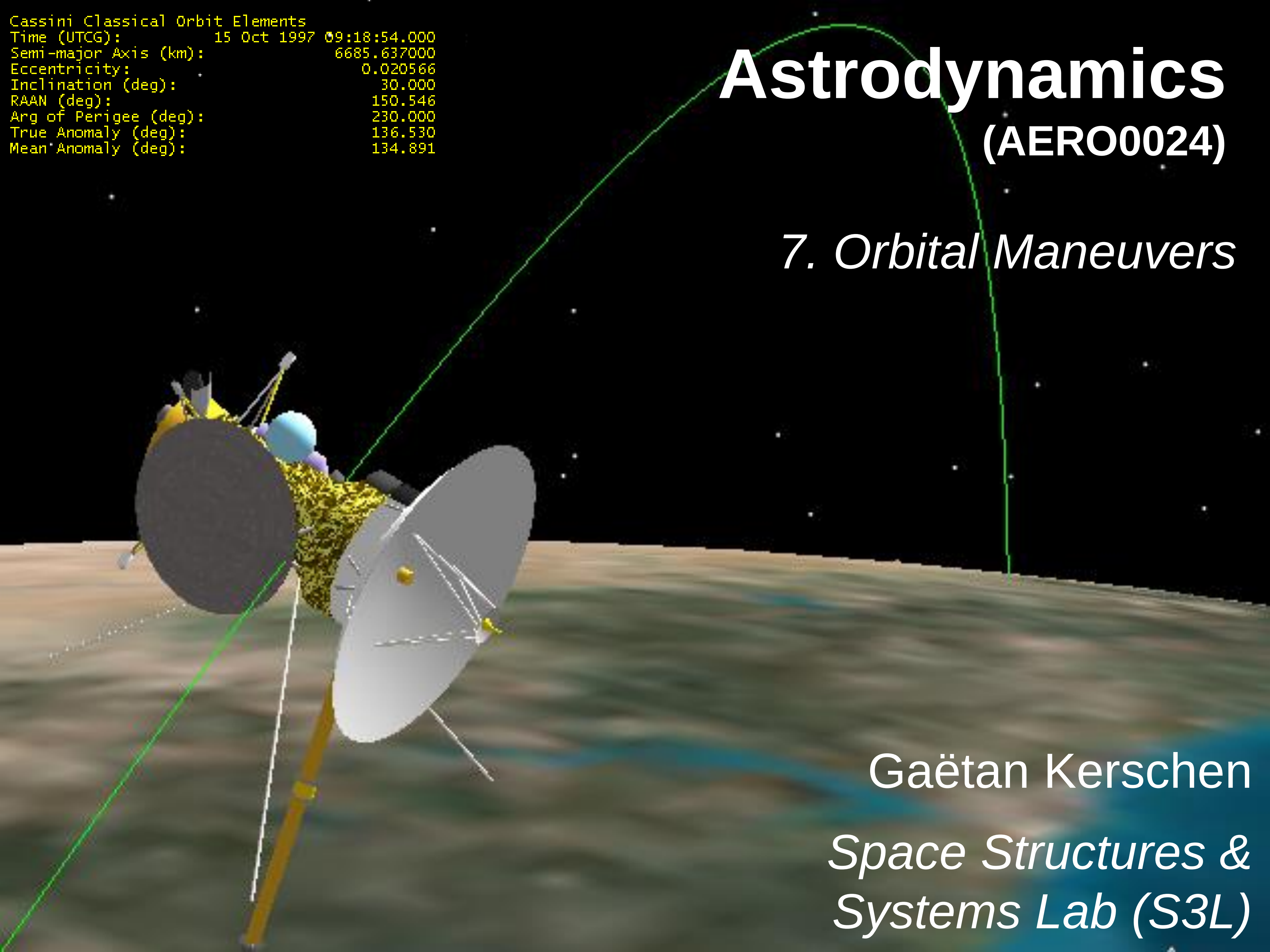


Astrodynamics

(AERO0024)

7. Orbital Maneuvers

Gaëtan Kerschen

Space Structures & Systems Lab (S3L)

Motivation

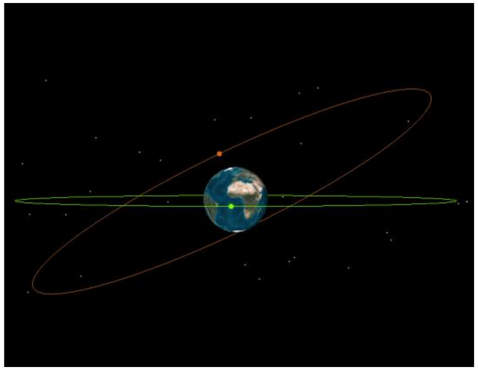
Coplanar transfers have wide applicability (e.g., phasing maneuvers and orbit raising). They alter 4 orbital elements:

1. Semi-major axis.
2. Eccentricity.
3. True anaomaly.
4. Argument of perigee.

Noncoplanar transfers requires applying Δv out of the orbit plane and can therefore alter the last two elements:

1. Inclination.
2. RAAN.

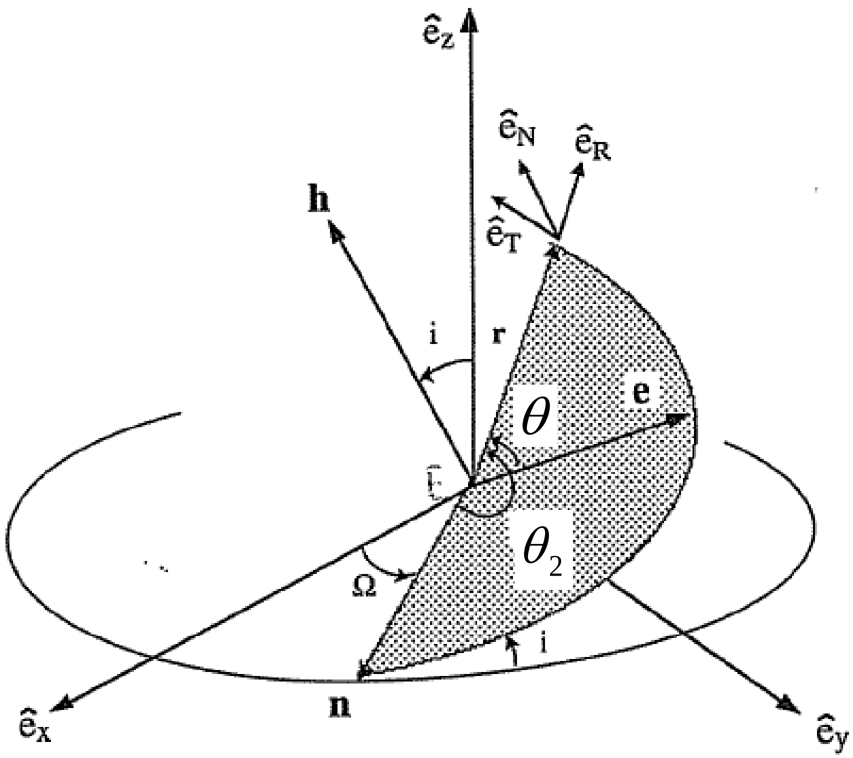
7. Orbital Maneuvers



Noncoplanar transfers

Rendez-vous

Gaussian VOP (Lecture 4)



$$\mathbf{F} = R\,\hat{\mathbf{e}}_R + T\hat{\mathbf{e}}_T + N\hat{\mathbf{e}}_N$$

$$\dot{i} = \sqrt{\frac{a(1-e^2)}{\mu}}\,\frac{N\cos\theta_2}{(1+e\cos\theta)} \qquad \dot{\Omega} = \sqrt{\frac{a(1-e^2)}{\mu}}\,\frac{N\sin\theta_2}{\sin i\,(1+e\cos\theta)}$$

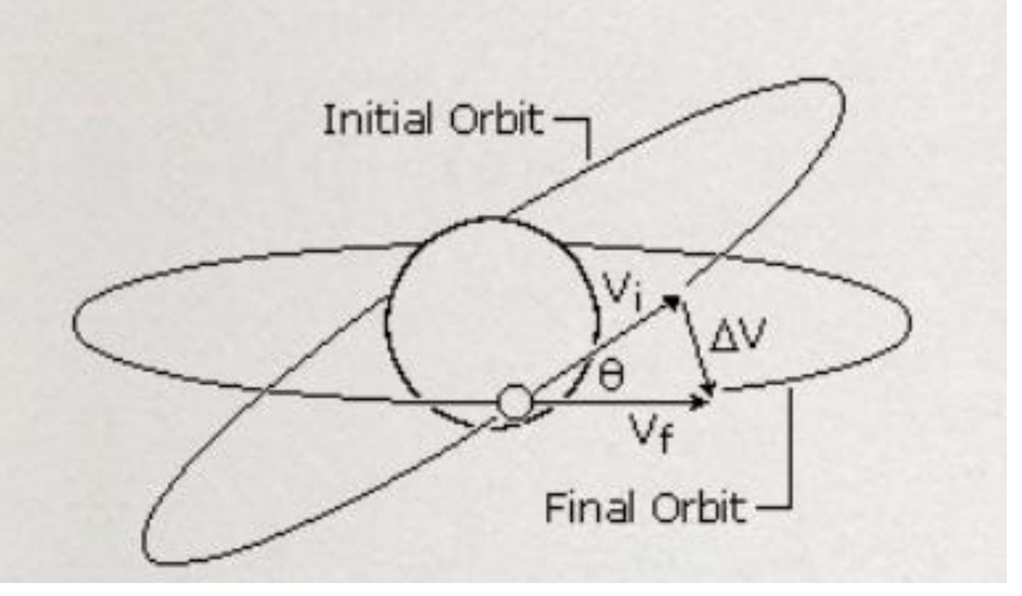
Where ?

