

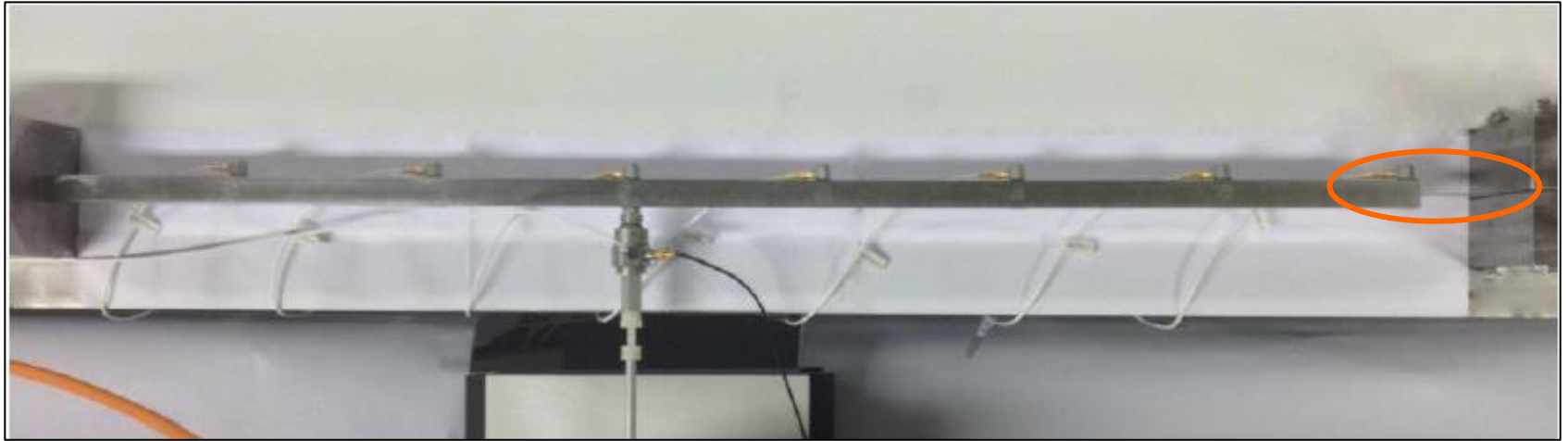
Nonlinear Vibrations of Aerospace Structures

