted average of the time kept by about 300 atomic clocks in over 50 national laboratories worldwide.

Atomic Clocks: Stability and Accuracy

The caesium clock has high accuracy and good long-term stability.

The hydrogen maser has the best stability for periods of up to a few hours.

Is Atomic Time the Adequate Solution ?

No connection with the motion of the sun across the sky !

Physical and Astronomical Times

Astronomical clocks:

$\Rightarrow$ Related to everyday life.

$\Rightarrow$ Not consistent; the length of one second of UT is not constant. Typical accuracies ~10^{-8}.

Atomic clocks:

$\Rightarrow$ Consistent. Typical accuracies ~10^{-14}.

$\Rightarrow$ Not related to everyday life. If no adjustment is made, then within a millennium, local noon (i.e., the local time associated with the Sun’s zenith position) would occur at 13h00 and not 12h00.

Coordinated Universal Time (UTC)

The good practical compromise between atomic and universal times: it is the international standard on which civil time is based.

Its **time interval** corresponds to atomic time TAI:

$\Rightarrow$ It is accurate.

Its **epoch** differs by no more than 0.9 sec from UT1:

$\Rightarrow$ The mean sun is overhead on the Greenwich meridian at noon.

Leap Seconds

Leap seconds were introduced in 1971 to reconcile astronomical time, which is based on the rotation of the Earth, and physical time, which can be measured with great accuracy using atomic clocks.

Leap seconds are introduced to account for the fact that the Earth currently runs slow at 2 milliseconds per day and they ensure that the Sun continues to be overhead on the Greenwhich meridian at noon to within 1s.

Le Soir Article

Le Nouvel An en retard d'une seconde cette année

Rédaction en ligne

mercredi 31 décembre 2008, 09:19

Une seconde sera ajoutée à la dernière heure de 2008 pour refléter le ralentissement de la rotation de la Terre, sur fond de débat entre partisans de deux systèmes de mesure : le Greenwich Mean Time (« heure moyenne de Greenwich », GMT), une institution britannique, et le Temps atomique international (TAI), calculé près de Paris.

Le 31 décembre, à 23 heures, 59 minutes et 59 secondes en temps universel coordonné (UTC), une seconde supplémentaire sera ajoutée.

Not So Simple…

Record de vitesse de rotation de la Terre : "On pourrait être amené à retirer une seconde. Cela n'est jamais arrivé"

Le mercredi 29 juin 2022, la Terre a battu son record de vitesse de rotation.

