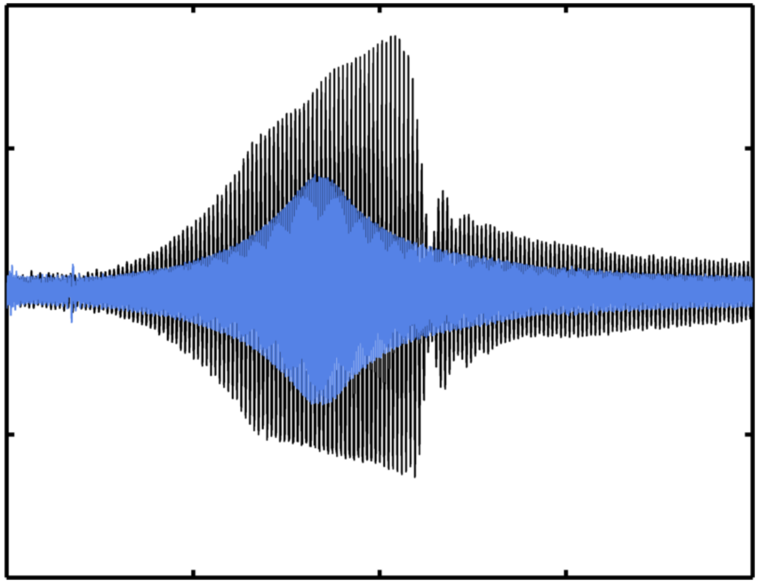
Nonlinear Vibrations of Aerospace Structures

University of Liège, Belgium

L07 Nonlinearity Detection

- Introduction to NSI
- Excitation signals
- Detection



Ghislain Raze | g.raze@uliege.be
Jean-Philippe Noël

System Identification in Structural Dynamics?

Construction of an accurate mathematical model of the dynamics of a real structure based on input and output measurements.

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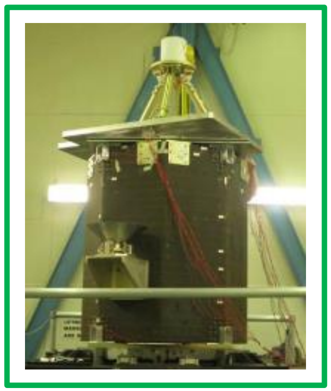
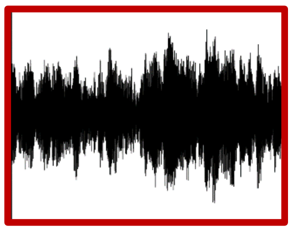
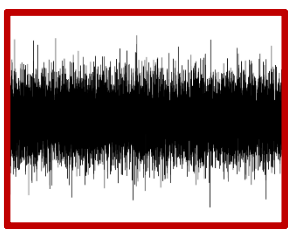


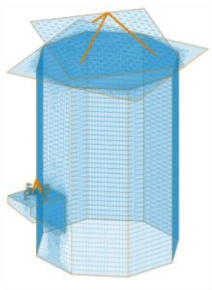




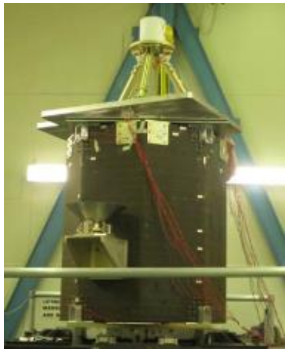
Cost function

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{g}(\mathbf{x}, \mathbf{u}, \mathbf{y}, \boldsymbol{\theta}) \\ \mathbf{y}(t) = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{y}, \boldsymbol{\theta}) \end{cases}$$

NSI in the Design Cycle of Engineering Structures



Computer-aided modelling (FEM, …)



MEASURE → IDENTIFY → MODEL → UNDERSTD UNCOVER → DESIGN

Outline of Lecture 7

Why is nonlinear system identification difficult?

Driving factors of progress over the past decade.

NSI: a three-step process.

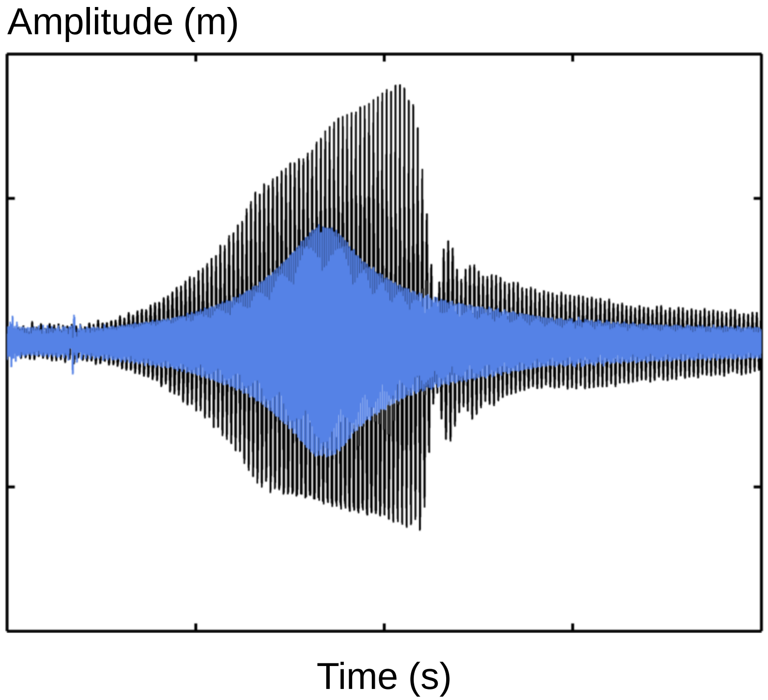
Two good excitation signals.

Nonlinearity detection in sine and broadband conditions.

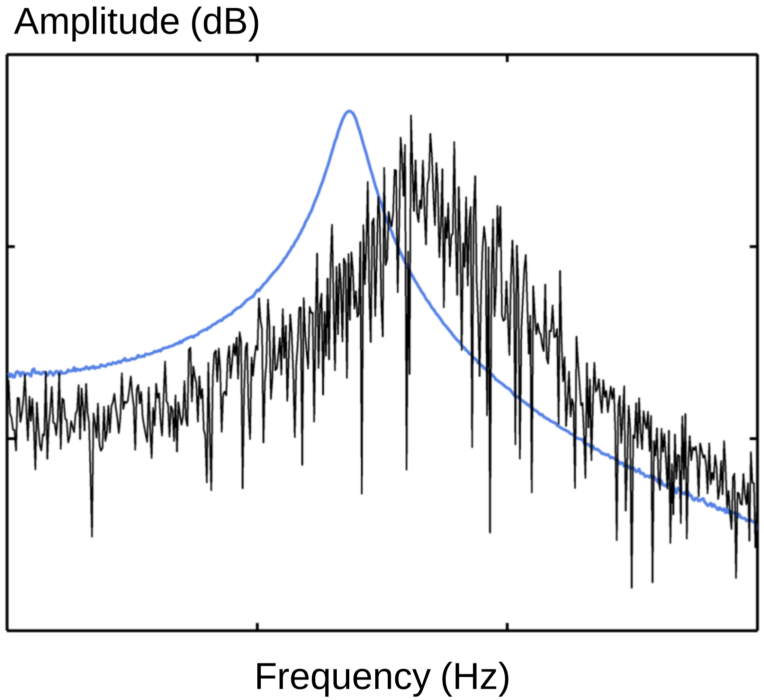
Why Is Nonlinear System Identification Difficult?

Challenge #1: Sensitivity to the type of input signal.

Distorted resonance
under sine excitation.



Distorted resonance
under random excitation.



Why Is Nonlinear System Identification Difficult?

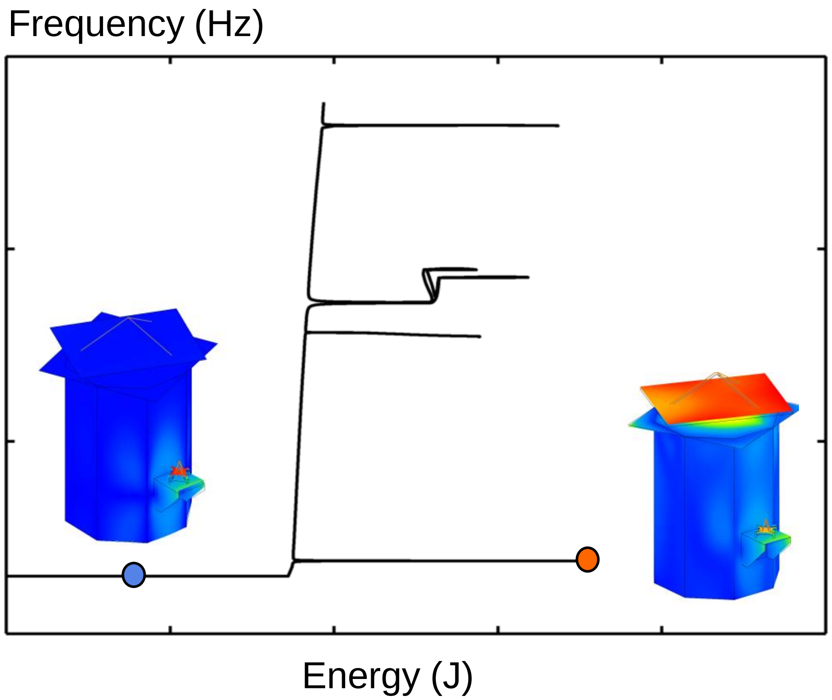
Challenge #2: Specific and demanding test campaigns.

- Increase the sampling frequency.
- Record throughput time histories.
- Measure the force signal.
- Instrument potential nonlinearities with sensors on both sides.
- Consider multiple levels of excitations/types of excitations.
- Study multiple sets of initial conditions, and reverse the sweep.

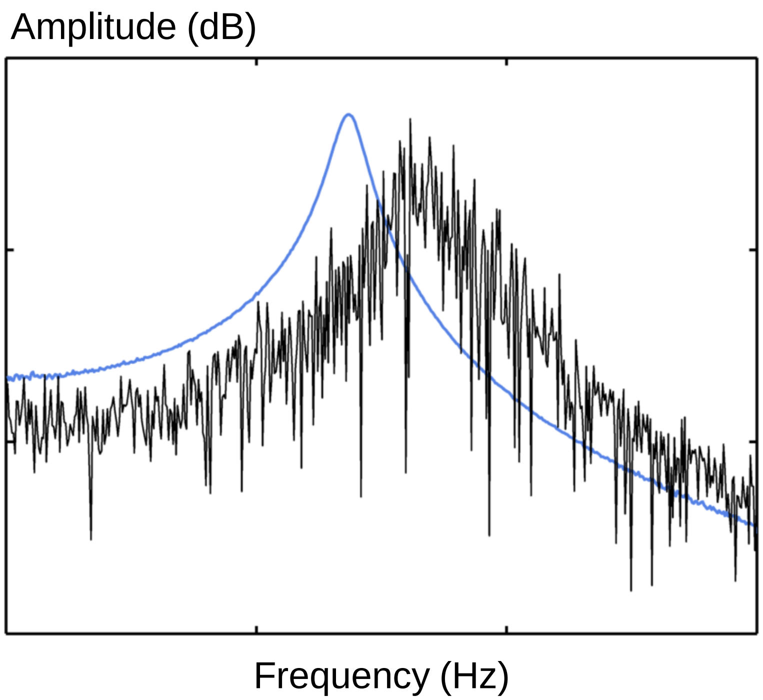
Why Is Nonlinear System Identification Difficult?

Challenge #3: Inapplicability of linear concepts.

Modal properties are not invariant.

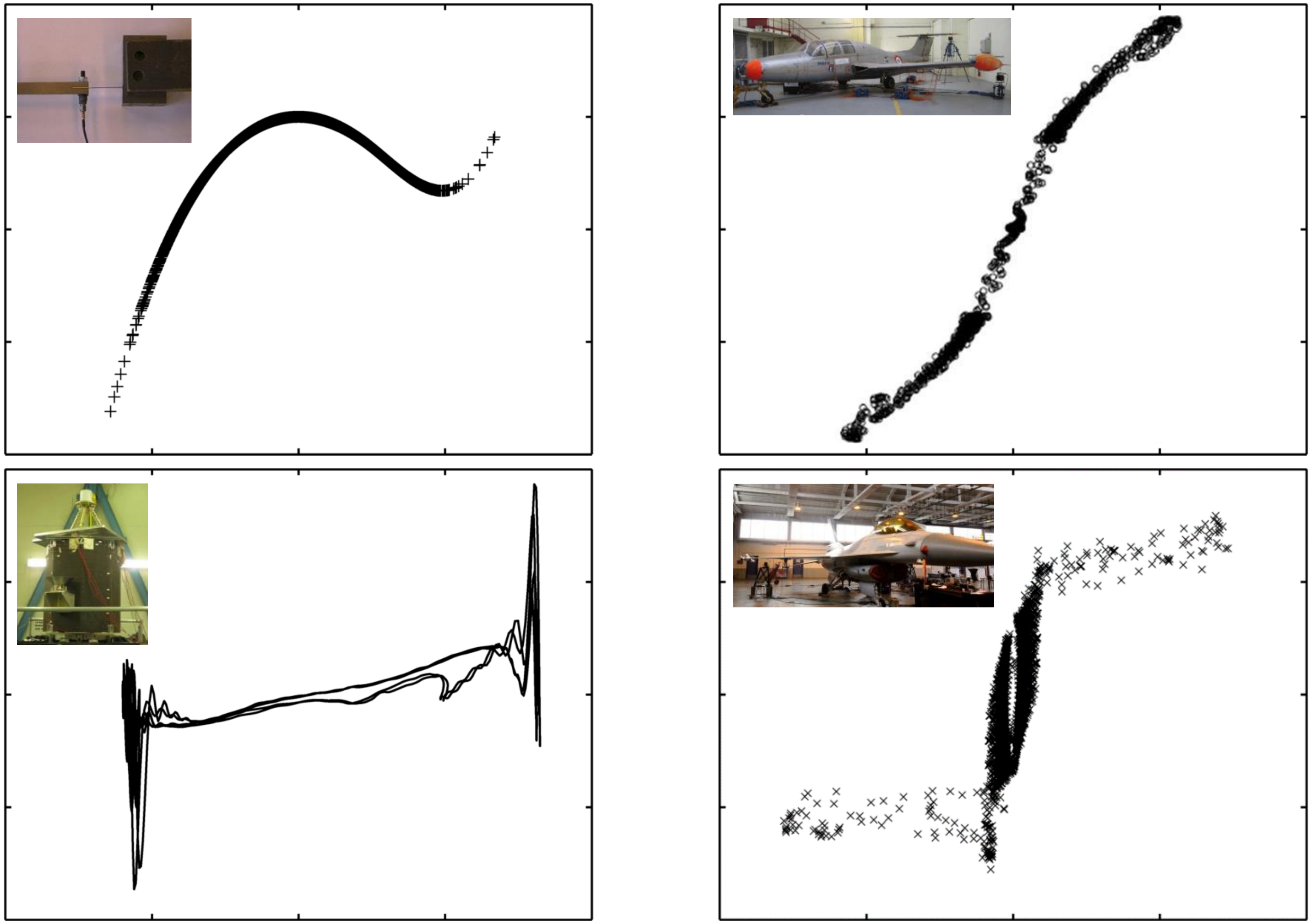


FRFs are not invariant.



Why Is Nonlinear System Identification Difficult?

Challenge #4: Individualistic nature of structural nonlinearities.



Why Is Nonlinear System Identification Difficult?

Challenge #5: Limited amount of prior knowledge.

