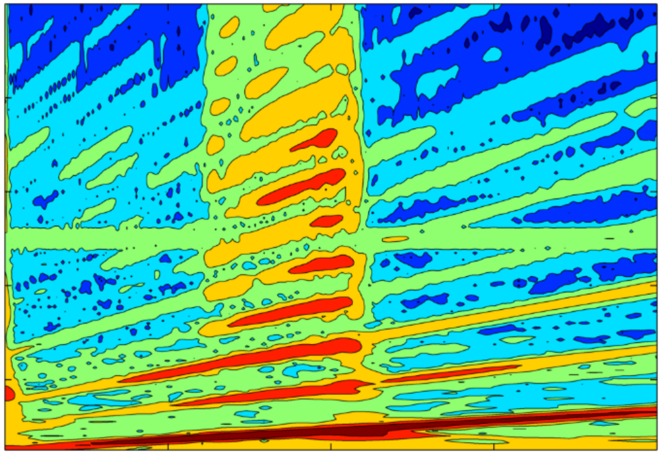
Nonlinear Vibrations of Aerospace Structures

University of Liège, Belgium

L08 Nonlinear Characterisation



Time-Frequency Analysis

Acceleration Surface Method

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Jean-Philippe Noël

Nonlinear System Identification: a Three-Step Process

1 Detection

Do I observe nonlinear effects? *Yes.*

Should I build a nonlinear model? *Yes.*

2 Characterisation

Where is the nonlinearity located? *At the joint.*

What is the underlying physics? *Dry friction.*

How to model its effects? $f_{nl}(q, \dot{q}) = c\ sign(\dot{q})$.

This lecture

3 Estimation

Model parameters? $c = 5.47$.

How uncertain are they? $c = \mathcal{N}(5.47, 1)$.

Information vs. complexity

Computer-Aided Modelling Is Useful but…

Paris aircraft,
ONERA, France.

Bolted connection between wing tip and fuel tank.

Complex geometry.
Multi-scale physics.
Model parameters?
Applied torque?

Objective of this Lecture

Infer from experimental data a suitable nonlinearity model.

This is challenging:

Prior knowledge is most often very limited.

Physical mechanisms resulting in nonlinearity are extremely diverse.

Nonlinearity may translate into a plethora of dynamic phenomena.

This is crucial:

The success of the parameter estimation step is conditional upon an accurate characterisation of all observed nonlinearities.

Typical Questions to be Answered

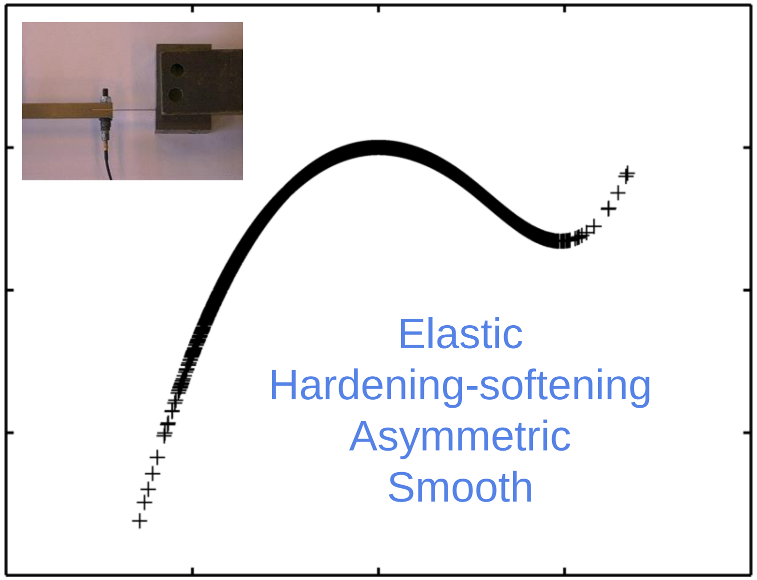
Is nonlinearity elastic or dissipative?

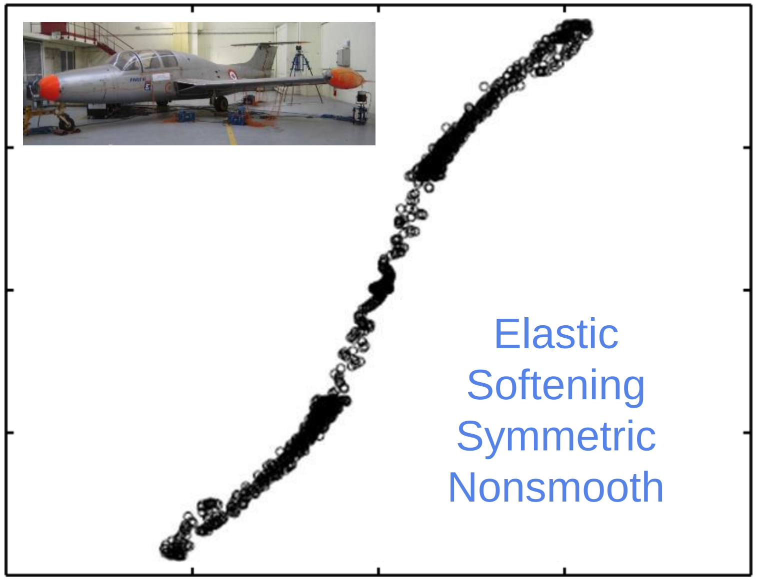
Is nonlinearity hardening or softening?

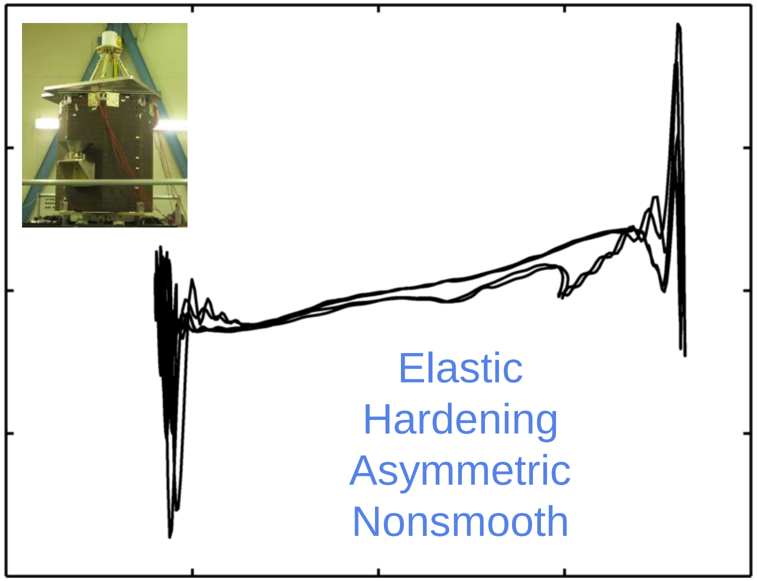
Is nonlinearity symmetric or asymmetric?

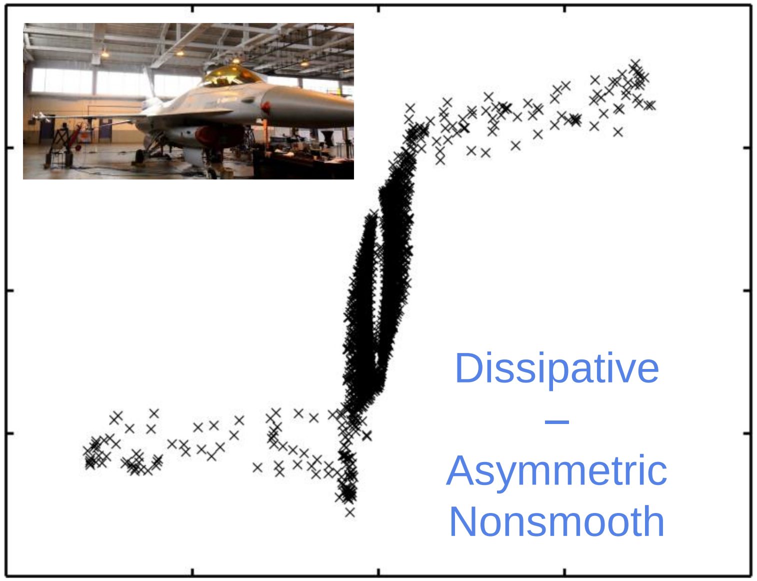
Is nonlinearity smooth or nonsmooth?

Reminder: Individualistic Nature of Structural Nonlinearities









Importance of the Toolbox Philosophy

Different methods bring different perspectives to the dynamics.

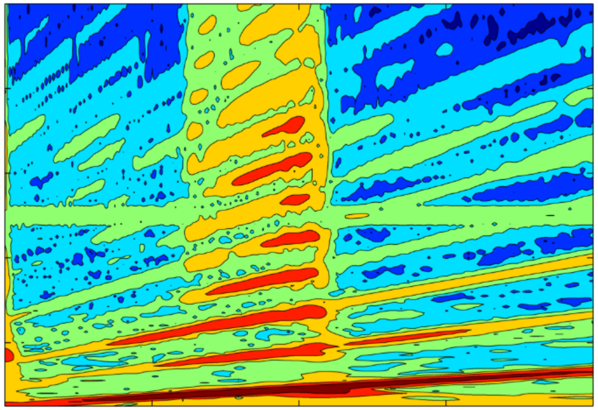
1. Time-frequency analysis:

   Reveals the frequency-amplitude dependence of NL oscillations.