Cranking maneuver: with a single Δv maneuver, one wants to change only the inclination of the orbit plane.

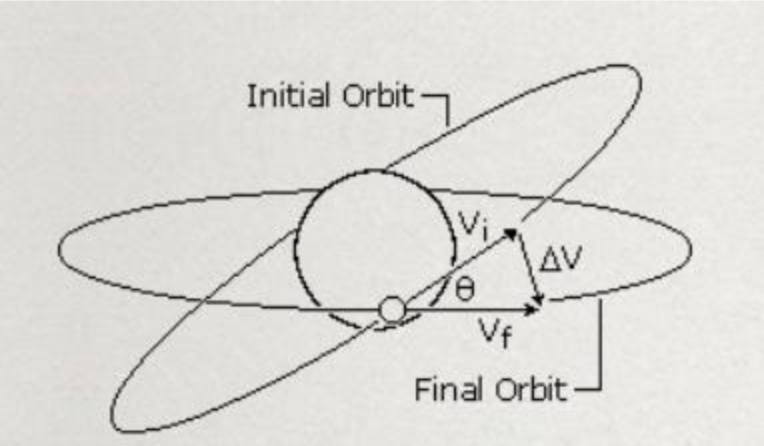
Where should we apply this maneuver ?

Where ? At an Equator Crossing

The two orbit planes intersect in the equatorial plane. We can therefore change the inclination at an equator crossing, by a simple rotation of the velocity vector.



ΔV Computation for Unchanged Circular Orbit

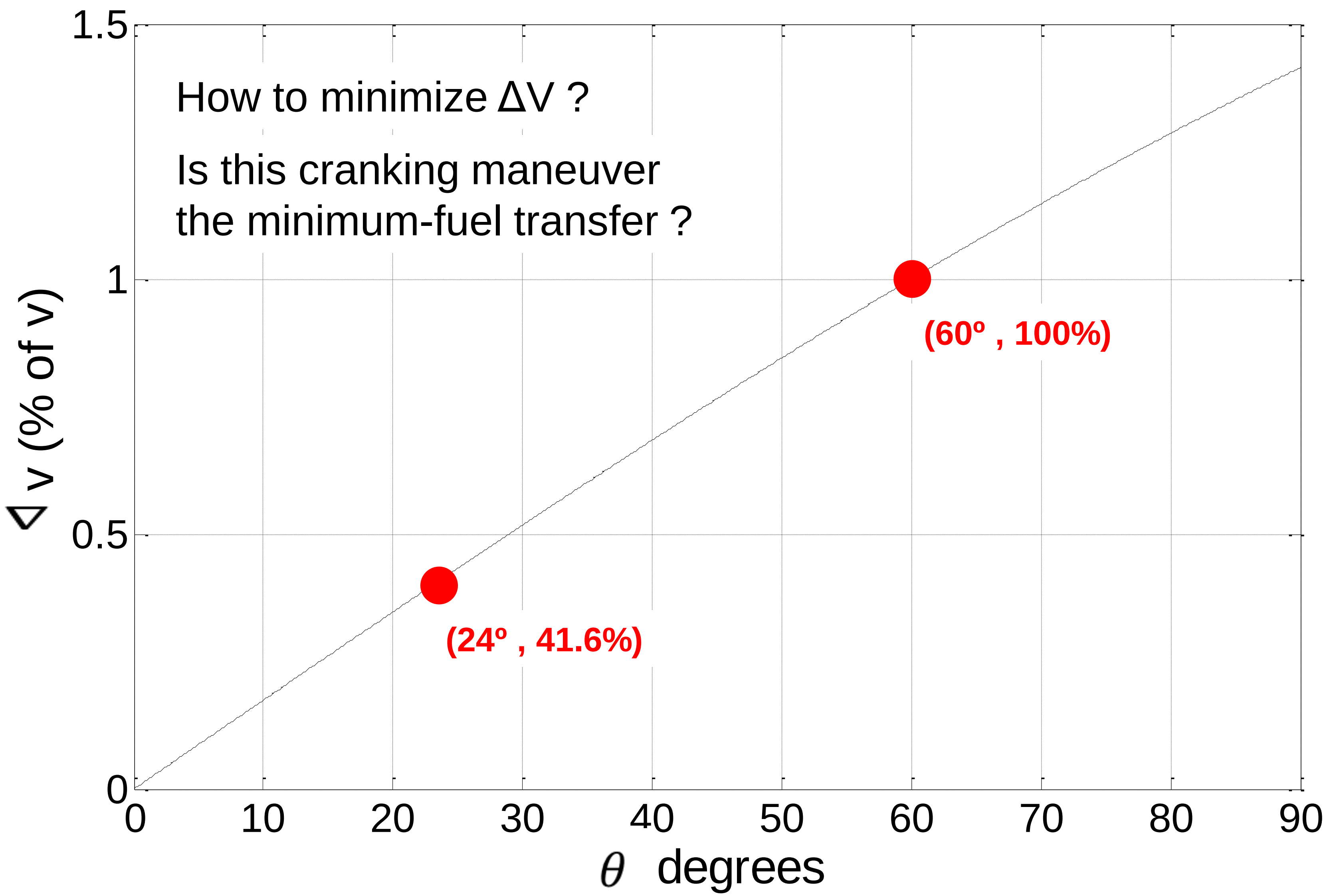


$$\left\|\overrightarrow{V_i}\right\| = V_i = V_f = \left\|\overrightarrow{V_f}\right\|$$

$$\Delta V = \sqrt{{V_i}^2 + {V_f}^2 - 2V_iV_f cos\theta} = V_i\sqrt{2(1 - cos\theta)} = 2V_i sin\frac{\theta}{2}$$

$$\boxed{\Delta V = 2V sin\frac{\theta}{2}}$$

Inclination Changes Are Very Expensive



Real-Life Examples: Space Shuttle and Hubble

The Space Shuttle is capable of a plane change in orbit of only about 3º, a maneuver which would exhaust its entire fuel capacity.

What is Hubble space telescope's inclination ?

28.46 (HST) vs. 28.35 (KSC)…

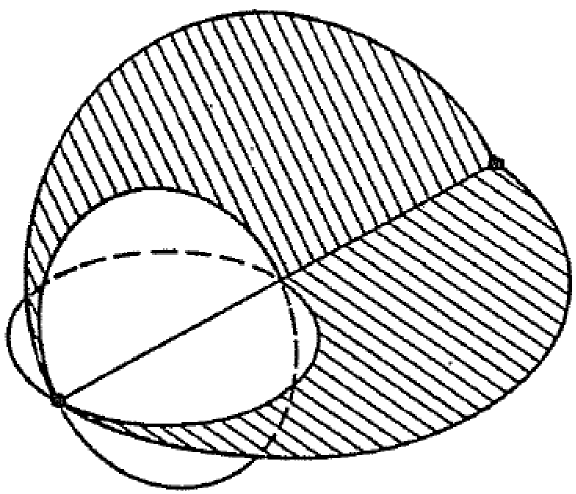
What's Unique with this Picture ?

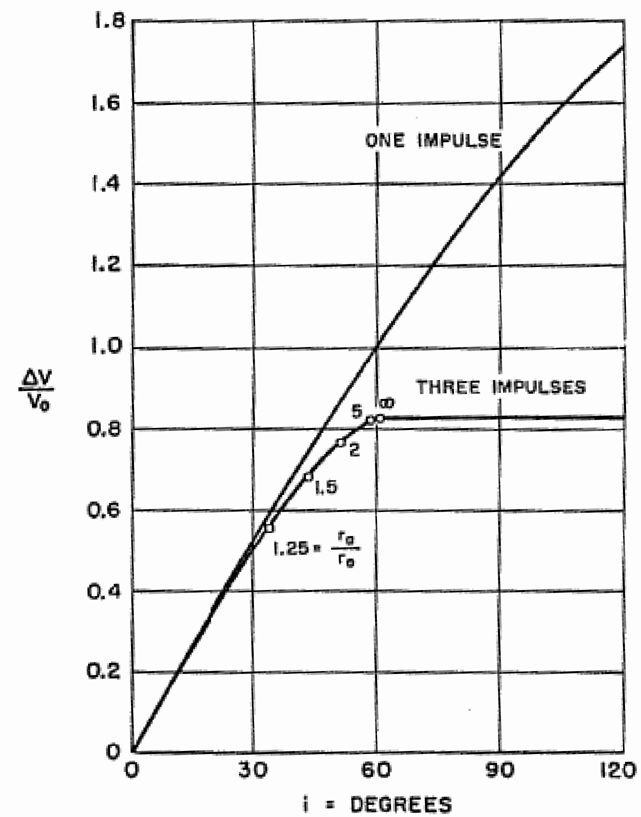
Can Hubble be Moved to the International Space Station?[1]

Too costly ! Instead, NASA has chosen to have another shuttle ready to lift off to retrieve the astronauts if needed.

Minimum-Fuel Transfer: Bi-Elliptic Transfer

One can always save fuel over a one-impulse maneuver by using three-impulses: noncoplanar bi-elliptic transfer.





