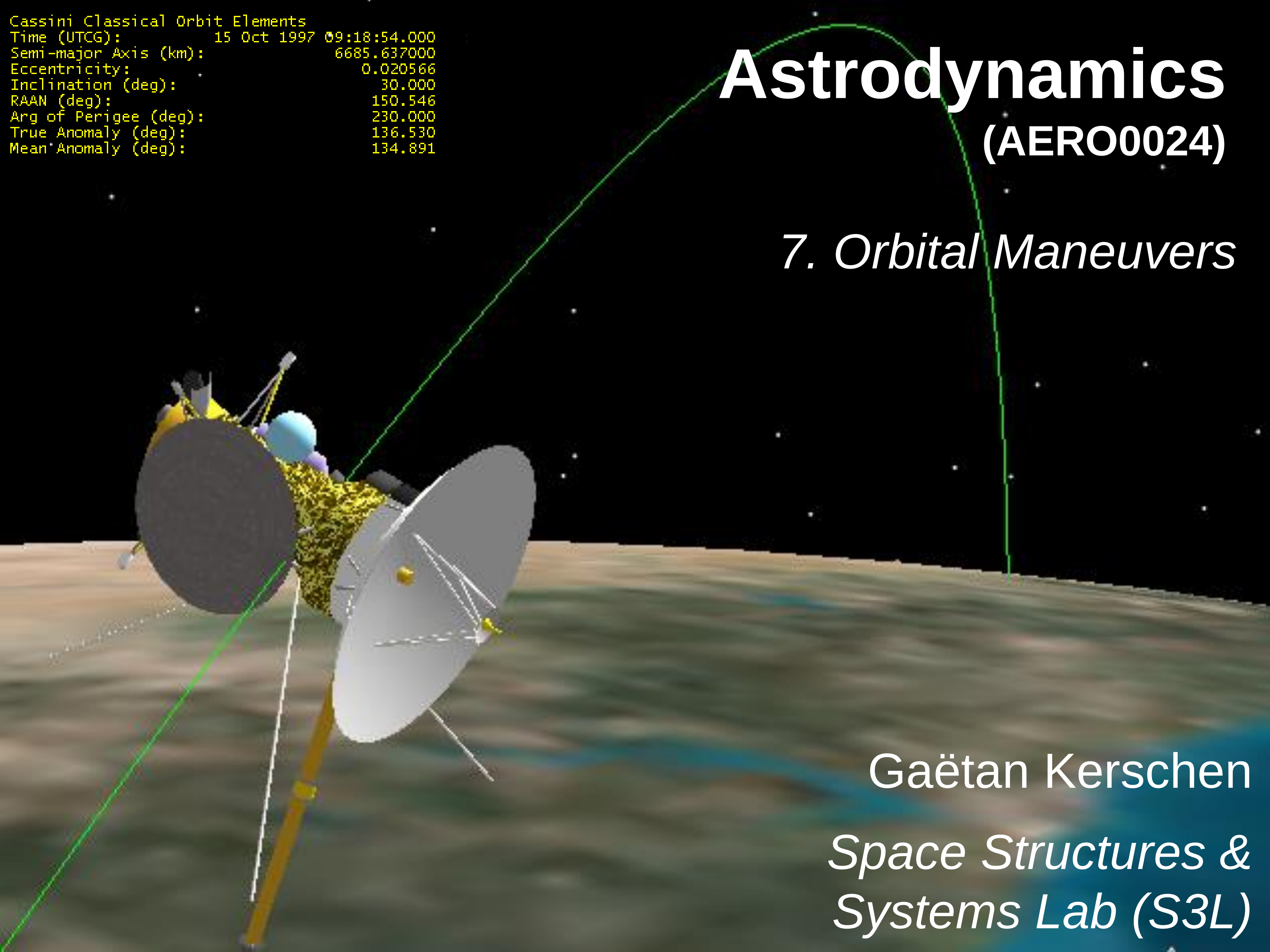


Astrodynamics

(AERO0024)

7. Orbital Maneuvers

Gaëtan Kerschen

Space Structures & Systems Lab (S3L)

Course Outline

PART 1: ORBITAL DYNAMICS

Lecture 02: The Two-Body Problem

Lecture 03: The Orbit in Space and Time

Lecture 04: Three-Body Problem

Lecture 05: Non-Keplerian Motion

Lecture 06: Orbital Propagation

PART 2: ORBIT CONTROL

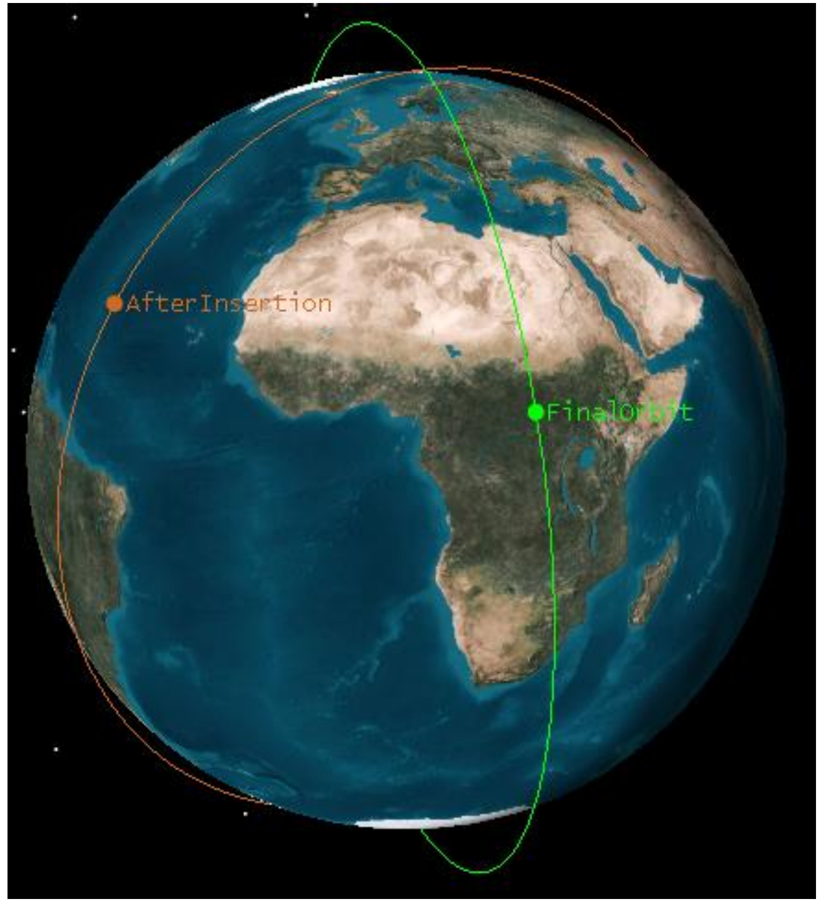
Lecture 07: Orbital Maneuvers

Lecture 08: Interplanetary Trajectories

Definition of Orbital Maneuvering

It encompasses all orbital changes after insertion required to place a satellite in the desired orbit.

This lecture focuses on satellites in Earth orbit.



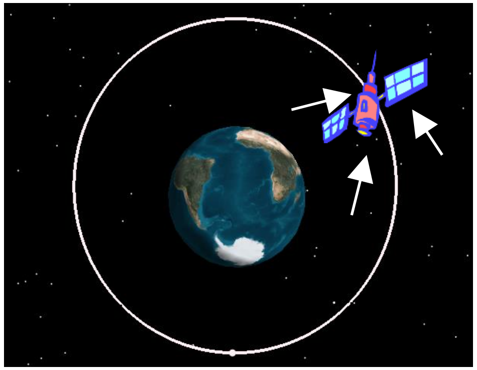
Motivation

Without maneuvers, satellites could not go beyond the close vicinity of Earth.

For instance, a GEO spacecraft is usually placed on a transfer orbit (LEO or GTO).

7. Orbital Maneuvers

Introduction

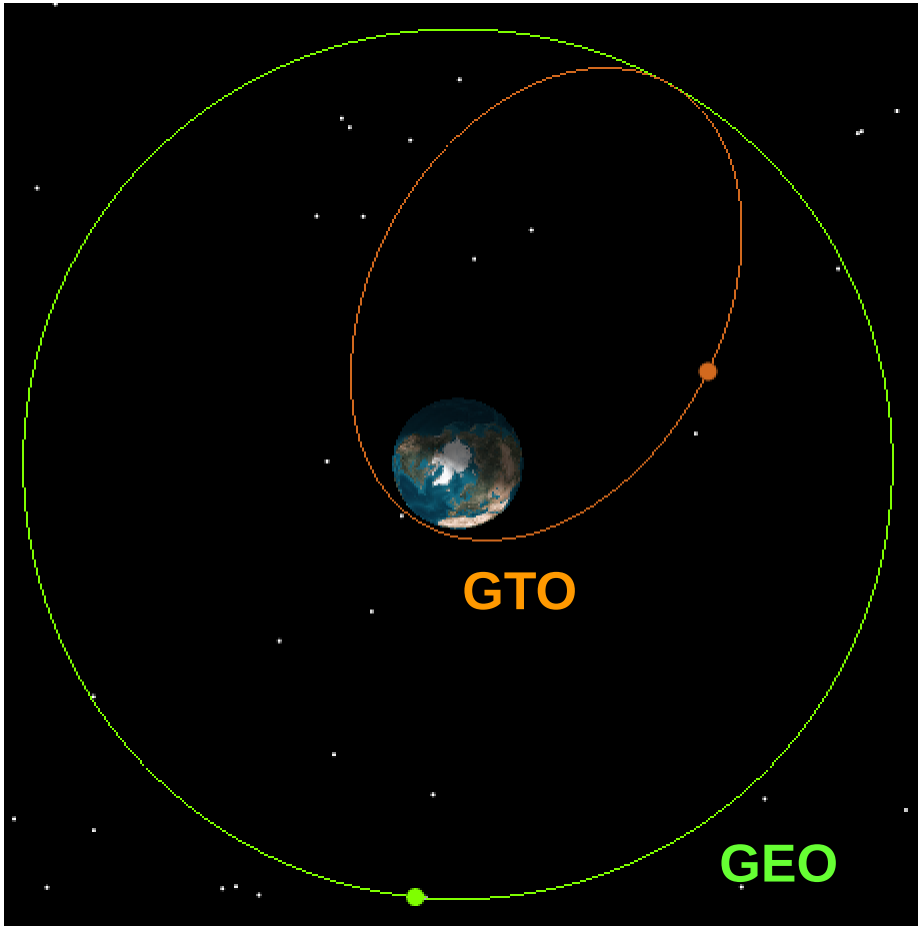


Coplanar maneuvers

Orbit Circularization

Ariane V is able to place heavy GEO satellites in GTO:

perigee: 200-650 km
apogee: ~35786 km.



GTO and GEO

For an orbit with a perigee at 320 km and an apogee at 35786 km, what is the velocity increment required to reach the geostationary orbit ?

GTO	GEO
$a = \dfrac{r_a + r_p}{2} = 24430\,\mathrm{km}$	$v_{circ} = \sqrt{\dfrac{398000}{35786 + 6378}}$
$v_p = 10.13\,\mathrm{km/s}$	$= 3.07\,\mathrm{km/s}$
$v_a = 1.61\,\mathrm{km/s}$	

Answer: 1.46 km/s
(apogee motor)

Orbit Raising: Reboost

ISS reboost due to atmospheric drag (ISS, Shuttle, Progress, ATV).