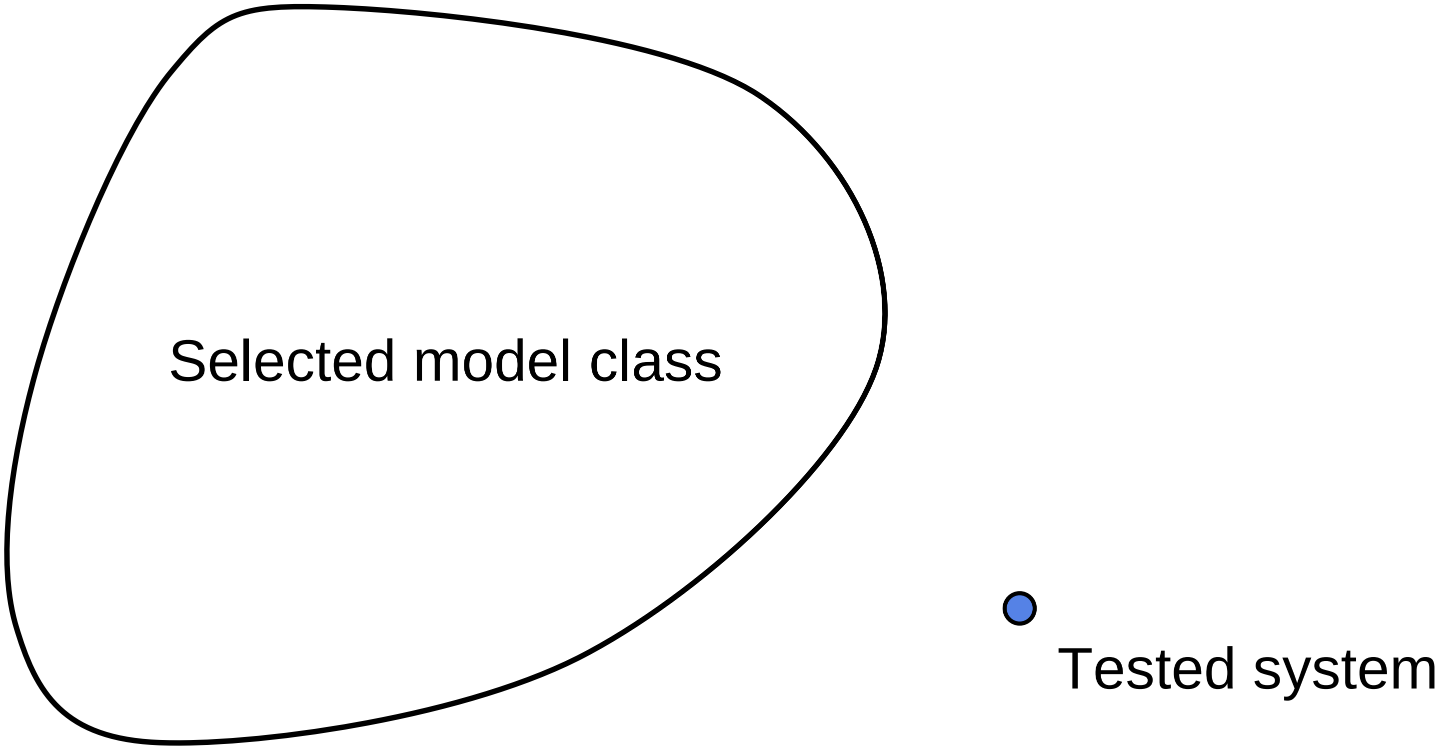
$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C}_v\,\dot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} + \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{p}$$

?

Determining nonlinear functional forms is an integral part of nonlinear system identification (and arguably the most difficult).

Why Is Nonlinear System Identification Difficult?

Challenge #6: True system may be outside the model class.



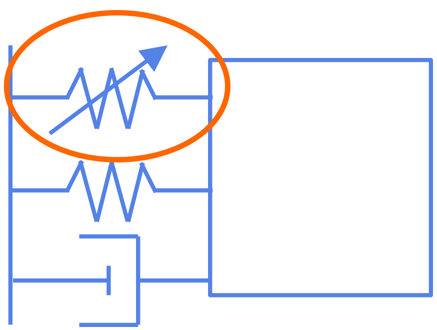
Where Do We Stand?

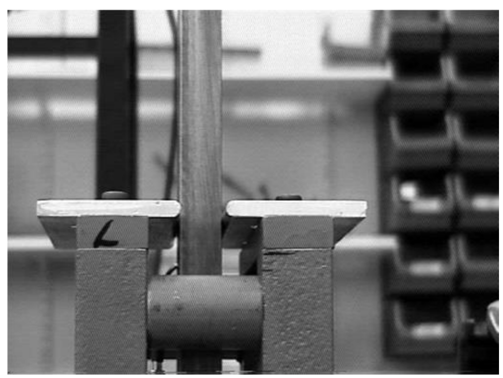
First analyses of SDOF systems.

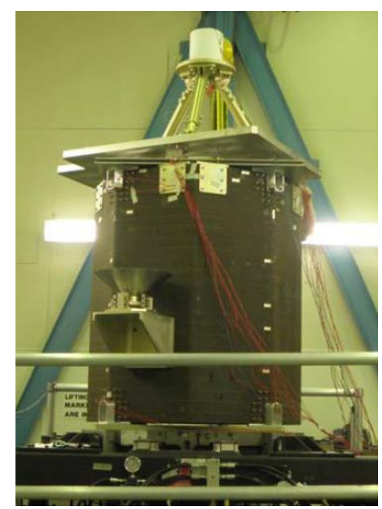
MDOF systems and continuous structures with localised nonlinearities.

Real-life structures with strong nonlinearities, nonlinear modal analysis, numerical model updating.

1980s | 1990s | 2000s | Today







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#1: Real Structures Are Nonlinear…

Large
deformations

Friction and
clearance in
actuators

Fluid-structure
interactions

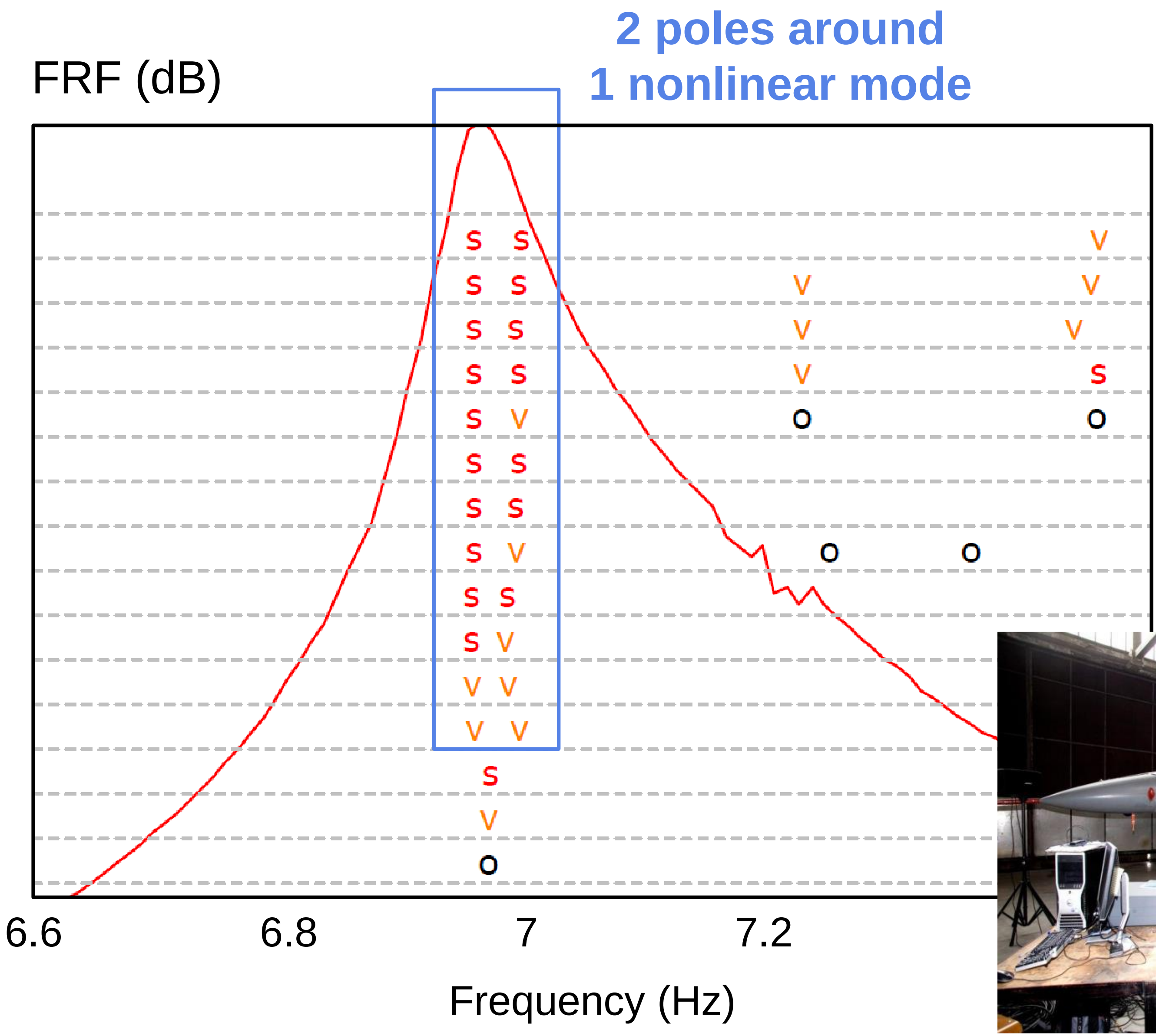
APUs in
tail-cone

Landing gears

Wing-engine pylons,
engine mounts

New materials
(*e.g.*, composites)

…and (Linear) Commercial Software Fail to Identify Them!



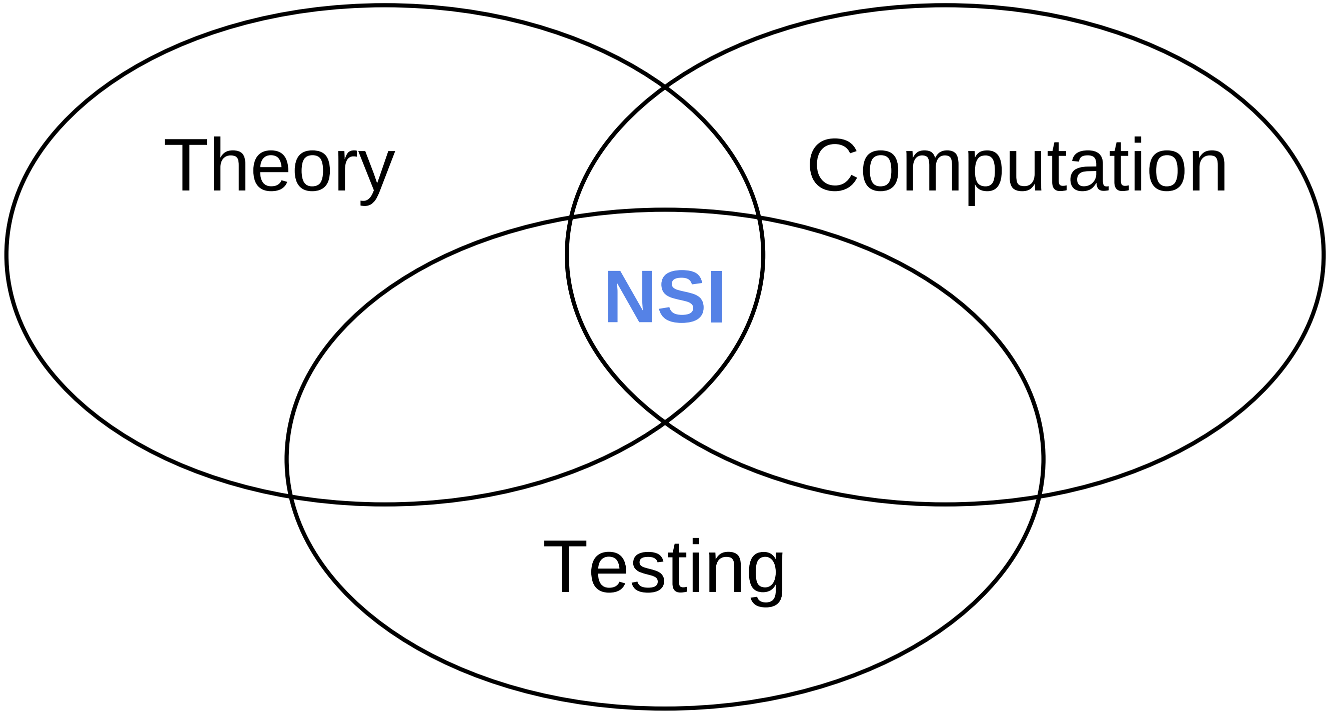
F16 Fighter

Peeters et al., IMAC 2011.

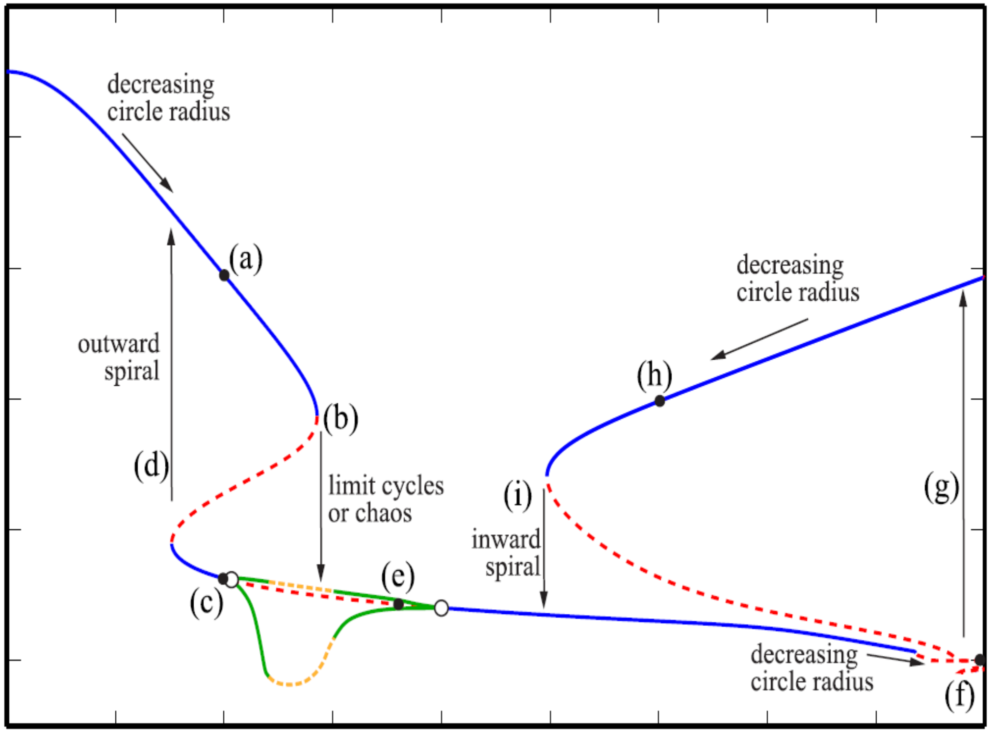
#2: Environmental Constraints Boost Technological Progress

In 2050 technologies and procedures available allow a 75% reduction in CO2 emissions per passenger kilometre […] and a 90% reduction in NOx emissions. The perceived noise emission […] is reduced by 65%.

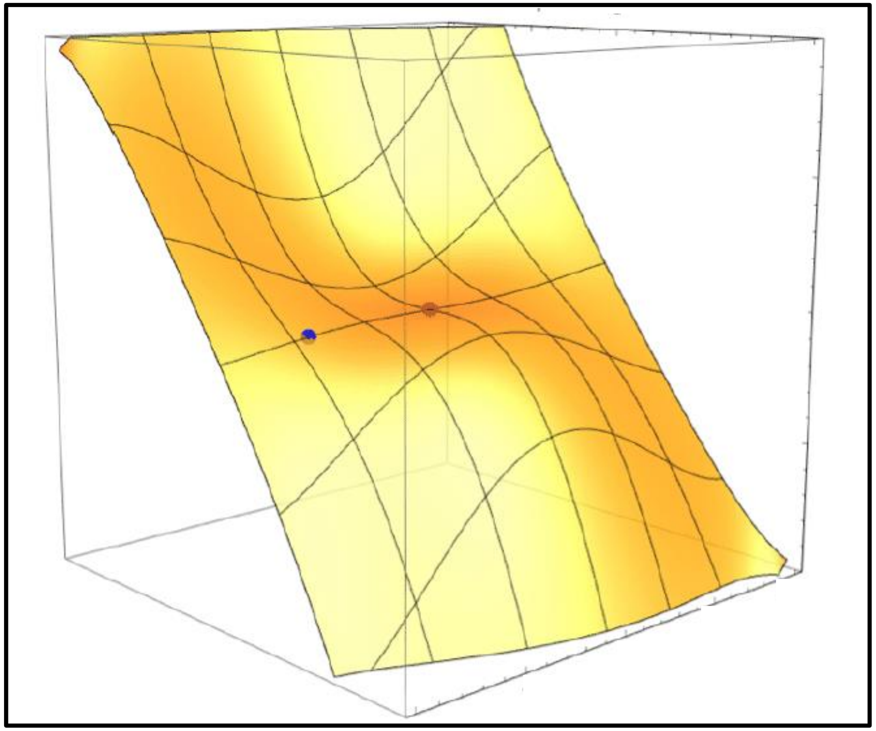
#3: Increased Maturity of Connected Fields



Bifurcation theory and nonlinear normal modes (NNMs).

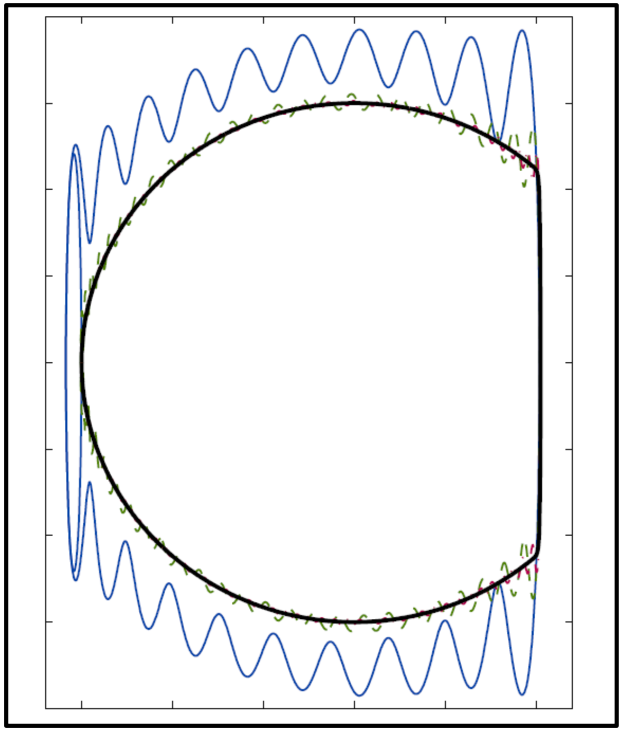


Single-aisle aircraft bifurcation diagram
[B. Krauskopf, 2015].