J.E. Prussing, B.A. Conway, *Orbital Mechanics*, Oxford University Press

7. Orbital Maneuvers

Rendez-vous

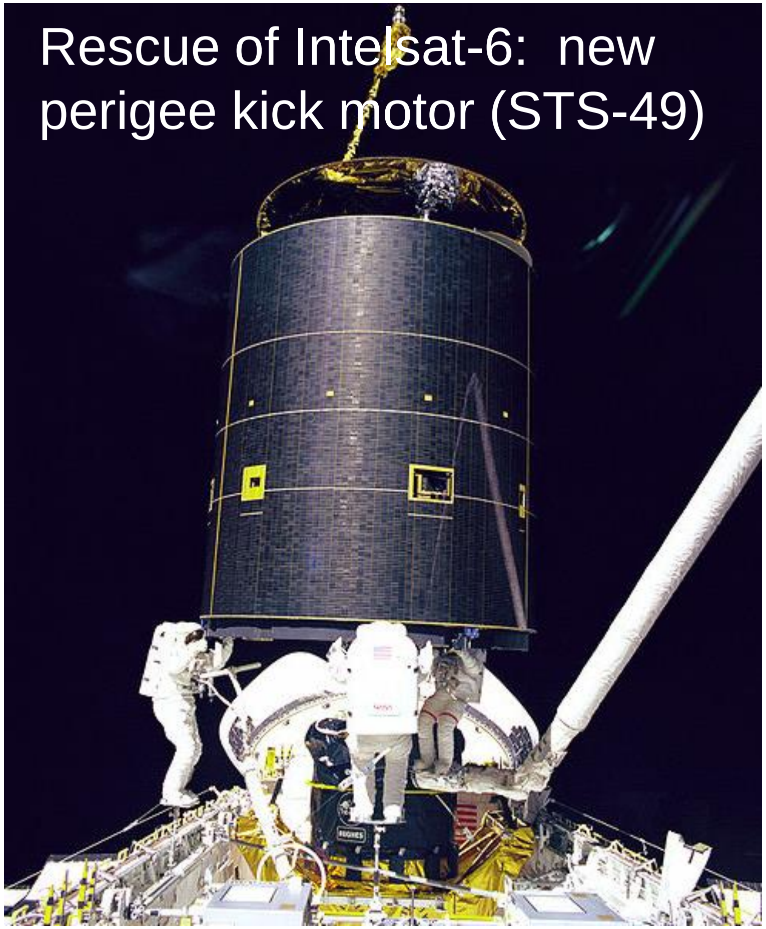
Rendez-vous: Two Successive Phases

1. Long-distance navigation: basic maneuvers bring the spacecraft in a vicinity (<100 km) of the target in the same orbit plane.

2. Terminal rendez-vous: the interest is in the **relative motion**, i.e. the motion of the maneuvering spacecraft in relation to the target (spacecraft or neighboring orbit).

$\Rightarrow$ Focus of the present discussion.


ISS & ATV


Rescue of Intelsat-6: new
perigee kick motor (STS-49)


STS & HST

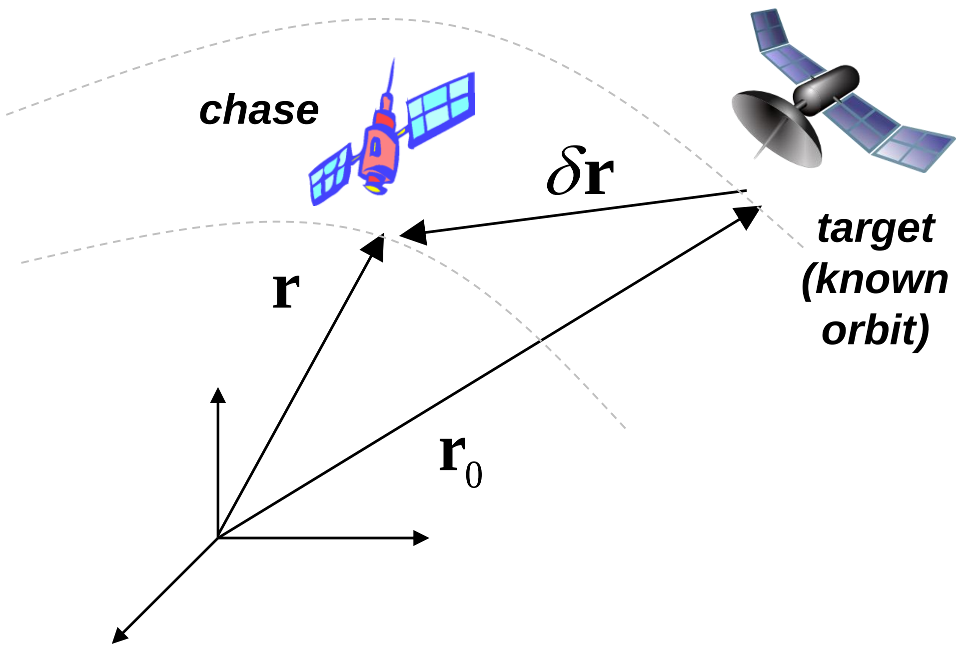
Basic Assumption

The rendez-vous can be solved using the “classical” 2-body problem. But nonlinear equations of motion have to be solved and one should resort to numerical methods (e.g., Lambert’s problem).

Can we use approximate equations considering that distances are small relative to the dimensions of the orbit ?

Yes ! Use linearization procedures.

Problem Statement



$$\mathbf{r} = \mathbf{r}_0 + \delta\mathbf{r}, \quad \frac{\delta r}{r_0} <<< 1$$

What is the equation governing the relative motion $\delta\mathbf{r}$?

A rendez-vous occurs when $\delta\mathbf{r} = 0$.

$$\mathbf{r} = \mathbf{r}_0 + \delta\mathbf{r}$$

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} \quad \Rightarrow \quad \delta\ddot{\mathbf{r}} = -\ddot{\mathbf{r}}_0 - \mu \frac{\mathbf{r}_0 + \delta\mathbf{r}}{r^3} \ ?$$

$$r^{-3} = r_0^{-3}\left(1 - \frac{3\mathbf{r}_0.\delta\mathbf{r}}{r_0^2}\right)$$

See next page

$$\delta\ddot{\mathbf{r}} = -\ddot{\mathbf{r}}_0 - \mu\left(\frac{1}{r_0^3} - \frac{3\mathbf{r}_0.\delta\mathbf{r}}{r_0^5}\right)(\mathbf{r}_0 + \delta\mathbf{r})$$

Higher-order terms neglected

$$\delta\ddot{\mathbf{r}} = -\ddot{\mathbf{r}}_0 - \mu \frac{\mathbf{r}_0}{r_0^3} - \frac{\mu}{r_0^3}\left(\delta\mathbf{r} - \frac{3\mathbf{r}_0.\delta\mathbf{r}}{r_0^2}\mathbf{r}_0\right)$$

$$\ddot{\mathbf{r}}_0 = -\mu \frac{\mathbf{r}_0}{r^3}$$

$$\delta\ddot{\mathbf{r}} = -\frac{\mu}{r_0^3}\left(\delta\mathbf{r} - \frac{3\mathbf{r}_0.\delta\mathbf{r}}{r_0^2}\mathbf{r}_0\right)$$

Linear ODE. Observer in an inertial frame

Expression of r^{-3}

$$r^2 = (\mathbf{r}_0 + \delta\mathbf{r}).(\mathbf{r}_0 + \delta\mathbf{r}) = r_0^2 + 2\mathbf{r}_0.\delta\mathbf{r} + \delta r^2$$

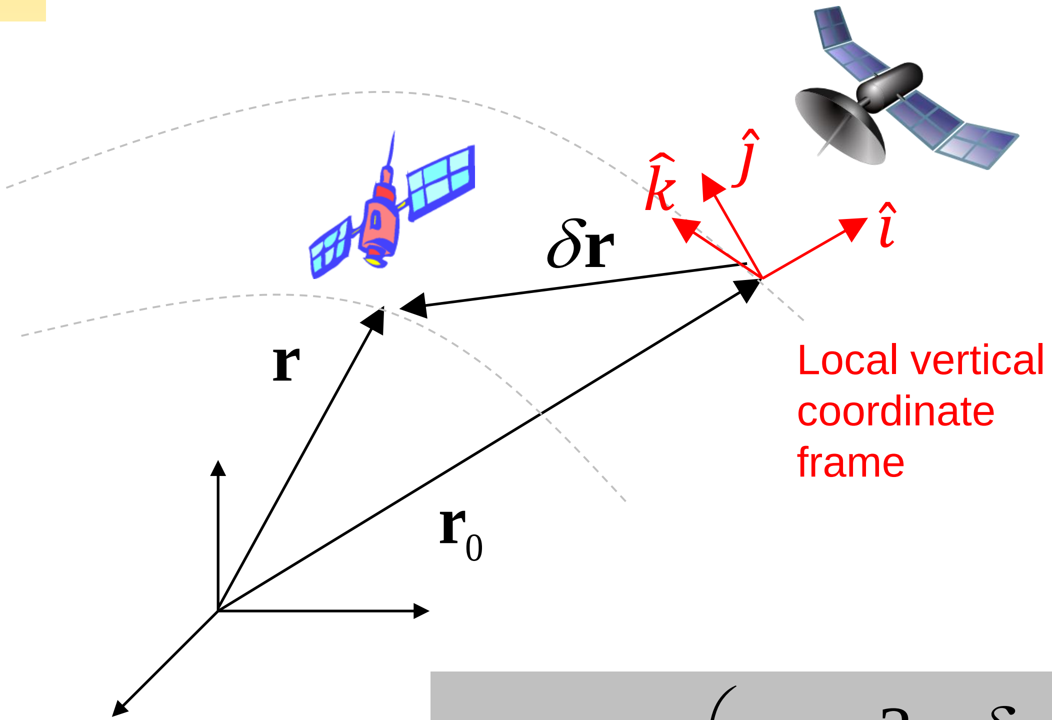
$$r^3 = (r_0^2 + 2\boldsymbol{r}_0.\delta\boldsymbol{r})^{3/2} = r_0^3\left(1 + \frac{2\boldsymbol{r}_0.\delta\boldsymbol{r}}{r_0^2}\right)^{3/2}$$

$$r^{-3} = r_0^{-3}\left(1 + \frac{2\mathbf{r}_0.\delta\mathbf{r}}{r_0^2}\right)^{\frac{-3}{2}}$$