Orbit Raising: Evasive Maneuvers

ATV CARRIES OUT FIRST DEBRIS AVOIDANCE MANOEUVRE FOR THE ISS

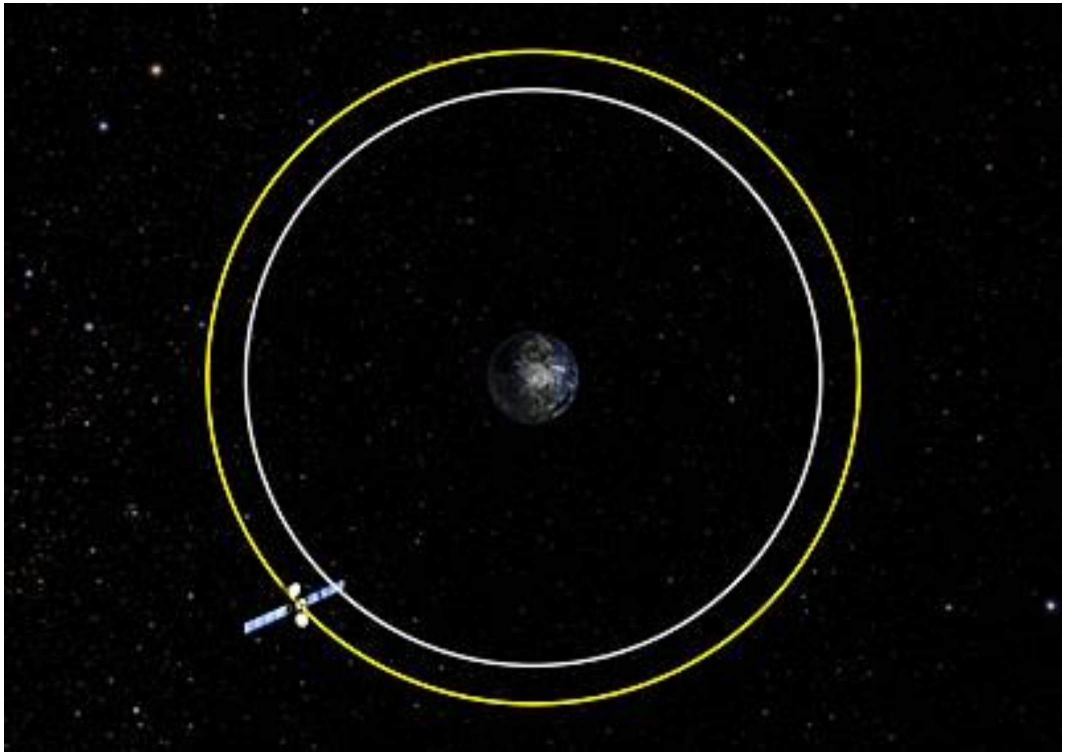
Jules Verne ATV is docked to the aft end of Zvezda

28 August 2008 The Automated Transfer Vehicle, Europe's ISS logistics spacecraft, was used to perform its first debris avoidance manoeuvre for the International Space Station. The manoeuvre was started yesterday at 18:11 CEST (16:11 UT) and finished 5 minutes 2 seconds later.

In the current Station configuration the Automated Transfer Vehicle(ATV), which is docked to the aft end of the Russian Zvezda Service module at the back of the Station, is the only vehicle that can carry out this kind of manoeuvre.

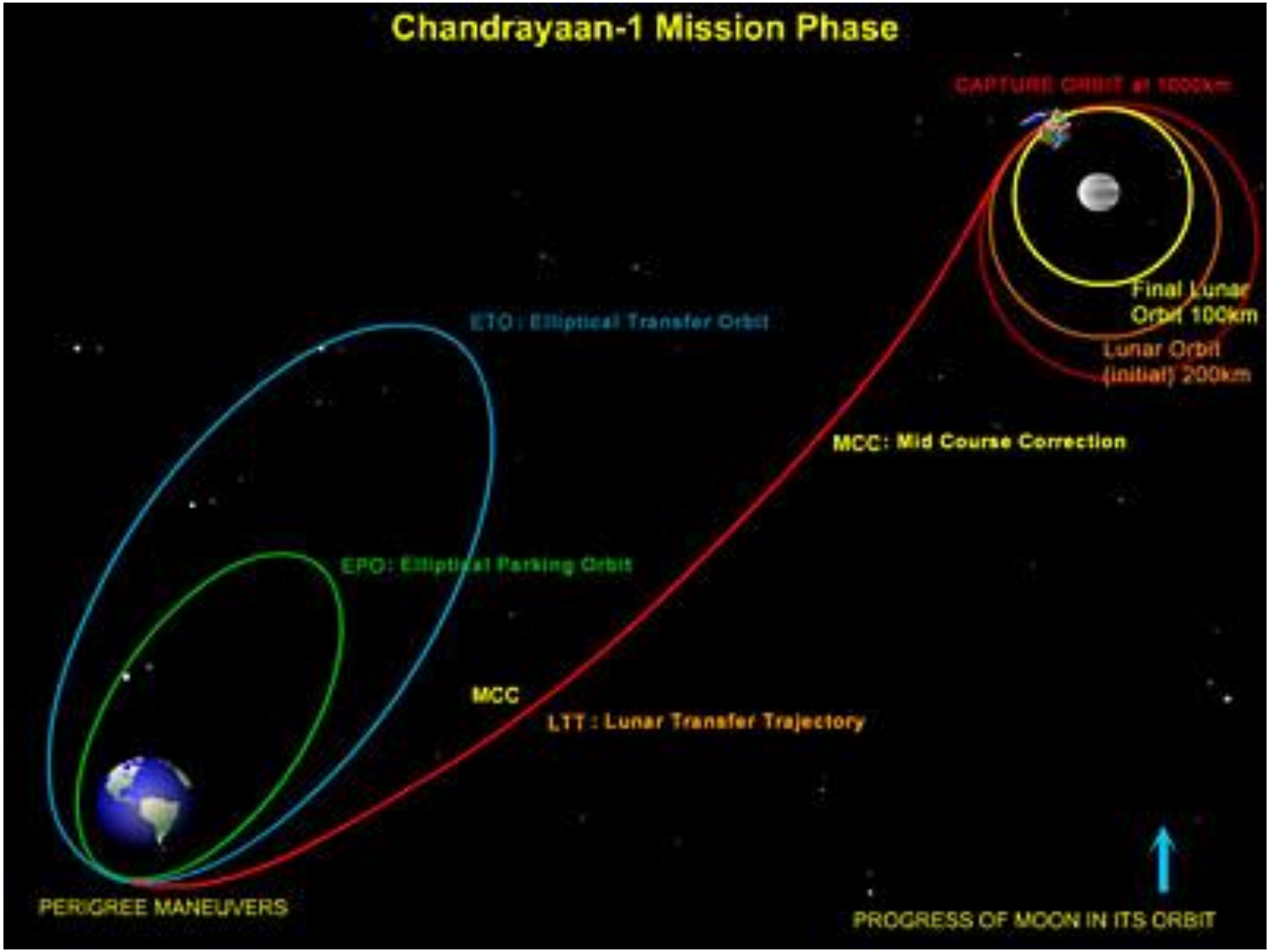
See also *www.esa.int/SPECIALS/Operations/SEM64X0SAKF_0.html*

Orbit Raising: Deorbiting GEO Satellites



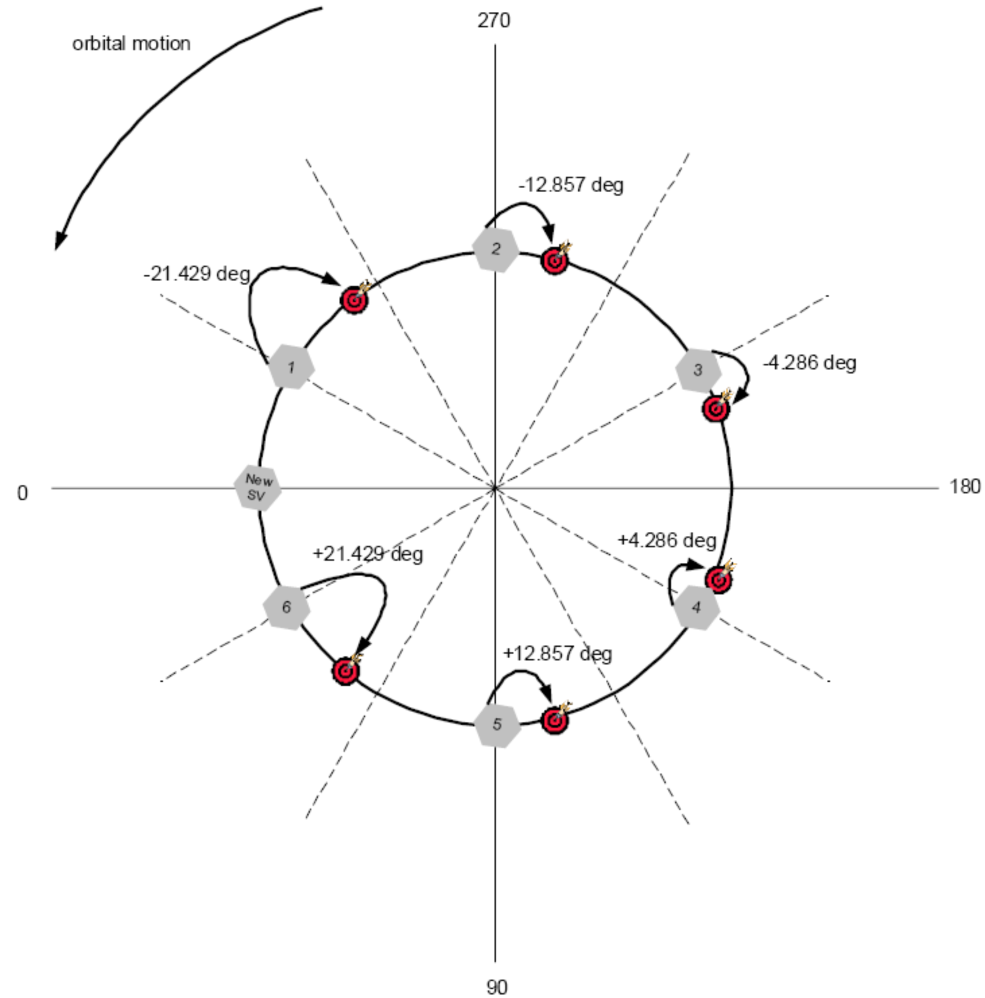
Graveyard orbit: to eliminate collision risk, satellites should be moved out of the GEO ring at the end of their mission. Their orbit should be raised by about 300 km to avoid future interference with active GEO spacecraft.

Orbit Lowering



Orbit Phasing

Replacement of a failed satellite of a constellation by an existing on-orbit spare.



Final Rendezvous

The crew of Gemini 6 took this photo of Gemini 7 when they were about 7 meters apart.

Plane Change

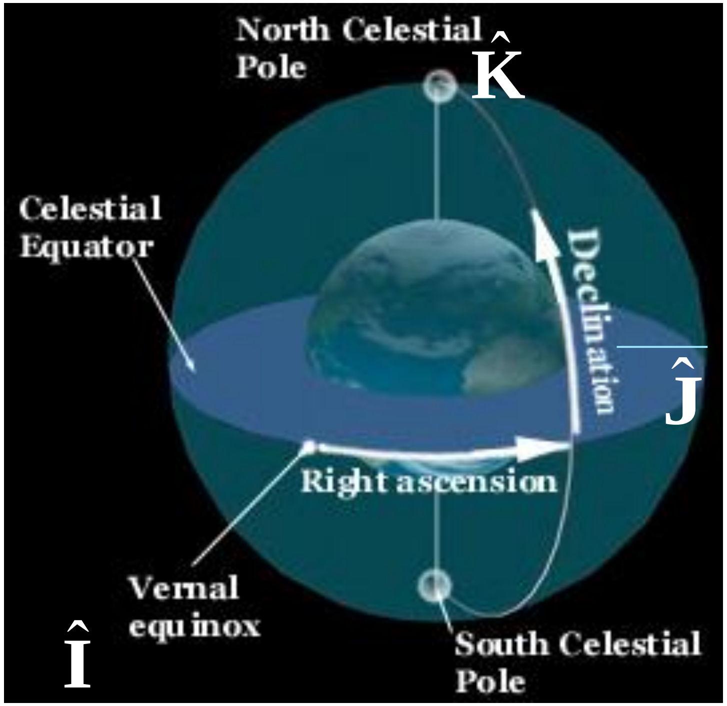
A launch site location restricts the initial orbit inclination for a satellite.

Which one is correct ? For a direct launch

1. *launch site latitude $\leq$ desired inclination.*

2. *launch site latitude $\geq$ desired inclination.*

Proof



$$i = \cos^{-1}\left(\frac{h_z}{h}\right) = \cos^{-1}\left(\frac{(\mathbf{r}\times\mathbf{v}).\hat{\mathrm{K}}}{\|\mathbf{r}\times\mathbf{v}\|}\right)$$

At the equator, horizontal velocity $\rightarrow \boldsymbol{r} \times \boldsymbol{v}$ is parallel to $\widehat{\boldsymbol{K}}$

At the equator, vertical velocity $\rightarrow \boldsymbol{r} \times \boldsymbol{v}$ is perpendicular to $\widehat{\boldsymbol{K}}$

At the pole, horizontal velocity $\rightarrow \boldsymbol{r} \times \boldsymbol{v}$ is perpendicular to $\widehat{\boldsymbol{K}}$

At the pole, vertical velocity $\rightarrow \boldsymbol{r} \times \boldsymbol{v}$ is zero

And Launch Errors !

2.5. Injection accuracy

The following table gives the typical standard deviation (1 sigma) for standard GTO and for SSO.