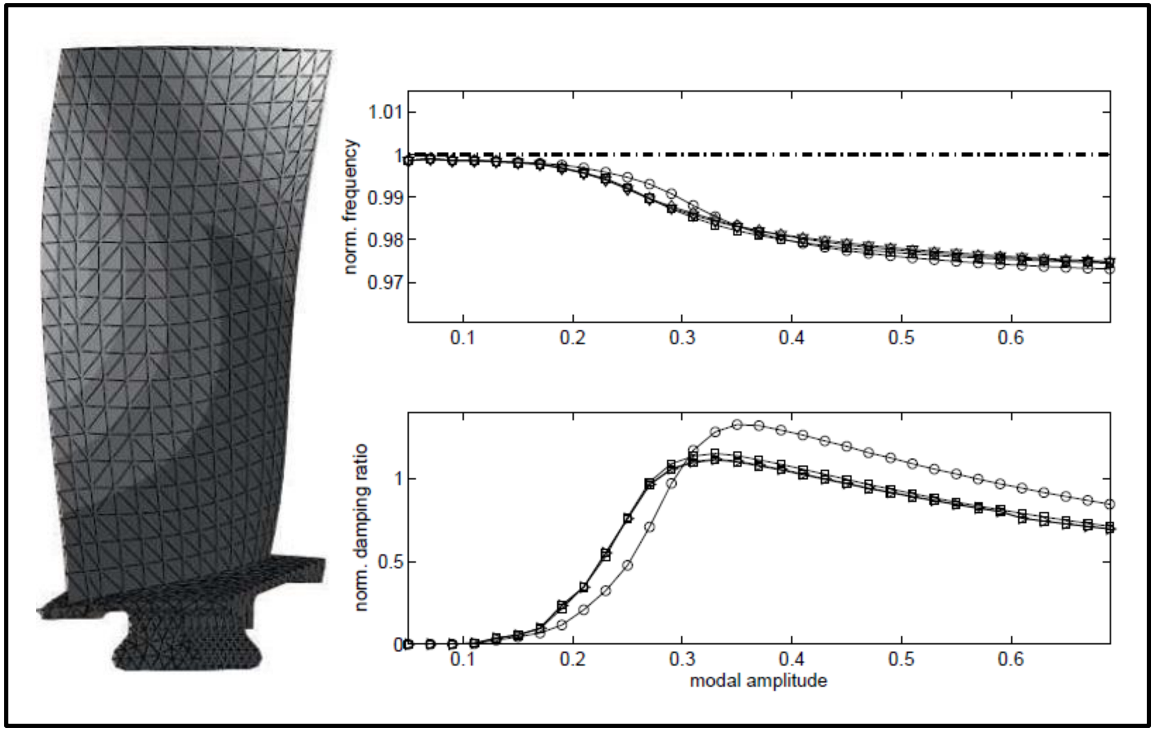


Spectral submanifold of a 2-DOF system
[G. Haller, 2016].

Tailored shooting, harmonic balance and collocation algorithms.



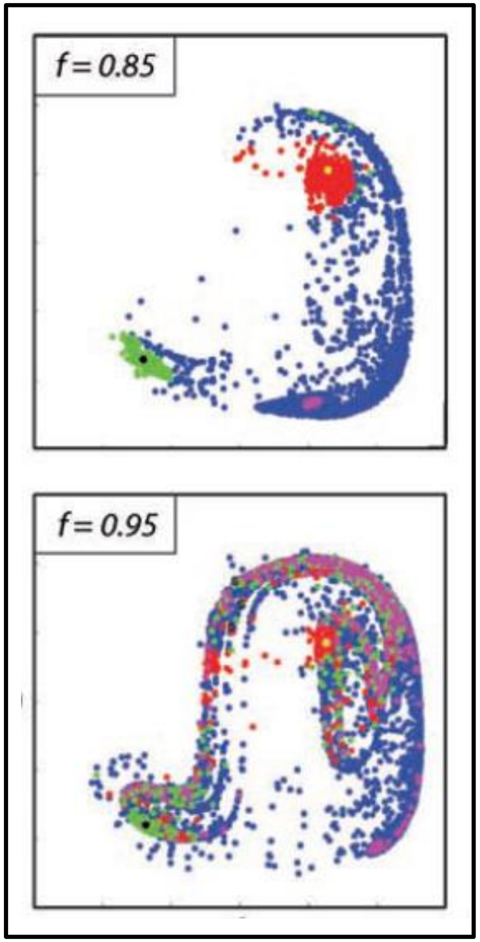
Phase diagram of a vibro-impact system [B. Cochelin, 2014].



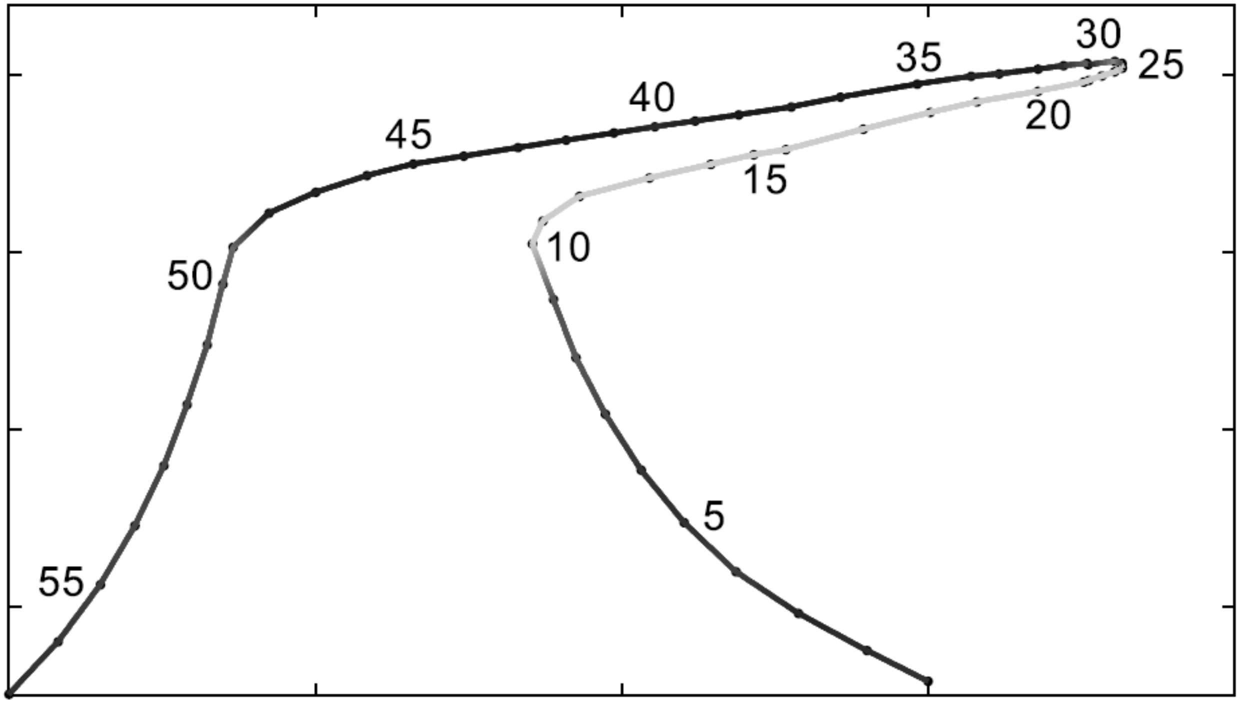
NNM of a compressor blade with friction [F. Thouverez, 2009].

Recent Progress in Testing Nonlinear Structures

Basins of attraction and experimental continuation.



Stochastic interrogation [L.N. Virgin, 2014].



Experimental impact oscillator [J.J. Thomsen, 2014].

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Importance of the Toolbox Philosophy in NSI

1 Detection

Do I observe nonlinear effects? *Yes.*

Should I build a nonlinear model? *Yes.*

2 Characterisation

Where is the nonlinearity located? *At the joint.*

What is the underlying physics? *Dry friction.*

How to model its effects? $f_{nl}(q, \dot{q}) = c \; sign(\dot{q})$.

3 Estimation

Model parameters? $c = 5.47$.

How uncertain are they? $c = \mathcal{N}(5.47,1)$.

↓

Information vs. complexity

Nonlinearity Detection

Evidence nonlinear behaviour in experimental data.

This is not particularly challenging:

- A wide variety of techniques exists in the literature.
- Harmonic and random excitations can be addressed.
- A single excitation level is generally sufficient.

This is crucial:

- Detection supports an important decision as building a nonlinear model demands more time, capabilities and efforts than a linear model.

Nonlinearity Characterisation

Infer a suitable nonlinearity model from experimental data.

This is challenging:

- Prior knowledge is most often very limited.
- Physical mechanisms resulting in nonlinearity are extremely diverse.
- Nonlinearity may translate into a plethora of dynamic phenomena.

This is crucial:

- The success of the parameter estimation step is conditional upon an accurate characterisation of all observed nonlinearities.

Nonlinear Parameter Estimation

Fit a mathematical model to experimental data.

This is challenging:

- If characterisation was not successfully completed (black-box models).
- If nonlinearities were not instrumented.
- If the input signals were not measured (operational NSI).

This is crucial:

- Quantitative models are required to carry out response prediction, numerical model updating and validation, design optimisation, …

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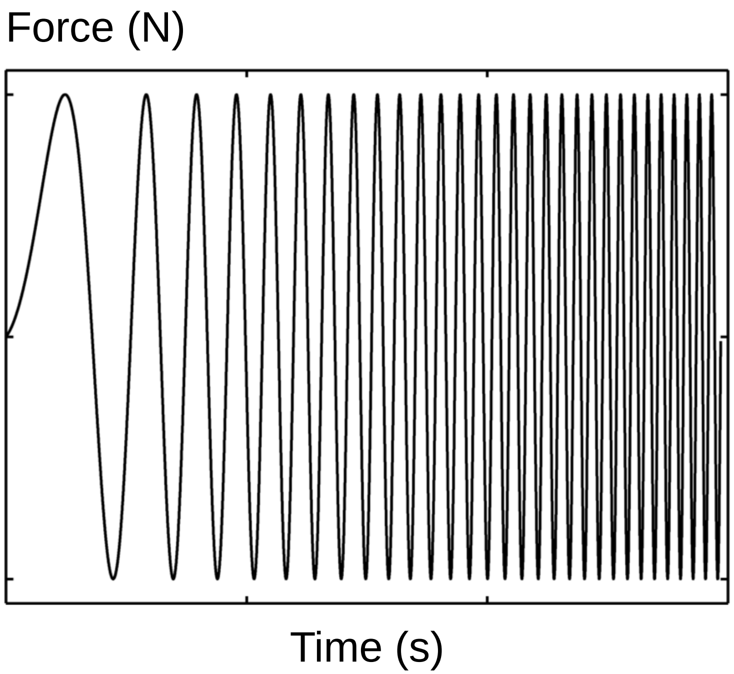
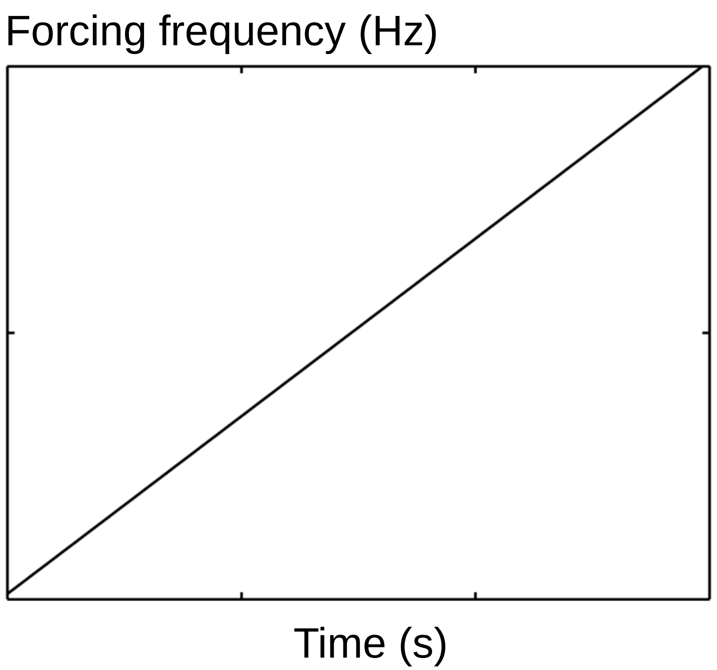
Two "Good" Input Signals: Sine Sweep and Random Multisine

Linear sine sweep:

$$u(t) = A\,sin\left(2\pi f_0(t-t_0) + 2\pi\frac{r}{2}(t-t_0)^2 + \varphi_0\right)$$





Two “Good” Input Signals: Sine Sweep and Random Multisine

Linear sine sweep:

$$u(t) = A\, sin\left(2\pi f_0(t - t_0) + 2\pi \frac{r}{2}(t - t_0)^2 + \varphi_0\right)$$

Fully deterministic signal, so easy to interpret data visually.

Strongly activates nonlinearities, as energy is concentrated.

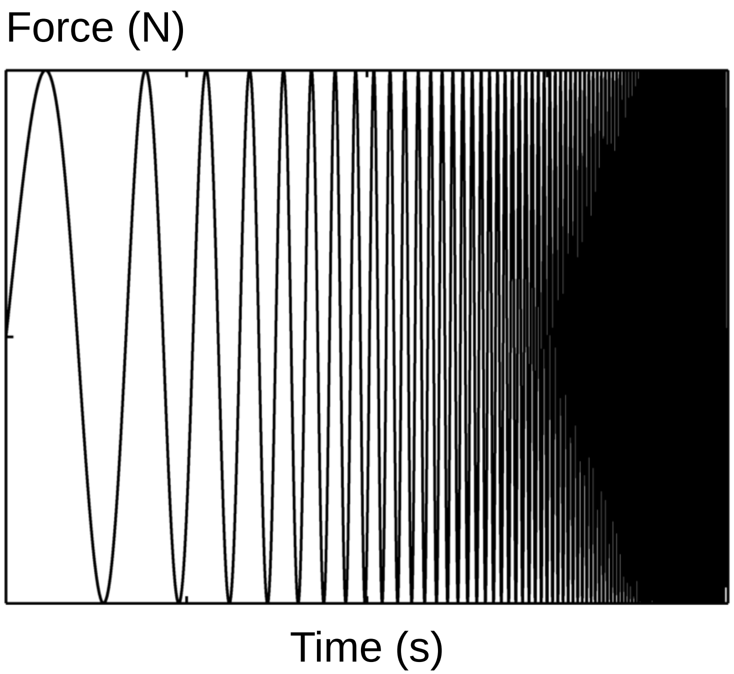
Very useful in nonlinearity characterisation.

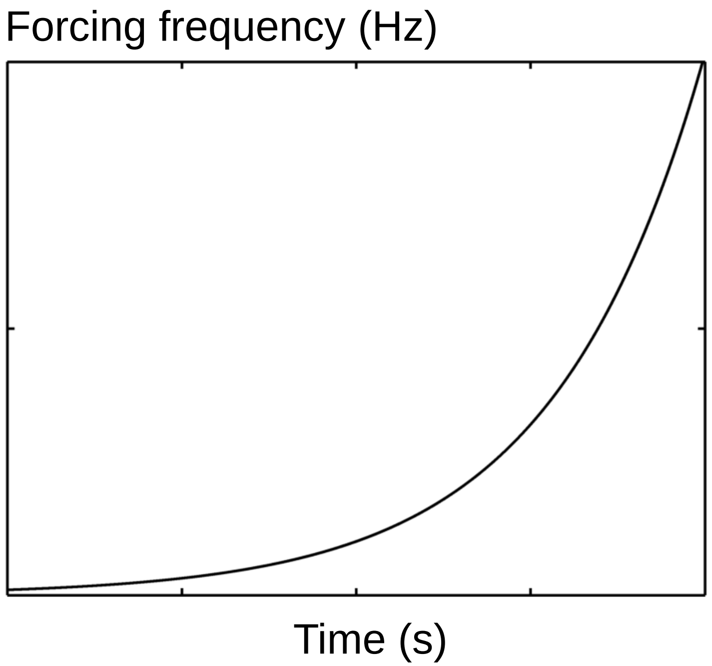
Logarithmic sweep may be used to focus on low frequencies.