Binomial theorem

$$r^{-3} = r_0^{-3}\left(1 - \frac{3\mathbf{r}_0.\delta\mathbf{r}}{r_0^2}\right)$$

Co-Moving Frame



Our focus is on a rotating observer in the local frame (e.g., an ISS astronaut watching the ATV)

$$\delta\ddot{\mathbf{r}} = -\frac{\mu}{r_0^3}\left(\delta\mathbf{r} - \frac{3\mathbf{r}_0.\delta\mathbf{r}}{r_0^2}\mathbf{r}_0\right)$$

$$\delta\mathbf{r} = \delta x\,\hat{\mathbf{i}} + \delta y\,\hat{\mathbf{j}} + \delta z\,\hat{\mathbf{k}}$$

$$\boldsymbol{r}_0 = r_0\hat{\boldsymbol{\imath}}$$

?

$$= -n^2\left(\delta x\,\hat{\mathbf{i}} + \delta y\,\hat{\mathbf{j}} + \delta z\,\hat{\mathbf{k}} - \frac{3r_0.\delta x}{r_0^2}r_0\hat{\mathbf{i}}\right)$$

Assumption of a Circular Orbit

$$v = \sqrt{\frac{\mu}{r_0}} = \Omega r_0$$

$$\boldsymbol{\Omega} = n\hat{\mathbf{k}}$$

$$\Omega = n = \sqrt{\frac{\mu}{{r_0}^3}}$$

Differentiation in A Rotating Frame

1.6 RELATIVE MOTION

Let P be a particle in arbitrary motion. The absolute position vector of P is $\mathbf{r}$ and the position of P relative to the moving frame is $\mathbf{r}_{rel}$. If $\mathbf{r}_O$ is the absolute position of the origin of the moving frame, then it is clear from Figure 1.10 that

$$\mathbf{r} = \mathbf{r}_O + \mathbf{r}_{rel} \tag{1.33}$$

Since $\mathbf{r}_{rel}$ is measured in the moving frame,

$$\mathbf{r}_{rel} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \tag{1.34}$$

where x, y and z are the coordinates of P relative to the moving reference.

The absolute velocity $\mathbf{v}$ of P is $d\mathbf{r}/dt$, so that from Equation 1.33 we have

$$\mathbf{v} = \mathbf{v}_O + \frac{d\mathbf{r}_{rel}}{dt} \tag{1.35}$$

where $\mathbf{v}_O = d\mathbf{r}_O/dt$ is the (absolute) velocity of the origin of the xyz frame. From Equation 1.28, we can write

$$\frac{d\mathbf{r}_{rel}}{dt} = \mathbf{v}_{rel} + \mathbf{\Omega} \times \mathbf{r}_{rel} \tag{1.36}$$

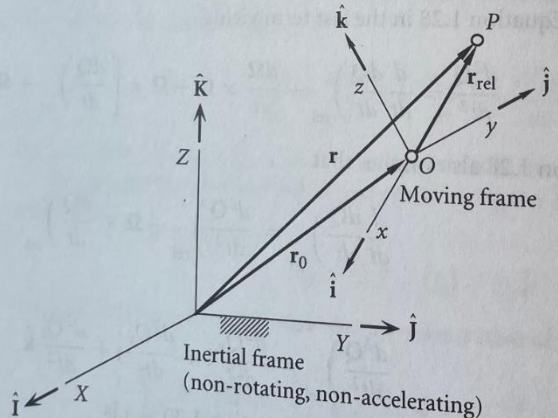


Figure 1.10 Absolute and relative position vectors.

Differentiation in A Rotating Frame

where $\mathbf{v}_{\text{rel}}$ is the velocity of P relative to the xyz frame:

$$\mathbf{v}_{\text{rel}} = \left.\frac{d\mathbf{r}_{\text{rel}}}{dt}\right)_{\text{rel}} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} \tag{1.37}$$

Substituting Equation 1.36 into Equation 1.35 yields

$$\mathbf{v} = \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{\text{rel}} + \mathbf{v}_{\text{rel}} \tag{1.38}$$

The absolute acceleration $\mathbf{a}$ of P is $d\mathbf{v}/dt$, so that from Equation 1.35 we have

$$\mathbf{a} = \mathbf{a}_O + \frac{d^2\mathbf{r}_{\text{rel}}}{dt^2} \tag{1.39}$$

where $\mathbf{a}_O = d\mathbf{v}_O/dt$ is the absolute acceleration of the origin of the xyz frame. We evaluate the second term on the right using Equation 1.32:

$$\frac{d^2\mathbf{r}_{\text{rel}}}{dt^2} = \left.\frac{d^2\mathbf{r}_{\text{rel}}}{dt^2}\right)_{\text{rel}} + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{\text{rel}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{\text{rel}}) + 2\boldsymbol{\Omega} \times \left.\frac{d\mathbf{r}_{\text{rel}}}{dt}\right)_{\text{rel}} \tag{1.40}$$

Since $\mathbf{v}_{\text{rel}} = d\mathbf{r}_{\text{rel}}/dt)_{\text{rel}}$ and $\mathbf{a}_{\text{rel}} = d^2\mathbf{r}_{\text{rel}}/dt^2)_{\text{rel}}$, this can be written

$$\frac{d^2\mathbf{r}_{\text{rel}}}{dt^2} = \mathbf{a}_{\text{rel}} + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{\text{rel}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{\text{rel}}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rel}} \tag{1.41}$$

Upon substituting this result into Equation 1.39, we find

$$\mathbf{a} = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{\text{rel}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{\text{rel}}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}} \tag{1.42}$$

The cross product $2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rel}}$ is called the Coriolis acceleration after Gustave Gaspard de Coriolis (1792–1843), the French mathematician who introduced this term (Coriolis, 1835). For obvious reasons, Equation 1.42 is sometimes referred to as the five-term acceleration formula.

Differentiation in A Rotating Frame

Circular

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_0 + \frac{d\mathbf{\Omega}}{dt}\times\delta\mathbf{r} + \mathbf{\Omega}\times\left(\mathbf{\Omega}\times\delta\mathbf{r}\right) + 2\mathbf{\Omega}\times\delta\mathbf{v}_{rel} + \delta\mathbf{a}_{rel} = \ddot{\mathbf{r}}_0 + \delta\ddot{\mathbf{r}}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{b}(\mathbf{a}\cdot\mathbf{c}) - \mathbf{c}(\mathbf{a}\cdot\mathbf{b}).$$

$$\delta\ddot{\mathbf{r}} = \mathbf{\Omega}\left(\mathbf{\Omega}.\delta\mathbf{r}\right) - \Omega^2\delta\mathbf{r} + 2\mathbf{\Omega}\times\delta\mathbf{v}_{rel} + \delta\mathbf{a}_{rel}$$

$$\delta\mathbf{r} = \delta x\,\hat{\mathbf{i}} + \delta y\,\hat{\mathbf{j}} + \delta z\,\hat{\mathbf{k}}$$

$$\delta\mathbf{v}_{rel} = \delta\dot{x}\,\hat{\mathbf{i}} + \delta\dot{y}\,\hat{\mathbf{j}} + \delta\dot{z}\,\hat{\mathbf{k}}$$

$$\delta\mathbf{a}_{rel} = \delta\ddot{x}\,\hat{\mathbf{i}} + \delta\ddot{y}\,\hat{\mathbf{j}} + \delta\ddot{z}\,\hat{\mathbf{k}}$$

$$\mathbf{\Omega} = n\hat{\mathbf{k}}$$

Interpretation of the terms ?

$$\delta\ddot{\mathbf{r}} = (-n^2\delta x - 2n\delta\dot{y} + \delta\ddot{x})\hat{\mathbf{i}} + (-n^2\delta y + 2n\delta\dot{x} + \delta\ddot{y})\,\hat{\mathbf{j}} + \delta\ddot{z}\,\hat{\mathbf{k}}$$

Centrifugal

Coriolis

Finally

$$\delta\ddot{\mathbf{r}} = (-n^2\delta x - 2n\delta\dot{y} + \delta\ddot{x})\hat{\mathbf{i}} + (-n^2\delta y + 2n\delta\dot{x} + \delta\ddot{y})\hat{\mathbf{j}} + \delta\ddot{z}\hat{\mathbf{k}}$$

$$= -n^2\left(\delta x\hat{\mathbf{i}} + \delta y\hat{\mathbf{j}} + \delta z\hat{\mathbf{k}} - \frac{3r_0.\delta x}{r_0^2} r_0\hat{\mathbf{i}}\right)$$

$$\delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} = 0$$

$$\delta\ddot{y} + 2n\delta\dot{x} = 0$$

$$\delta\ddot{z} + n^2\delta z = 0$$

Hill – Clohessy – Wiltshire equations: acceleration relative to a rotating observer fixed in the moving frame.

An Obvious Solution

$$\delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} = 0 \qquad \delta x = ?$$

$$\delta\ddot{y} + 2n\delta\dot{x} = 0 \qquad \delta y = ?$$

$$\delta\ddot{z} + n^2\delta z = 0 \qquad \delta z = ?$$

$\delta x=0$, $\delta y=$constant.

$\delta z=$harmonic motion (the out-of plane motion is decoupled from the in-plane motion).

Physical interpretation ?