Standard GTO

a	semi-major axis (km)	40
e	eccentricity	$4.5\ 10^{-4}$
i	inclination (deg)	0.02
ωp	argument of perigee (deg)	0.2
Ω	ascending node (deg)	0.2

Leading to:

- standard deviation on apogee altitude 80 km
- standard deviation on perigee altitude 1.3 km

Ariane V User's Manual

And Launch Errors !

Due to a malfunction in Ariane V's upper stage, Artemis was injected into an abnormally low transfer orbit.

Artemis could still be placed, over a period of 18 months, into its intended operating position in GEO:

1. Several firings of the satellite's apogee kick motor raised the apogee and circularized the orbit at about 31000 km.

2. An unforeseen use of the ion engine was used to maneuver into GEO.

3. A final trim maneuver nudged Artemis into its originally intended trajectory.

Rocket Engines

Maneuvers are performed using firings of onboard rocket motors.

Chemical rocket engines:

Assumption of impulsive thrust in this lecture: because the burn times are small compared with the time intervals between burns, the thrust can be idealized as having infinitely small duration (no thrust included in the equation of motion).

Electric propulsion:

Not covered herein (continuous and low thrust).

Rocket Engines: Monopropellant

Astrium CHT 1 N:

Hydrazine

Burn life: 50h

Length: 17cm

Attitude and orbit control of small satellites and deep space probes.

Herschel, Globalstar

Astrium CHT 400 N:

Hydrazine

Burn life: 30m

Length: 32cm

Ariane V attitude control system

Rocket Engines: Bipropellant

Astrium S 10 N:

MMH (Fuel)
N2O4-MON1-MON3 (Oxidizers)

Attitude and orbit control of large satellites and deep space probes

Venus Express, Arabsat

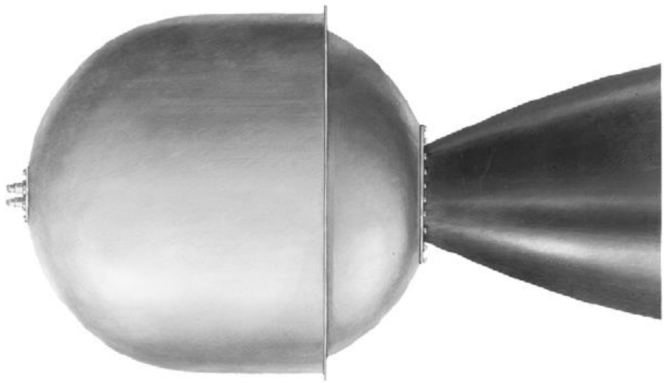
Astrium S 400 N:

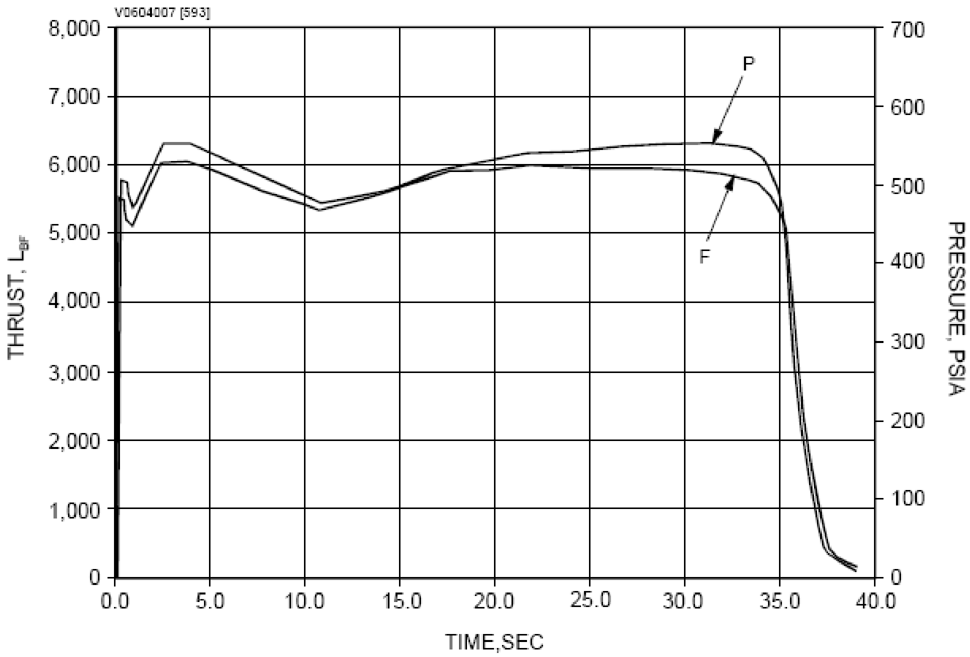
MMH (Fuel),
N2O4-MON1-MON3 (Oxidizers)

For apogee orbit injection of GEO satellites and for planetary orbit maneuvers of of deep space probes

Venus Express, Artemis

Rocket Engines: Solid





ATK Star 27 (TE-M-616) 27 kN:

Burn time: 34s

Length: 1.3m

Gross mass: 361 kg

Apogee motor (GOES,GPS)

Rocket Engines: Low-Thrust

Astrium RITA 150 mN:

Xenon

Beam voltage: 1200V

Burn time: >20000h

Gross mass: 154 kg

Stationkeeping, orbit transfer, deep space trajectories

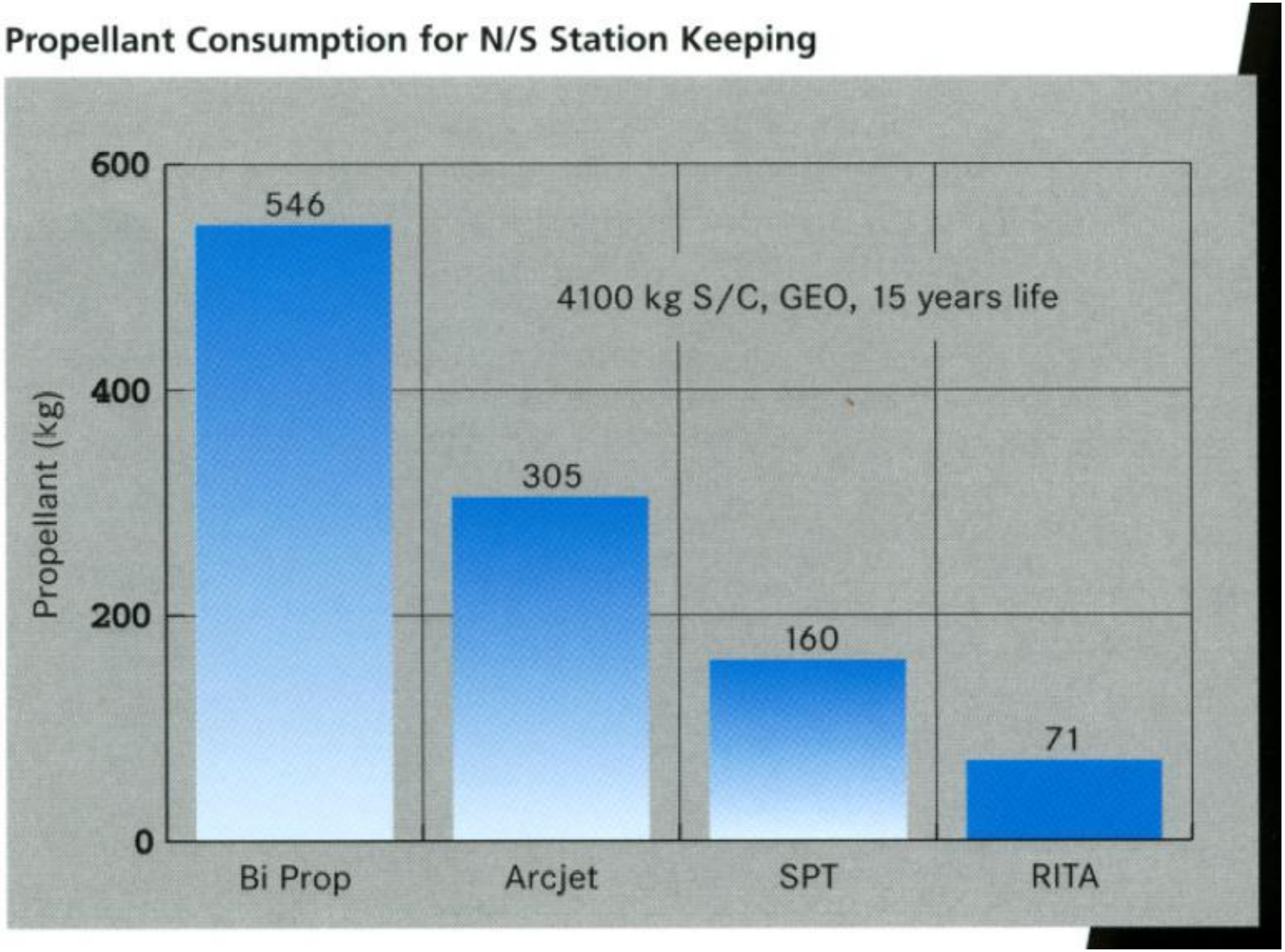
RITA-10 (Artemis)

Specific Impulse, Isp

It is a measure of the performance of a propulsion system.

Astrium CHT 1N: Astrium CHT 400N:	210s 220s	*Monopropellant*
Astrium S 10N: Astrium S 400N:	291s 318s.	*Bipropellant*
ATK STAR 27:	288s	*Solid*
Astrium RITA-150:	3000-5000s	*Electric*
[Cold gas: Liquid oxygen/liquid hydrogen	~50s 455s	]

Rocket Engines: Isp



RITA, Astrium – The Ion Propulsion System for the Future

Goal: Efficiency

Use a minimum amount of fuel.

Do not take too much time.

Delta-V

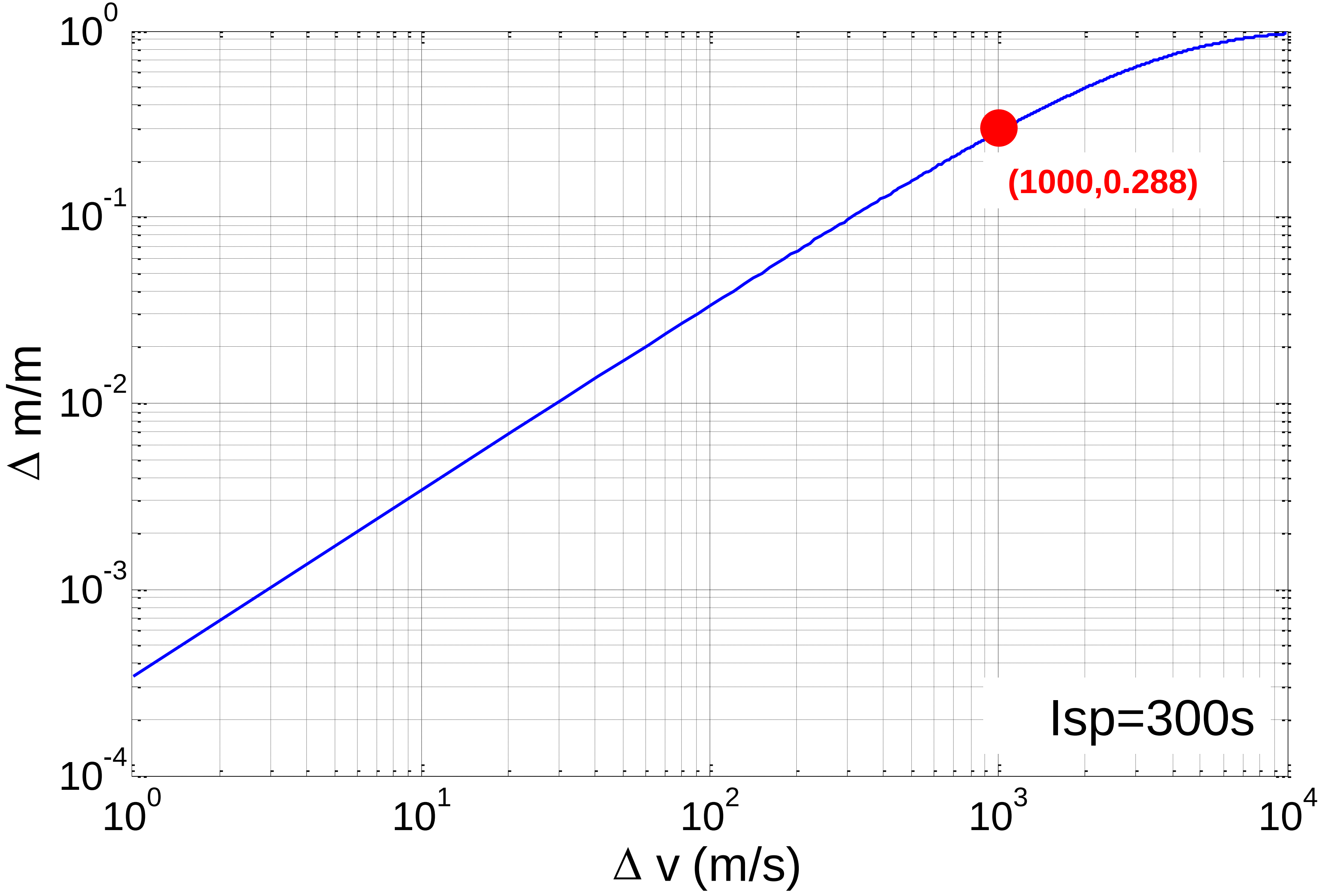
Each impulsive maneuver results in a change Δv, an indicator of how much propellant will be required.