Two “Good” Input Signals: Sine Sweep and Random Multisine

Log sine sweep:

$$u(t) = A\,sin\left(\frac{2\pi\, f_0}{\log(2)\;r}\left(2^{r\,(t-t_0)} - 1\right) + \varphi_0\right)$$

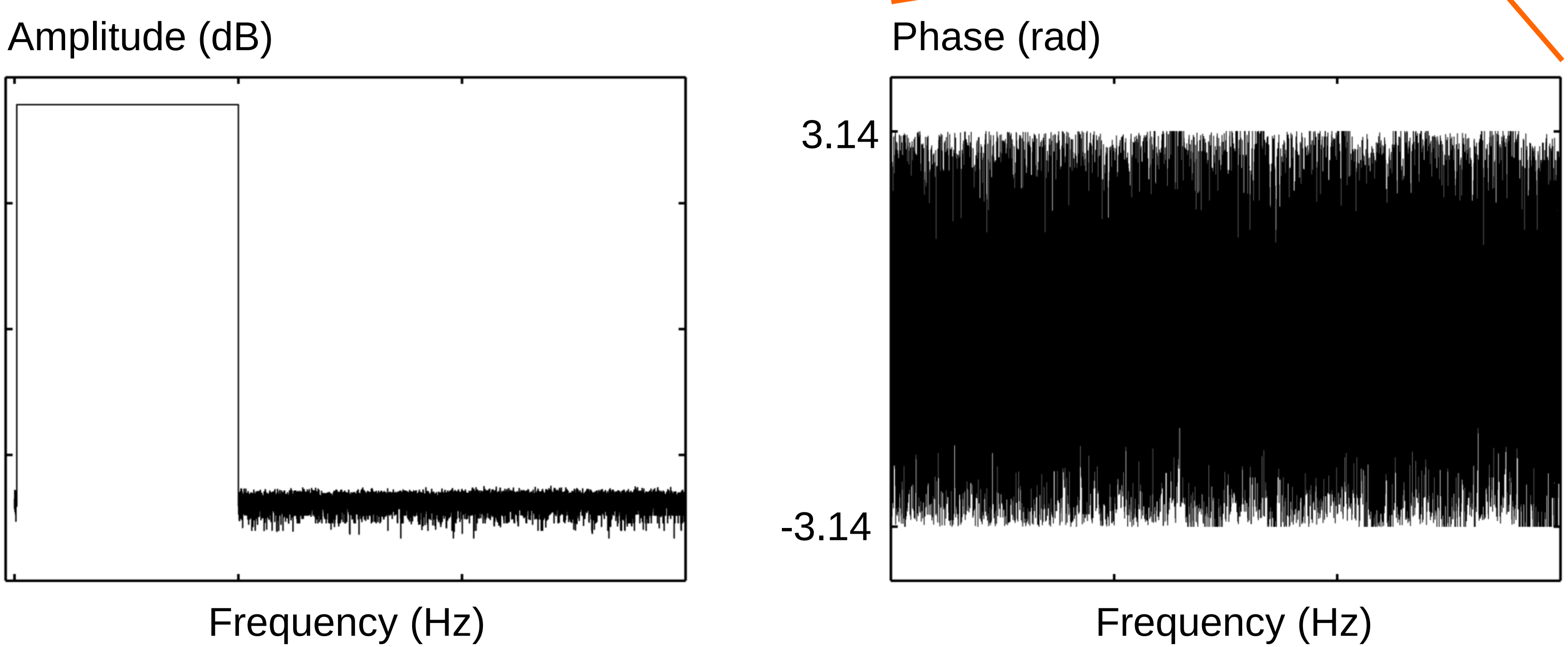




Two “Good” Input Signals: Sine Sweep and Random Multisine

Multisine with uniformly-distributed random phases:

$$u(t) = N^{-1/2} \sum_k U_k \, exp\left(j2\pi \, k \frac{f_s}{N} t + j \, \varphi_k \right)$$



Two “Good” Input Signals: Sine Sweep and Random Multisine

Multisine with uniformly-distributed random phases:

$$u(t) = N^{-1/2} \sum_{k} U_k \, exp\left(j2\pi \, k \frac{f_s}{N} t + j \, \varphi_k \right)$$

Random appearance in TD, but deterministic amplitudes in FD.

All frequencies excited throughout the test.

Periodicity allows to remove transients and characterise noise.

Gaussian signal for a sufficiently large number of frequencies.

Very useful in nonlinear parameter estimation.

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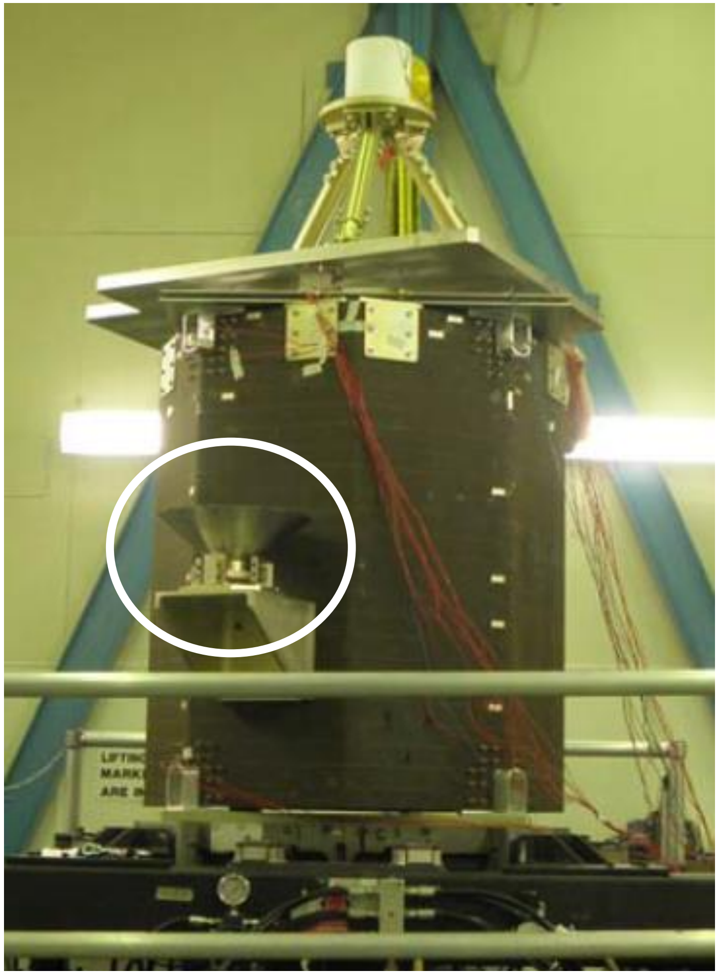
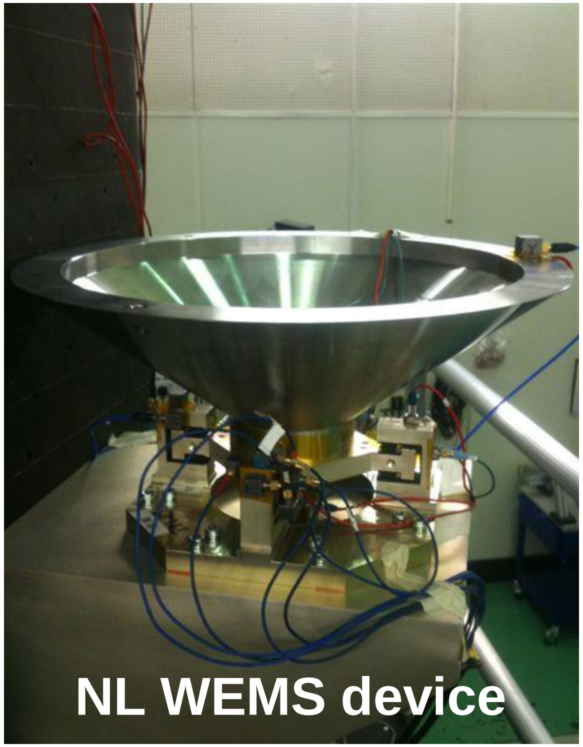
A First Test Case: the SmallSat Spacecraft from Airbus DS

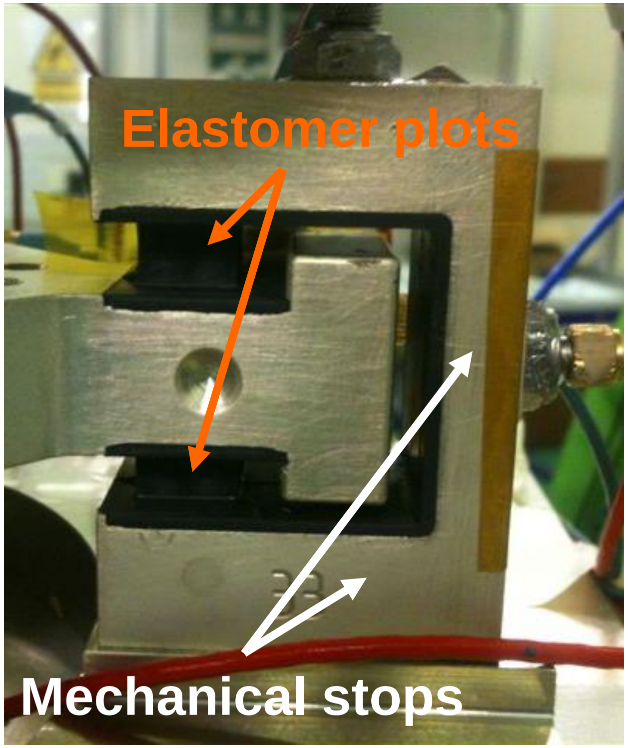




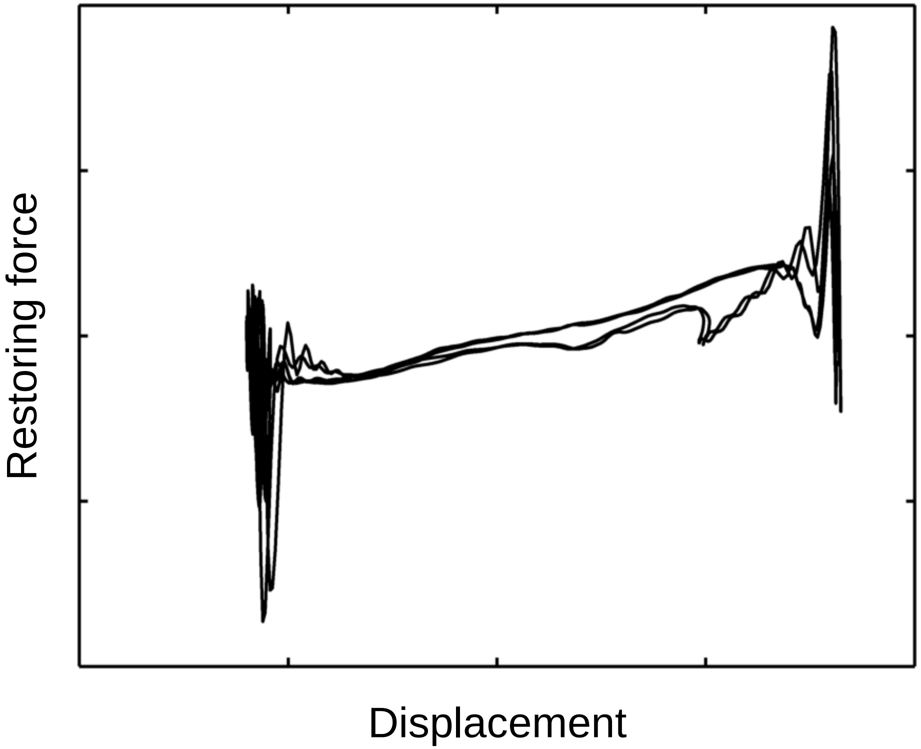
Challenges:

- Nonsmooth nonlinearities
- High damping
- Gravity-induced asymmetry
- Viscoelastic components

WEMS Device: Piecewise-linear Behaviour

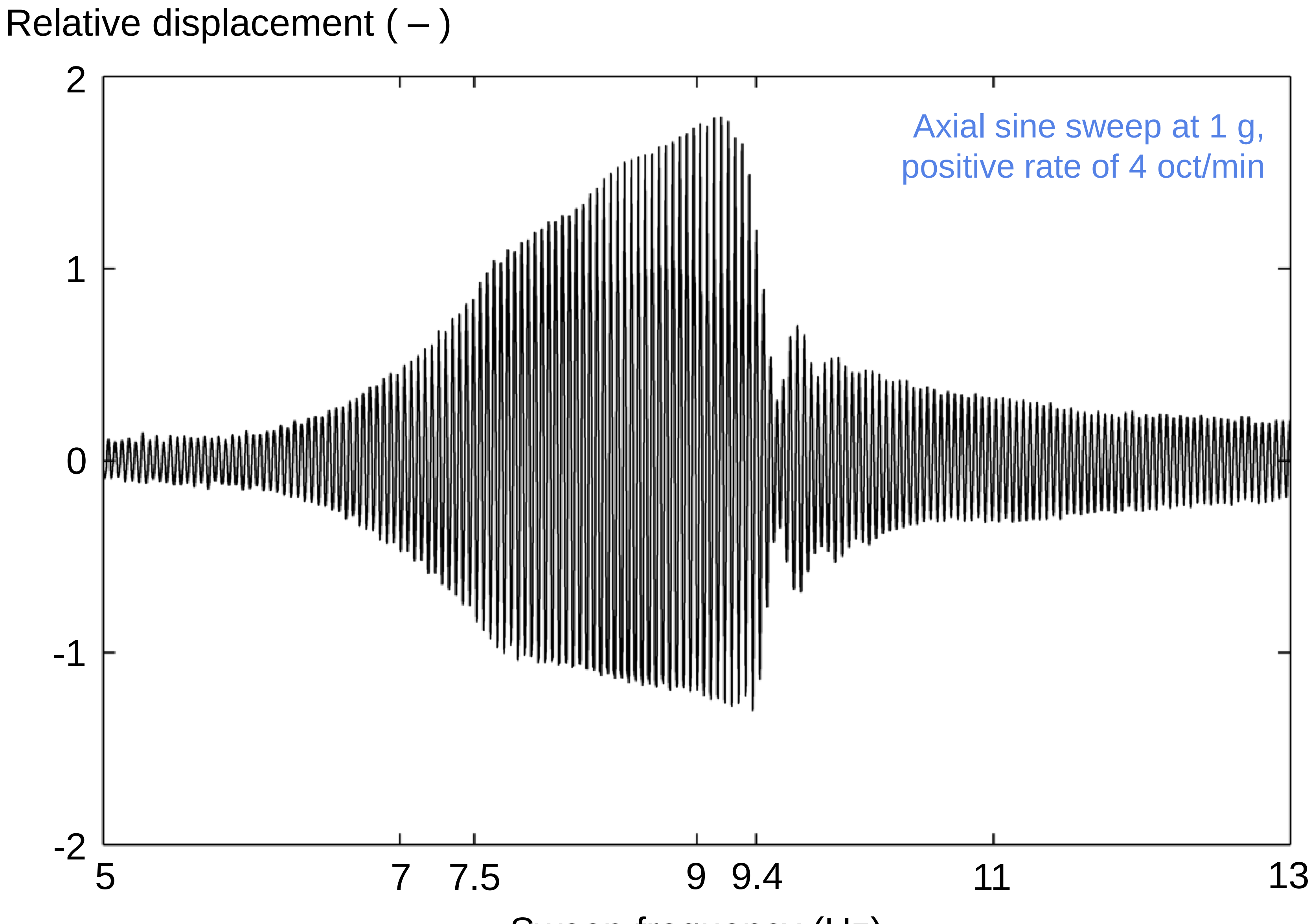


Isolation and protection device.

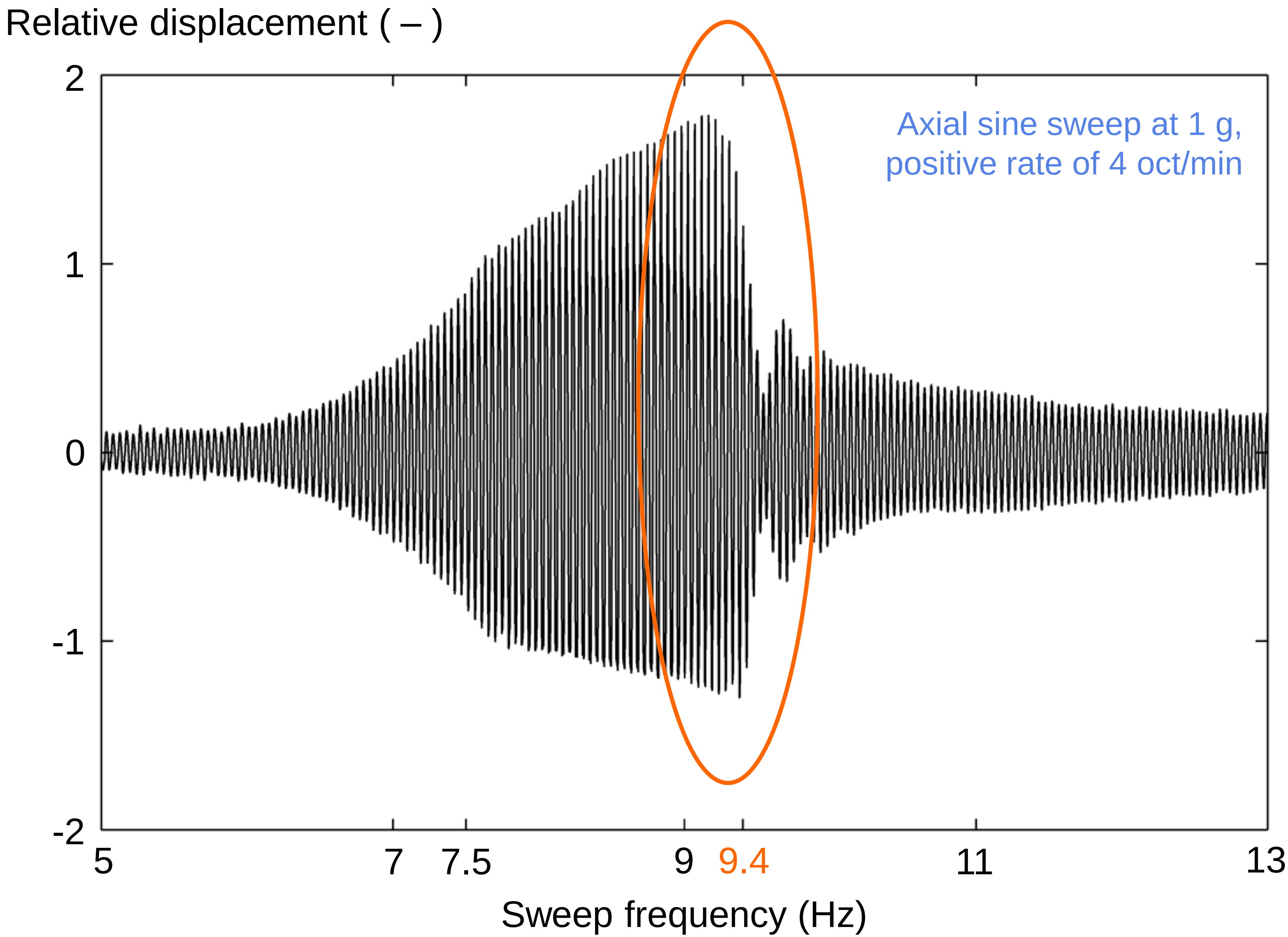


Experimental stiffness curve.