$\Rightarrow$ Two spacecraft on the same orbit.

In Matrix Form

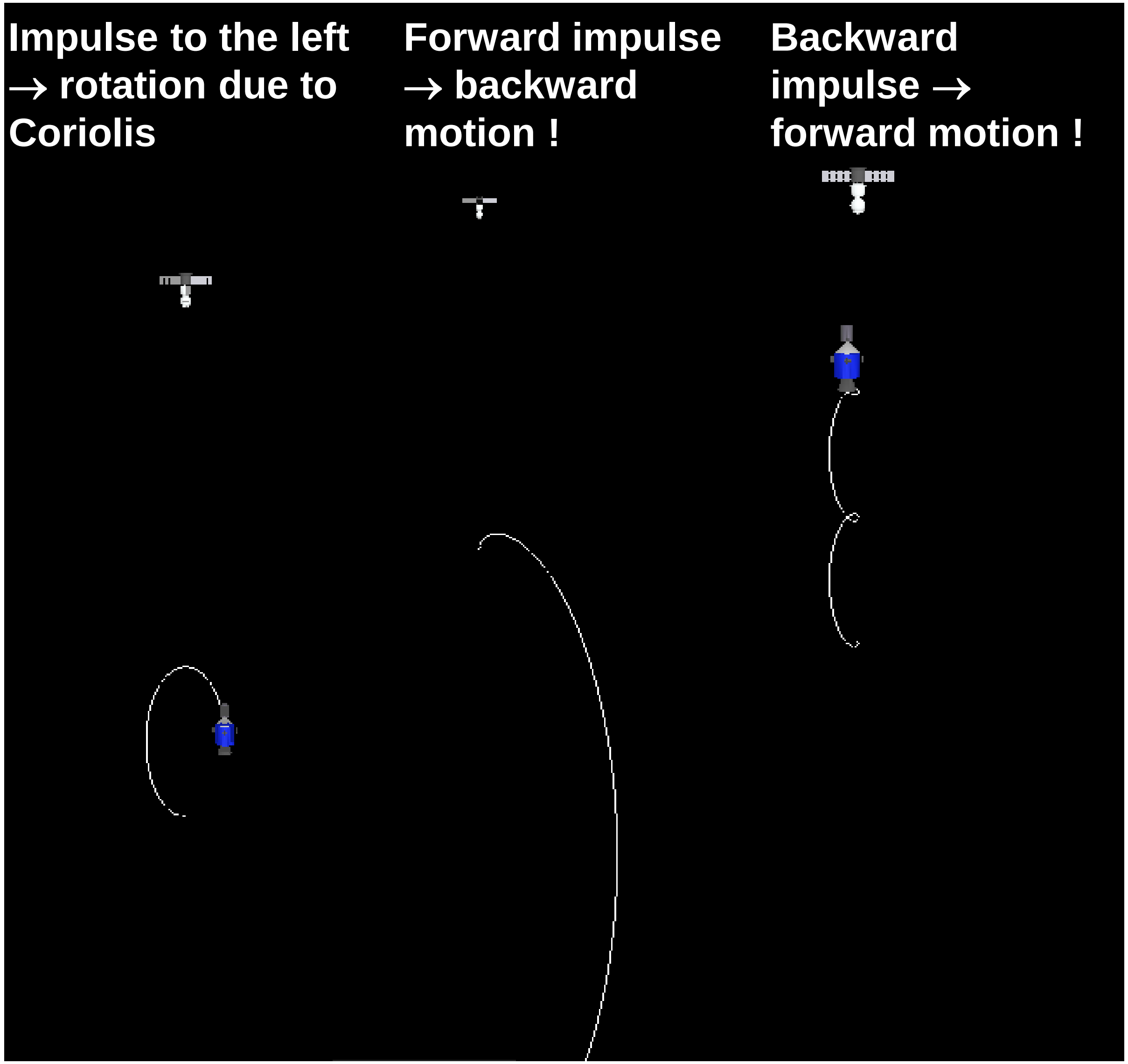
Linear ODEs with constant coefficient: we can solve them !

$$\begin{pmatrix} \delta\ddot{x} \\ \delta\ddot{y} \\ \delta\ddot{z} \end{pmatrix} - \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \delta\dot{x} \\ \delta\dot{y} \\ \delta\dot{z} \end{pmatrix} - \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix} \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Gyroscopic damping: imposes a rotation if there is a relative velocity

Centrifugal + gravitational terms: move away from the target if not on the same orbit

Interpretation



Solution

$$\delta\ddot{y} + 2n\delta\dot{x} = 0 \rightarrow \delta\dot{y} + 2n\delta x = Cte = \delta\dot{y}_0 + 2n\delta x_0$$

$$\delta\ddot{x} - 3n^2\delta x - 2n\delta\dot{y} = 0 \rightarrow \delta\ddot{x} + n^2\delta x = 2n\delta\dot{y}_0 + 4n^2\delta x_0$$

$$\delta x = A\sin nt + B\cos nt + \frac{2}{n}\delta\dot{y}_0 + 4\delta x_0$$

…

Solution: Overall

$$\underbrace{\begin{pmatrix}\delta x\\ \delta y\\ \delta z\end{pmatrix}}_{\delta\mathbf{r}(t)}=\underbrace{\begin{bmatrix}4-3\cos nt & 0 & 0\\ 6(\sin nt-nt) & 1 & 0\\ 0 & 0 & \cos nt\end{bmatrix}}_{\boldsymbol{\Phi}_{rr}(t)}\underbrace{\begin{pmatrix}\delta x_0\\ \delta y_0\\ \delta z_0\end{pmatrix}}_{\delta\mathbf{r}_0(t)}+\underbrace{\begin{bmatrix}\dfrac{\sin nt}{n} & \dfrac{2(1-\cos nt)}{n} & 0\\ \dfrac{2(\cos nt-1)}{n} & \dfrac{(4\sin nt-3nt)}{n} & 0\\ 0 & 0 & \dfrac{\sin nt}{n}\end{bmatrix}}_{\boldsymbol{\Phi}_{rv}(t)}\underbrace{\begin{pmatrix}\delta \dot{x}_0\\ \delta \dot{y}_0\\ \delta \dot{z}_0\end{pmatrix}}_{\delta\mathbf{v}_0(t)}$$

$$\underbrace{\begin{pmatrix}\delta \dot{x}\\ \delta \dot{y}\\ \delta \dot{z}\end{pmatrix}}_{\delta\mathbf{v}(t)}=\underbrace{\begin{bmatrix}3n\sin nt & 0 & 0\\ 6n(\cos nt-1) & 0 & 0\\ 0 & 0 & -n\sin nt\end{bmatrix}}_{\boldsymbol{\Phi}_{vr}(t)}\underbrace{\begin{pmatrix}\delta x_0\\ \delta y_0\\ \delta z_0\end{pmatrix}}_{\delta\mathbf{r}_0(t)}+\underbrace{\begin{bmatrix}\cos nt & 2\sin nt & 0\\ -2\sin nt & 4\cos nt-3 & 0\\ 0 & 0 & \cos nt\end{bmatrix}}_{\boldsymbol{\Phi}_{vv}(t)}\underbrace{\begin{pmatrix}\delta \dot{x}_0\\ \delta \dot{y}_0\\ \delta \dot{z}_0\end{pmatrix}}_{\delta\mathbf{v}_0(t)}$$

H. Curtis, *Orbital Mechanics for Engineering Students*, Elsevier.

Example: HST and STS

The HST is released from the Space Shuttle, which is in a circular orbit of 590 km altitude. The relative velocity of ejection is 0.1 m/s down, 0.04 m/s backwards and 0.02 m/s to the right. Find the position of HST after 5,10,20 minutes.

$$n = \sqrt{\frac{398600}{6968^3}} = 0.0011 \text{ rad/s}$$

$$\begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix} = \begin{bmatrix} 4-3\cos nt & 0 & 0 \\ 6(\sin nt - nt) & 1 & 0 \\ 0 & 0 & \cos nt \end{bmatrix} \begin{pmatrix} \delta x_0 \\ \delta y_0 \\ \delta z_0 \end{pmatrix} + \begin{bmatrix} \frac{\sin nt}{n} & \frac{2(1-\cos nt)}{n} & 0 \\ \frac{2(\cos nt - 1)}{n} & \frac{(4\sin nt - 3nt)}{n} & 0 \\ 0 & 0 & \frac{\sin nt}{n} \end{bmatrix} \begin{pmatrix} \delta \dot{x}_0 \\ \delta \dot{y}_0 \\ \delta \dot{z}_0 \end{pmatrix}$$