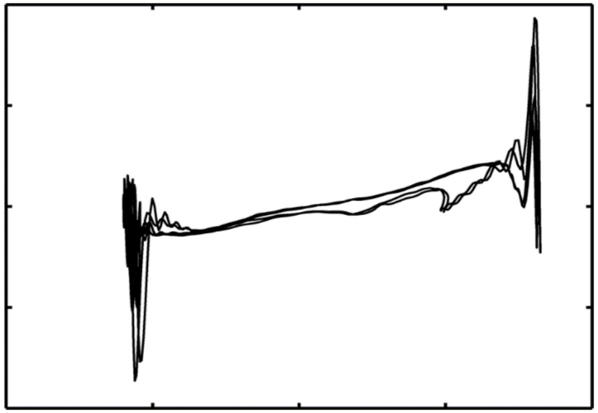
2. Restoring force plots:

   Provide a direct visualisation of NL stiffness and damping curves.

Outline of Lecture 8



Time-frequency analysis using the wavelet transform (WT).



Acceleration surface method (ASM).

Nonlinear Frequency Spectra Are Highly Informative

Is nonlinearity elastic or dissipative?

Resonance frequencies are not affected much by dissipative NLs.

Is nonlinearity hardening or softening?

Resonance frequencies increase or decrease with amplitude.

Is nonlinearity symmetric or asymmetric?

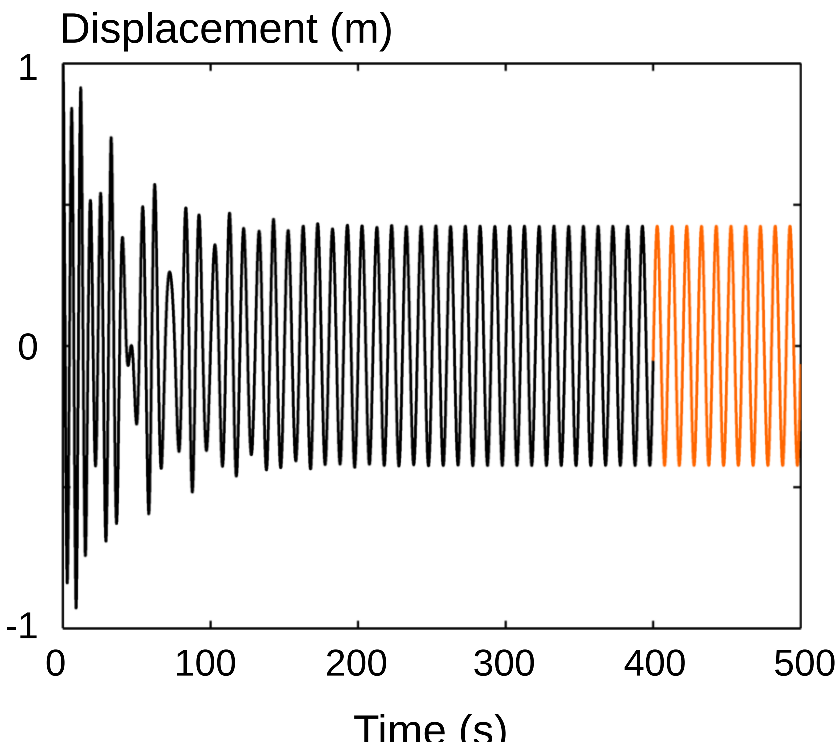
Asymmetries generate important even harmonic components.

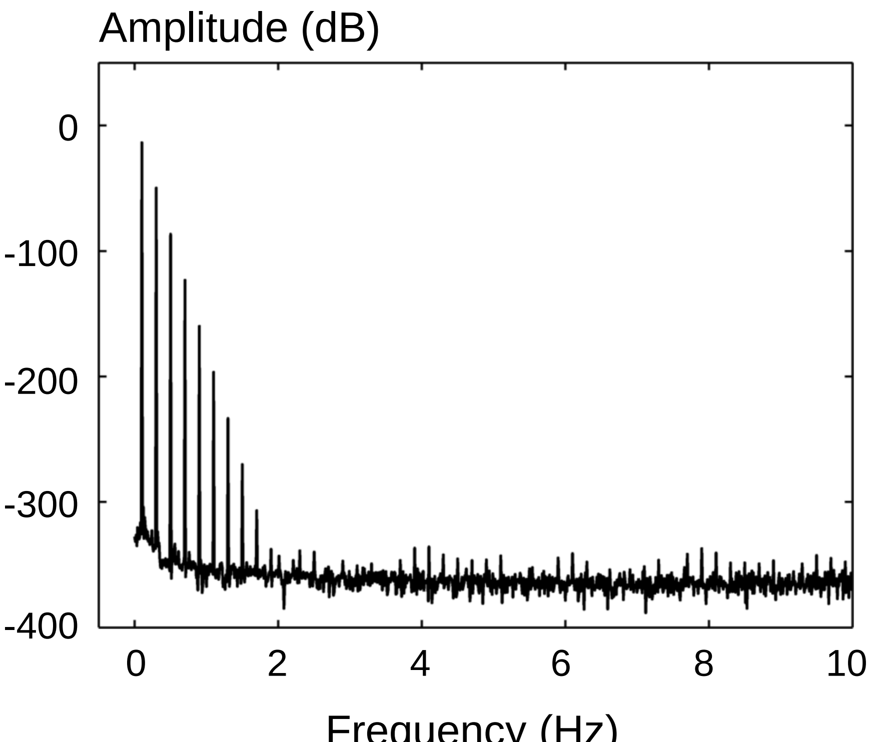
Is nonlinearity smooth or nonsmooth?

Nonsmoothness generates wideband frequency components.

Reminder: the Fourier Transform (FT)

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

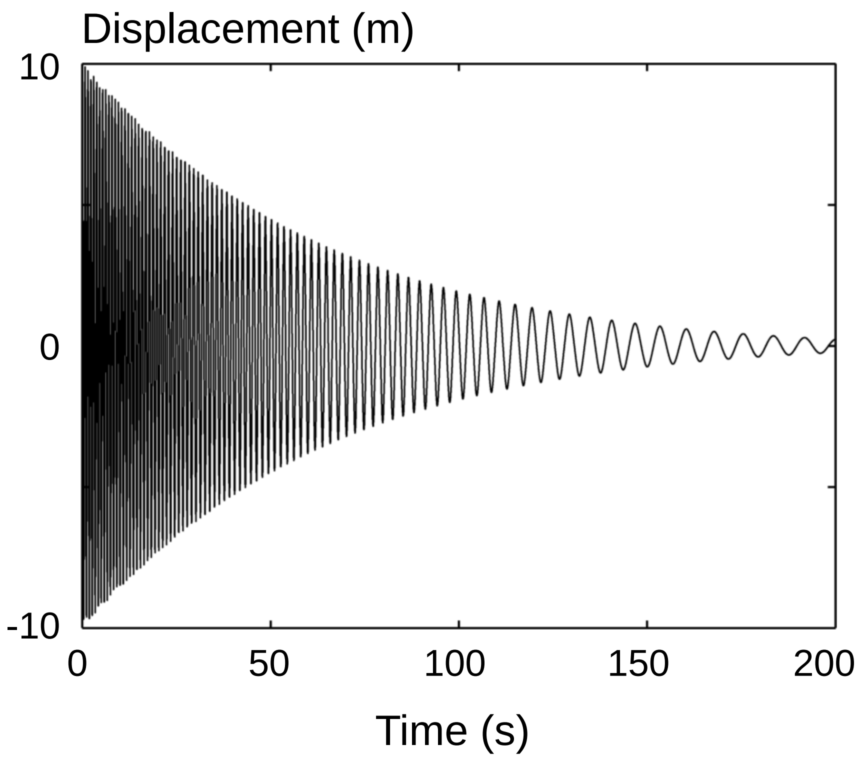


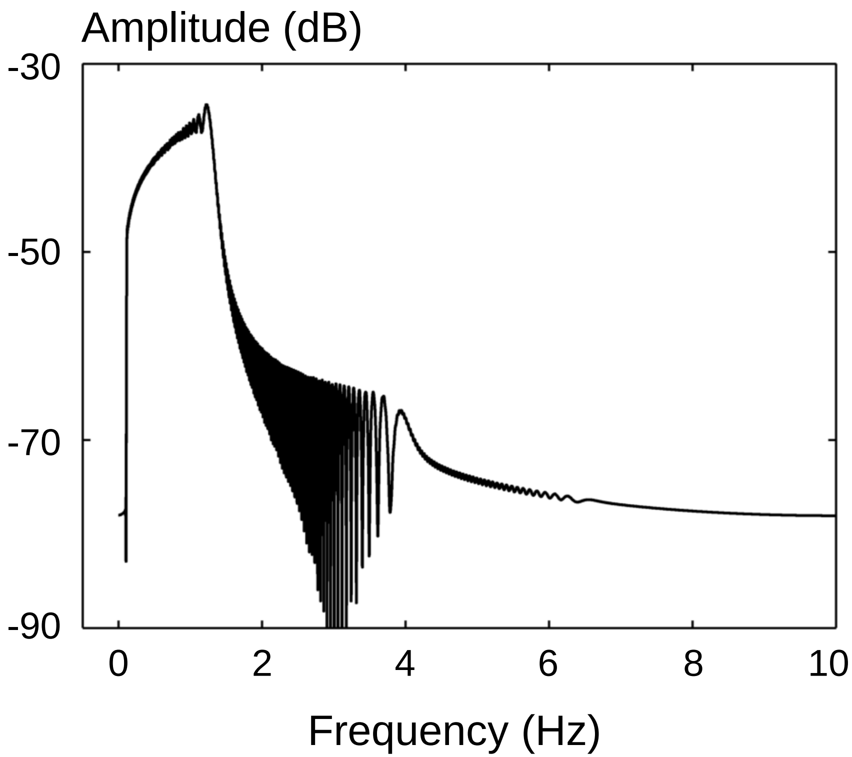


FT Fails to Capture Time-Varying Frequencies

$$1\,\ddot{q} + 0.05\,\dot{q} + 0.5\,q + 1\,q^3 = 0$$

with $q_0 = 10$ and $\dot{q}_0 = 0$





The Short-Time Fourier Transform (STFT)

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$\boxed{X(\omega, \tau)} = \int_{-\infty}^{+\infty} x(t) \boxed{w(t - \tau)} e^{-j\omega t} dt$$

Time-frequency representation.