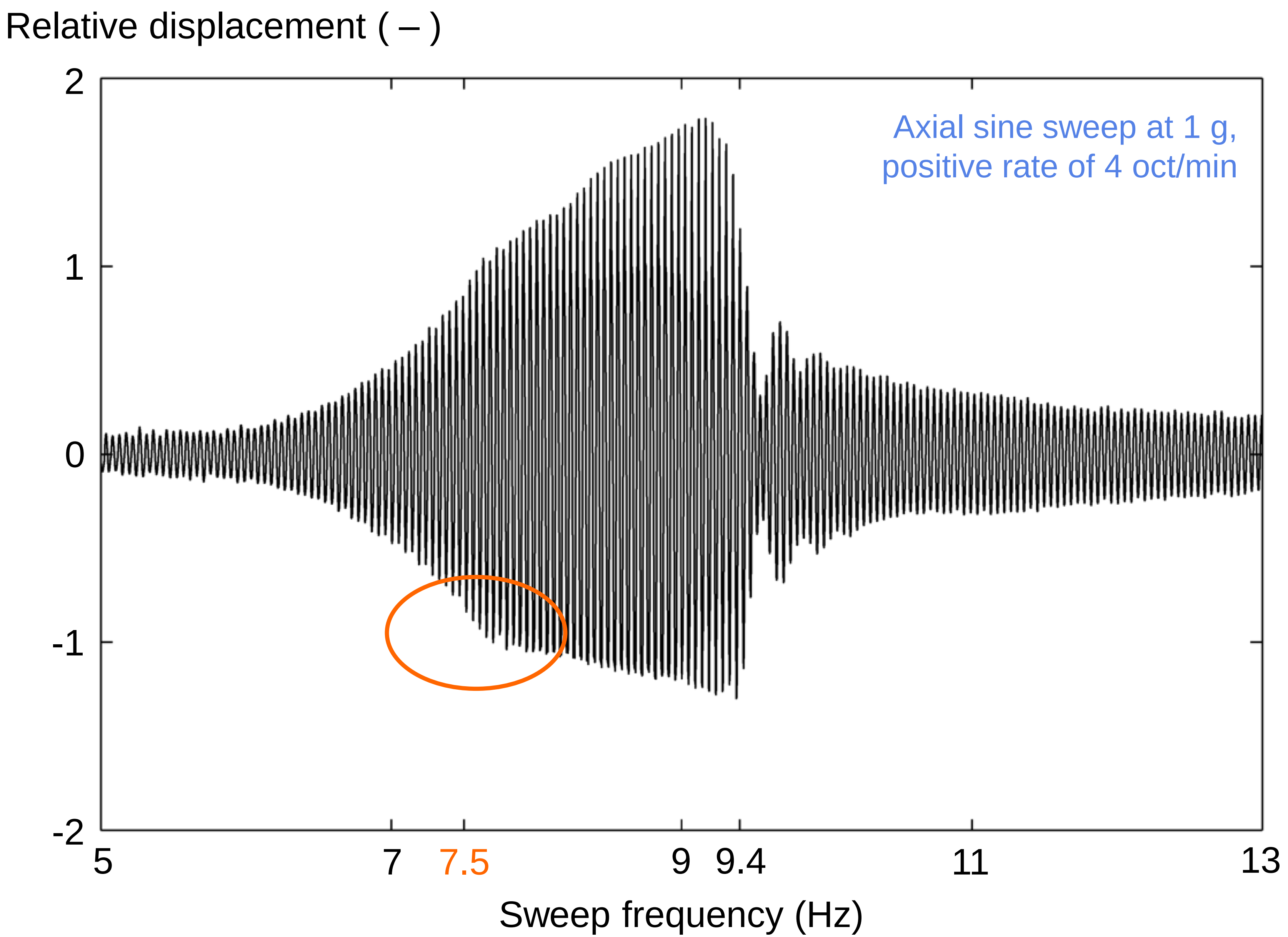
Envelope-based Analysis of a Raw Time History



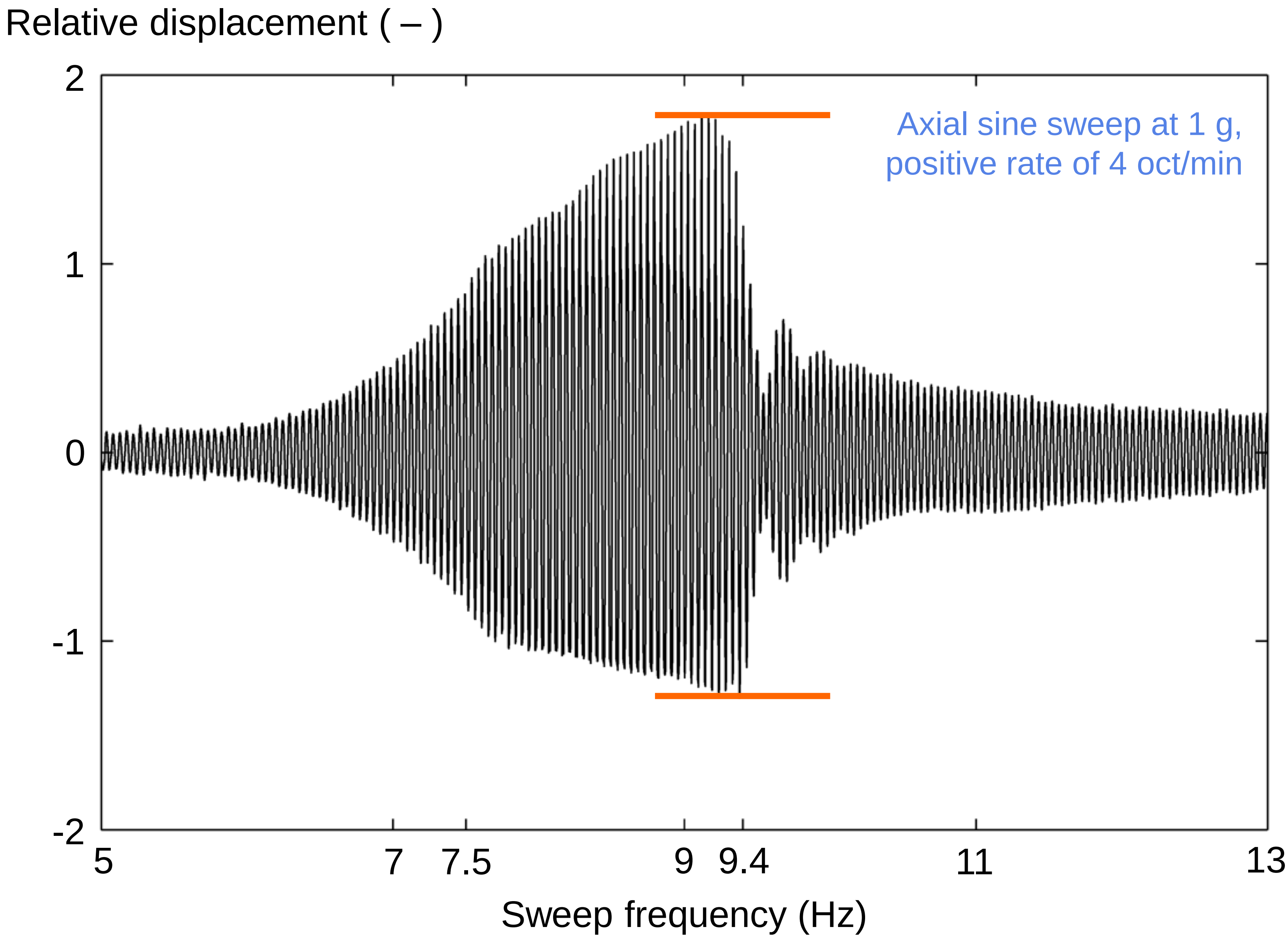
Skewness and Nonsmoothness Result in a Jump



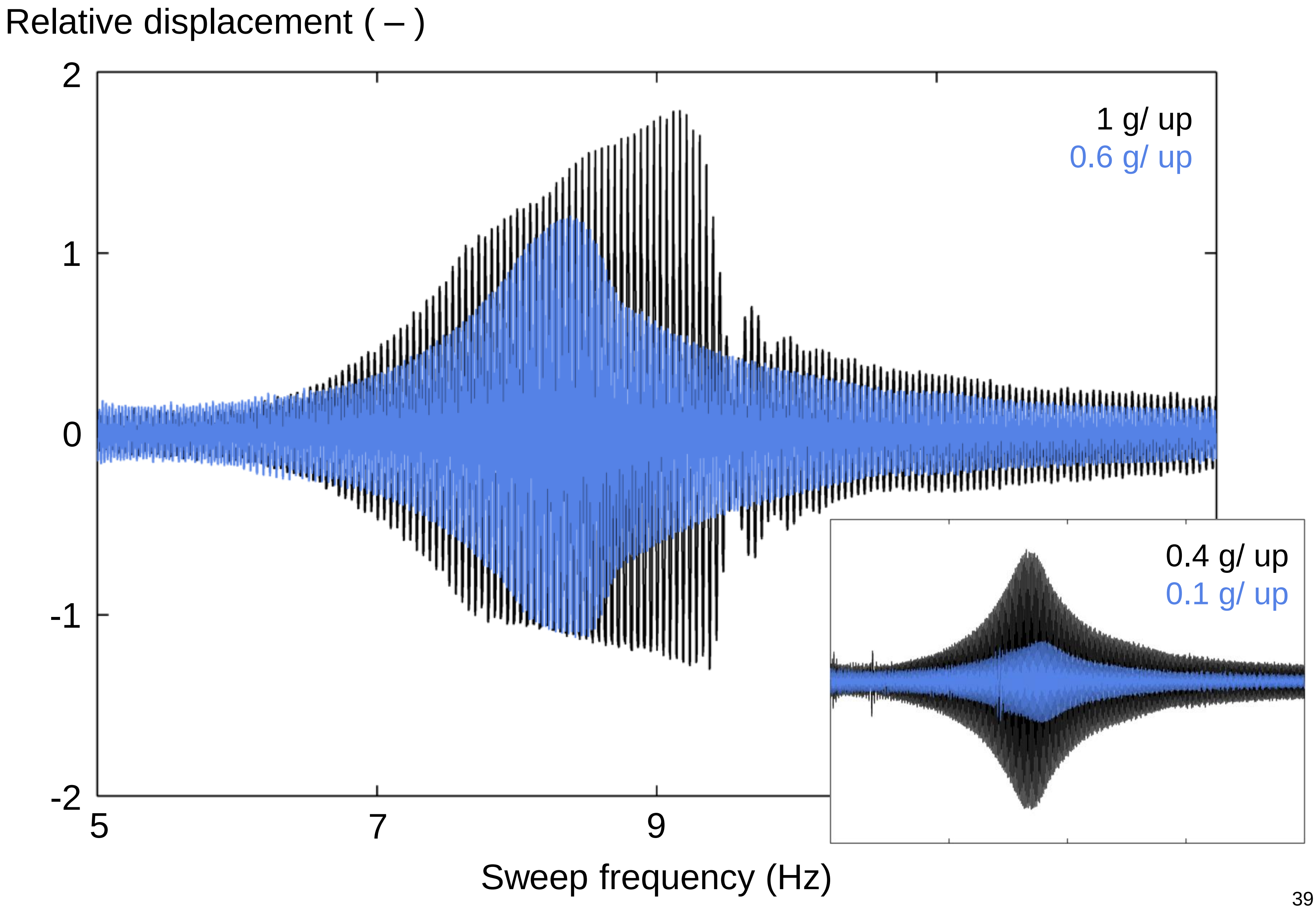
Discontinuity in Slope due to a Clearance in the System



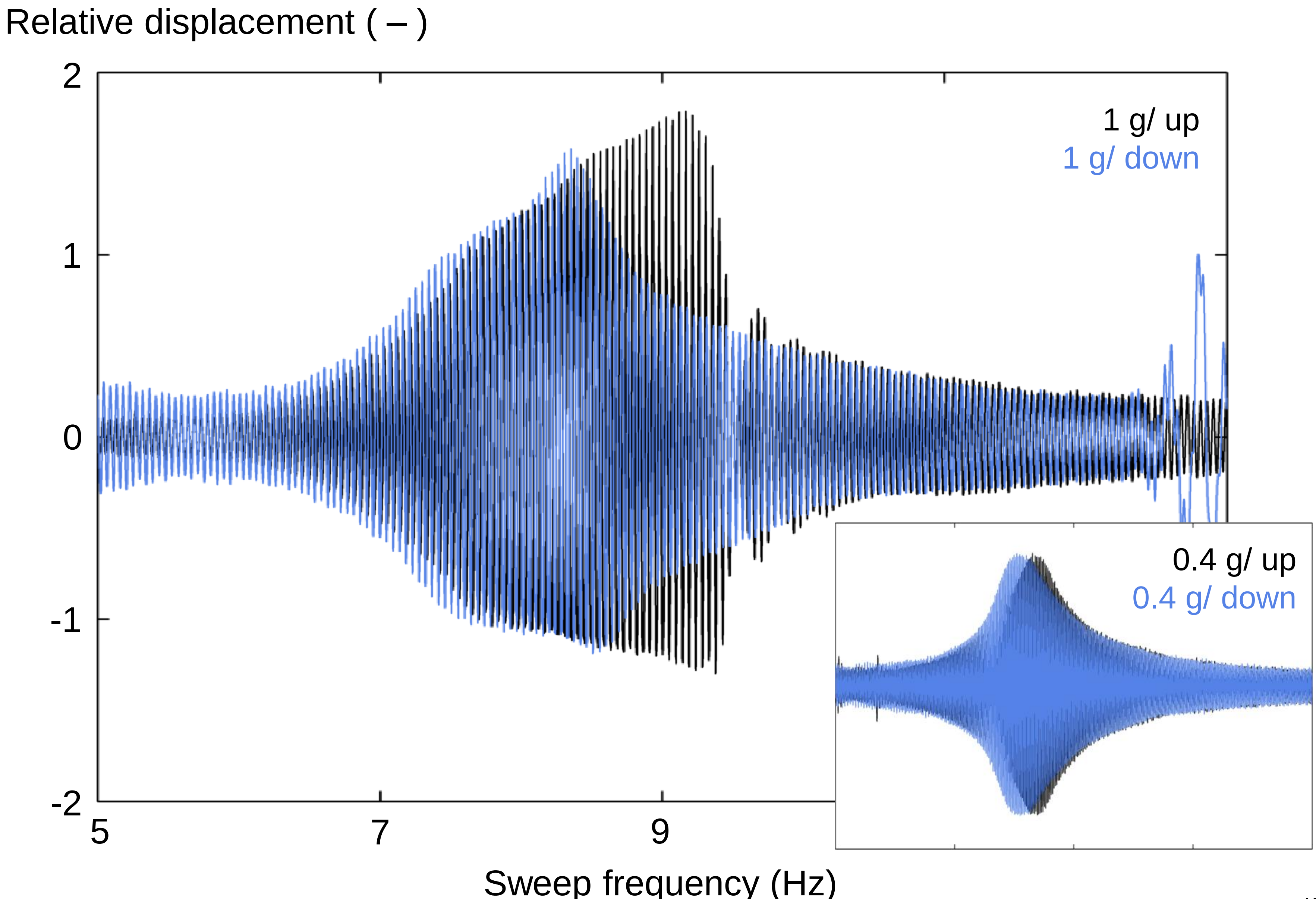
Asymmetry owing to Gravity-induced Prestress