$(-0.1;-0.04;-0.02)^T$

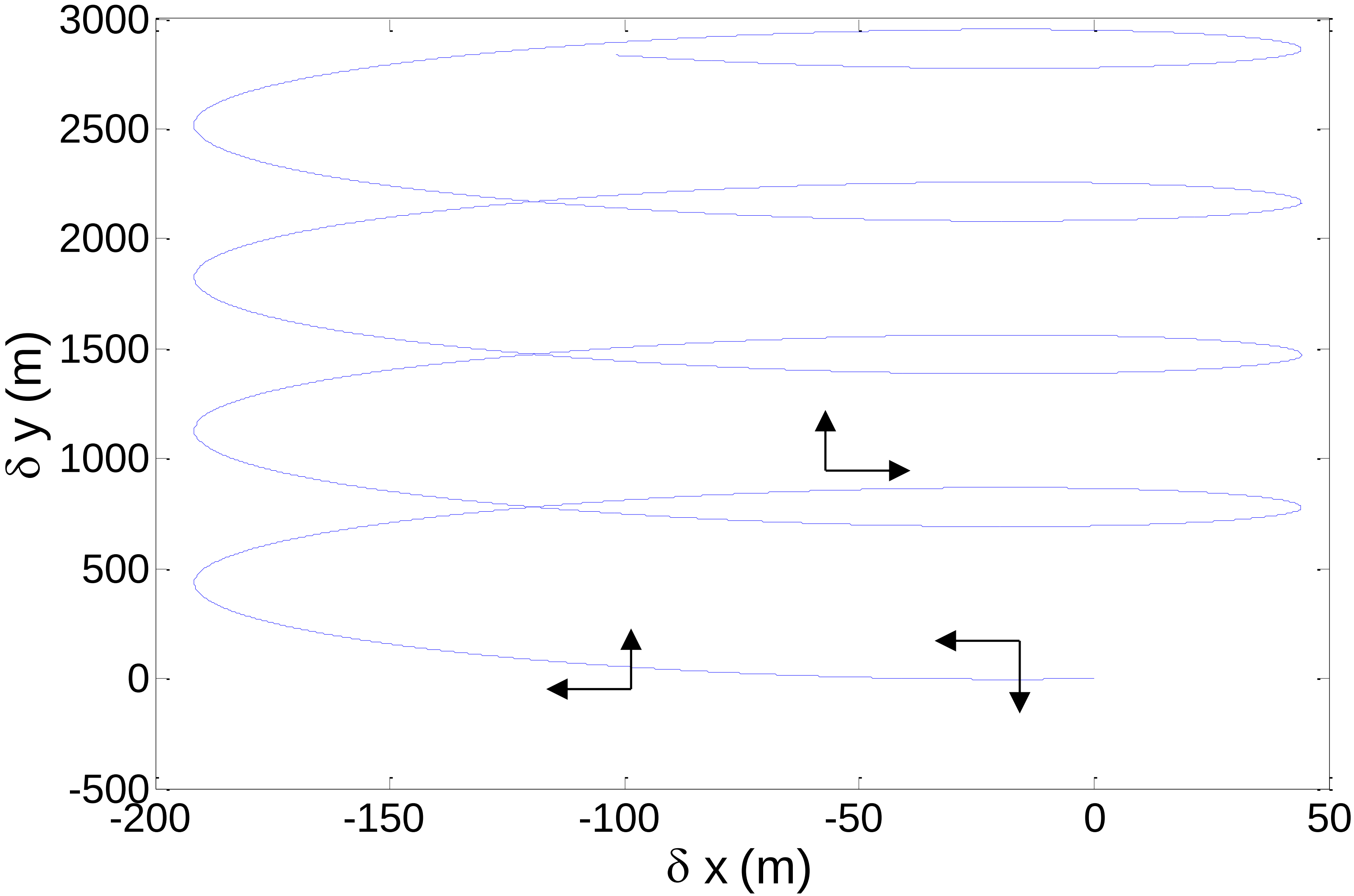
Example: HST and STS

5 minutes: $(-33.345 ; -1.473 ; -5.894)^T$

10 minutes: $(-70.933 ; 20.357 ; -11.170)^T$

20 minutes: $(-143.000 ; 137.279 ; -17.766)^T$

Example: HST and STS

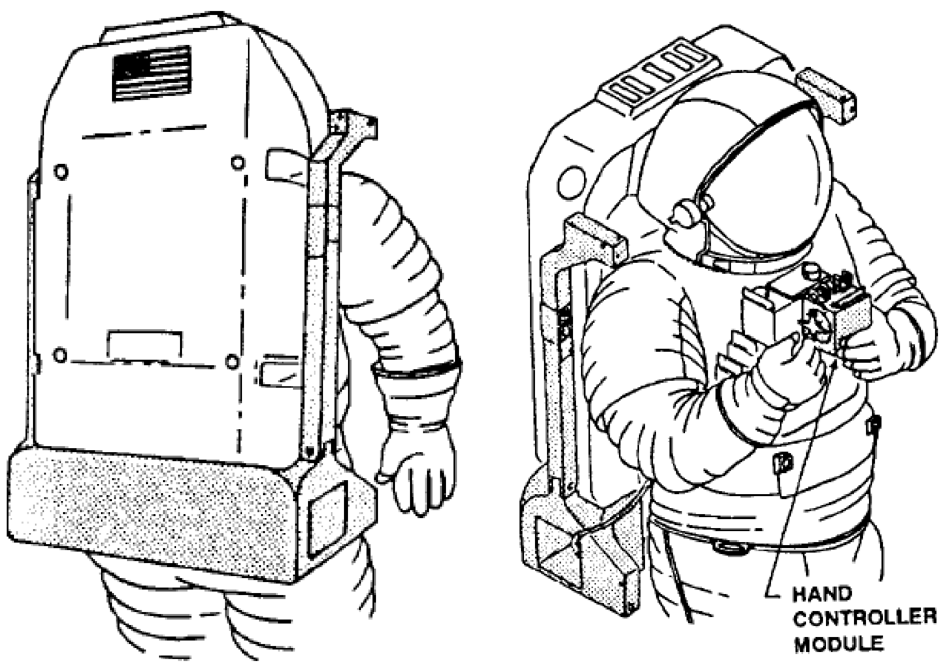


Further Reading on the Web Site

SELF-RESCUE STRATEGIES FOR EVA CREWMEMBERS EQUIPPED WITH THE SAFER BACKPACK

Trevor Williams[1] and David Baughman[2]

initiated, as a consequence of orbital effects. A very important practical question is then whether the total Δv of SAFER is adequate to perform self-rescue for worst case values of separation speed, time to detumble and time for the astronaut to visually acquire the station.



They use Hill – Clohessy – Wiltshire equations !

Solution: Overall

$$\delta\mathbf{r}(t) = \mathbf{\Phi}_{rr}(t)\delta\mathbf{r}_0 + \mathbf{\Phi}_{rv}(t)\delta\mathbf{v}_0$$
$$\delta\mathbf{v}(t) = \mathbf{\Phi}_{vr}(t)\delta\mathbf{r}_0 + \mathbf{\Phi}_{vv}(t)\delta\mathbf{v}_0$$

Linear analog of Kepler's equation

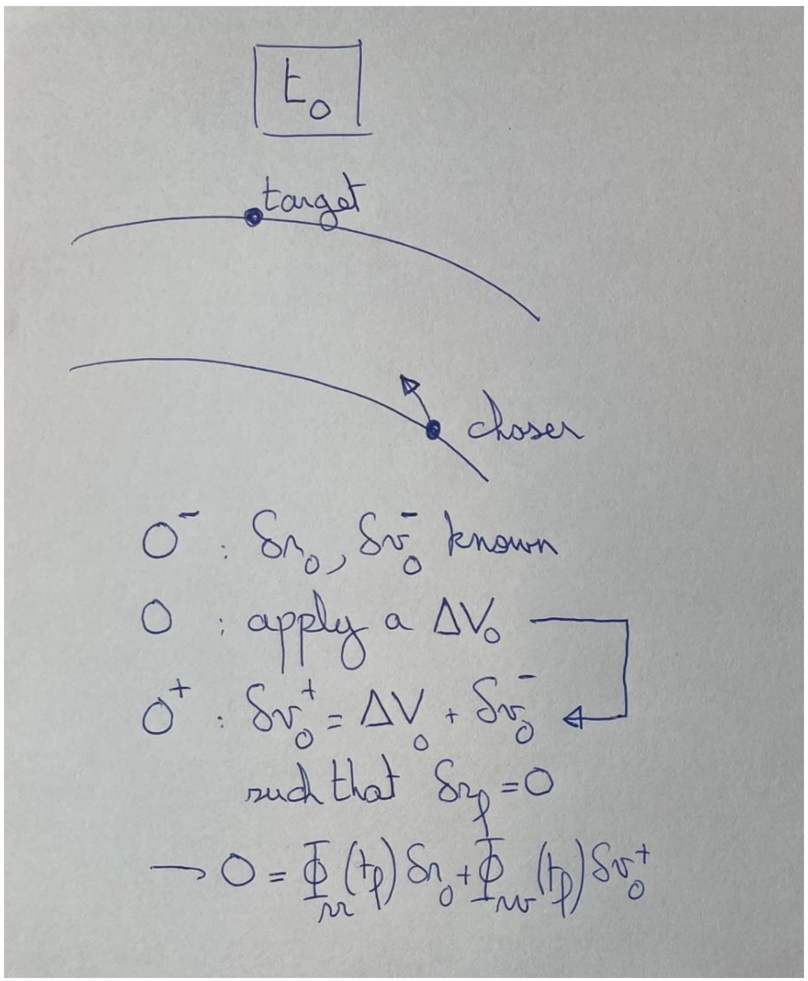
Assumptions:

1. The two satellites are within a few kms of each other.
2. The target is in a circular orbit.
3. There are no external forces on the interceptor.

Starting from t=0, how to exploit these equations for rendez-vous after a time t_f ?