Tutorial 02

Nonlinear Symptoms



Nonlinear Model of the 1st Beam Mode

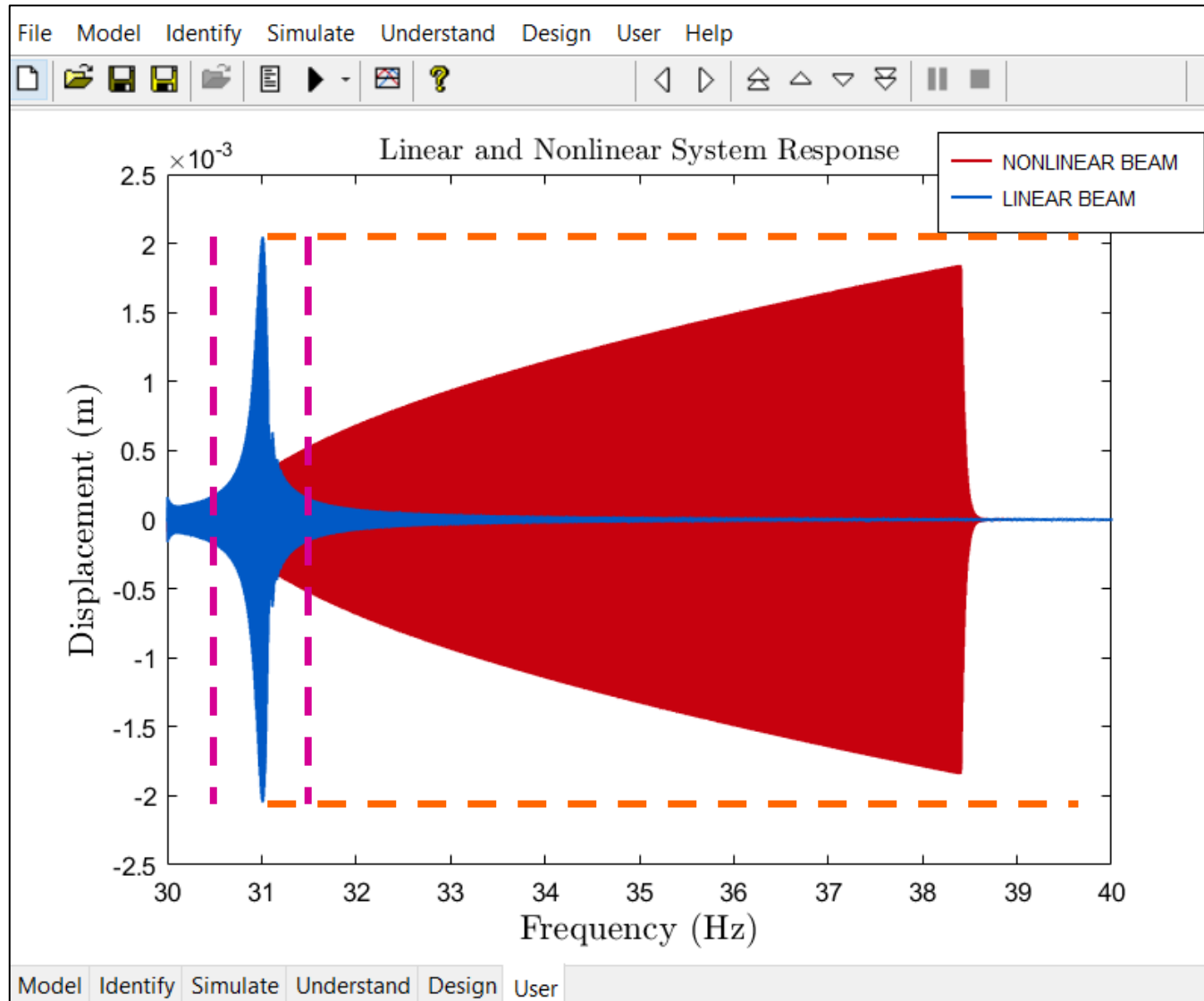


Linear model identified at low level (31 Hz, 0.12%):

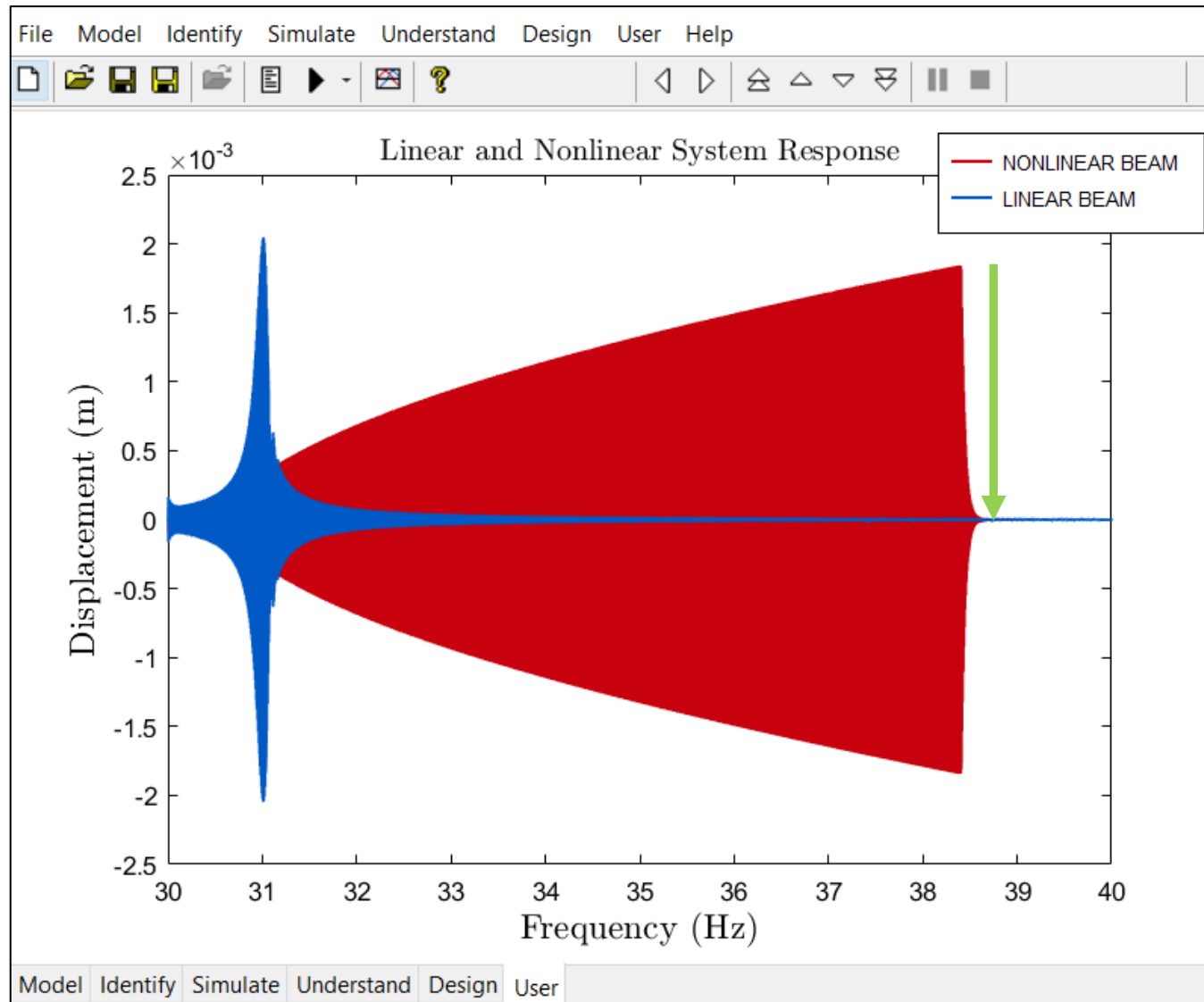
$$0.289\ddot{x} + 0.1357\dot{x} + 11009x = F \sin \omega t$$