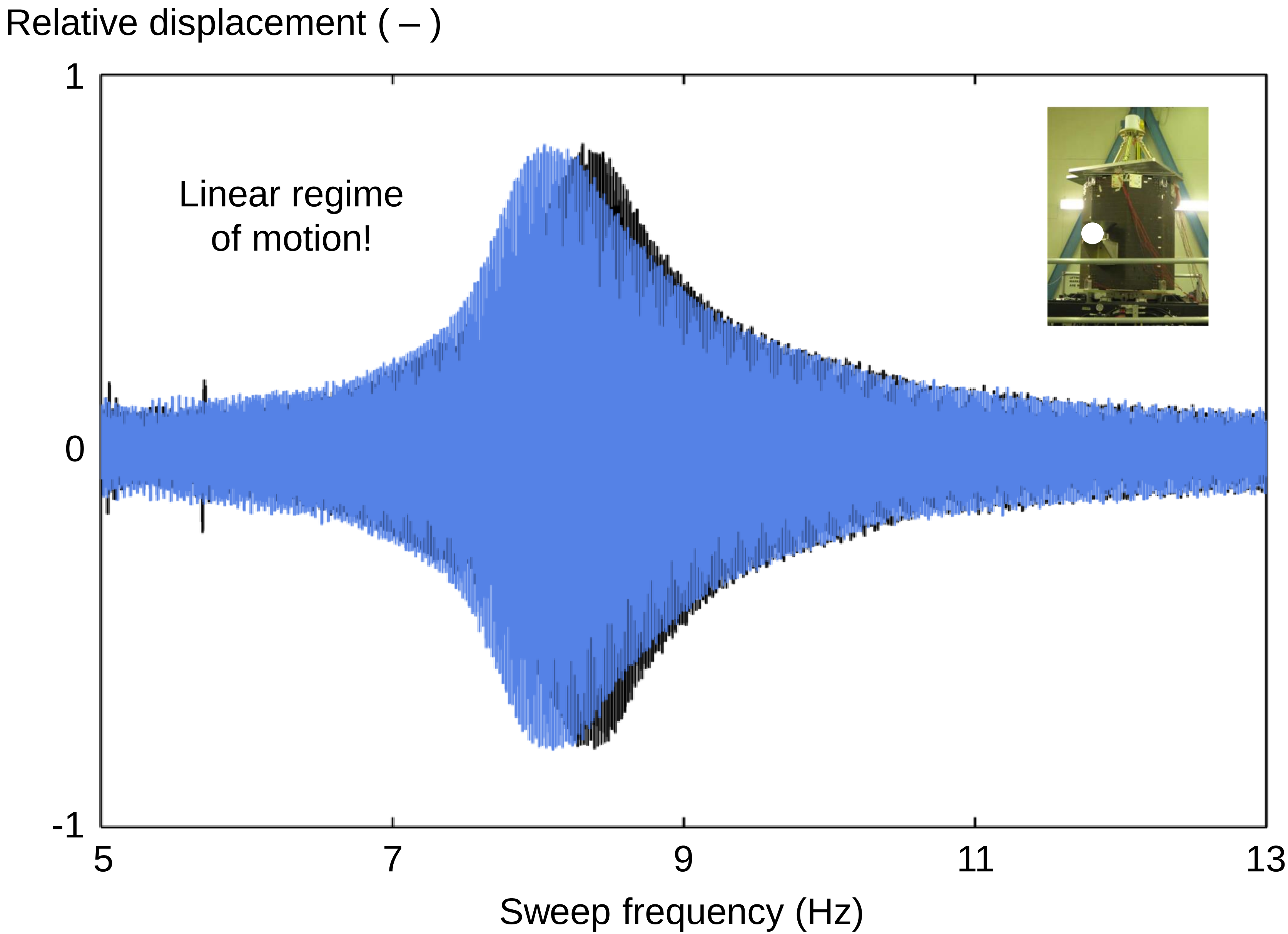
Breakdown of the Principle of Superposition



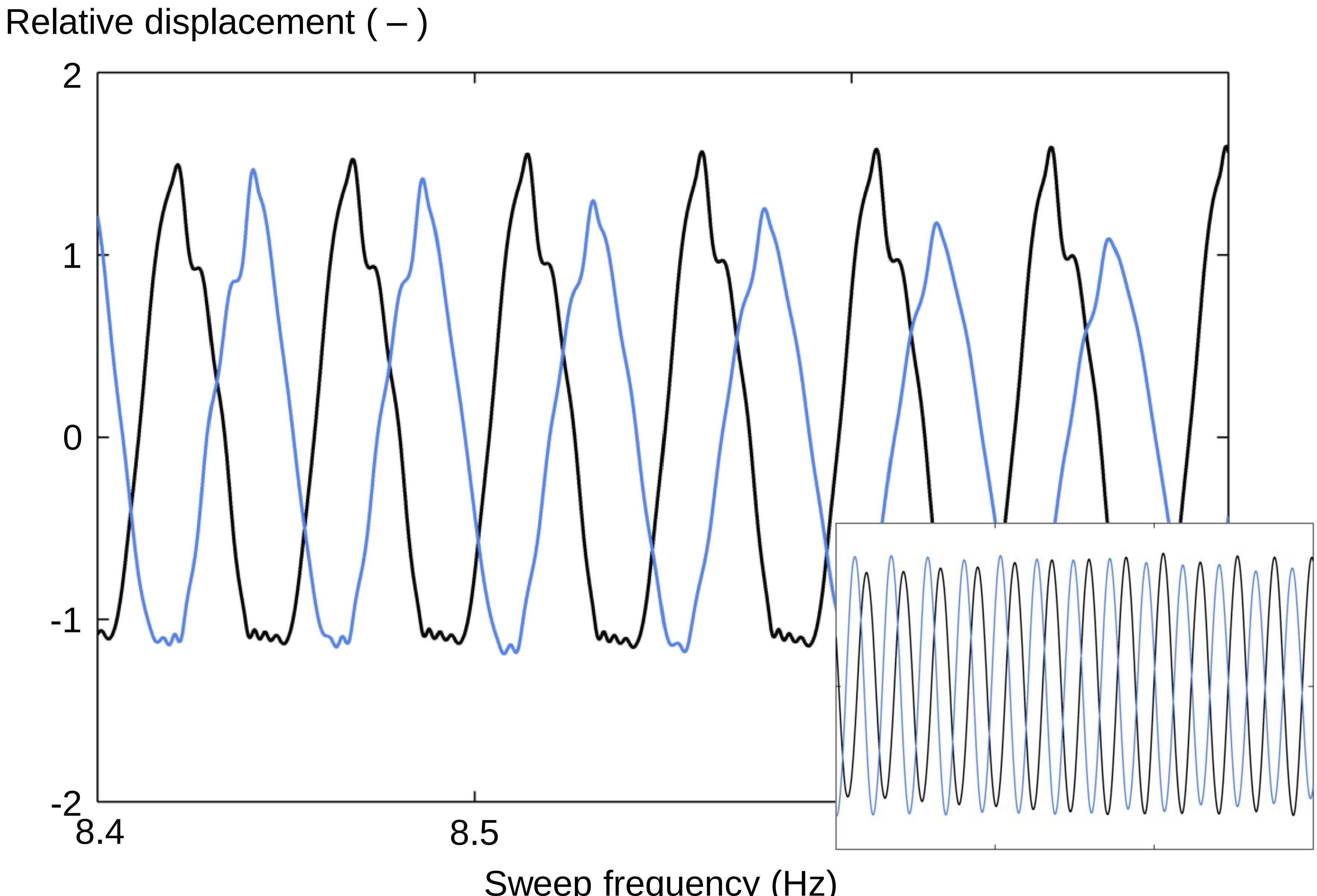
Existence of Nonlinear Hysteresis



Transient Effect in Sweep-Up and Sweep-Down Testing

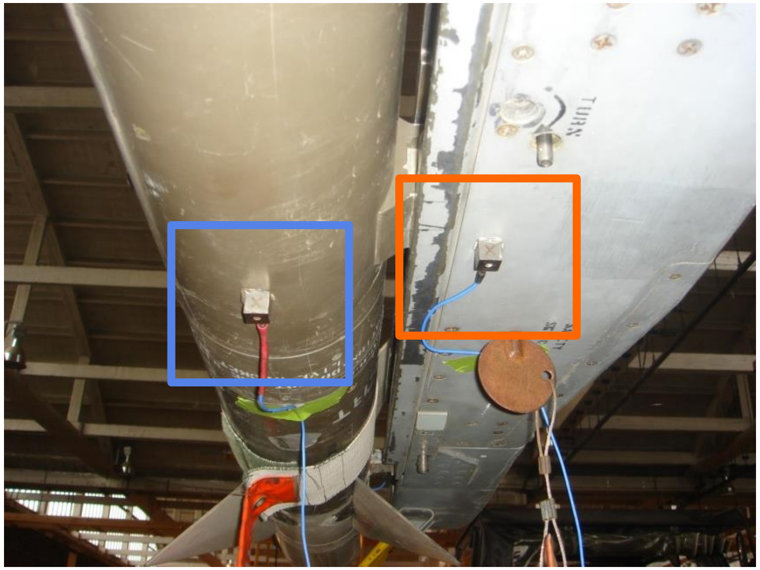


A Close-up Reveals the Appearance of Strong Harmonics



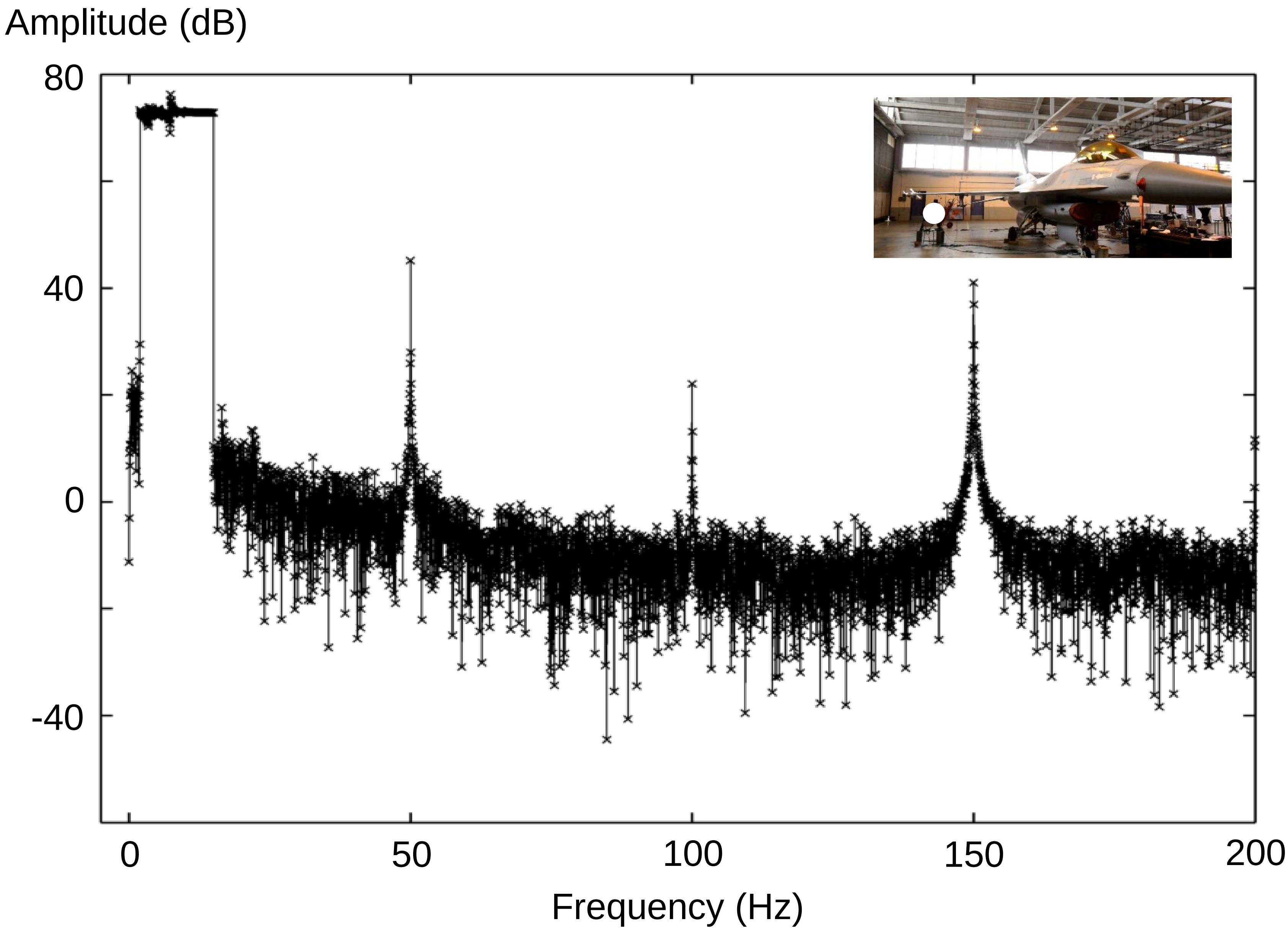
A Second Test Case: an F-16 Fighter Aircraft

F16 aircraft,
Saffraanberg,
Belgium.

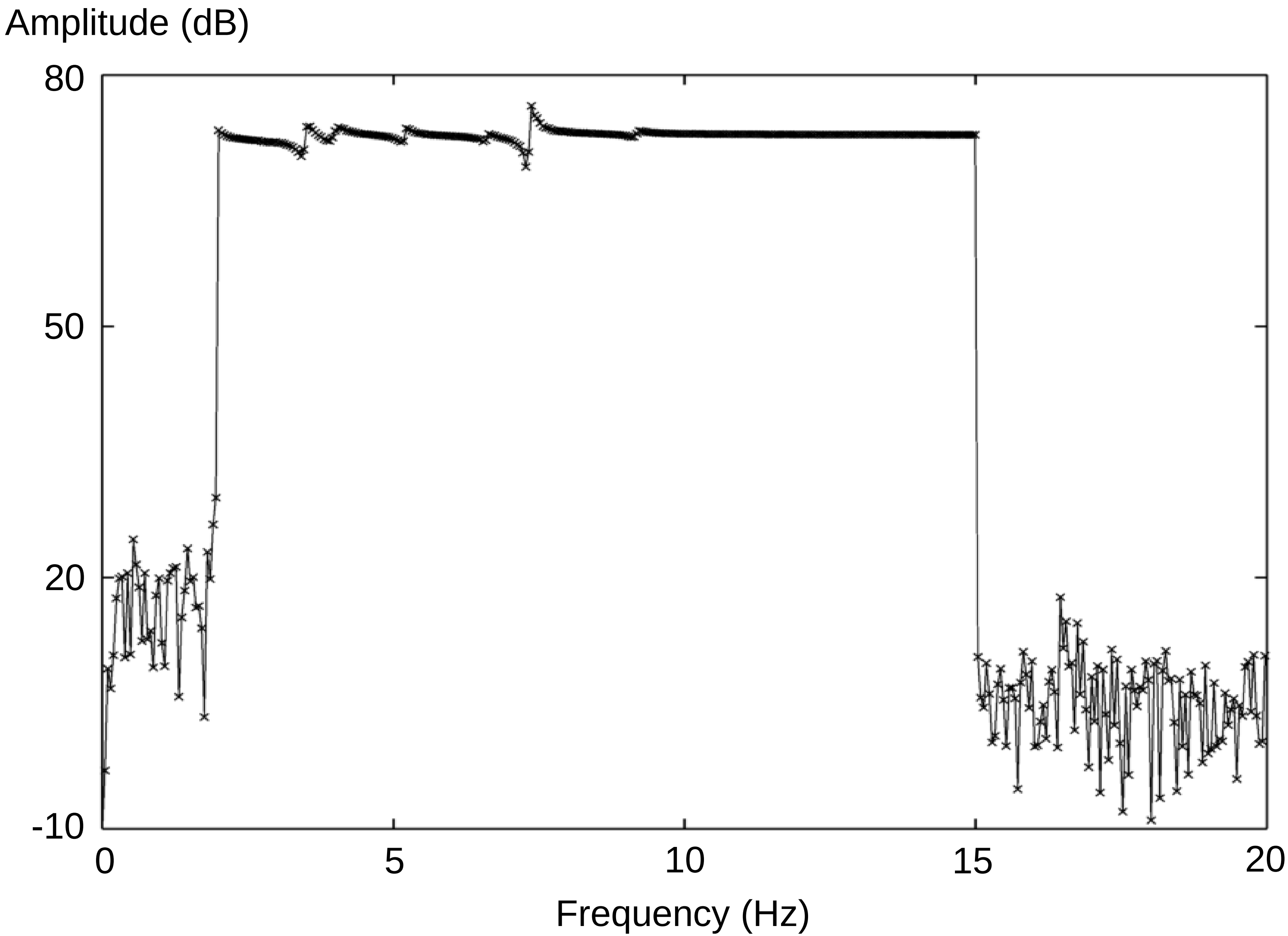


Joints between substructures are generic sources of NLs.