Two-Impulse Rendez-vous Maneuvers

The initial velocity change places the target on a trajectory which will intercept the chaser at time t_f.



$$\delta \mathbf{r}(t) = \mathbf{\Phi}_{rr}(t)\delta \mathbf{r}_0 + \mathbf{\Phi}_{rv}(t)\delta \mathbf{v}_0$$

Compute $\delta \mathbf{v}_0^+$ for rendez-vous in a time t_f

⇩

$$0 = \mathbf{\Phi}_{rr}(t_f)\delta \mathbf{r}_0 + \mathbf{\Phi}_{rv}(t_f)\delta \mathbf{v}_0^+$$

⇩

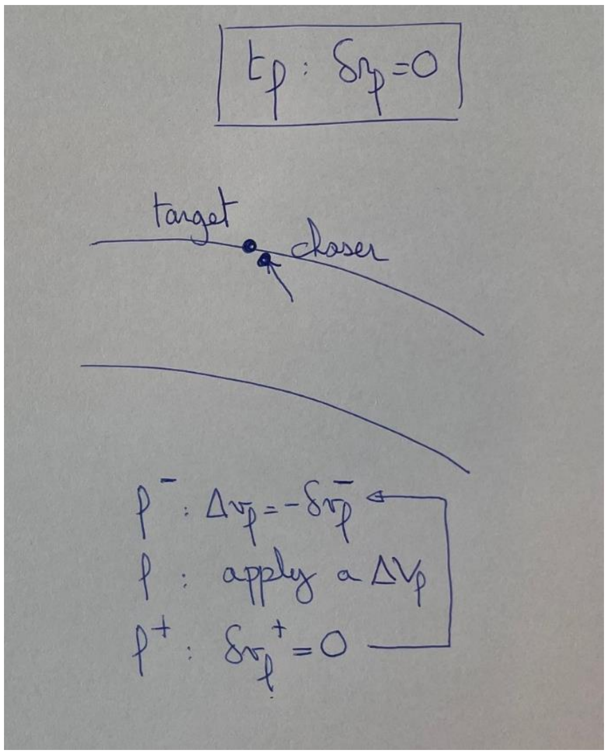
$$\delta \mathbf{v}_0^+ = \mathbf{\Phi}_{rv}^{-1}(t_f)\mathbf{\Phi}_{rr}(t_f)\delta \mathbf{r}_0$$

⇩

$$\Delta \mathbf{v}_0 = \delta \mathbf{v}_0^+ - \delta \mathbf{v}_0^-$$

Two-Impulse Rendez-vous Maneuvers

The final velocity change cancels the arrival velocity of the target and completes the rendez-vous.



$$\delta \mathbf{v}(t) = \mathbf{\Phi}_{vr}(t)\delta \mathbf{r}_0 + \mathbf{\Phi}_{vv}(t)\delta \mathbf{v}_0$$

⇩

$$\delta \mathbf{v}_f^- = \mathbf{\Phi}_{vr}(t_f)\delta \mathbf{r}_0 + \mathbf{\Phi}_{vv}(t_f)\delta \mathbf{v}_0^+$$

⇩

$$= \mathbf{\Phi}_{vr}(t_f)\delta \mathbf{r}_0 + \mathbf{\Phi}_{vv}(t_f)\mathbf{\Phi}_{rv}^{-1}(t_f)\mathbf{\Phi}_{rr}(t_f)\delta \mathbf{r}_0$$

$$\delta \mathbf{v}_f^+ = 0 \quad \Rightarrow \quad \Delta \mathbf{v}_f = -\delta \mathbf{v}_f^-$$

Example: HST and STS

After 10 minutes, the STS astronauts want to retrieve HST. Compute the δv_0^+ to rendez-vous in 5 and 15 minutes.

5 minutes: $(0.2742 ; 0.0135 ; 0.0359)^T$

15 minutes: $(0.1356 ; 0.0753 ; 0.0082)^T$

Real Data: Gemini 10 and Agena 8

A 4-orbit rendez-vous plan similar to that of Gemini 6-7:

O1: no maneuvers to allow spacecraft checkout.

N_H O2: height adjustment at perigee (0.2 m/s for orbital decay)

N_{CI} phase adjustment maneuver at apogee (17 m/s).

N_{SR} O3: switch to a circular orbit in the same plane as Agena (16 m/s).

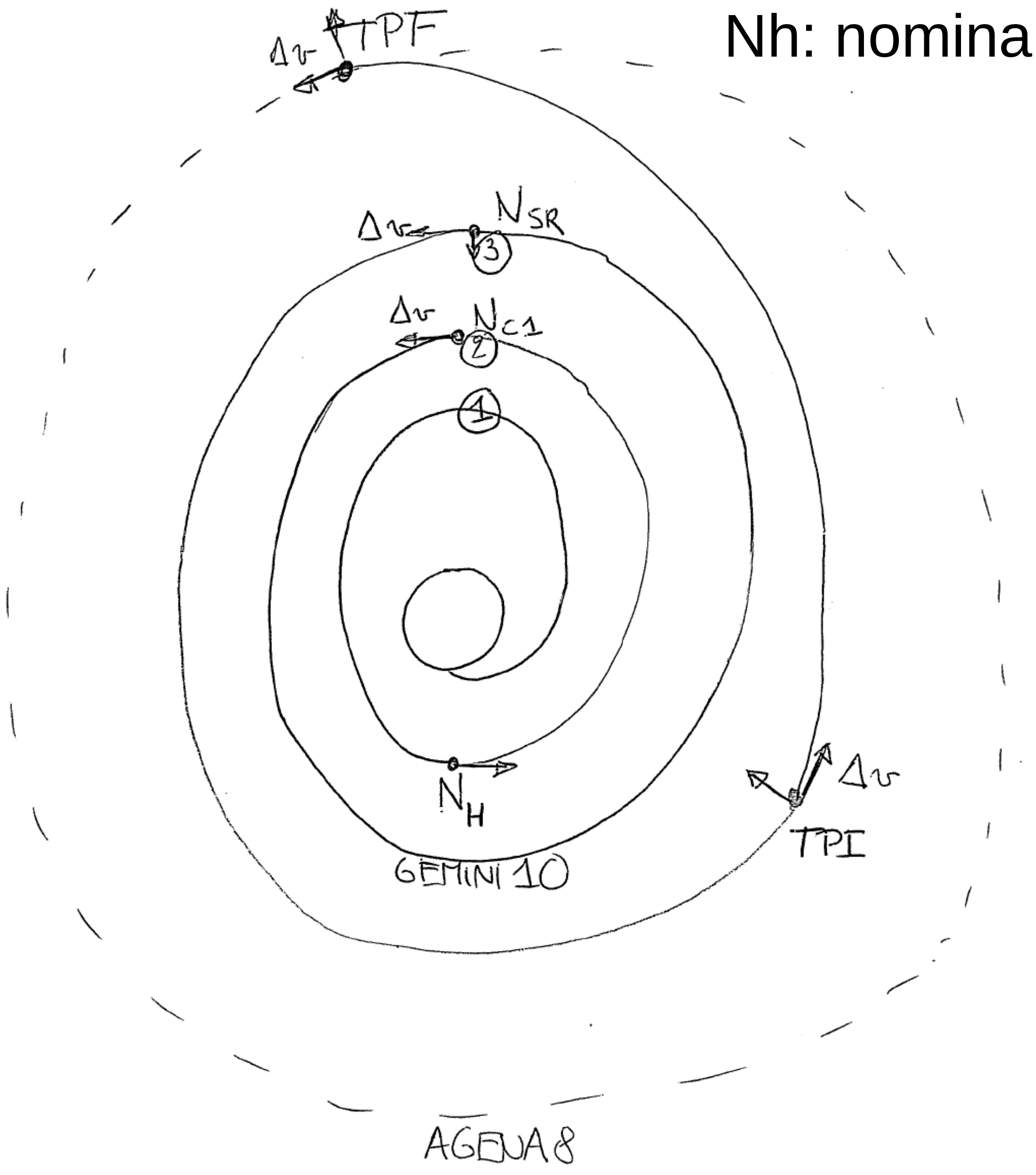
TPI O4: terminal phase initiation (10 m/s)

TPF velocity matching (13 m/s)

Total: $\pm$ 56 m/s

Nc: nominal corrective burn

Nh: nominal height adjust burn



Real Data: Gemini 10 and Agena 8

SUMMARY OF GEMINI RENDEZVOUS EXPERIENCE

by

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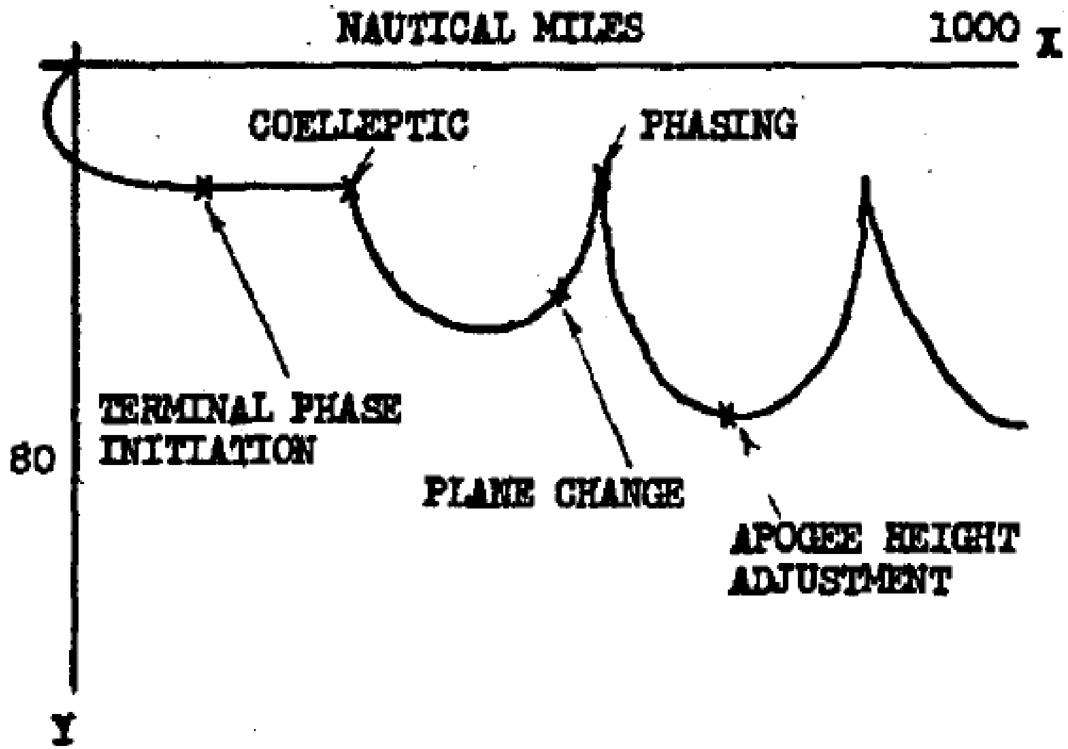


The maneuver sequence is described in relative motion coordinates in Figure 5.