La Rédaction

Publié le 05-08-2022 à 11h20 - Mis à jour le 05-08-2022 à 15h11

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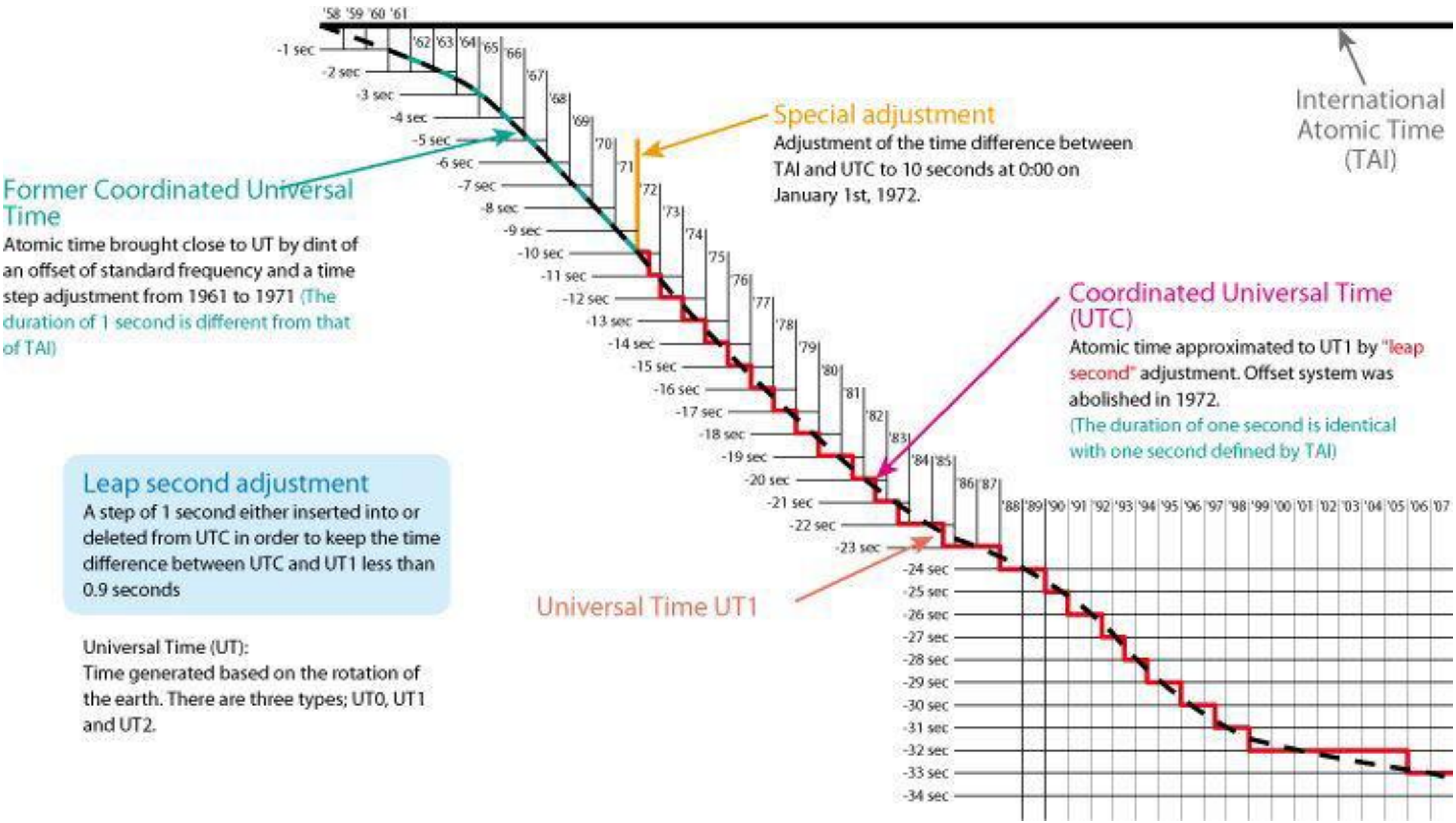
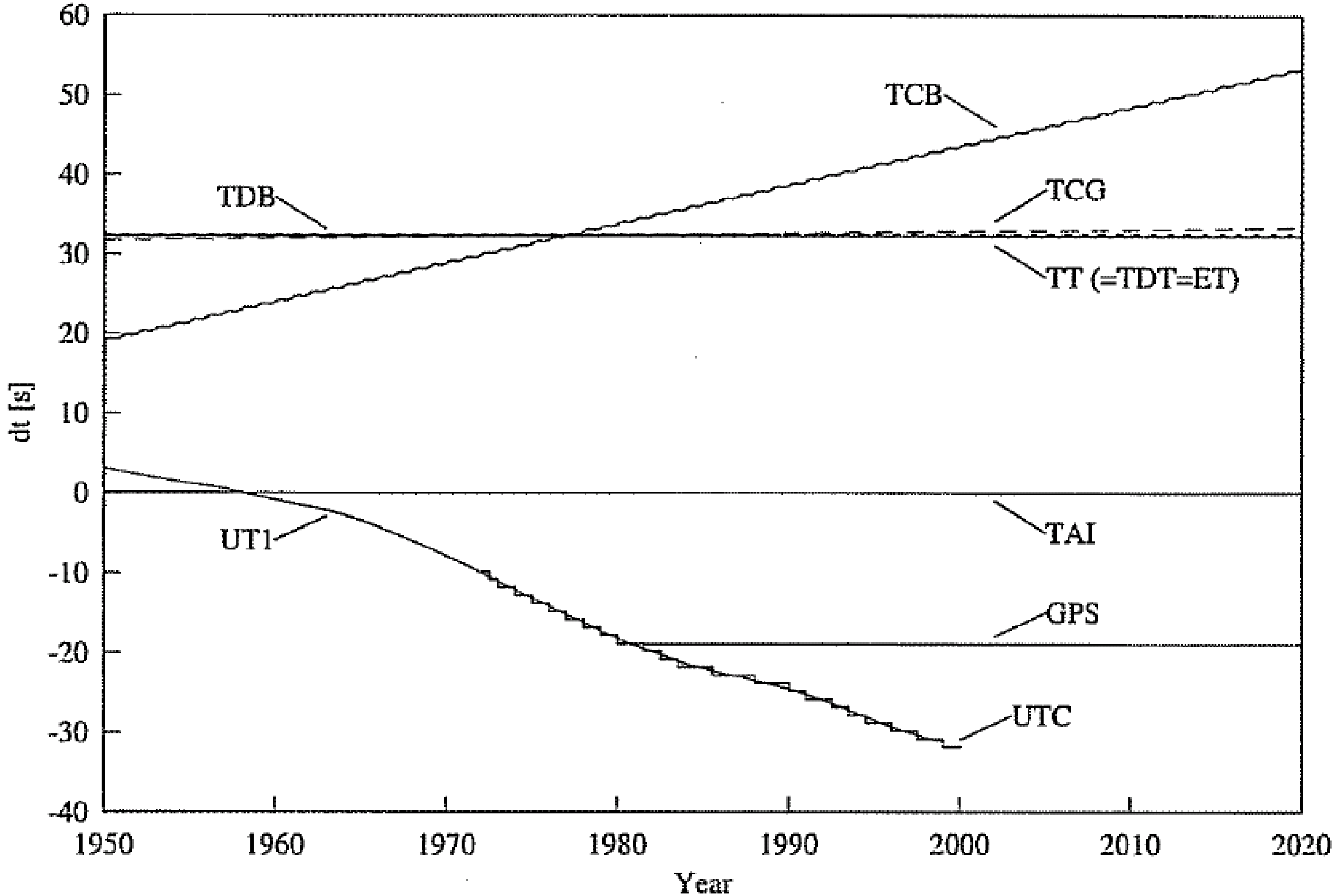
rotation de la Terre au journal Ouest-France, "*depuis 2016, on s'aperçoit que la vitesse de rotation de la Terre s'accélère, et donc la durée du jour diminue*". Avant cela, "*depuis 1930, raconte l'astronome,on observait une baisse de la vitesse de rotation de la Terreet donc une augmentation de la durée du jour. Cette tendance s'est inversée depuis sept ans sans qu'on puisse l'expliquer*".