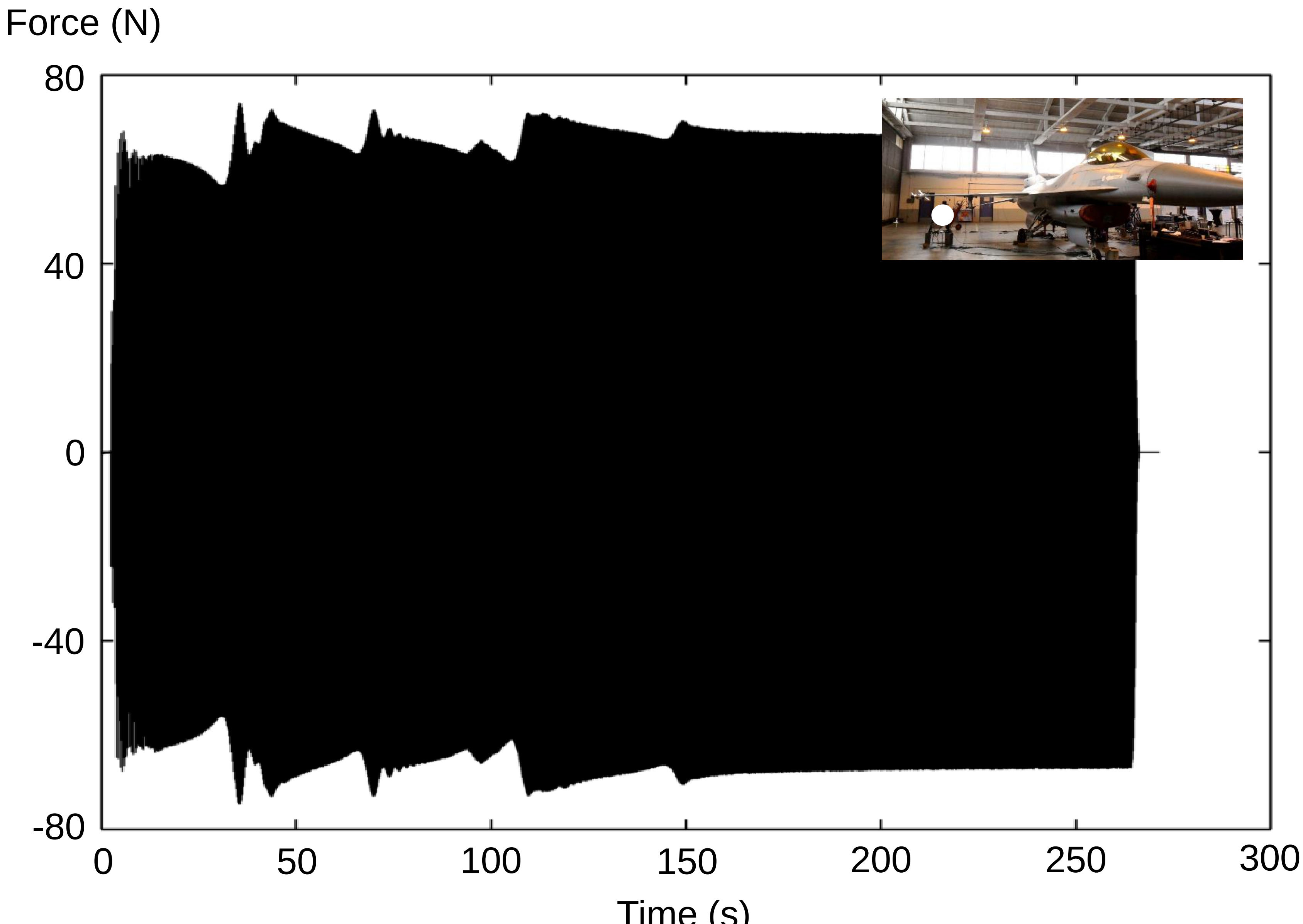
Force Spectrum May Exhibit Drop-off at Resonance



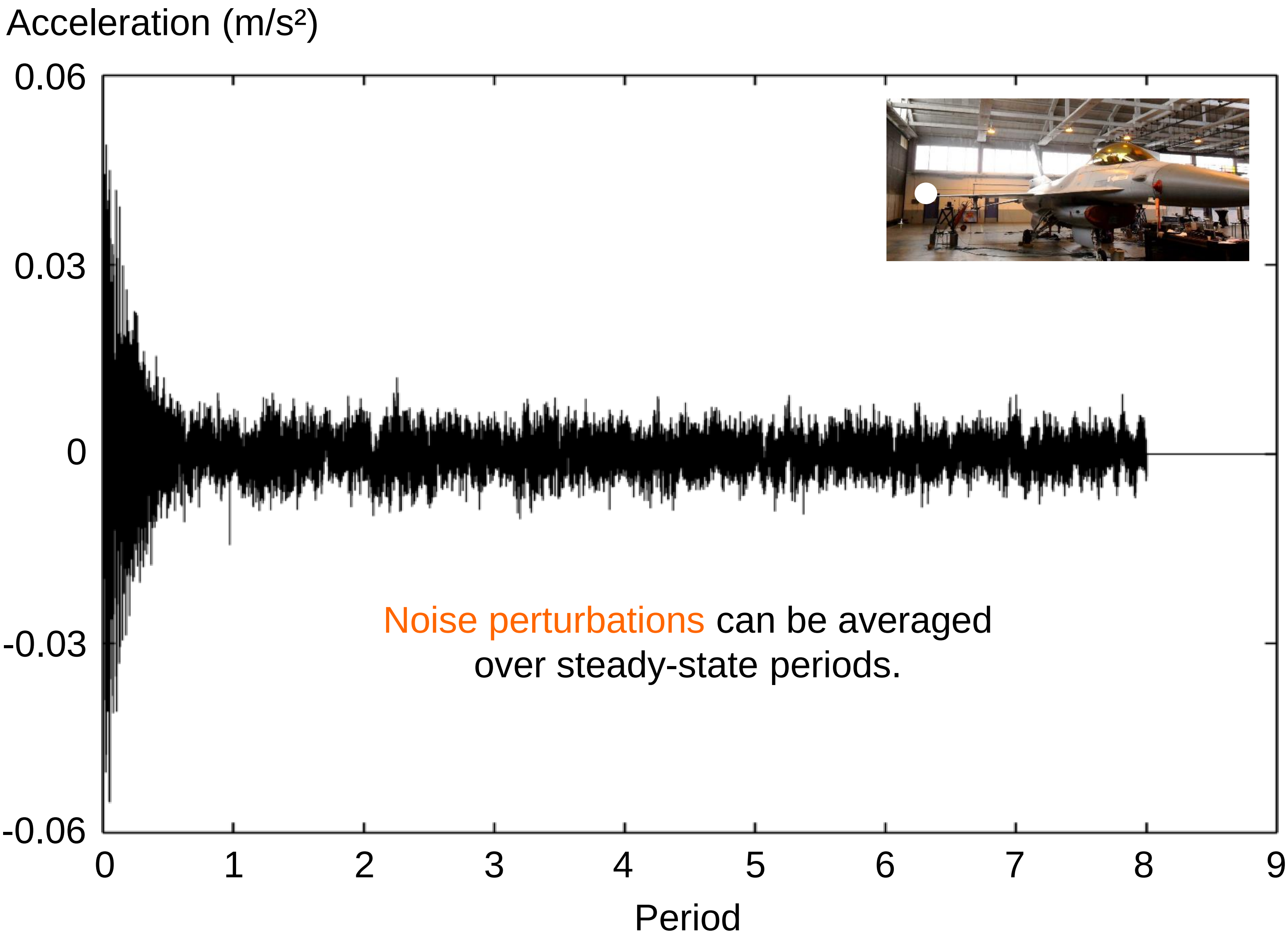
The Input Spectrum Is Accurately Realised



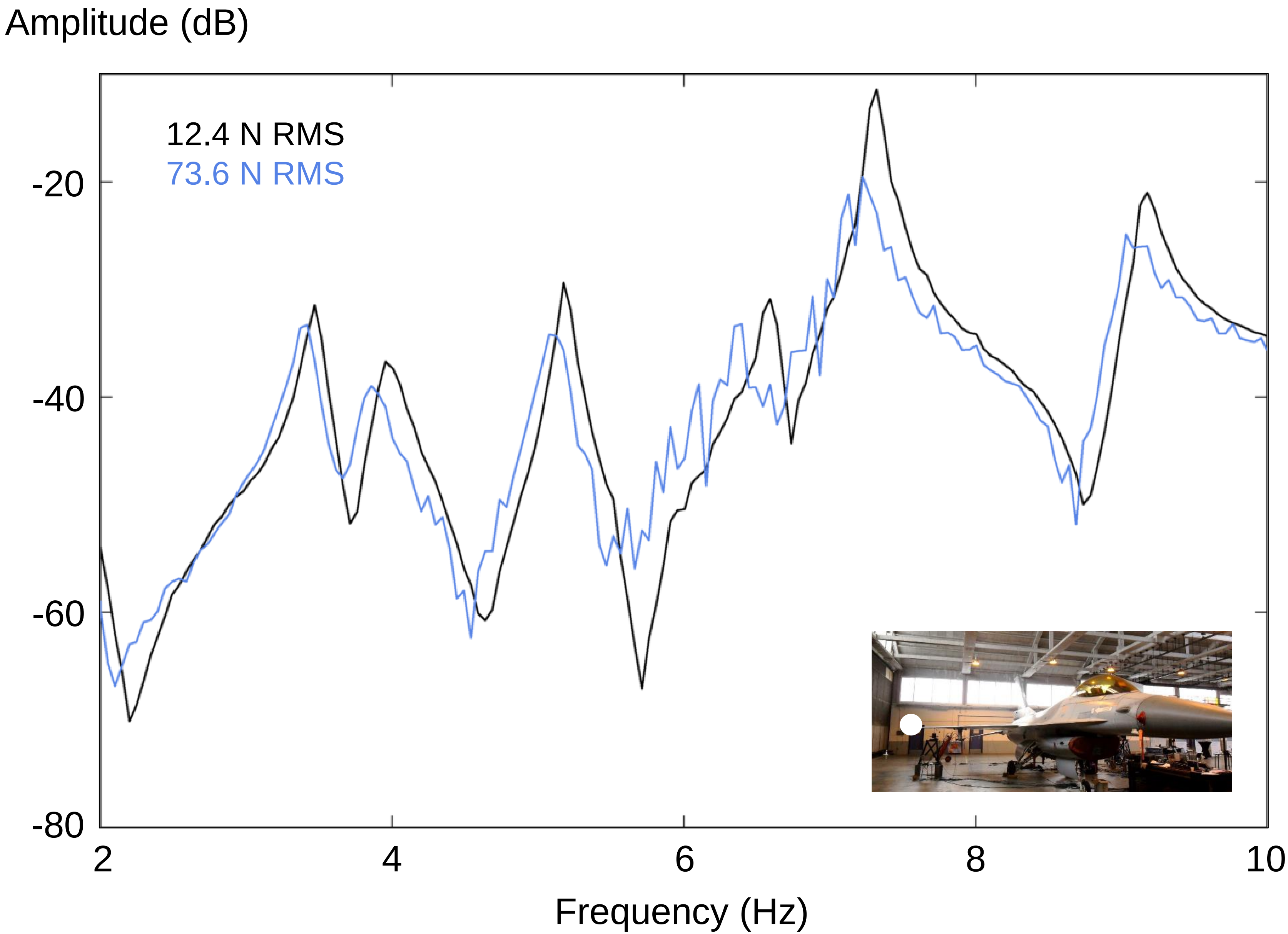
Digression: Force Realisation in Sine-Sweep Testing



Transient Decay Over Multiple Periods of Measurement

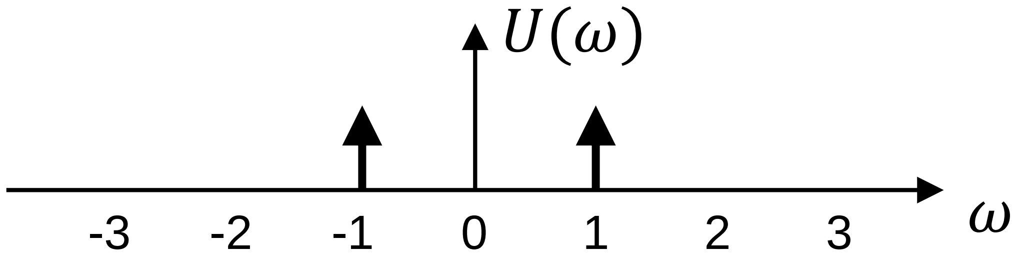


Superposed FRFs do not Match at Two Excitation Levels



Digression: Frequency Mixing in Nonlinear Systems

$$u(t) = 2\cos(\omega t) = e^{j\omega t} - e^{-j\omega t}, \omega = 1$$



Output of a cubic nonlinearity:

$$\begin{aligned} y(t) &= u^3(t) \\ &= (e^{j\omega t} - e^{-j\omega t})(e^{j\omega t} - e^{-j\omega t})\,(e^{j\omega t} - e^{-j\omega t}) \end{aligned}$$

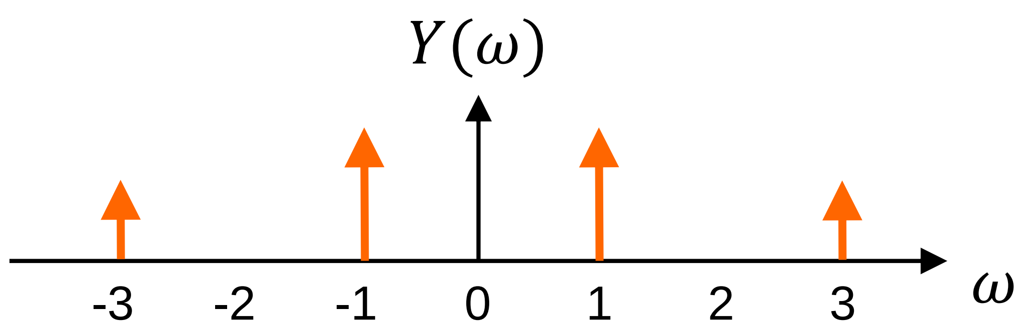
Digression: Frequency Mixing in Nonlinear Systems (2)

Output of a cubic nonlinearity:

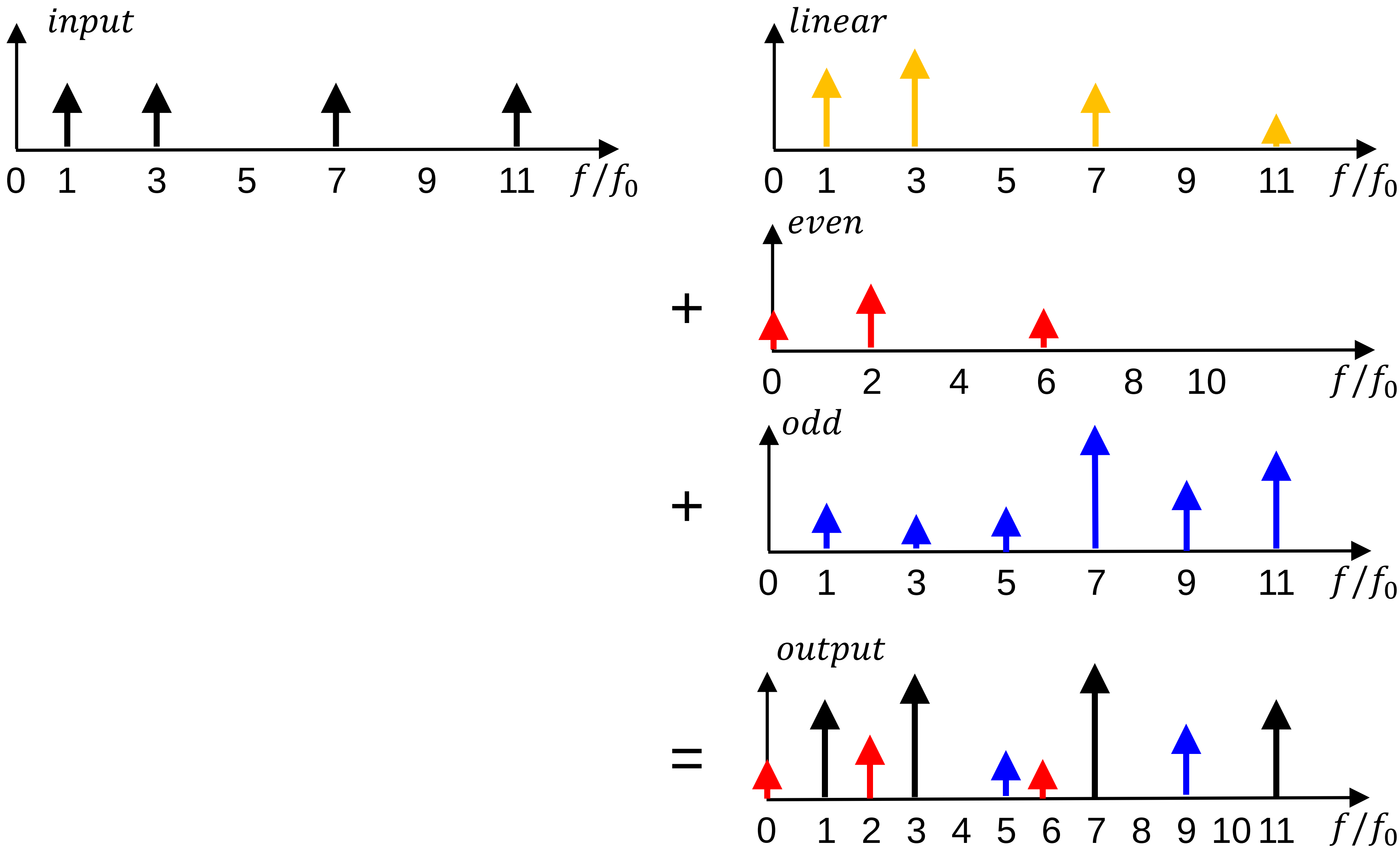
$$
\begin{aligned}
y(t) &= u^3(t) \\
&= (e^{j\omega t} - e^{-j\omega t})(e^{j\omega t} - e^{-j\omega t})\,(e^{j\omega t} - e^{-j\omega t})
\end{aligned}
$$

All possible combinations, 3 by 3, of the frequencies -1 and 1.