$$\Delta v = \int_{t_o}^{t_1} \frac{|T(t)|}{m(t)} dt = \int_{t_o}^{t_1} \frac{I_{sp} g_0 |\dot{m}|}{m} dt = -I_{sp} g_0 \int_{t_o}^{t_1} \frac{dm}{m}$$

$$= -I_{sp} g_0 \ln \frac{m_1}{m_0} = -I_{sp} g_0 \ln \frac{m_0 - \Delta m}{m_0}$$

$$\boxed{\frac{\Delta m}{m_0} = 1 - e^{\frac{-\Delta v}{I_{sp} g_0}}}$$

Delta-V



(1000,0.288)
Isp=300s
Δ m/m
Δ v (m/s)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^0
10^1
10^2
10^3
10^4

Delta-V: Examples

Maneuver	Average Δv per year [m/s]	
Drag compensation (400–500 km)	<25	
Drag compensation (500–600 km)	< 5	
Stationkeeping GEO	50 – 55	(~90% N/S, ~10% E/W)
GTO $\Rightarrow$ GEO	1460	

Attitude control (3-axis)	2 – 6
First cosmic velocity	7900
Second cosmic velocity	11200
Space Ship One	1400

Delta-V Budget

It is the sum of the velocity changes required throughout the space mission life.

It is a good starting point for early design decisions. As important as power and mass budgets.

In some cases, it may become a principal design driver and impose complex trajectories to deep space probes (see Lecture 6) !

Delta-V Budget: GEO

Mission orbit	Geostationary	(Allowable deviation from nominal position 0,1 deg)
Launcher	Proton	
Launch in GTO		
Mission duration (yrs)	15	

Maneuvre	delta v/maneuvre (m/s)	cycle time (days)	no. of maneuvers (-)	delta v/yr (m/s)	total delta V (m/s)
Apogee kick	1836,49	*	1,0	*	1836,5
10 yr average NSSK	10,73	86,1	63,6	45,5	682,0
Worst Case NSSK	10,90	77,4	70,7	51,4	770,7
EWSK	0,13	35,3	155,3	1,33	19,9
Worst Case EWSK	NA	NA	NA	1,74	26,1
Orbit Maneuvres	0,00	*	0,0	*	0,0
Disposal	10,88	*	1,0	*	10,9
		Total Delta V (most favourable)			2549,3
		Total Delta V (worst case EWSK)			2555,5
		Total Delta V (worst case NSSK & EWSK)			2644,2

Time

Time is another key parameter, especially for manned missions.

Rendez-vous between the Space Shuttle and ISS cannot take more than a few days.

First Orbital Maneuvers

January 2, 1959, Luna 1:

The spacecraft missed the Moon by about 6000 km. But coming even this close required several maneuvers, including circularizing the initial launch orbit and doing midcourse corrections.

September 12, 1959, Luna 2:

Intentional crash into the lunar surface.

First Maneuvers for Manned Spacecraft

March 23, 1965, Gemini 3:

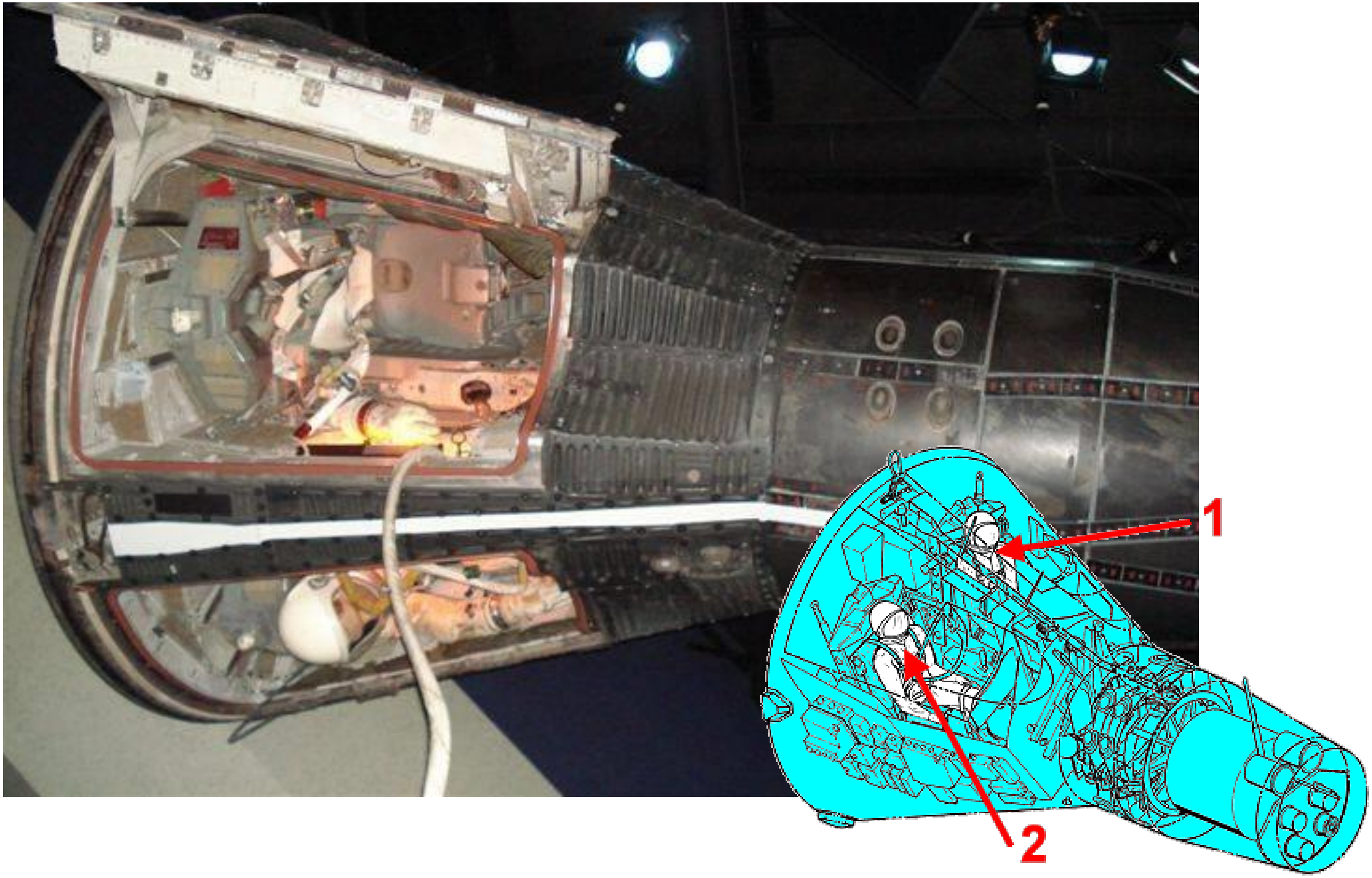
A 74s burn gave a ΔV of 15.5 meters per second. The orbit was changed from 161.2 km x 224.2 km to an orbit of 158 km x 169 km.

December 12, 1965: Gemini 6 and 7:

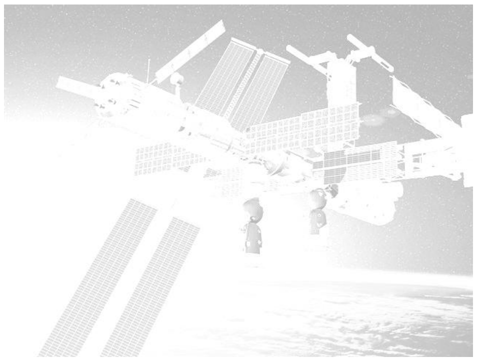
First rendezvous. The two Gemini capsules flew around each other, coming within a foot (0.3 meter) of each other but never touching.

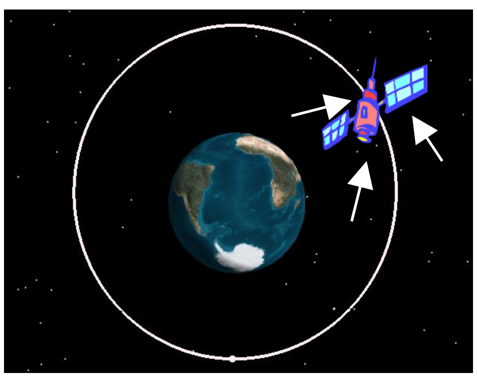
Gemini Program

Gemini Capsule



7. Orbital Maneuvers





Coplanar maneuvers

- One-impulse transfer
- Two-impulse transfer
- Three-impulse transfer
- Nontangential burns
- Phasing maneuvers

Perturbation Equations (Gauss)

$$\dot{\Omega} = \sqrt{\frac{a(1-e^2)}{\mu}} \frac{N \sin\theta_2}{\sin i\left(1+e\cos\theta\right)} \qquad \dot{a} = 2\sqrt{\frac{a^3}{\mu\left(1-e^2\right)}}\left[\mathrm{Re}\sin\theta + T\left(1+e\cos\theta\right)\right]$$

$$\dot{i} = \sqrt{\frac{a(1-e^2)}{\mu}} \frac{N\cos\theta_2}{\left(1+e\cos\theta\right)} \qquad \dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}}\left[R\sin\theta + T\left(\cos\theta + \cos E\right)\right]$$

$$\dot{\omega} = -\dot{\Omega}\cos i + \frac{1}{e}\sqrt{\frac{a(1-e^2)}{\mu}}\left[-R\cos\theta + \frac{T\sin\theta\left(2+e\cos\theta\right)}{1+e\cos\theta}\right]$$

$$M = nt - \chi, \text{with } \dot{\chi} = \sqrt{\frac{a}{\mu}} \frac{(1-e^2)\left[R\left(2e-\cos\theta - e\cos^2\theta\right) + T\sin\theta\left(2+e\cos\theta\right)\right]}{e\left(1+e\cos\theta\right)}$$

J.E. Prussing, B.A. Conway, *Orbital Mechanics*, Oxford University Press

Different Types of Maneuvers

Coplanar / noncoplanar:

$\Rightarrow$ coplanar maneuvers can change a, e, ω, θ.

WIDE APPLICABILITY !

Tangential / nontangential:

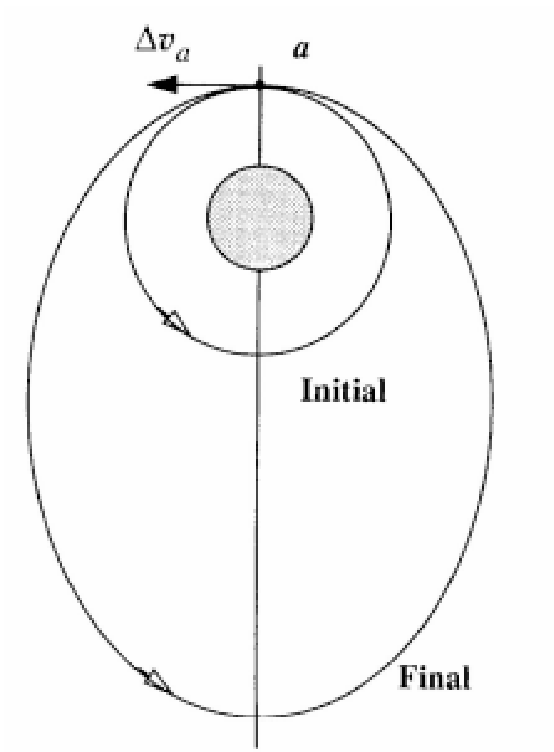
$\Rightarrow$ tangential burns occur only at apoapsis and periapsis or on circular orbit.