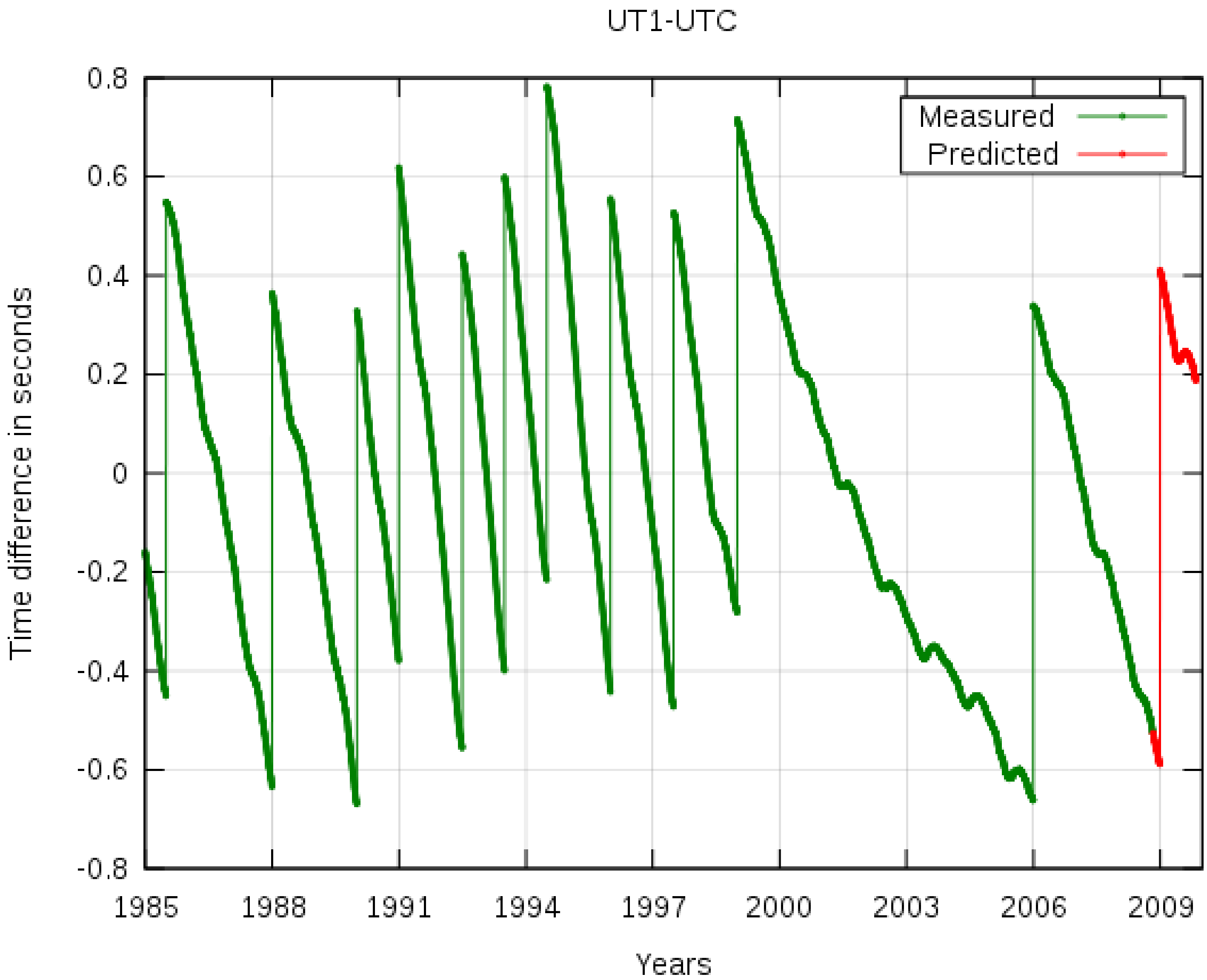


International Atomic Time (TAI)
Special adjustment
Adjustment of the time difference between TAI and UTC to 10 seconds at 0:00 on January 1st, 1972.
Former Coordinated Universal Time
Atomic time brought close to UT by dint of an offset of standard frequency and a time step adjustment from 1961 to 1971 (The duration of 1 second is different from that of TAI)
Coordinated Universal Time (UTC)
Atomic time approximated to UT1 by "leap second" adjustment. Offset system was abolished in 1972.
(The duration of one second is identical with one second defined by TAI)
Leap second adjustment
A step of 1 second either inserted into or deleted from UTC in order to keep the time difference between UTC and UT1 less than 0.9 seconds
Universal Time UT1
Universal Time (UT):
Time generated based on the rotation of the earth. There are three types; UT0, UT1 and UT2.
'58 '59 '60 '61 '62 '63 '64 '65 '66 '67 '68 '69 '70 '71 '72 '73 '74 '75 '76 '77 '78 '79 '80 '81 '82 '83 '84 '85 '86 '87 '88 '89 '90 '91 '92 '93 '94 '95 '96 '97 '98 '99 '00 '01 '02 '03 '04 '05 '06 '07
-1 sec -2 sec -3 sec -4 sec -5 sec -6 sec -7 sec -8 sec -9 sec -10 sec -11 sec -12 sec -13 sec -14 sec -15 sec -16 sec -17 sec -18 sec -19 sec -20 sec -21 sec -22 sec -23 sec -24 sec -25 sec -26 sec -27 sec -28 sec -29 sec -30 sec -31 sec -32 sec -33 sec -34 sec