Observation window is nonzero for a short period of time.

Drawback: the observation window is the same for all frequencies.

The Wavelet Transform (WT)

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$\boxed{X(a,b)} = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \boxed{\psi\left(\frac{t-b}{a}\right)} dt$$

Time-frequency representation.

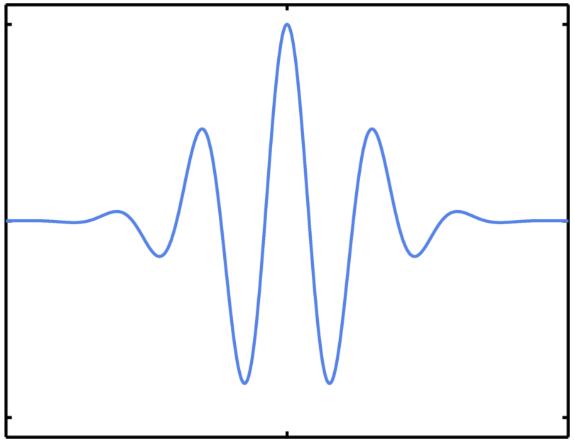
Mother wavelet = windowing strategy with variable resolution.

The Morlet Wavelet: a Gaussian-windowed Complex Sinusoid

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$X(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \boxed{\psi\left(\frac{t-b}{a}\right)} dt$$

$$\psi(t) = e^{-t^2/2}\, e^{j\sigma t}$$

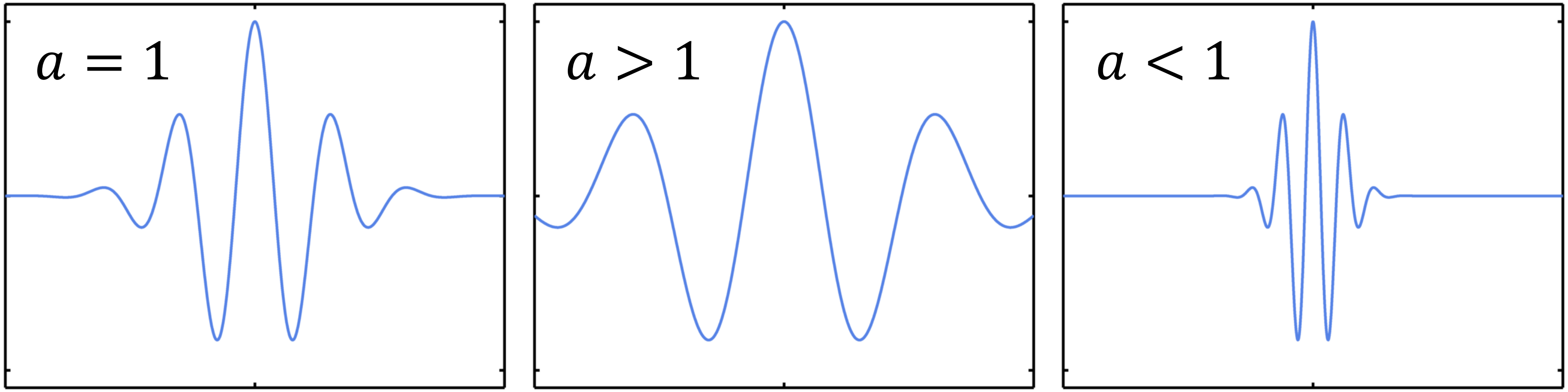


Windowing Strategy with Variable-Sized Regions

$$X(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\, \psi\left(\frac{t-b}{a}\right) dt$$

b locates of the observation window in the time domain.

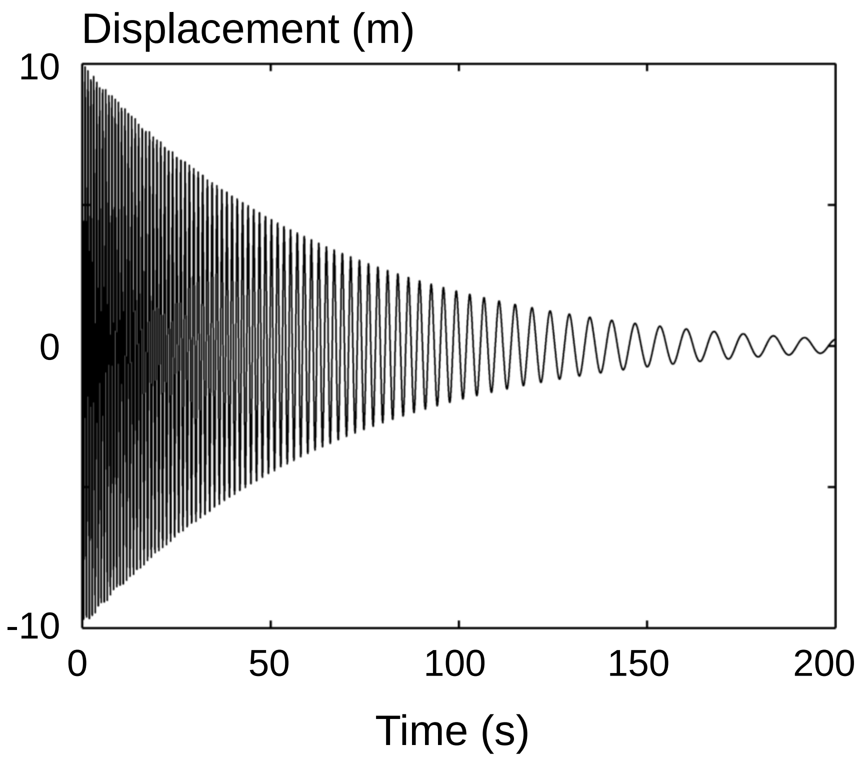
a defines the freq. resolution by expanding/contracting the window.