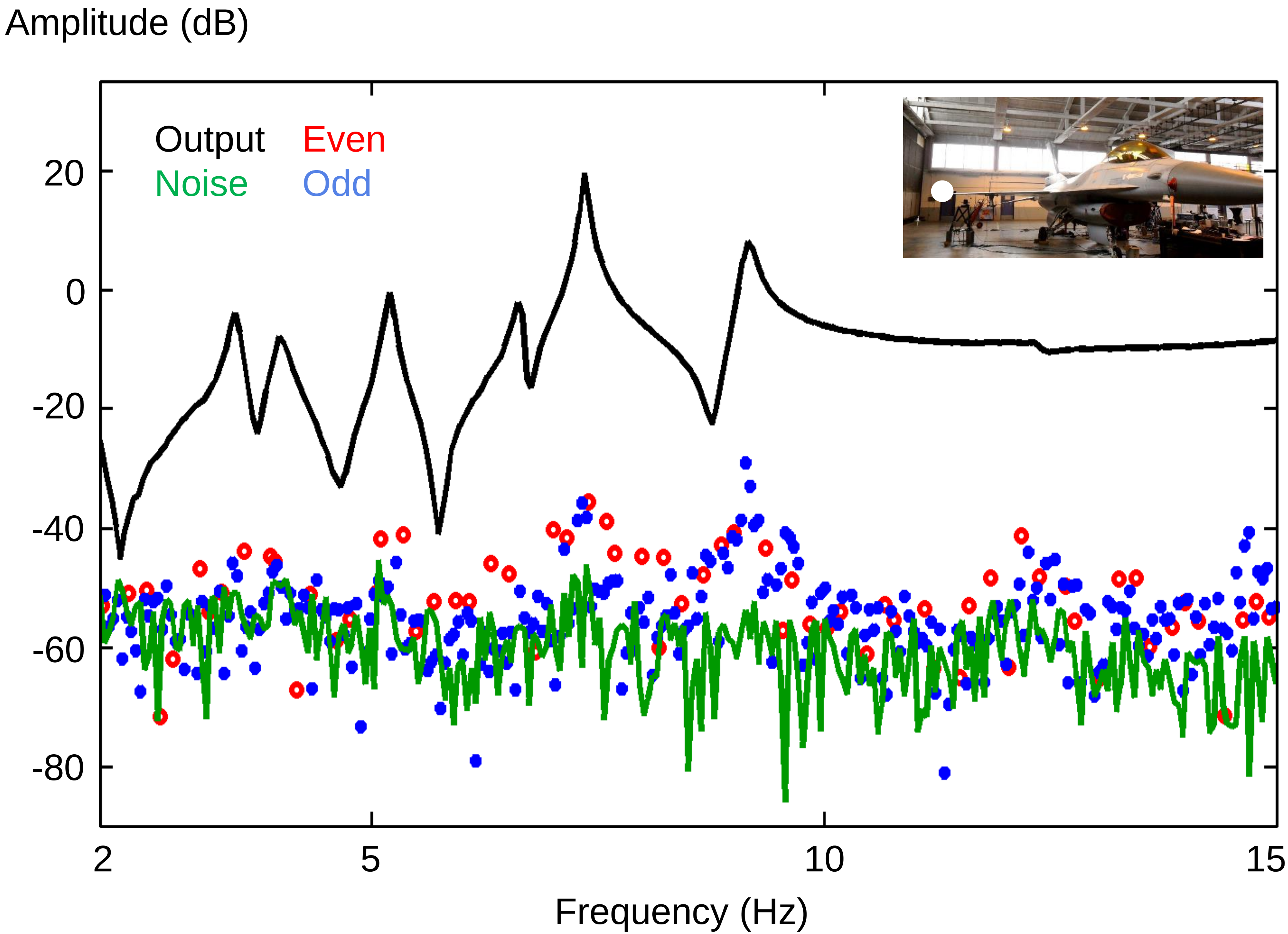
1 1 1	3
1 1 -1	1
1 -1 1	1
1 -1 -1	-1
-1 1 1	1
-1 1 -1	-1
-1 -1 1	-1
-1 -1 -1	-3



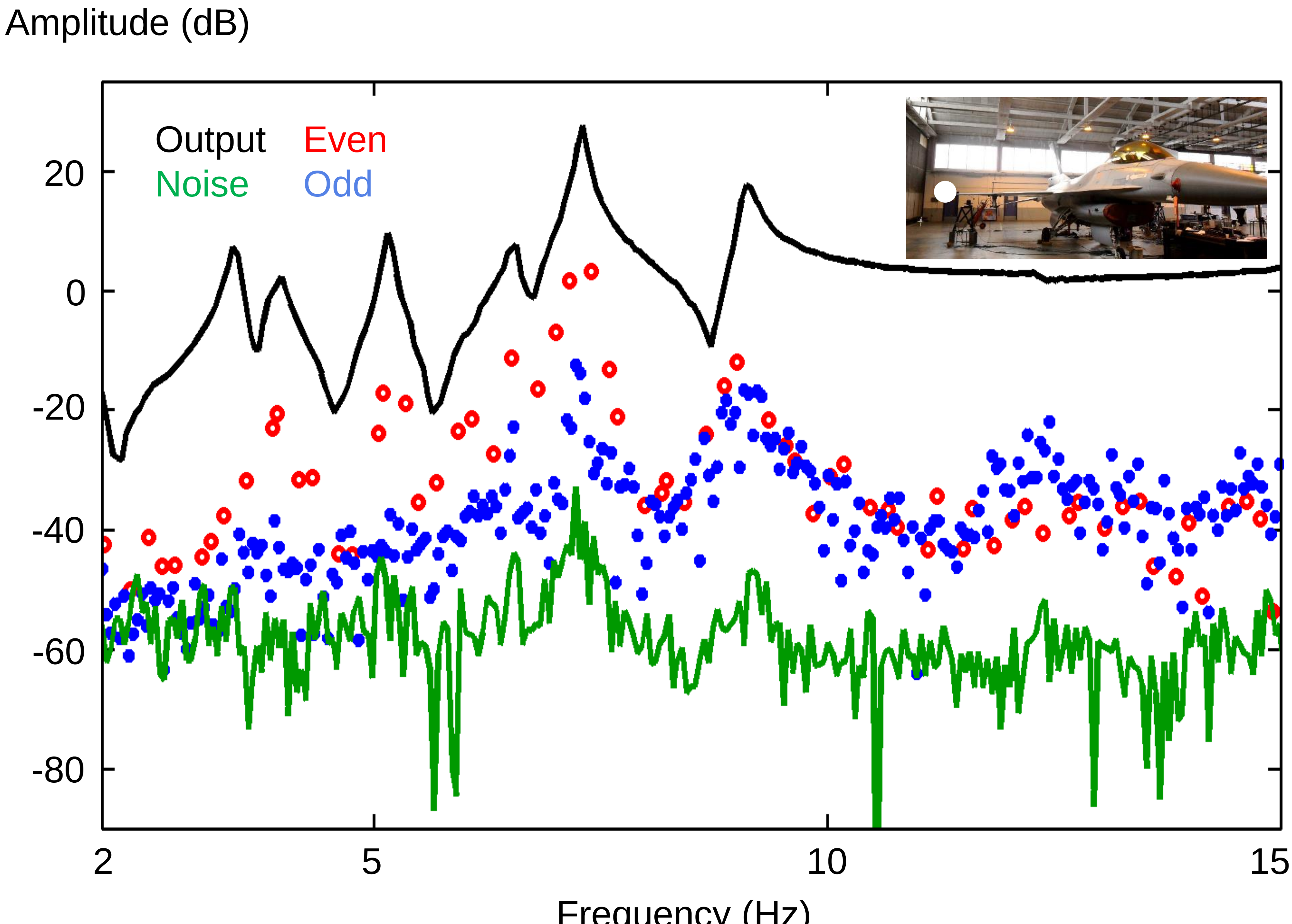
NL Detection using a Carefully-Selected Input Spectrum



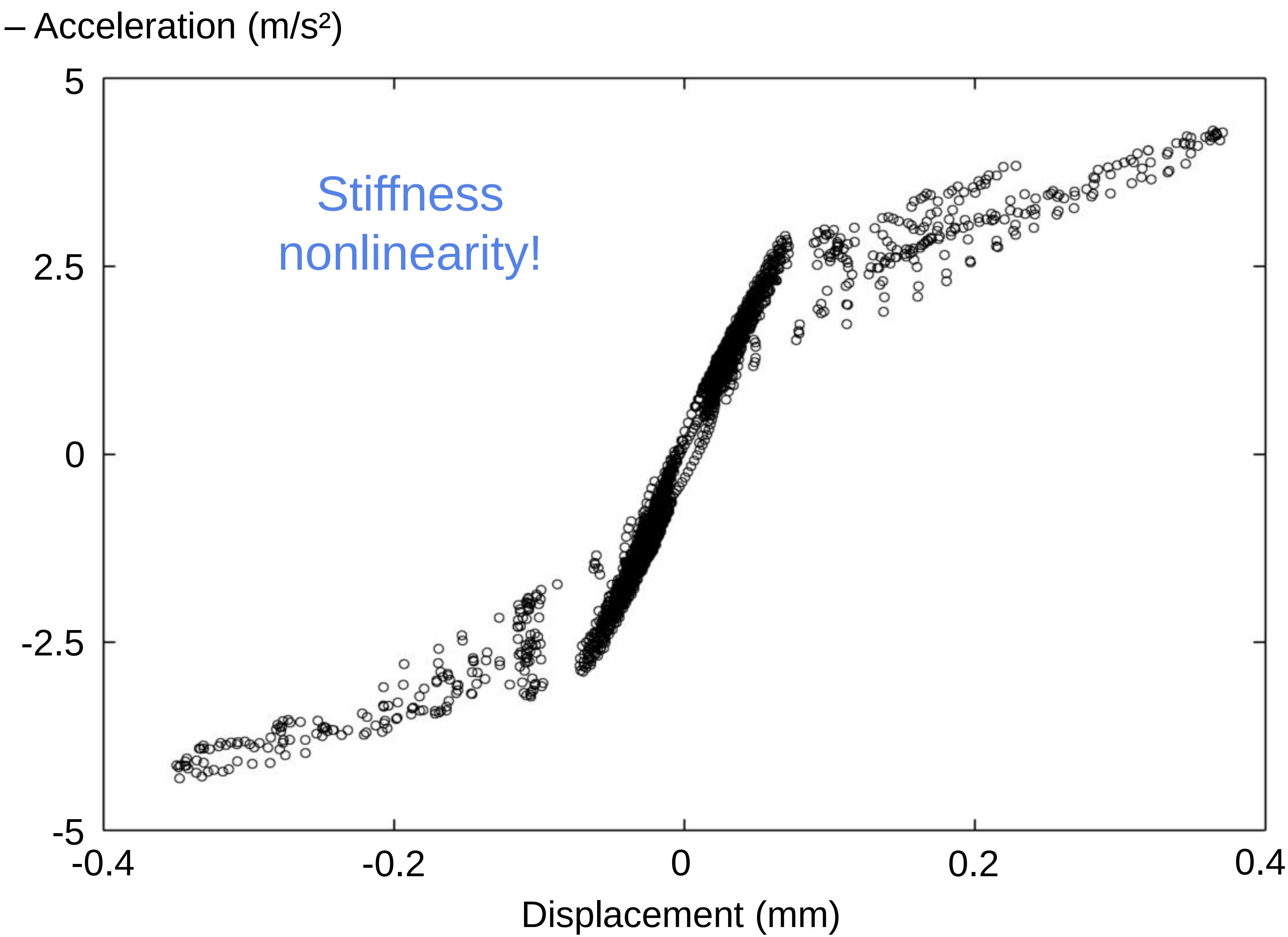
F-16 Measurement at Low Excitation Level



Odd and Even NLs Are Clearly Detected at High Level

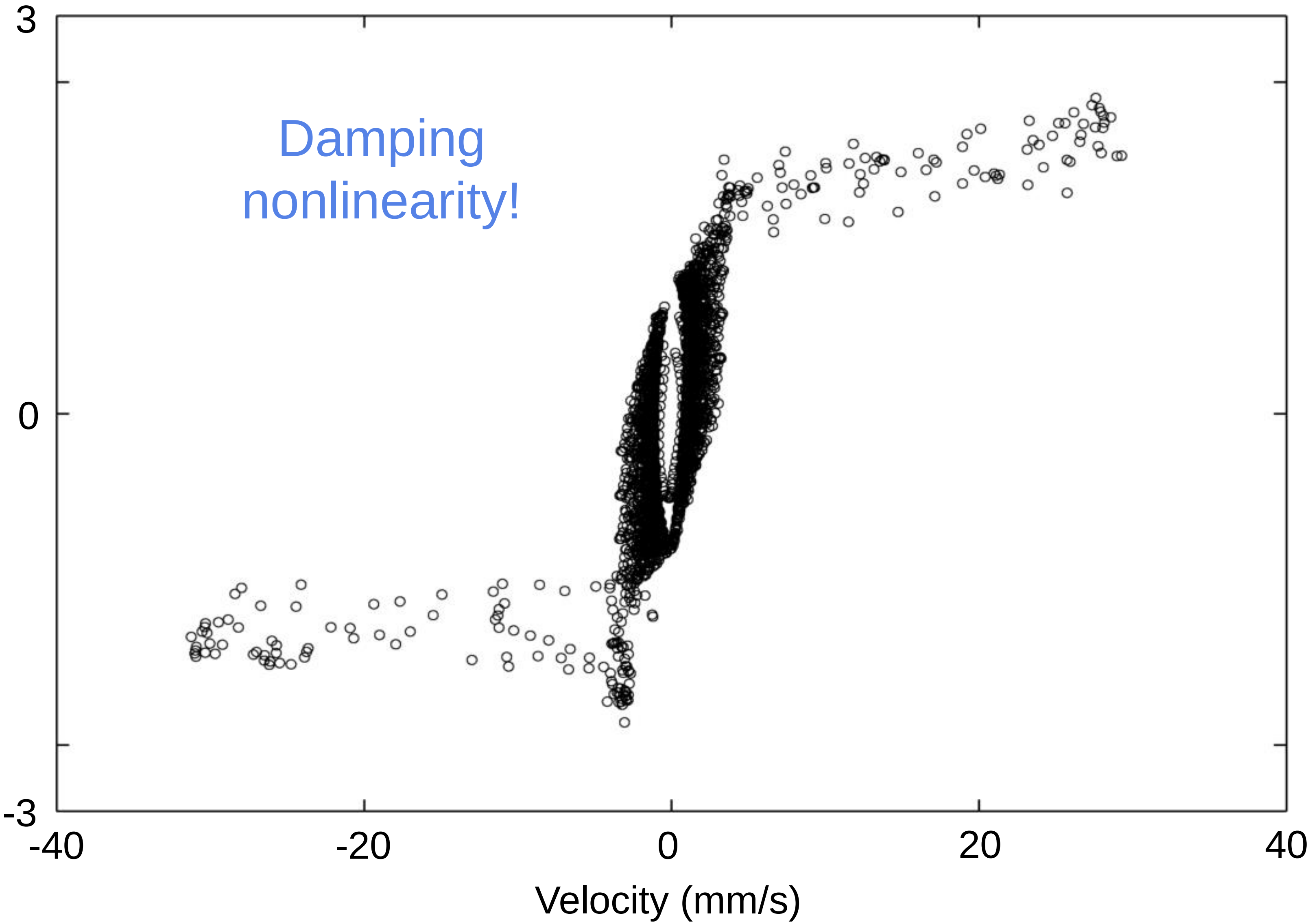


The Source of Nonlinearity … See Next Lecture



The Source of Nonlinearity … See Next Lecture

– Acceleration (m/s²)



Concluding Remarks

Nonlinear system identification is a three-step process.

Nonlinearity detection is the most straightforward step, but supports an important decision in the design cycle.

Sine and broadband excitations are treated separately (toolbox).

Next lecture is dedicated to characterising nonlinearity.

Further Readings

J.P. Noël, G. Kerschen, **Nonlinear system identification in structural dynamics: 10 more years of progress**, Mechanical Systems and Signal Processing, 83, 2-35, 2016.

J.P. Noël, L. Renson, G. Kerschen, **Complex dynamics of a nonlinear aerospace structure: Experimental identification and modal interactions**, Journal of Sound and Vibration, 333, 2588-2607, 2014.

J.P. Noël, A.F. Esfahani, G. Kerschen, J. Schoukens, **A nonlinear state-space approach to hysteresis identification**, Mechanical Systems and Signal Processing, In press, 2016.

T. Dossogne, J.P. Noël, C. Grappasonni, G. Kerschen, B. Peeters, J. Debille, M. Vaes, J. Schoukens, **Nonlinear ground vibration identification of an F-16 aircraft – Part II**, Proceedings of IFASD, Saint Petersburg, Russia, 2015.