60
50
40
30
20
10
0
-10
-20
-30
-40
dt [s]
1950
1960
1970
1980
1990
2000
2010
2020
Year
TCB
TDB
TCG
TT (=TDT=ET)
UT1
TAI
GPS
UTC

|DUT1 | =|UT1-UTC | <0.9s



Leap Seconds: Pros and Cons

This is currently the subject of intense debate (UT vs. TAI; i.e., UK vs. France).

+

Abandoning leap seconds would break sundials. In thousands of years, 16h00 would occur at 03h00. The British who have to wake up early in the morning to have tea…

Astronomers

-

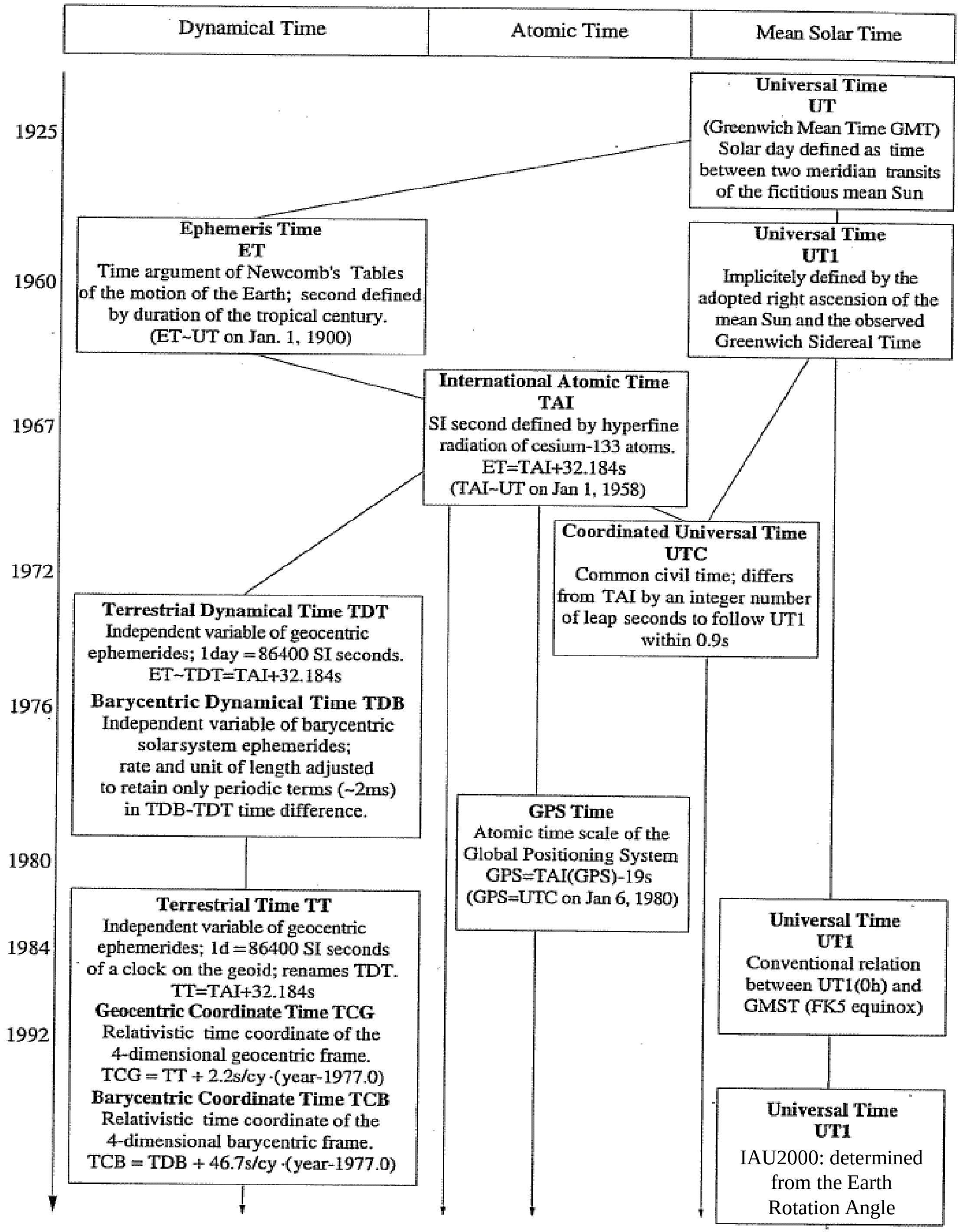
Leap seconds are a worry with safety-critical real-time systems (e.g., air-traffic control — GPS' internal atomic clocks race ahead of UTC).