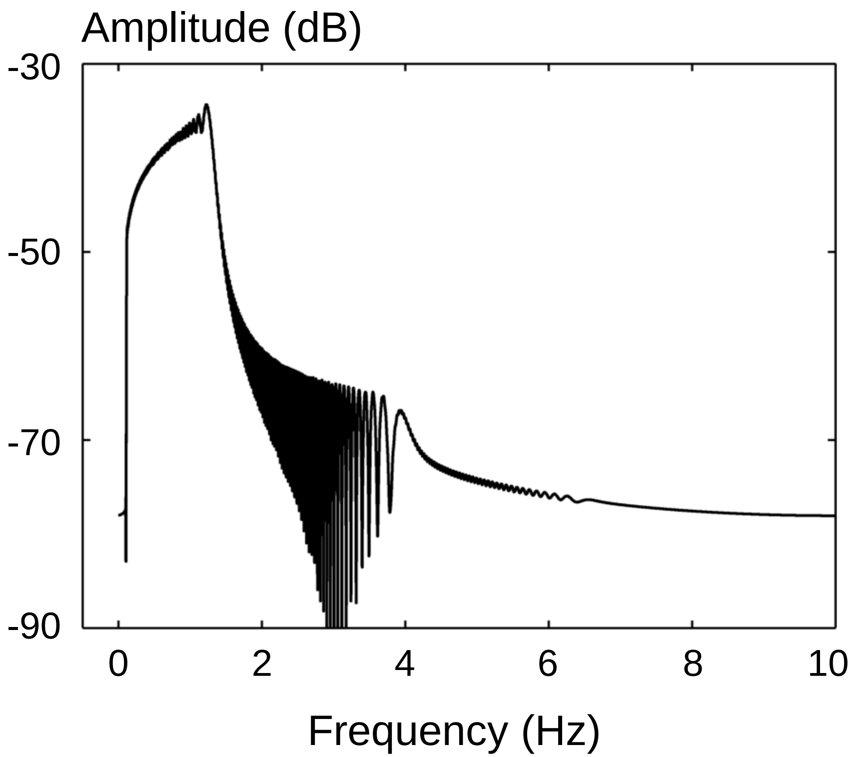


Reminder: Fourier Transform Applied to Free-Decay Data

$$1\,\ddot{q} + 0.05\,\dot{q} + 0.5\,q + 1\,q^3 = 0$$

with $q_0 = 10$ and $\dot{q}_0 = 0$

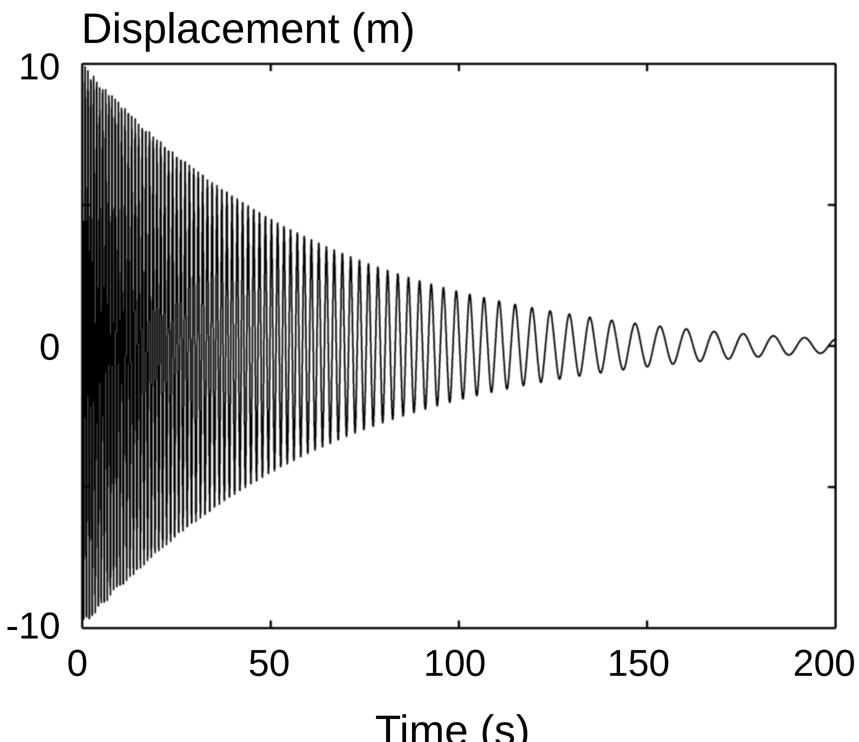


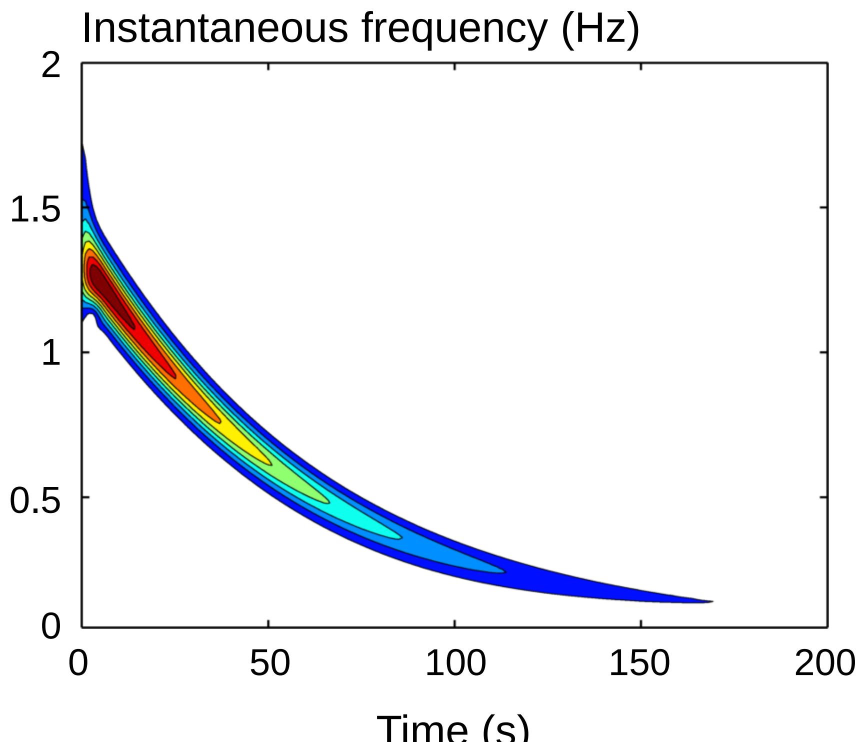


WT Highlights the Amplitude-Dependence of NL Oscillations

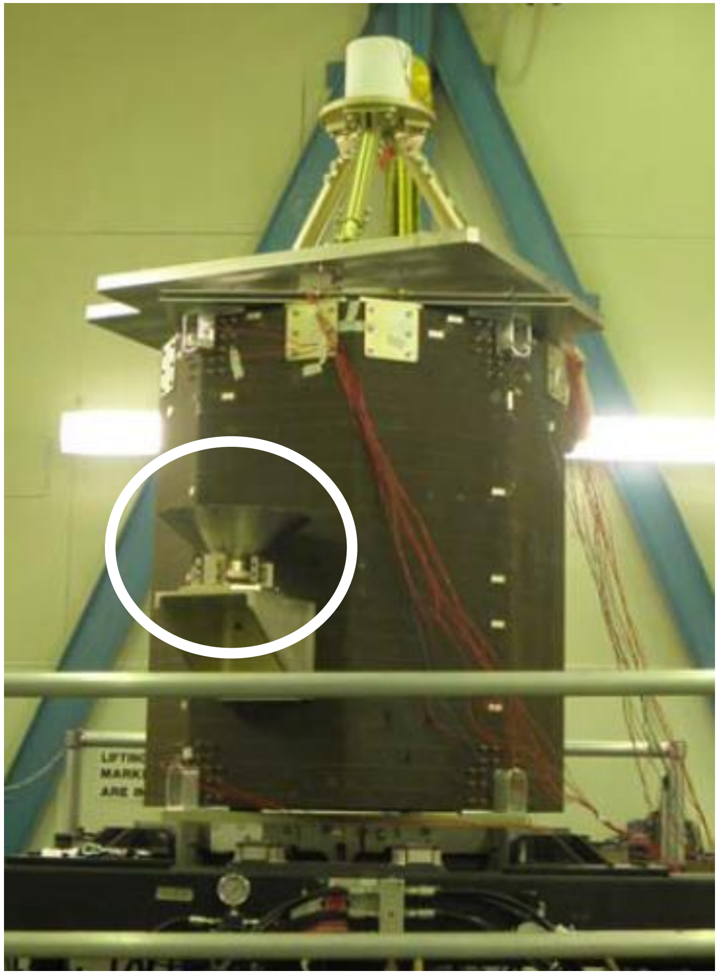
$$1\,\ddot{q} + 0.05\,\dot{q} + 0.5\,q + 1\,q^3 = 0$$