Impulsive / continuous:

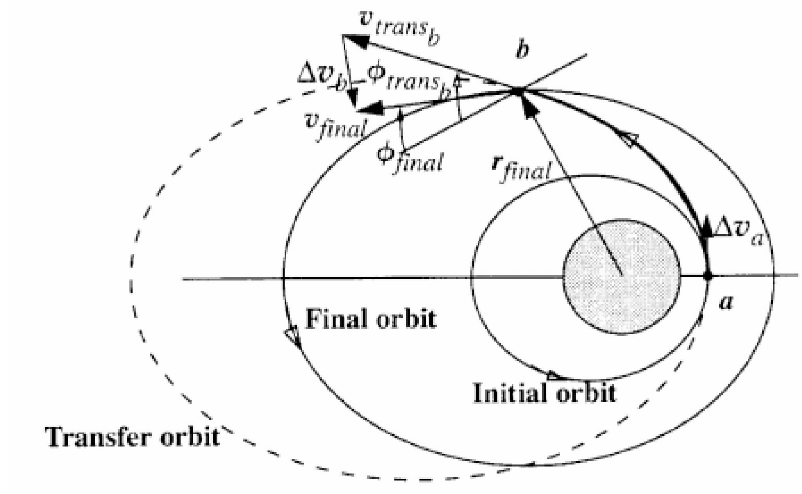
$\Rightarrow$ an impulsive maneuver corresponds to an instantaneous burn.

One-, two-, and three-impulse transfers:

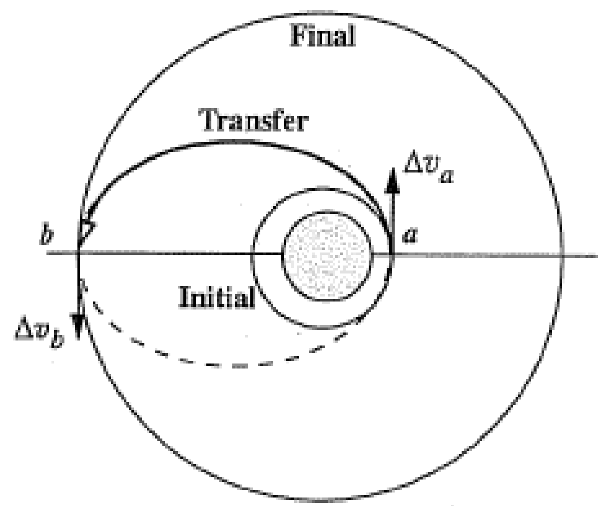
$\Rightarrow$ different purposes and efficiency.



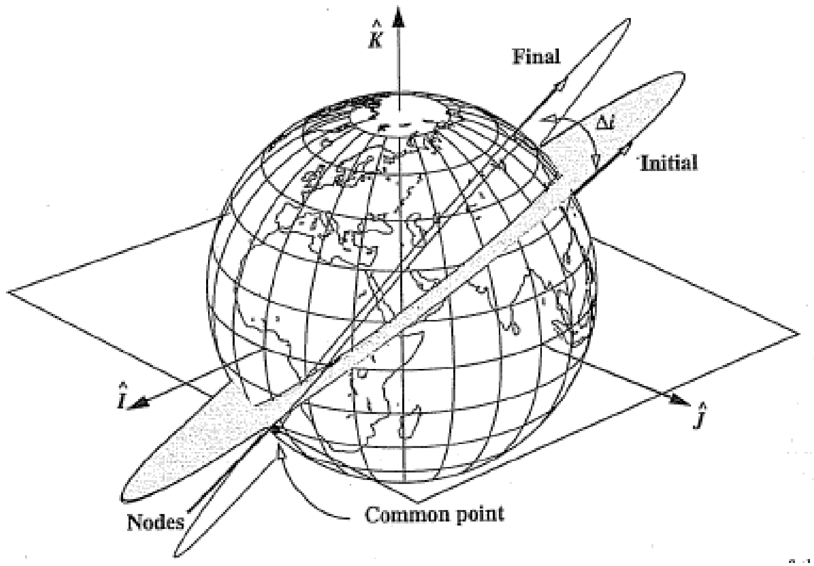
One-impulse burn (tangential, coplanar, impulsive)



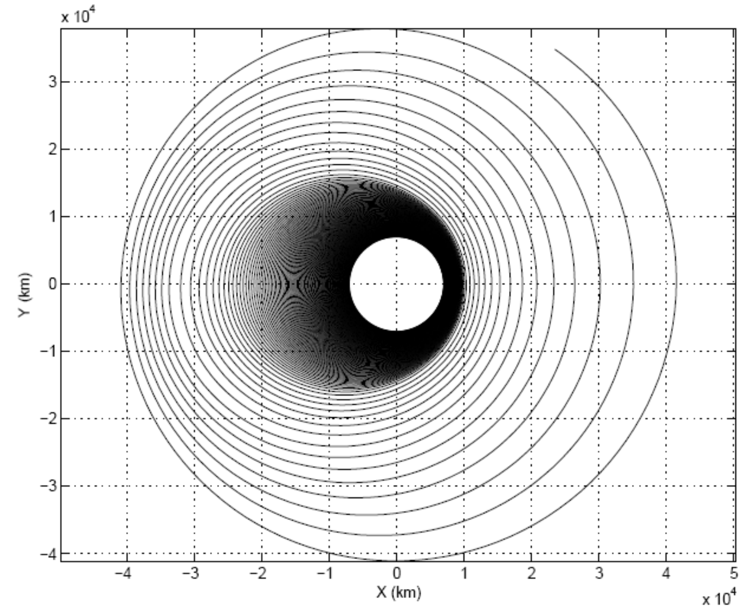
Two-impulse burn (nontangential, coplanar, impulsive)



Two-impulse burn (tangential, coplanar, impulsive)



One-impulse burn
(nontangential, noncoplanar,
impulsive)

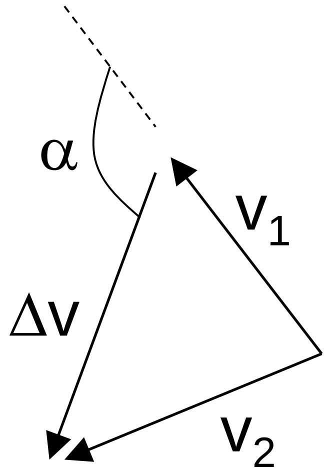


Continuous burn (low-
thrust orbit transfer)

Modifying the Semi-Major Axis

$$\frac{\mathrm{v}_1^2}{2}-\frac{\mu}{r_1}=\frac{-\mu}{2a_1},\quad \frac{\mathrm{v}_2^2}{2}-\frac{\mu}{r_2}=\frac{-\mu}{2a_2}$$

$$\mathbf{r}_1=\mathbf{r}_2,\ \mathbf{v}_2=\mathbf{v}_1+\mathbf{\Delta v},\ v_2^2=v_1^2+\Delta v^2+2v_1\Delta v\cos\alpha$$



$$\boxed{\frac{\mu}{2a_2}+\frac{-\mu}{2a_1}}=\frac{\mathrm{v}_1^2}{2}-\frac{\mathrm{v}_2^2}{2}=-\frac{1}{2}\left(\Delta v^2+2v_1\Delta v\cos\alpha\right)$$

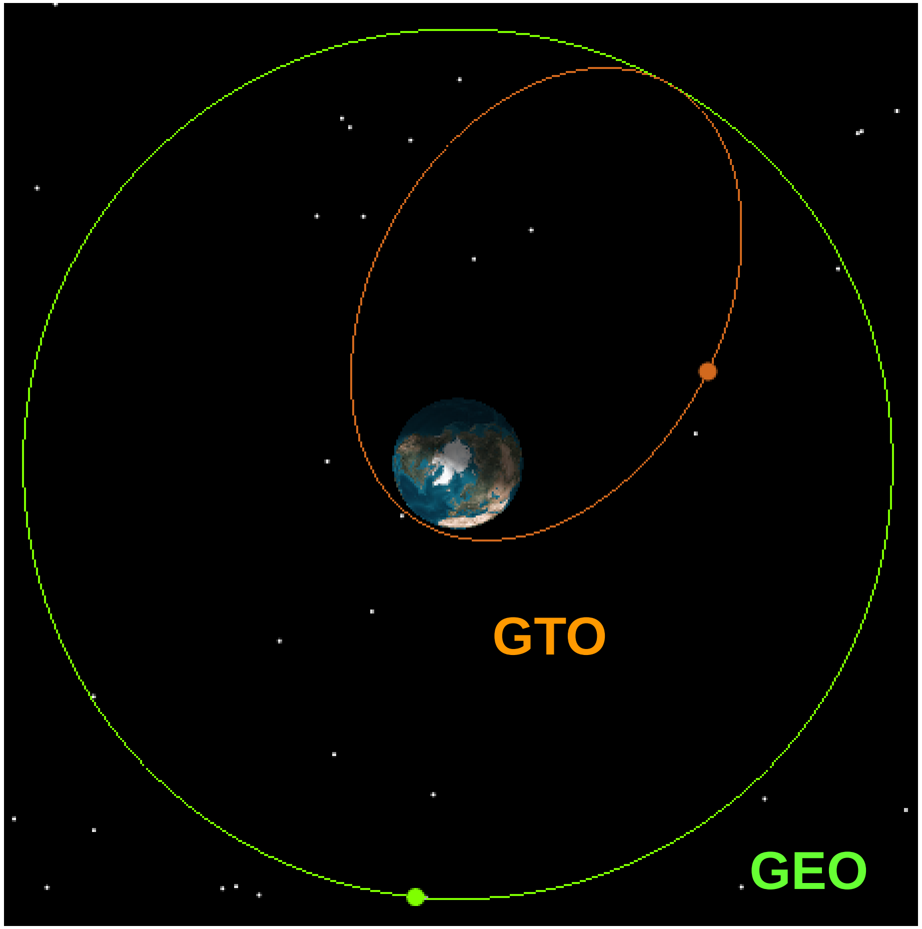
Fixed quantity

Δv minimum if α=0,v$_1$=max(v$_1$).

To get the most efficient burn, do the maneuver as close to perigee as possible in a direction collinear to the velocity.

From GTO to GEO

The impulse is necessarily applied at the apogee of the GTP, because we want to circularize the orbit.



The maneuver at apogee is in fact a combination of two maneuvers. Why ?

Hohmann Transfer

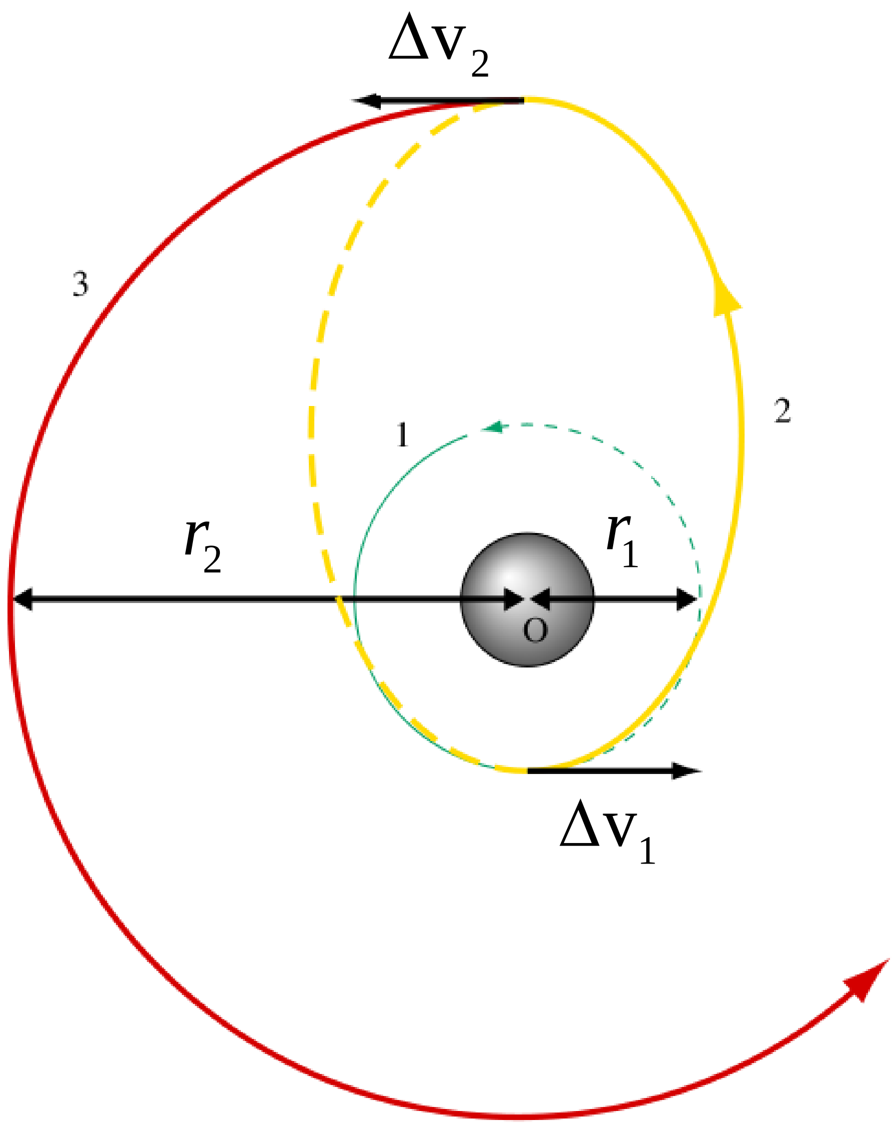
The transfer between two coplanar circular orbits requires at least two impulses Δv_1 and Δv_2.