Yet More Time Systems !

GPS time: running ahead of UTC but behind TAI (it was set in 1980 based on UTC, but leap seconds were ignored since then).

Time standards for planetary motion calculations:

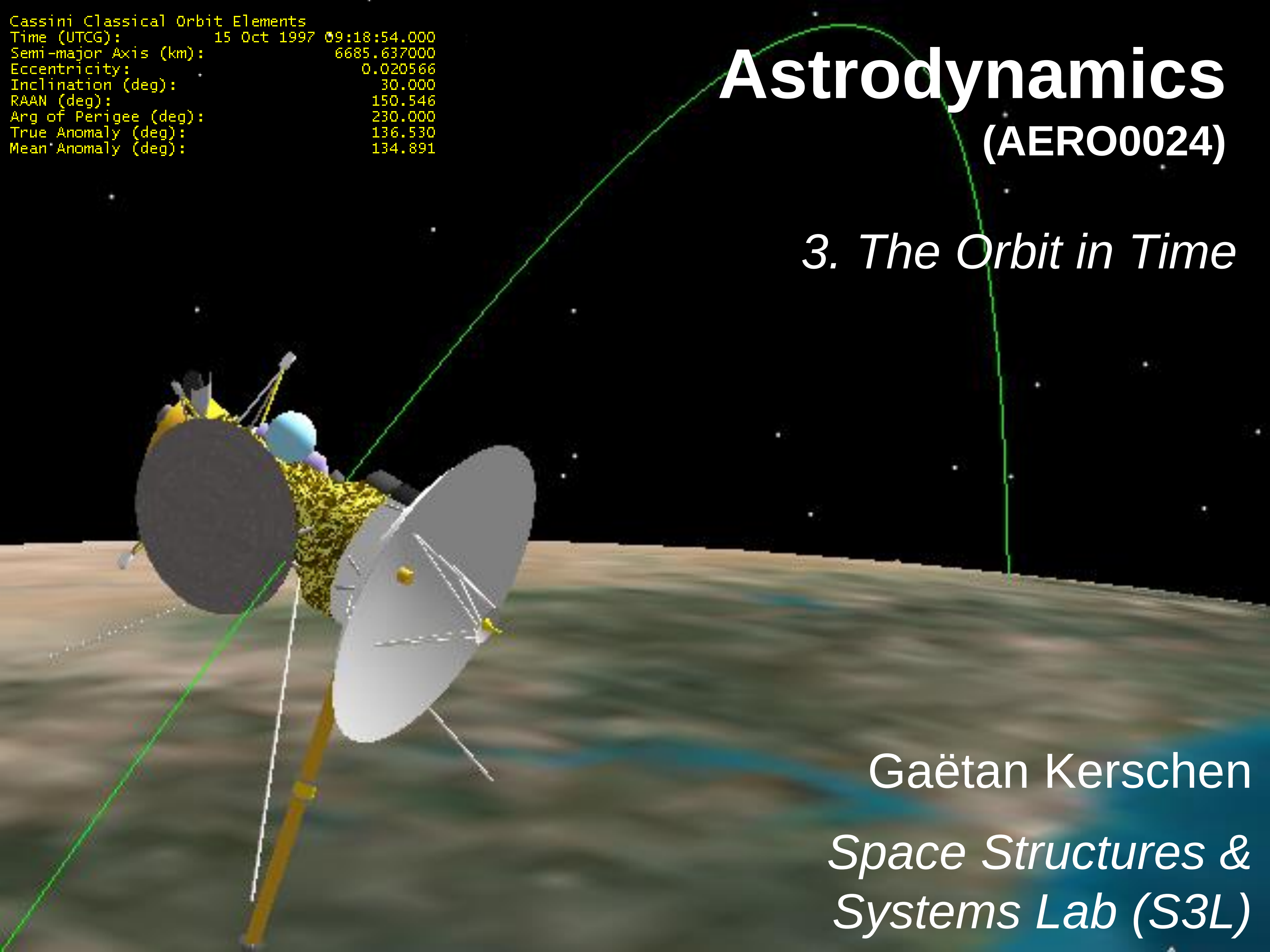
- ⇒ Terrestrial dynamic time: tied to TAI but with an offset of 32.184s to provide continuity with ephemeris time.
- ⇒ Barycentric dynamic time: similar to TDT but includes relativistic corrections that move the origin of the solar system barycenter.