with $q_0 = 10$ and $\dot{q}_0 = 0$

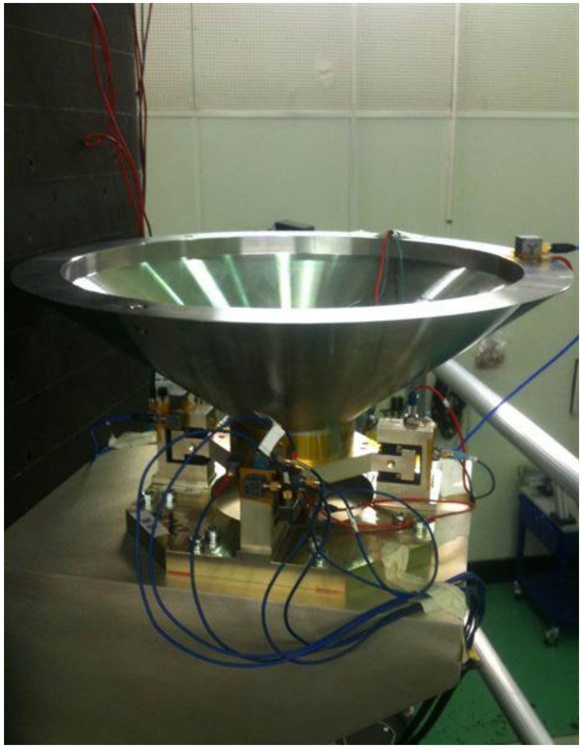




Wavelet Transform Applied to the SmallSat Spacecraft

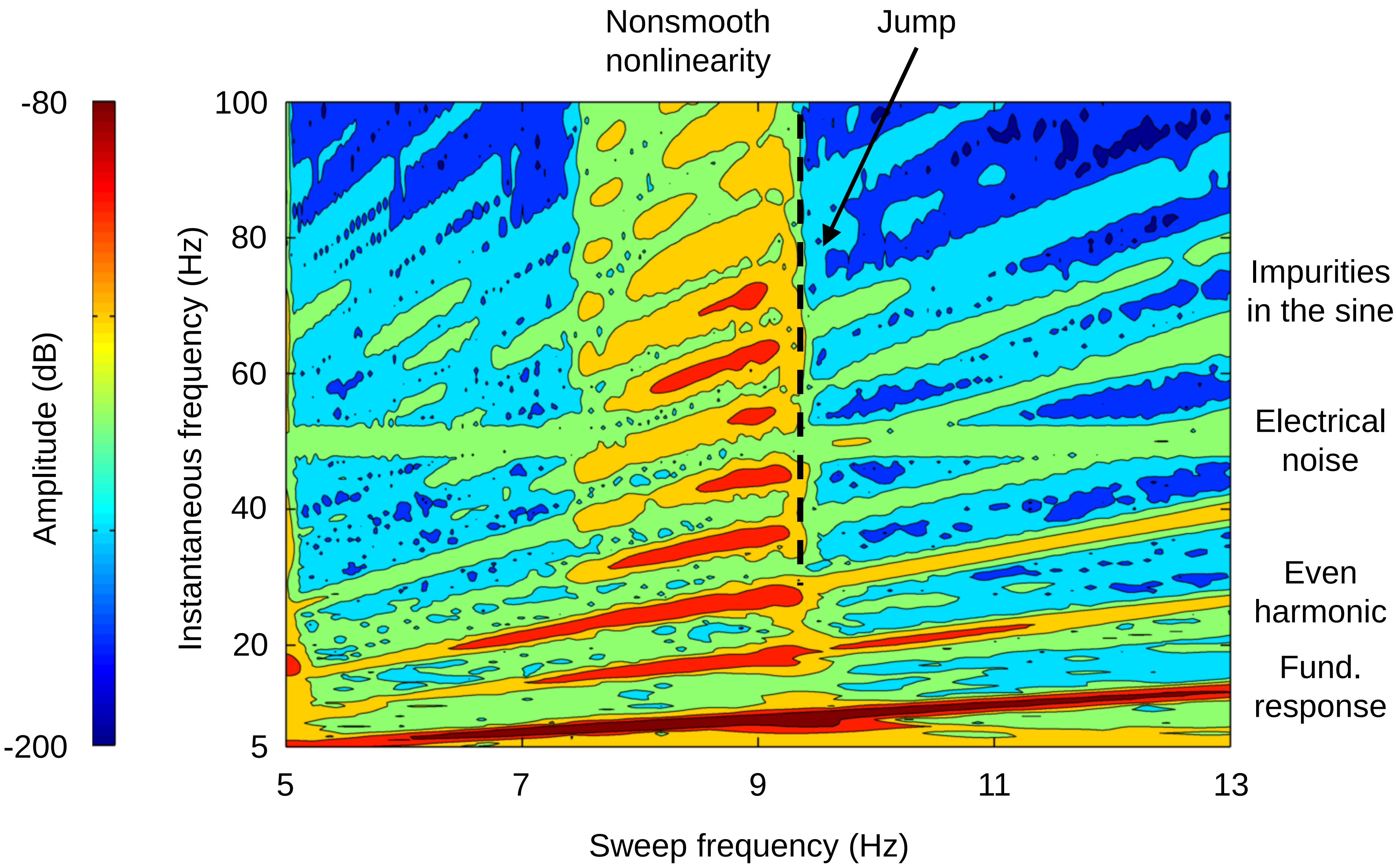


Test campaign in
Stevenage, UK.

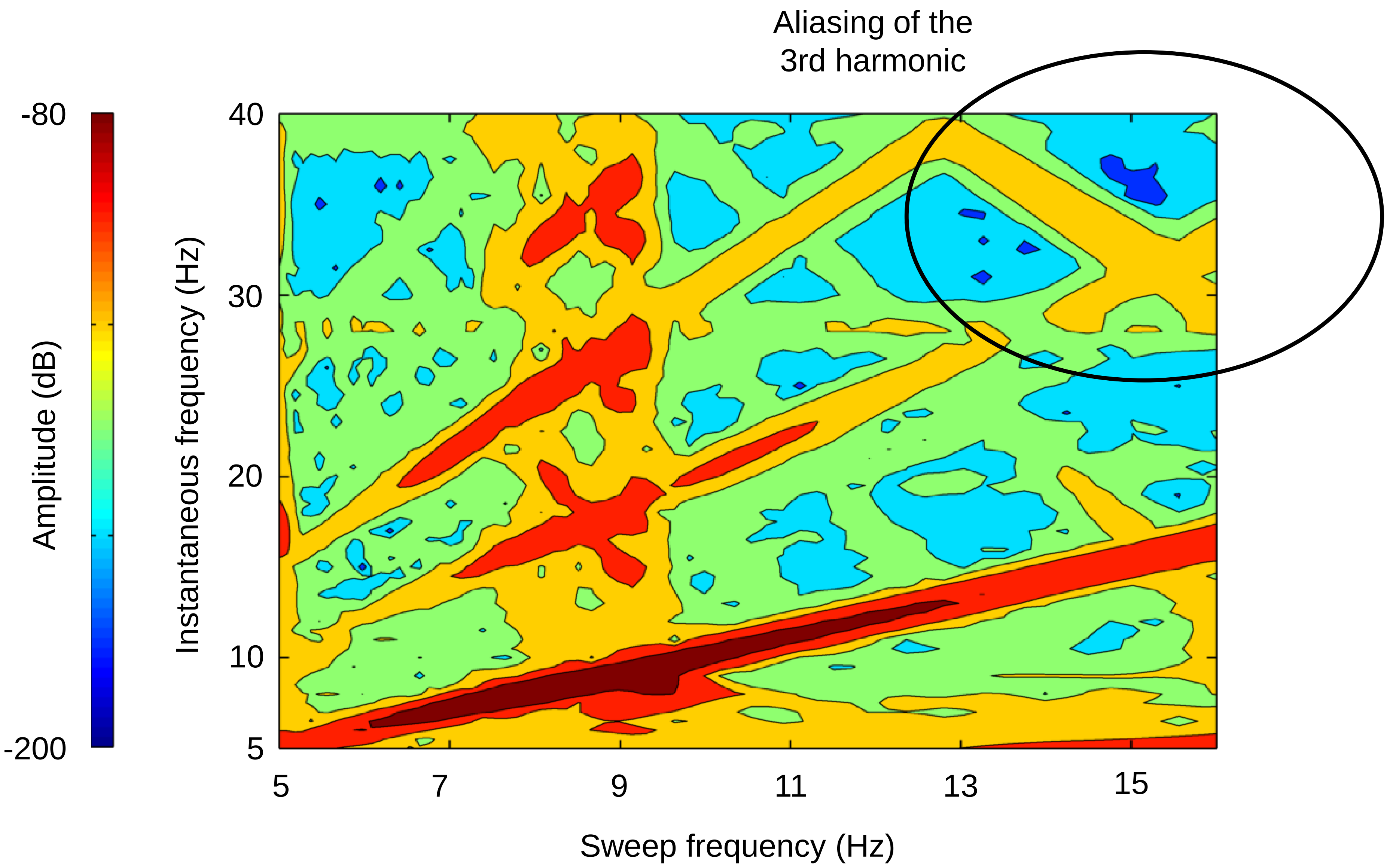


NL WEMS device.

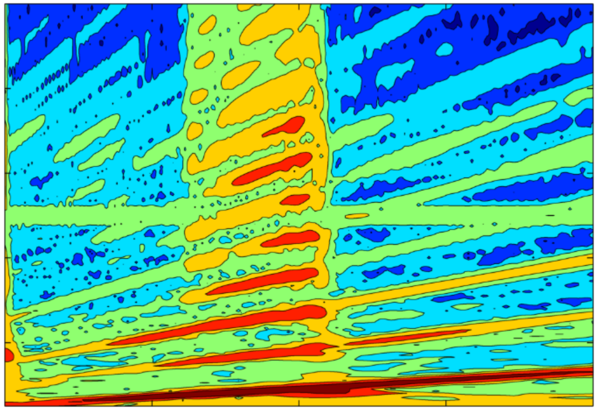
High-Level Data Convey Very Rich Information



Reminder: Choose a Sufficiently High Sampling Frequency



Outline of Lecture 8



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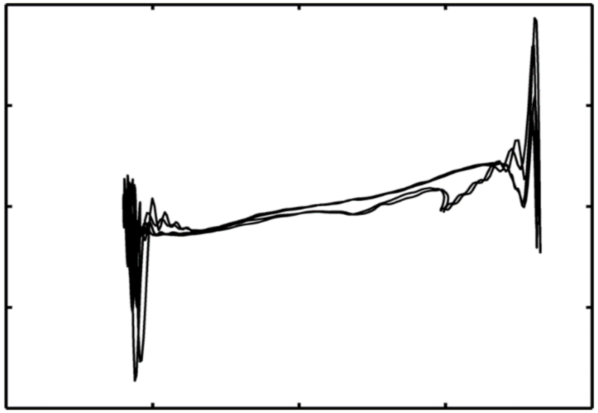


Acceleration surface method (ASM).

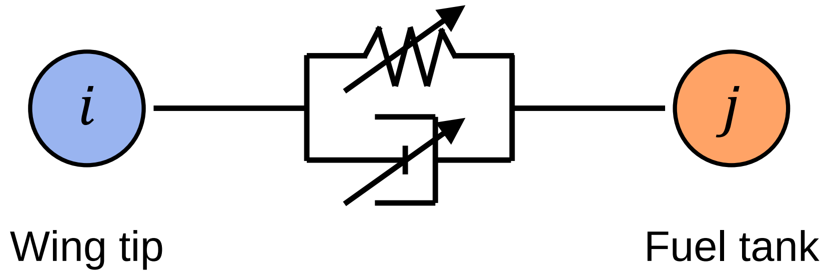
How Can We Visualise the NL Behaviour of this Connection?

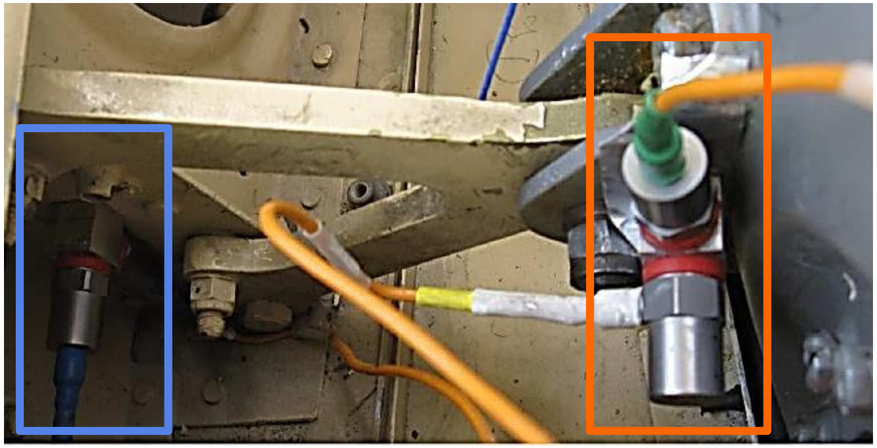
Paris aircraft, ONERA, France.