In 1925, Walter Hohmann conjectured that

The minimum-fuel impulsive transfer orbit is the elliptic orbit that is tangent to both orbits at its apse line.

The rigorous demonstration came some 40 years later !

Governing Equations



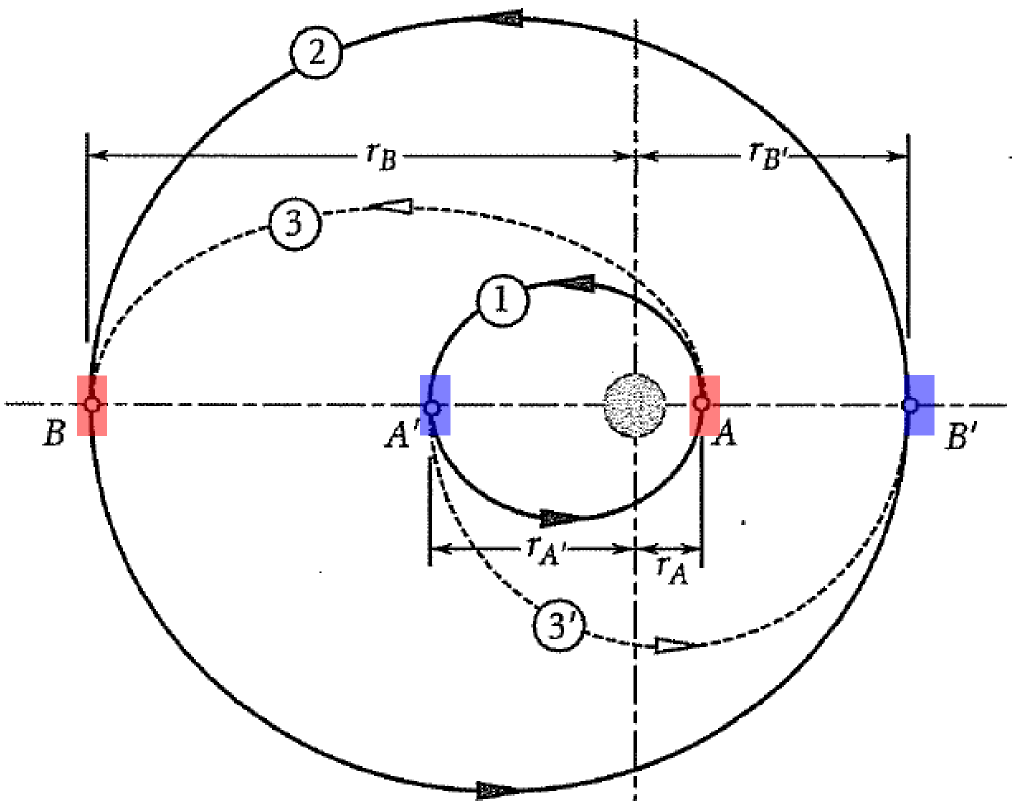
$$v_{circ} = \sqrt{\frac{\mu}{r}} \quad v_{ellip} = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$\Delta v_1 = \sqrt{\frac{2\mu r_2}{r_1\left(r_1 + r_2\right)}} - \sqrt{\frac{\mu}{r_1}}$$

$$\Delta v_2 = -\sqrt{\frac{2\mu r_1}{r_2\left(r_1 + r_2\right)}} + \sqrt{\frac{\mu}{r_2}}$$

Hohmann Transfer — Elliptical Orbits

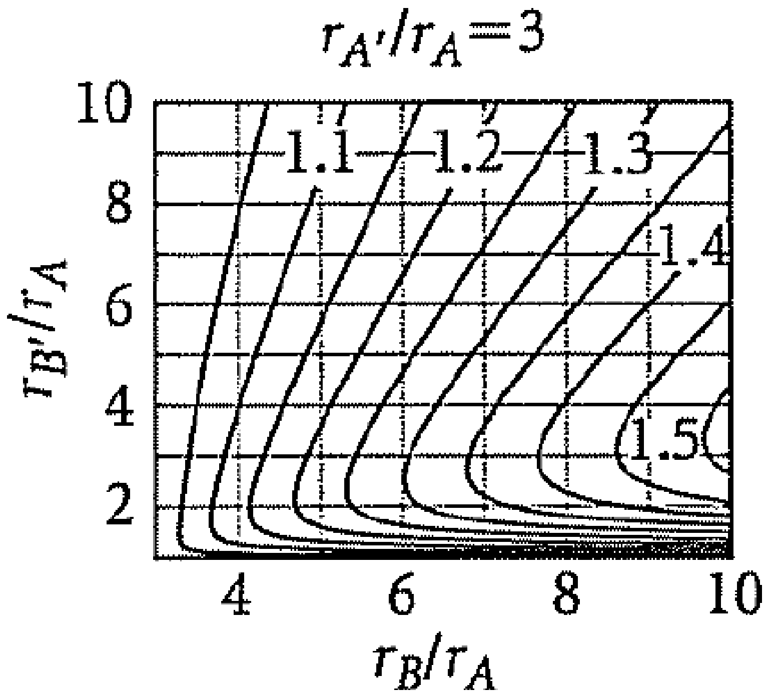
The transfer orbit between elliptic orbits with the same apse line must be tangent to both ellipses. But there are two such transfer orbits. Which one should we favor ?

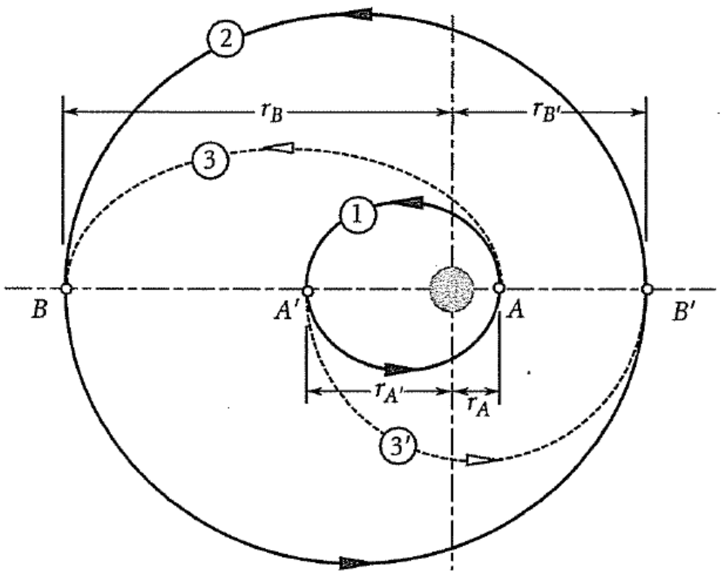


H. Curtis, *Orbital Mechanics for Engineering Students*, Elsevier.

Graphs of $\Delta v_{3'} / \Delta v_3$

The most efficient transfer is 3: it begins at the perigee on the inner orbit 1, where the kinetic energy is greatest, regardless of the shape of the outer target orbit.



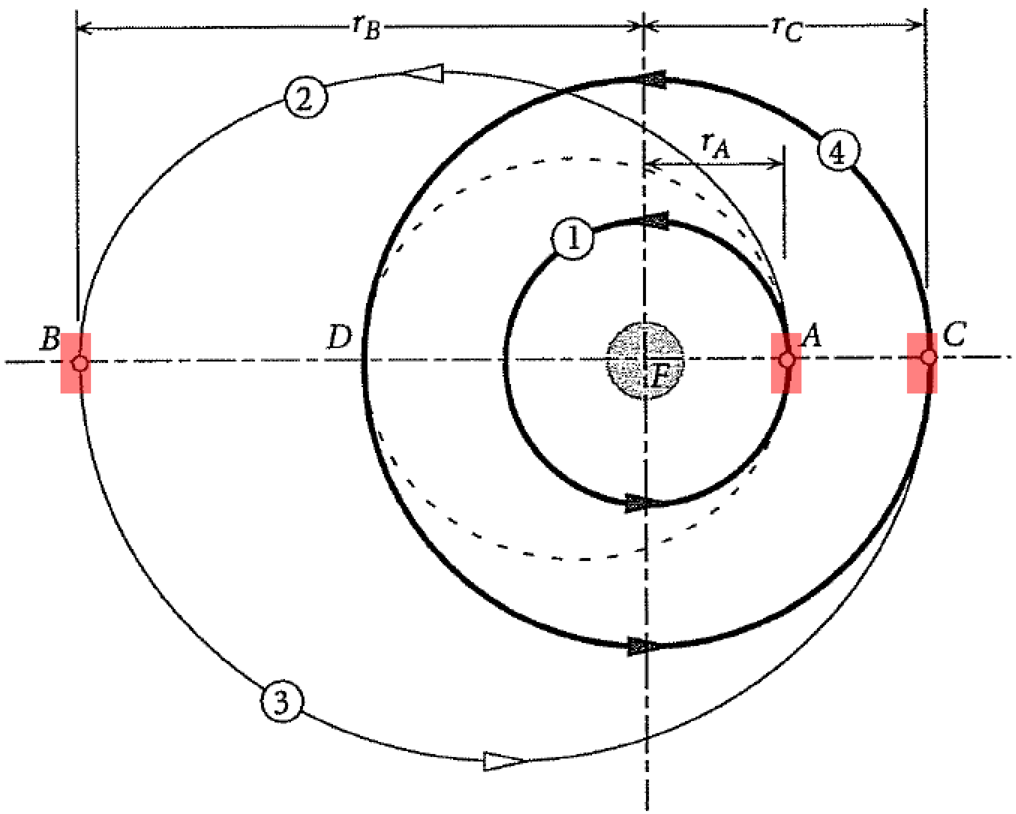


Inner elliptic orbit (A is the perigee)
outer elliptic orbit

Bi-Elliptic Transfer — Why ?