Real Data: Gemini 10 and Agena 8

ORBITAL AND RENDEZVOUS REPORT FOR GEMINI 10
REPORT IX

Prepared by
Mission Design Department
TRW Systems

Maneuver Vehicle	Vehicle	S/C Revolution Number	S/C GET Time of Maneuver Begin: hr:min:sec End: hr:min:sec	Incremental Velocity Components (Platform Coordinates) (ft/sec) ΔV_x	ΔV_y	ΔV_z	Thruster	Perigee/Apogee Altitude Above Spherical Earth (n mi)
SECO + 20 second separation from GLV	S/C	1	00:05:58 00:06:10	10.0	0.0	0.0	AFT	87/146
Height adjustment maneuver (N_H) N = 1.5	S/C	1	01:35:01 01:35:02	0.6	0.0	0.0	AFT	87/146
Phase adjustment maneuver (N_{C1}) N = 2.0	S/C	2	02:18:49 02:20:04	54.7	0.0	0.0	AFT	117/146
Co-elliptical maneuver (N_{SR}) N = 3.0	S/C	3	03:48:09 03:49:18	51.0	-4.0	0.0	AFT	145/147
Terminal Phase Initiation (TPI)	S/C	3	04:36:18 04:36:56	29.3	-15.4	-0.2	AFT	146/162
Velocity Match (TPF)	S/C	4	05:08:59 05:10:00	23.6	38.1	0.1	FWD	161/161

Real Data: Gemini 10 and Agena 8

a	e	i	ω_p	Ω_A	M	G.m.t
			GEMINI 10 INSERTION VECTOR			
21635602.50	.0091077124	28.88382458	111.7816705	150.4869956	.5407994613	22:17:18.5
			HEIGHT MANEUVER (N_H)			
21635581.00	.0091301698	28.88438296	112.5349903	150.0048580	.9923863560	23:46:10.7
			PHASE MANEUVER (N_{CI})			
21727314.25	.0032482316	28.88482356	113.8641033	149.7649955	180.0633697	24:30:33.9
			CIRCULARIZATION MANEUVER (N_{SR})			
21814210.00	.00078151549	28.87523651	283.1691055	149.2975559	11.40916025	25:59:52.5
			TERMINAL PHASE INITIATION (TPI)			
21867044.0	.0032772192	28.88197708	115.2505702	149.0380783	11.51484549	26:47:46
			TERMINAL PHASE FINALIZATION (TPF)			
21903004.25	.00068123432	28.87105154	265.4333839	148.9218177	350.9567184	27:20:07.9
			AGENA 8 AT AGENA 10 INSERTION			
22246183.50	.0014997875	28.86819243	176.7434349	150.7829933	2.069301992	20:39:31

STS-120 (Rendez-vous with ISS, Oct. 07)

Date	Time UTC	Name	Impulsion (m/s)	Orbite (km x km)
23	15:46:44	MECO	----	229 x 56
23	16:16:38	OMS-2	70,74	296 x 229
23	18:32:45	NC1	26,06	319 x 296
24	08:50:56	NC2	6,13	319 x 316
24	19:27:09	NC3	0,91	319 x 317
25	07:30:35	NH	7,01	344 x 319
25	08:24:45	NC4	2,74	345 x 327
25	09:55:43	Ti	2,71	347 x 332
25	11:12:45	MC1	0,64	347 x 334
25	11:26:20	MC2	0,64	348 x 334

Pour STS123: http://www.obsat.com/sts123tle.htm

STS-120 (Rendez-vous with ISS, Oct. 07)

Space Shuttle Rendezvous Maneuvers

OMS-1 (Orbit insertion) – Rarely used ascent burn.

OMS-2 (Orbit insertion) – Typically used to circularize the initial orbit following ascent, completing orbital insertion. For ground-up rendezvous flights, also considered a rendezvous phasing burn.

NC (Rendezvous phasing) – Performed to hit a range relative to the target at a future time.

NH (Rendezvous height adjust) – Performed to hit a delta-height relative to the target at a future time.

NPC (Rendezvous plane change) – Performed to remove planar errors relative to the target at a future time.

NCC (Rendezvous corrective combination) – First on-board targeted burn in the rendezvous sequence. Using star tracker data, it is performed to remove phasing and height errors relative to the target at Ti.

Ti (Rendezvous terminal intercept) – Second on-board targeted burn in the rendezvous sequence. Using primarily rendezvous radar data, it places the orbiter on a trajectory to intercept the target in one orbit.

MC-1, MC-2, MC-3, MC-4 (Rendezvous midcourse burns) – These on-board targeted burns use star tracker and rendezvous radar data to correct the post Ti trajectory in preparation for the final, manual proximity operations phase.

In Summary

Maneuvers can find many practical applications and may drive spacecraft design (fuel consumption).

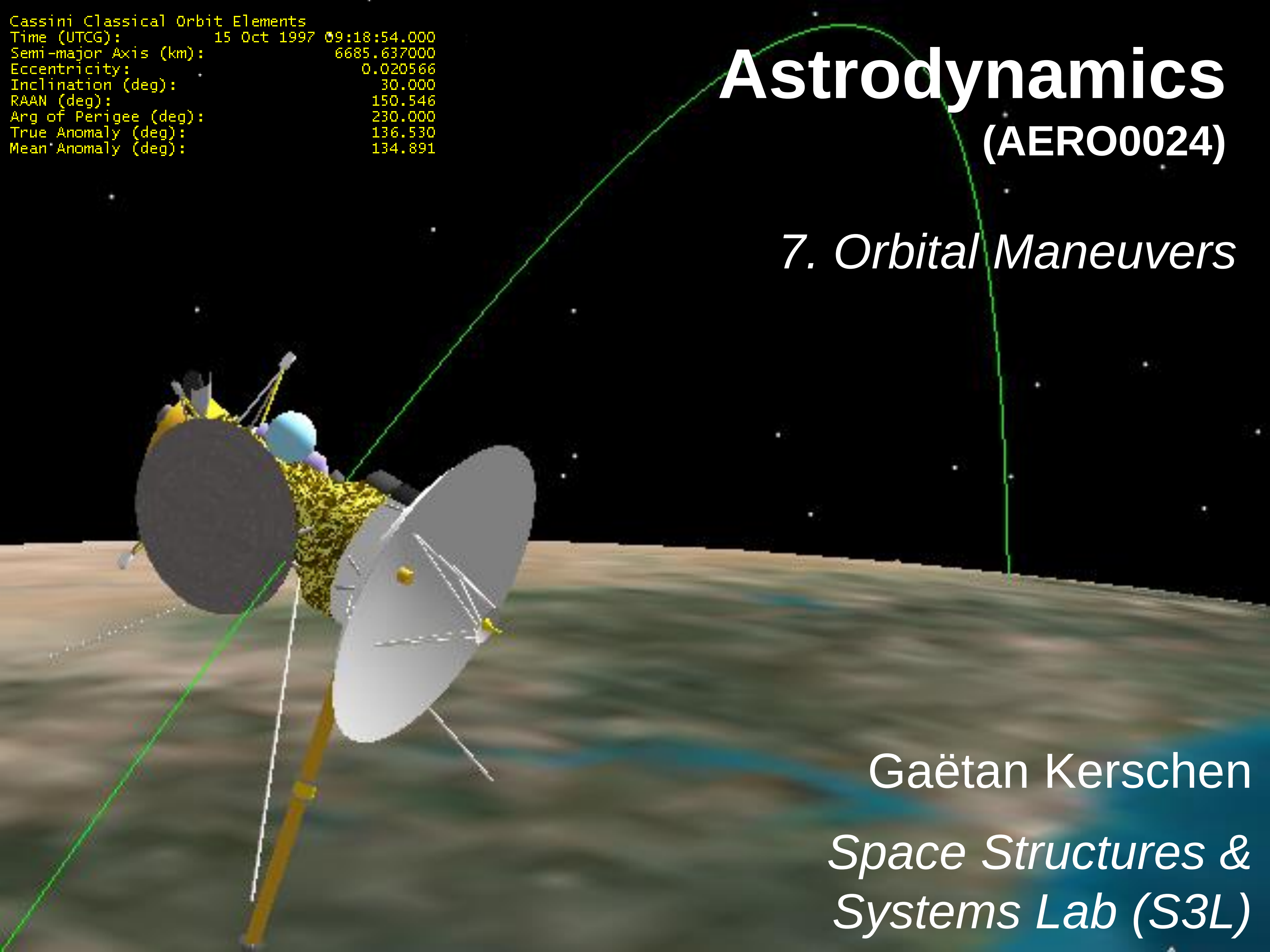
Focus on impulsive maneuvers (chemical propulsion).

Most important maneuvers in Earth's orbit have been described (plane change, circularization, phasing, rendez-vous).

Other maneuvers

Combined maneuvers (e.g., circularization and inclination change for a GEO satellite).

Fixed ΔV maneuvers.



Astrodynamics

(AERO0024)

7. Orbital Maneuvers

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