Bolted connection involving complex dynamics.

Physical insight is key to parametric modelling.

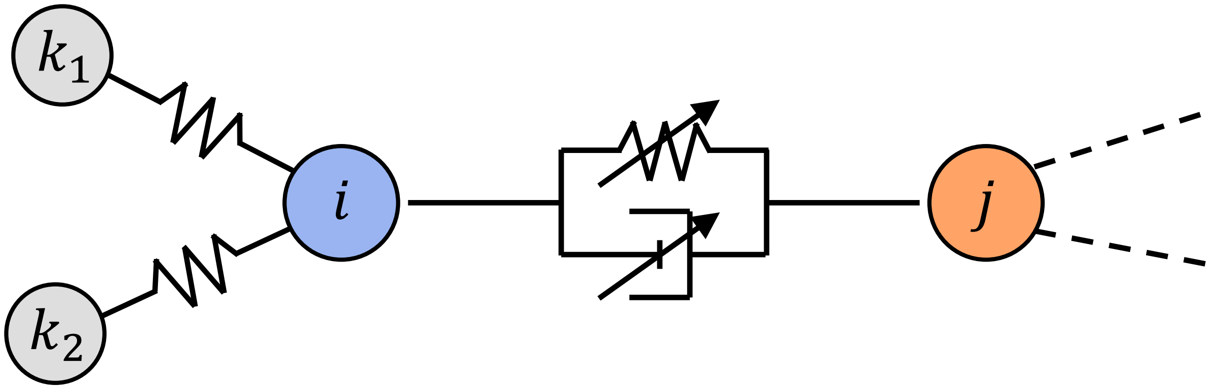
Macroscopic Idealisation as Lumped Spring and Dashpot





Potential nonlinear connections must be instrumented on both sides.

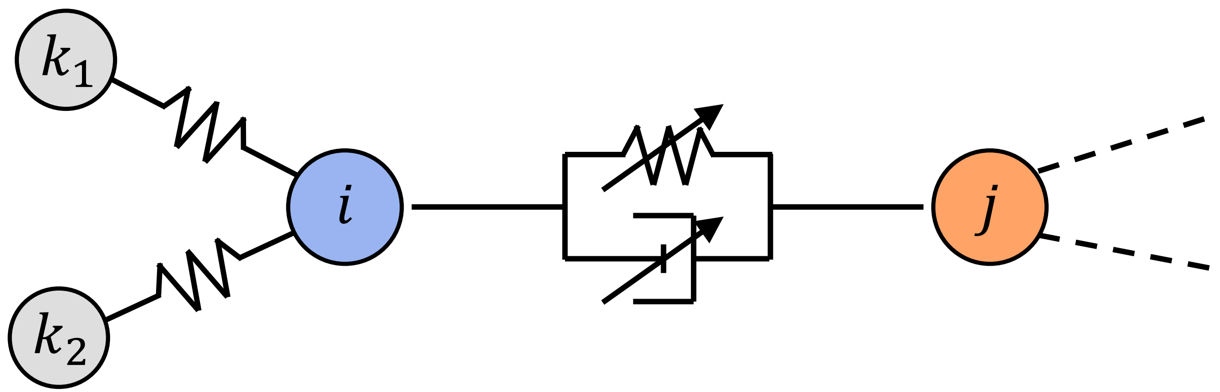
Newton's Second Law Written at Degree of Freedom 'i'



Linear connections
to neighbouring DOFs
(*e.g.*, bending stiffnesses in a wing)

$$\sum_k m_{i,k}\ \ddot{q}_k + g_i(q,\dot{q}) = p_i$$

Discard all Terms not Related to the Nonlinear Connection

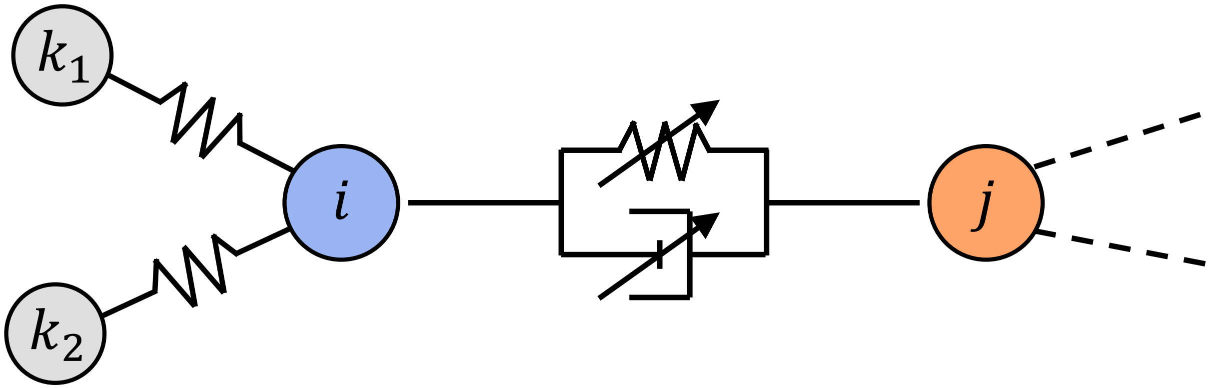


Linear connections
to neighbouring DOFs
(*e.g.*, bending stiffnesses in a wing)

$$\sum_k m_{i,k}\ \ddot{q}_k + g_i(q, \dot{q}) \ = p_i$$

$$m_{i,i}\ \ddot{q}_i + g_i(q_i - q_j, \dot{q}_i - \dot{q}_j) \cong p_i$$

Assume no Forcing Term and Drop the Mass Constant



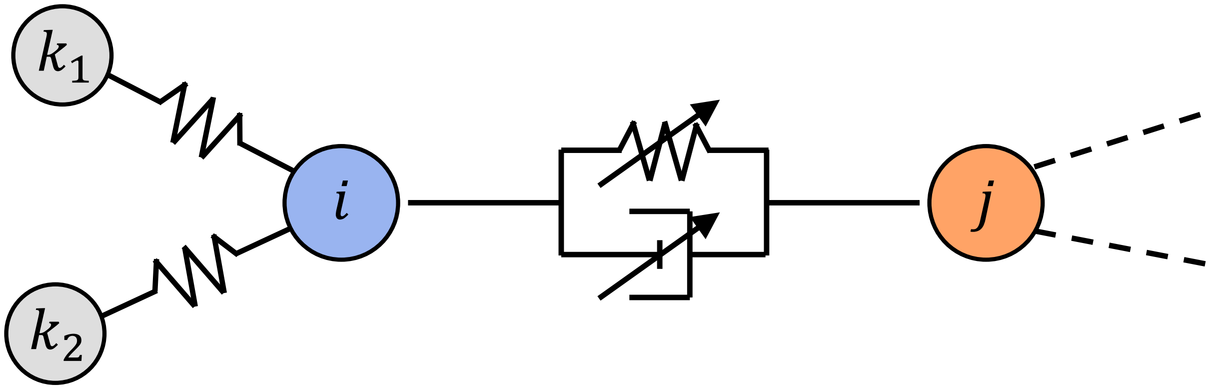
Linear connections
to neighbouring DOFs
(*e.g.*, bending stiffnesses in a wing)

$$\sum_k m_{i,k}\ \ddot{q}_k + g_i(q,\dot{q}) = p_i$$

$$g_i\left(q_i - q_j, \dot{q}_i - \dot{q}_j\right) \cong -\ddot{q}_i$$

NL can be visualised!

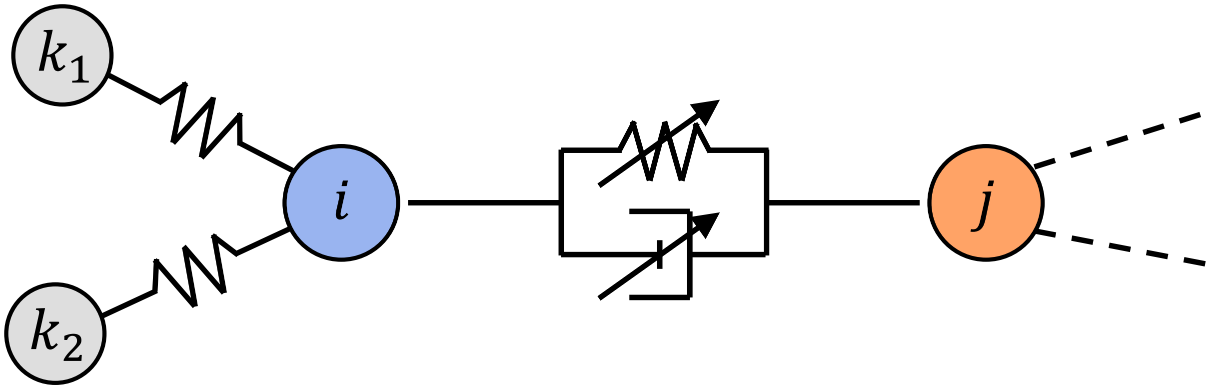
ASM in Summary: 4 Instrumentation and Processing Steps



$$g_i(q_i - q_j, \dot{q}_i - \dot{q}_j) \cong -\ddot{q}_i$$

1. Instrument the nonlinear connection with 2 accelerometers.
2. Integrate and filter to get displacement and velocity time series.
3. Calculate the 3D acceleration surface over a single mode.
4. Consider surface slices to obtain stiffness and damping curves.