Dynamical Time
Atomic Time
Mean Solar Time
1925
1960
1967
1972
1976
1980
1984
1992
Universal Time
UT
(Greenwich Mean Time GMT)
Solar day defined as time between two meridian transits of the fictitious mean Sun
Ephemeris Time
ET
Time argument of Newcomb's Tables of the motion of the Earth; second defined by duration of the tropical century.
(ET~UT on Jan. 1, 1900)
Universal Time
UT1
Implicitely defined by the adopted right ascension of the mean Sun and the observed Greenwich Sidereal Time
International Atomic Time
TAI
SI second defined by hyperfine radiation of cesium-133 atoms.
ET=TAI+32.184s
(TAI~UT on Jan 1, 1958)
Coordinated Universal Time
UTC
Common civil time; differs from TAI by an integer number of leap seconds to follow UT1 within 0.9s
Terrestrial Dynamical Time TDT
Independent variable of geocentric ephemerides; 1day = 86400 SI seconds.
ET~TDT=TAI+32.184s
Barycentric Dynamical Time TDB
Independent variable of barycentric solarsystem ephemerides;
rate and unit of length adjusted to retain only periodic terms (~2ms) in TDB-TDT time difference.
GPS Time
Atomic time scale of the Global Positioning System
GPS=TAI(GPS)-19s
(GPS=UTC on Jan 6, 1980)
Terrestrial Time TT
Independent variable of geocentric ephemerides; 1d = 86400 SI seconds of a clock on the geoid; renames TDT.
TT=TAI+32.184s
Geocentric Coordinate Time TCG
Relativistic time coordinate of the 4-dimensional geocentric frame.
TCG = TT + 2.2s/cy ·(year-1977.0)
Barycentric Coordinate Time TCB
Relativistic time coordinate of the 4-dimensional barycentric frame.
TCB = TDB + 46.7s/cy ·(year-1977.0)
Universal Time
UT1
Conventional relation between UT1(0h) and GMST (FK5 equinox)
Universal Time
UT1
IAU2000: determined from the Earth Rotation Angle

Further Reading

The leap second: its history and possible future

R. A. Nelson, D. D. McCarthy, S. Malys,
J. Levine, B. Guinot, H. F. Fliegel,
R. L. Beard and T. R. Bartholomew



Astrodynamics

(AERO0024)

3. The Orbit in Time

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