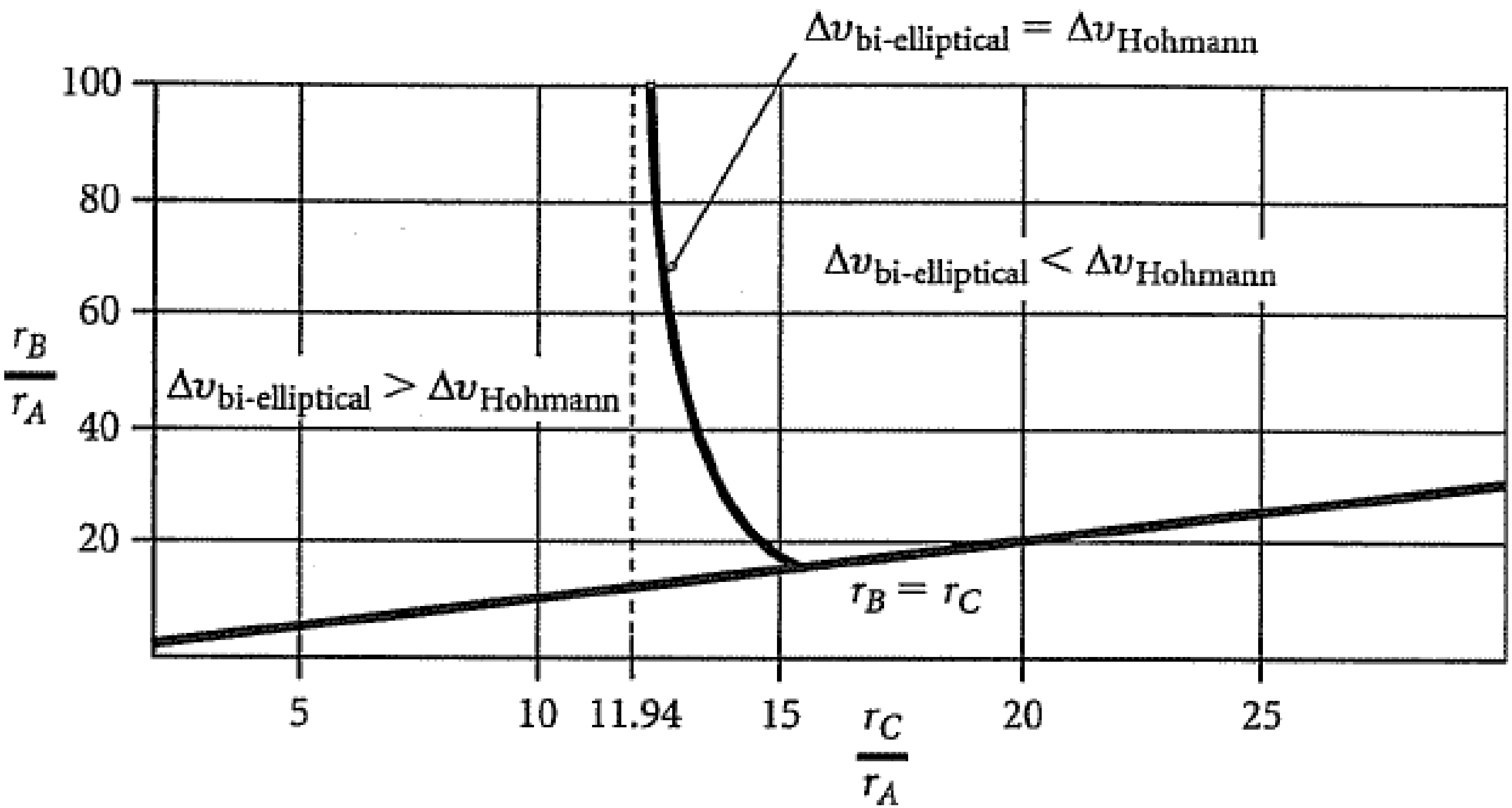
It is composed of two ellipses, separated by a midcourse tangential impulse (i.e., two Hohmann transfers in series).

A limiting case is the biparabolic transfer ($r_B \rightarrow \infty$).



H. Curtis, *Orbital Mechanics for Engineering Students*, Elsevier.

Two or Three-Impulse Transfer ?



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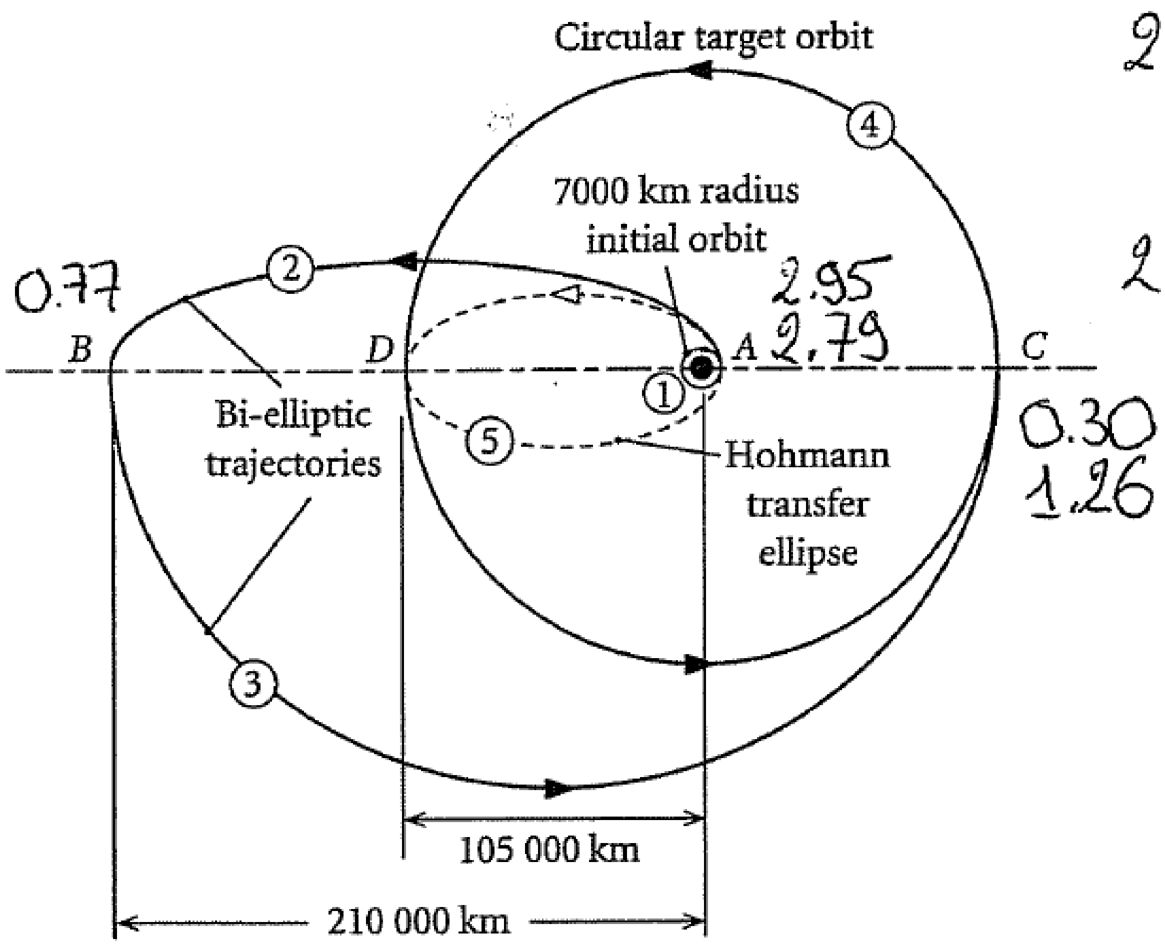
Two or Three-Impulse Transfer ?



2.95 + 0.77 + 0.30
= 4.02 km/s (5.66 jours)

2.79 + 1.26
= 4.05 km/s (0.76 jours)

H. Curtis, *Orbital Mechanics for Engineering Students*, Elsevier.

Two- or Three-Impulse Transfer ?

It depends on the ratio of the radii of the inner and outer orbits (threshold: $r_C / r_A = 11.94$).

For many practical applications (LEO to GEO), the two-impulse transfer is more economical. It is also the case for interplanetary transfers from Earth to all planets except the outermost three.

What is another important parameter to choose between two- or three- impulse transfer ?

Time of flight ! For instance, the bi-parabolic transfer requires an infinite transfer time.

Tangential Burns or Not ?

The major drawback to the Hohmann transfer is the long flight time.

Time of flight can be reduced at the expense of an acceptable increase in Δv.

A possible solution is a *one-tangent burn.* It comprises one tangential burn and one nontangential burn.

Tangential Burns or Not ?

	Initial Alt (km)	Final Alt (km)	ν_{trans_b}	Bi-elliptic Transfer Alt (km)	Δv (km/s)	τ_{trans} (h)
			Transfer to Geosynchronous			
Hohmann	191.344 11	35,781.35			3.935	5.256
One-tangent	191.344 11	35,781.35	160°		4.699	3.457
Bi-elliptic	191.344 11	35,781.35		47,836.00	4.076	21.944
			Transfer to the Moon			
Hohmann	191.344 11	376,310.00			3.966	118.683
One-tangent	191.344 11	376,310.00	175°		4.099	83.061
Bi-elliptic	191.344 11	376,310.00		503,873.00	3.904	593.919

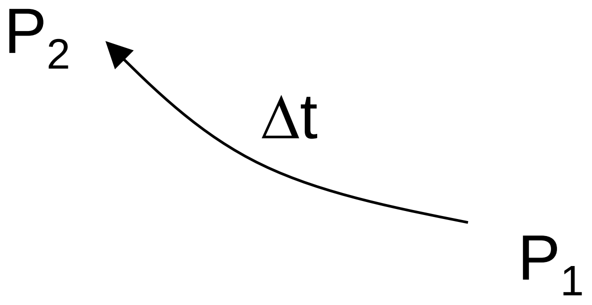
Vallado, *Fundamental of Astrodynamics and Applications*, Kluwer, 2001.

Two Nontangential Burns ?

Solve **Lambert's problem**: it gives a relationship between two positions of a spacecraft in an elliptical orbit and the time taken to traverse them:

The time required to traverse an elliptic arc between specified endpoints depends only on the semimajor axis, the chord length and the sum of the radii from the focus to the two points. It does not depend on eccentricity.

If two position vectors and the time of flight are known, then the orbit can be fully determined.



Lambert's Problem: Matlab Example

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Command Window

New to MATLAB? Watch this Video, see Demos, or read Getting Started.

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 Input data:

 Gravitational parameter (km^3/s^2) = 398600

 r1 (km) = [5000 10000 2100]
 r2 (km) = [-14600 2500 7000]
 Elapsed time (s) = 3600

 Solution:

 v1 (km/s) = [-5.99249 1.92536 3.24564]
 v2 (km/s) = [-3.31246 -4.19662 -0.385288]

 Orbital elements:
 Angular momentum (km^2/s) = 80466.8
 Eccentricity = 0.433488
 Inclination (deg) = 30.191
 RA of ascending node (deg) = 44.6002
 Argument of perigee (deg) = 30.7062
 True anomaly initial (deg) = 350.83
 True anomaly final (deg) = 91.1223
 Semimajor axis (km) = 20002.9
 Periapse radius (km) = 11331.9
 Period:
 Seconds = 28154.7
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>>
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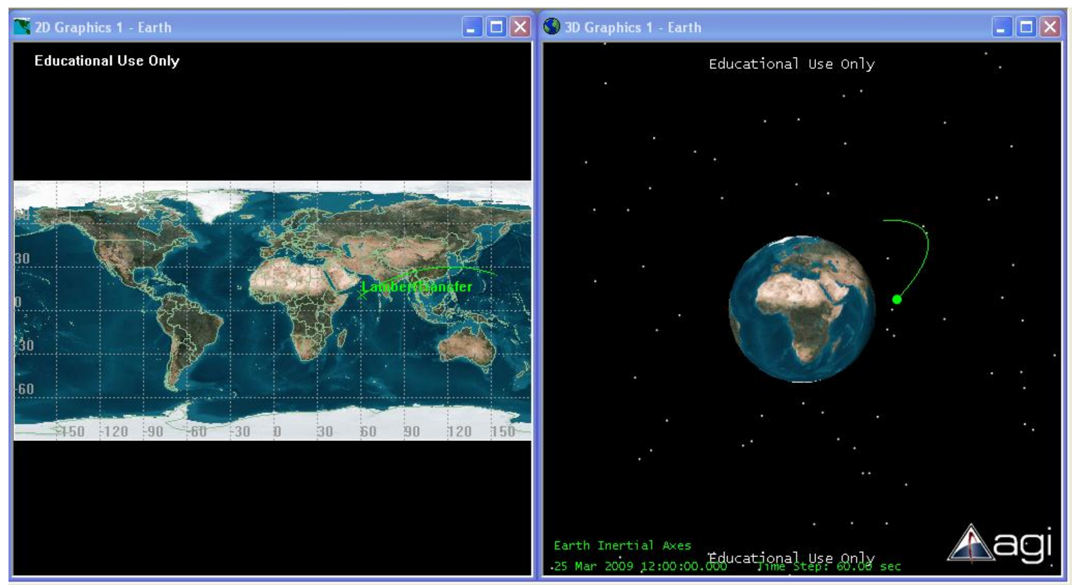
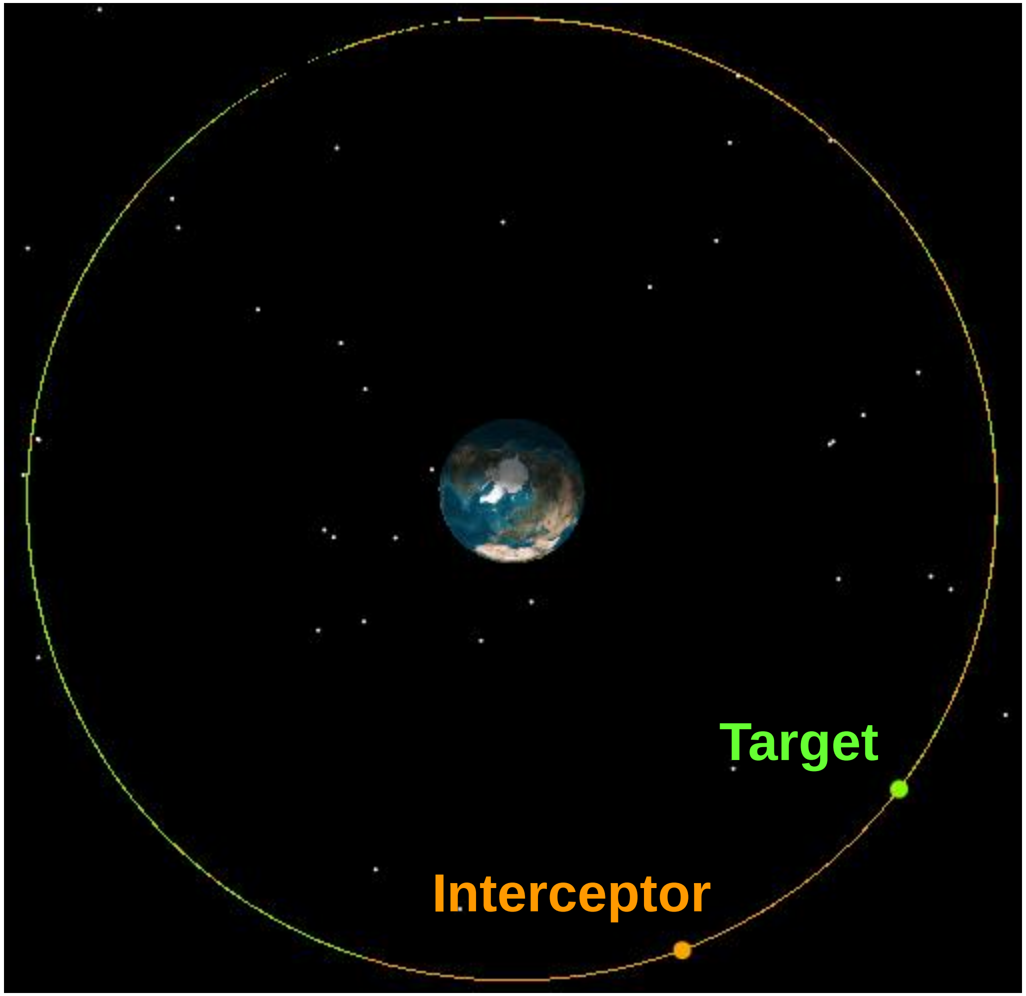