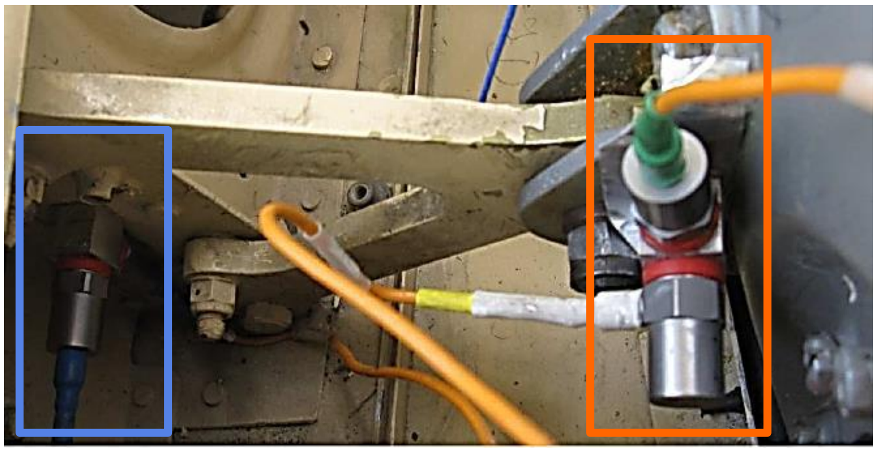
ASM in Summary: Assumptions and Strengths

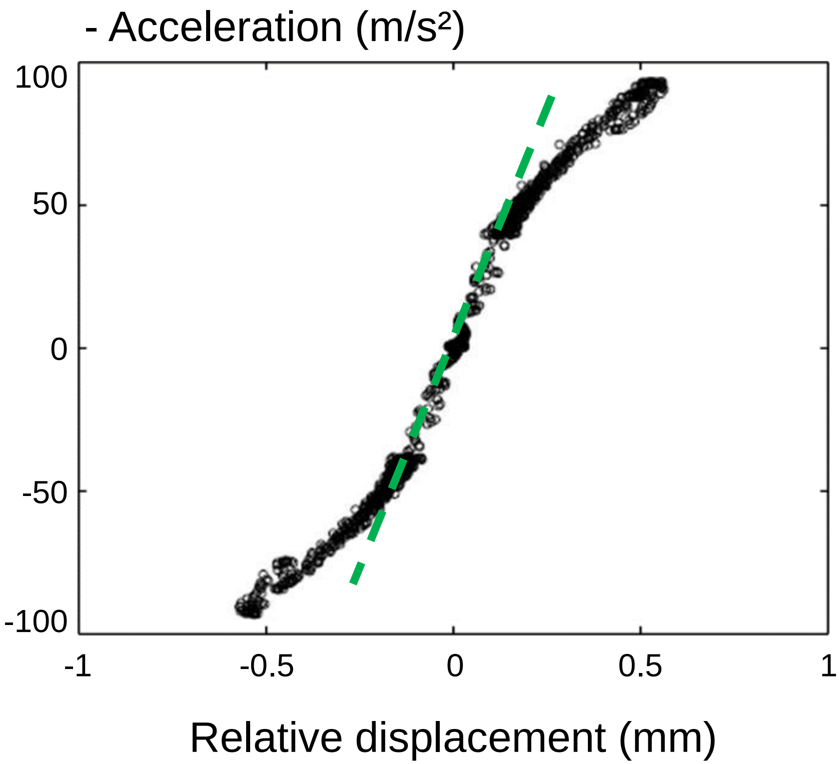


$$g_i(q_i - q_j, \dot{q}_i - \dot{q}_j) \cong -\ddot{q}_i$$

1. Exploits a SDOF (single-mode) simplification of the EOMs.
2. Works better with swept-sine (stepped-sine) excitations.
3. Relies exclusively on measured time series.
4. Can be easily understood.

A Softening, Symmetric, Nonsmooth Behaviour Is Revealed

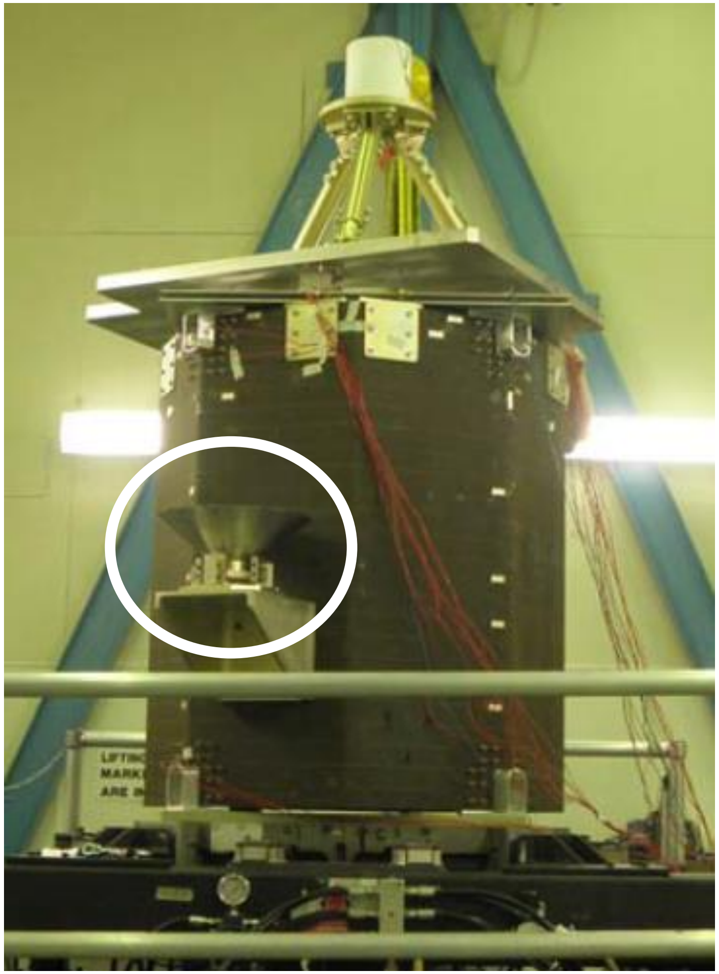




Note on instrumentation:
joints between substructures are generic candidates.

Opening of the connection translates into a sudden loss of stiffness.

ASM Applied to the SmallSat Spacecraft



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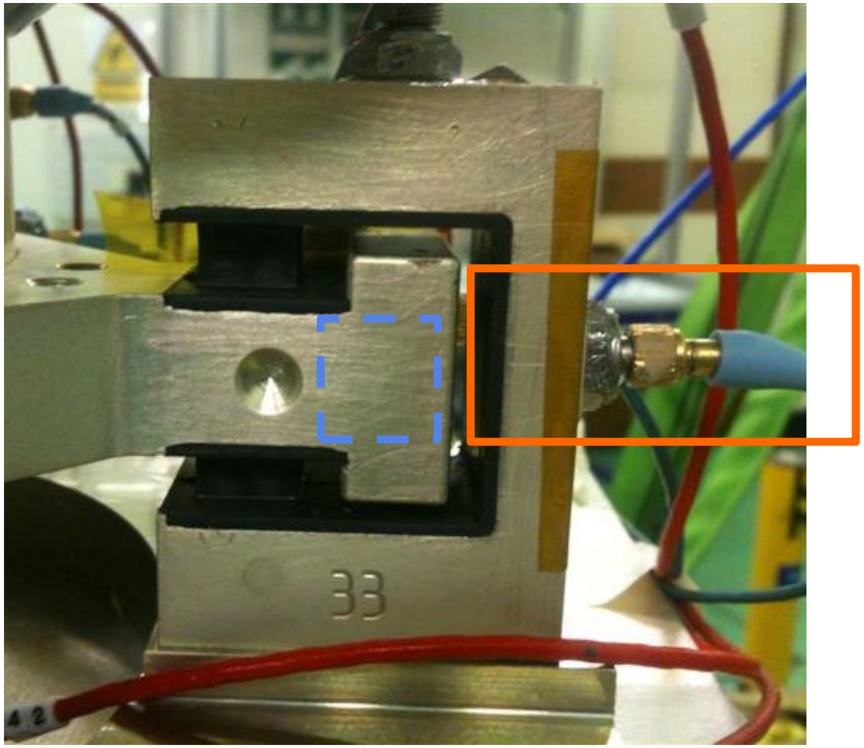
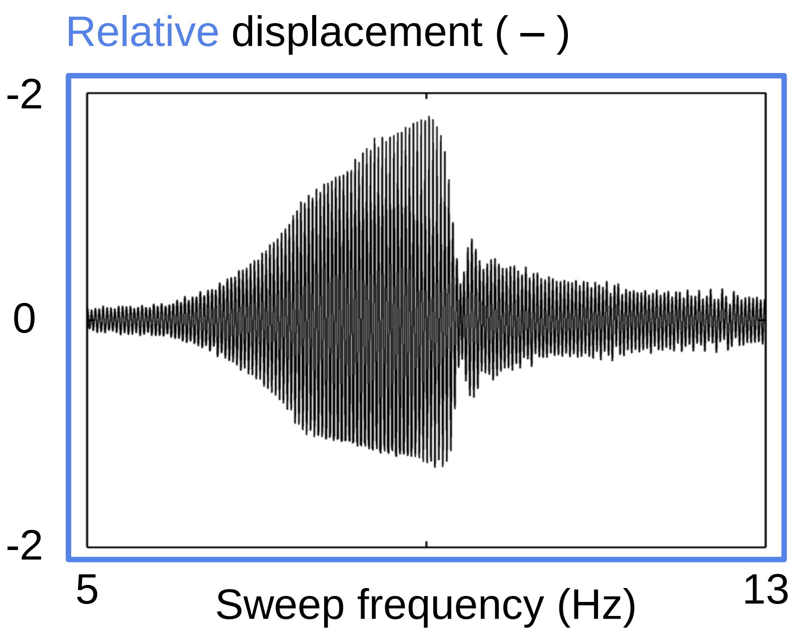
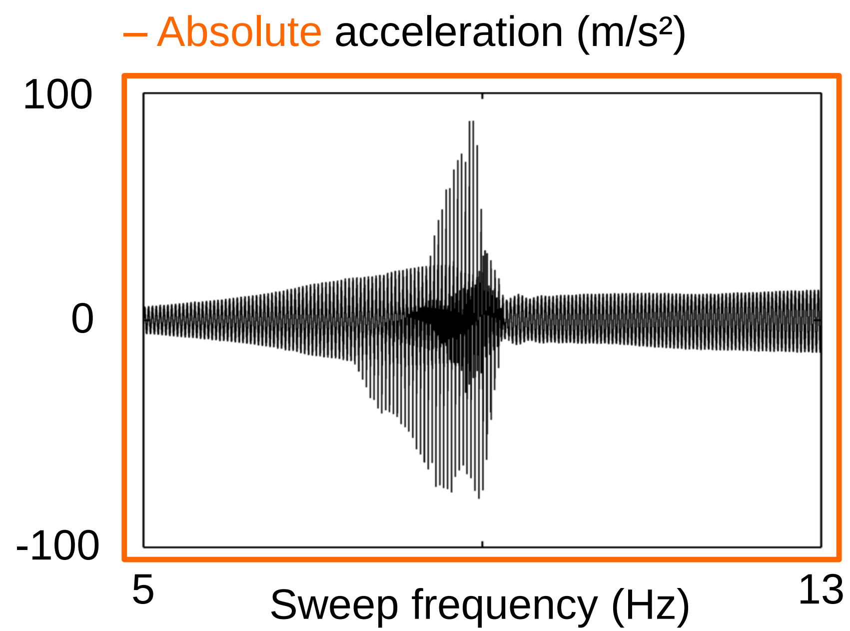


Accelerometers
positioned on both sides.

Stiffness Curve Is Obtained by Considering Small Velocities





$$g_i\left(\underline{q_i - q_j}, \underline{\dot{q}_i - \dot{q}_j}\right) \cong \underline{-\,\ddot{q}_i}$$

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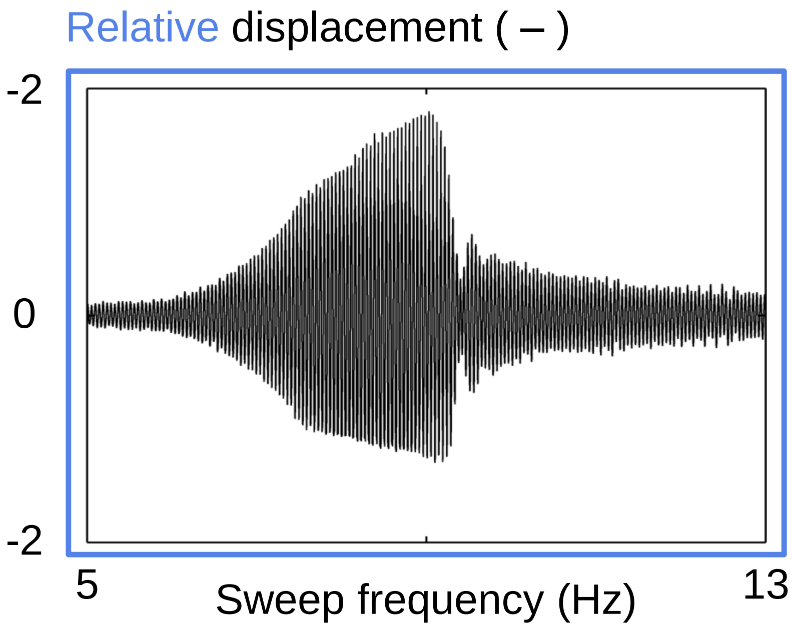
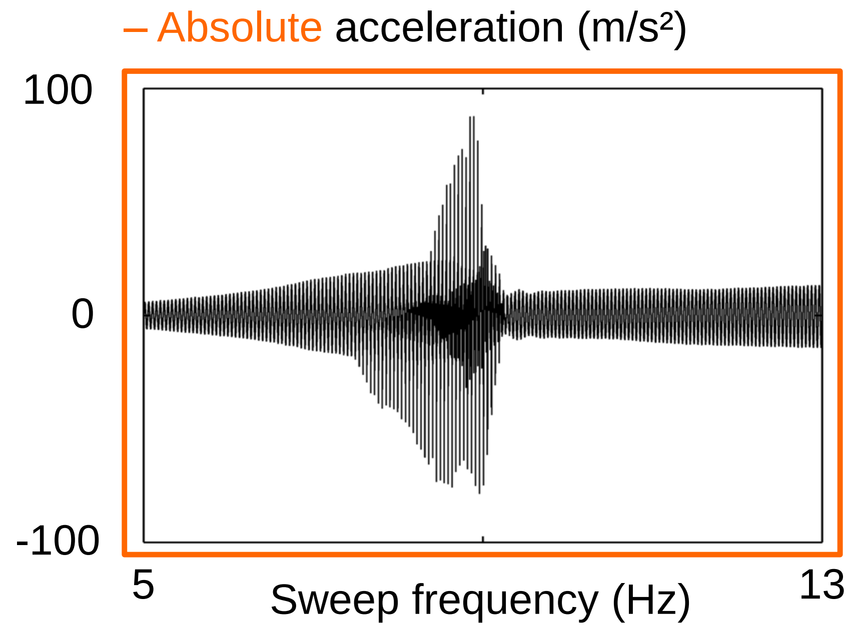
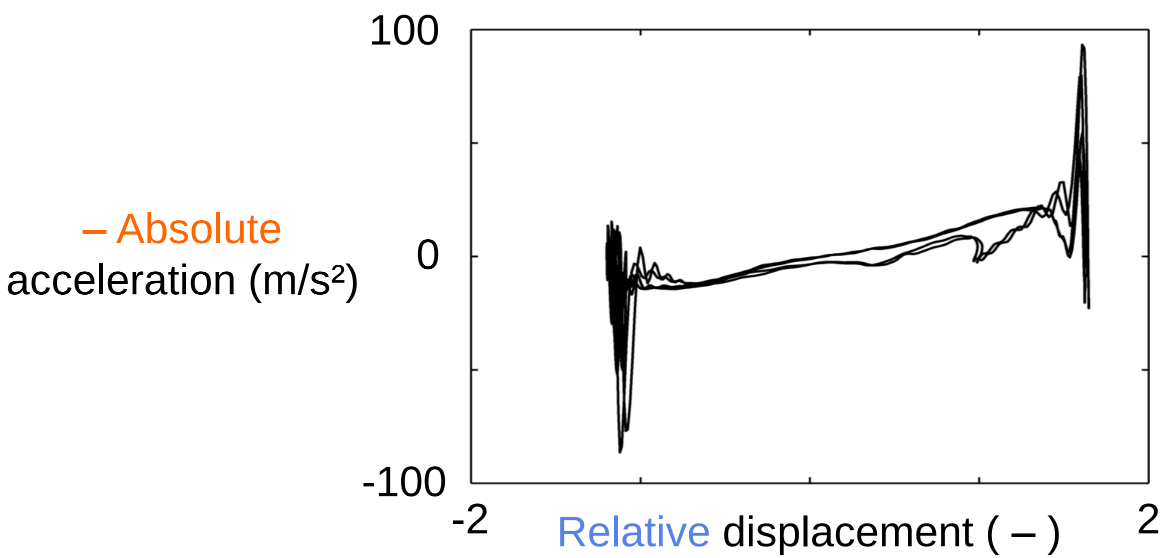
A Hardening, Asymmetric, Nonsmooth NL Is Visualised







Concluding Remarks and Learning Outcomes

Two complementary methods for nonlinearity characterisation. But, engineering insight and experience are equally important.

WT and ASM can be easily understood.

Instrument nonlinearities on both sides and apply sine excitations.

Damping characterisation remains a difficult endeavour.

Further Readings

J.P. Noël, G. Kerschen, **Nonlinear system identification in structural dynamics: 10 more years of progress**, Mechanical Systems and Signal Processing, 83, 2-35, 2016.

J.P. Noël, L. Renson, G. Kerschen, **Complex dynamics of a nonlinear aerospace structure: Experimental identification and modal interactions**, Journal of Sound and Vibration, 333, 2588-2607, 2014.