Phasing Maneuvers

Can we apply a tangential burn to intercept a target ?



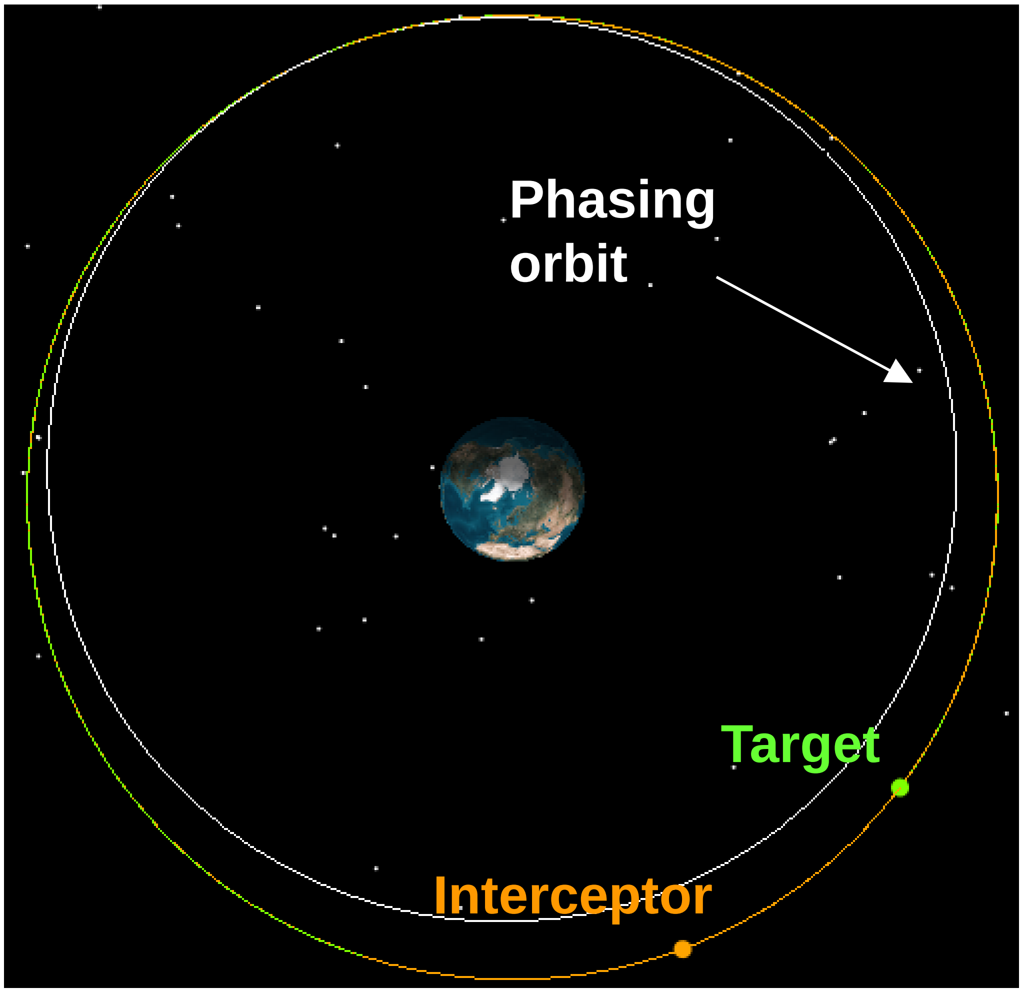
No !

Imagine that you take a bend with your car and that you want to catch the car in front of you…

Yes !

Can we exploit Hohmann transfer in a clever manner ?

GEO Repositioning



Δv=0.2 km/s for a longitude shift of 32º in one revolution.

Phasing Maneuver

It can take the form of a two-impulse Hohmann transfer from and back to the same orbit.

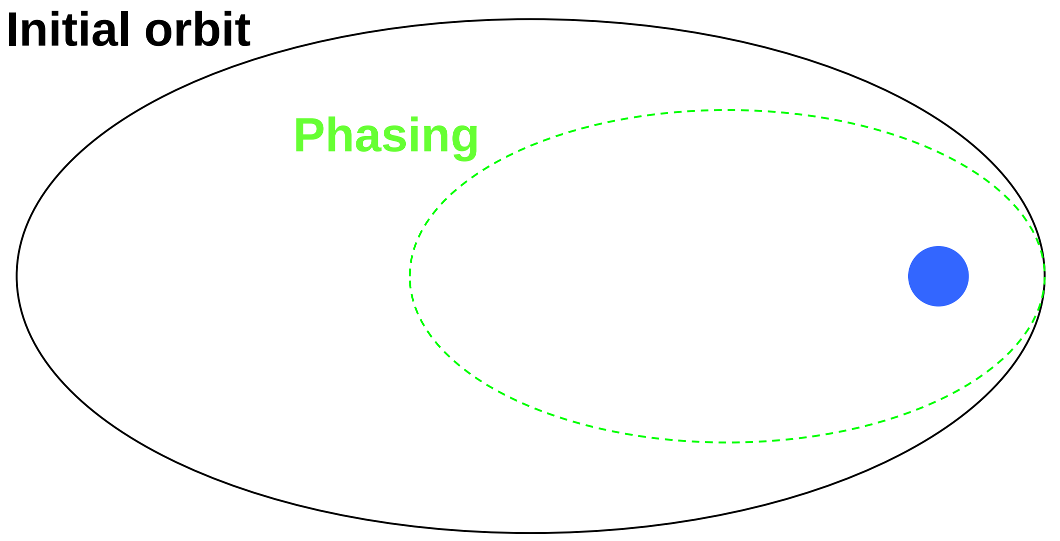
The target can be ahead or behind the chase vehicle.

Usefulness:

1. Constellation (deployment or replacement of a failed satellite)
2. GEO
3. First phase of a rendezvous procedure

Phasing Maneuver Design

$$\Delta\theta \rightarrow \Delta v$$



Phasing Maneuver Design

$$\Delta\theta \rightarrow \Delta t$$

$$T_{\text{phasing orbit}} = 2\pi\sqrt{\frac{a^3}{\mu}}$$ Lecture 2

$$a_{\text{phasing orbit}}$$

$$a = \frac{r_p + r_a}{2}, r_p \text{ is known}$$

$$r_a$$

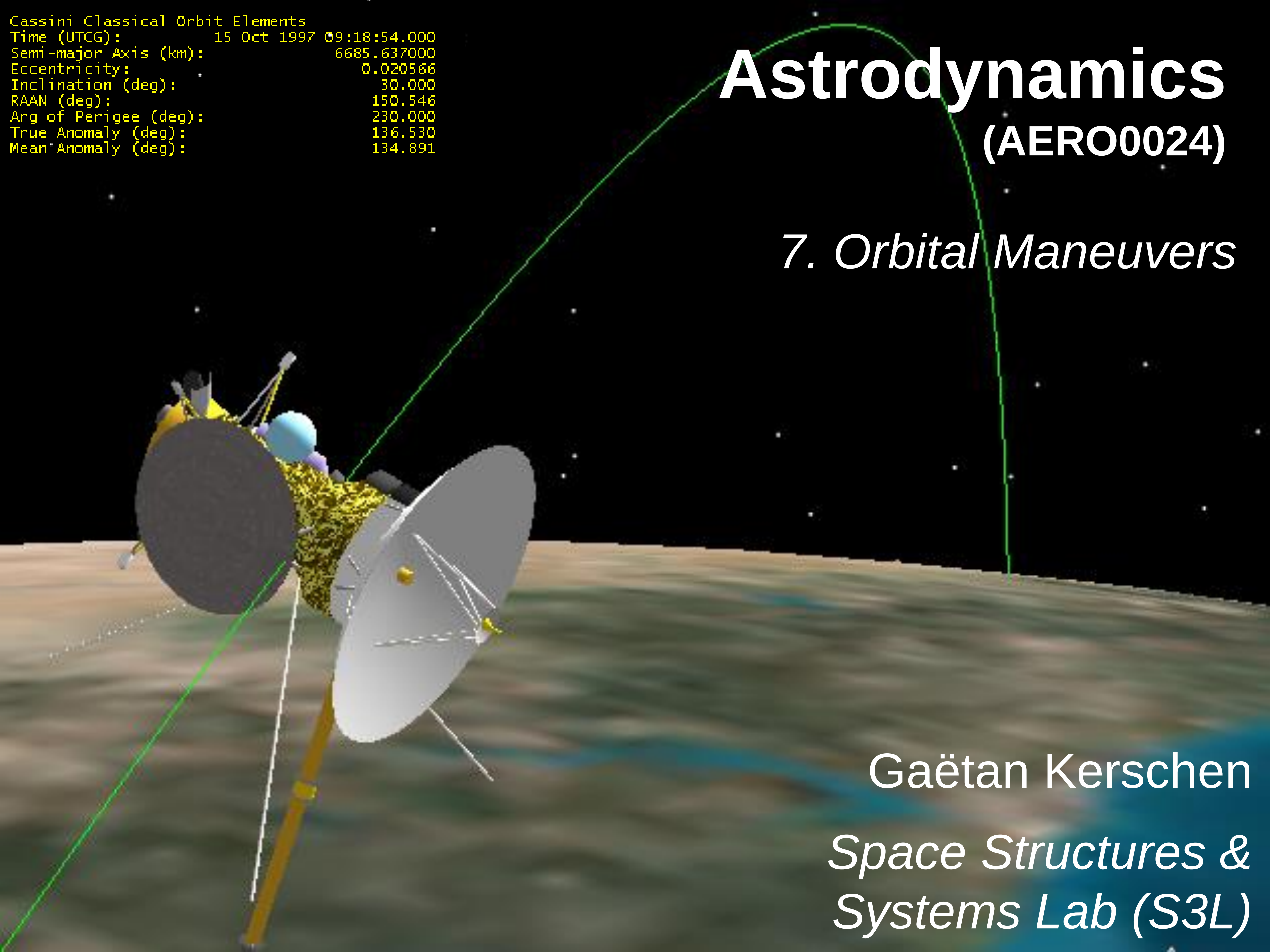
$$e = \frac{r_a - r_p}{r_a + r_p}$$

$$e$$

$$r_p = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}, v_p = \frac{h}{r_p}$$

$$\Delta v$$

$$h, v_P$$



Astrodynamics
(AERO0024)

7. Orbital Maneuvers

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