Nonlinearity identified at high level: $2.37 \cdot 10^9 x^3$

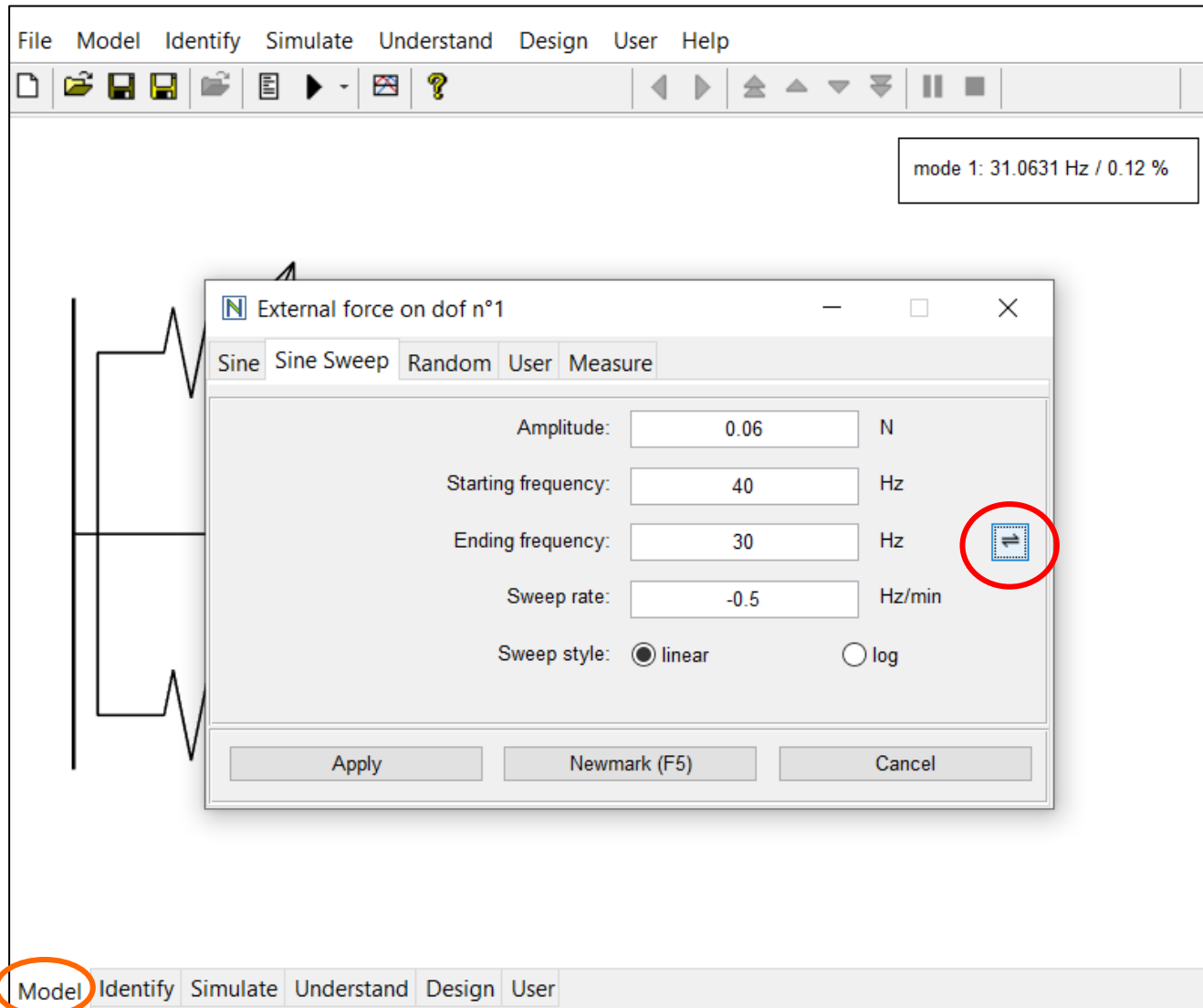
Nonlinearity Introduces a Fundamental Change



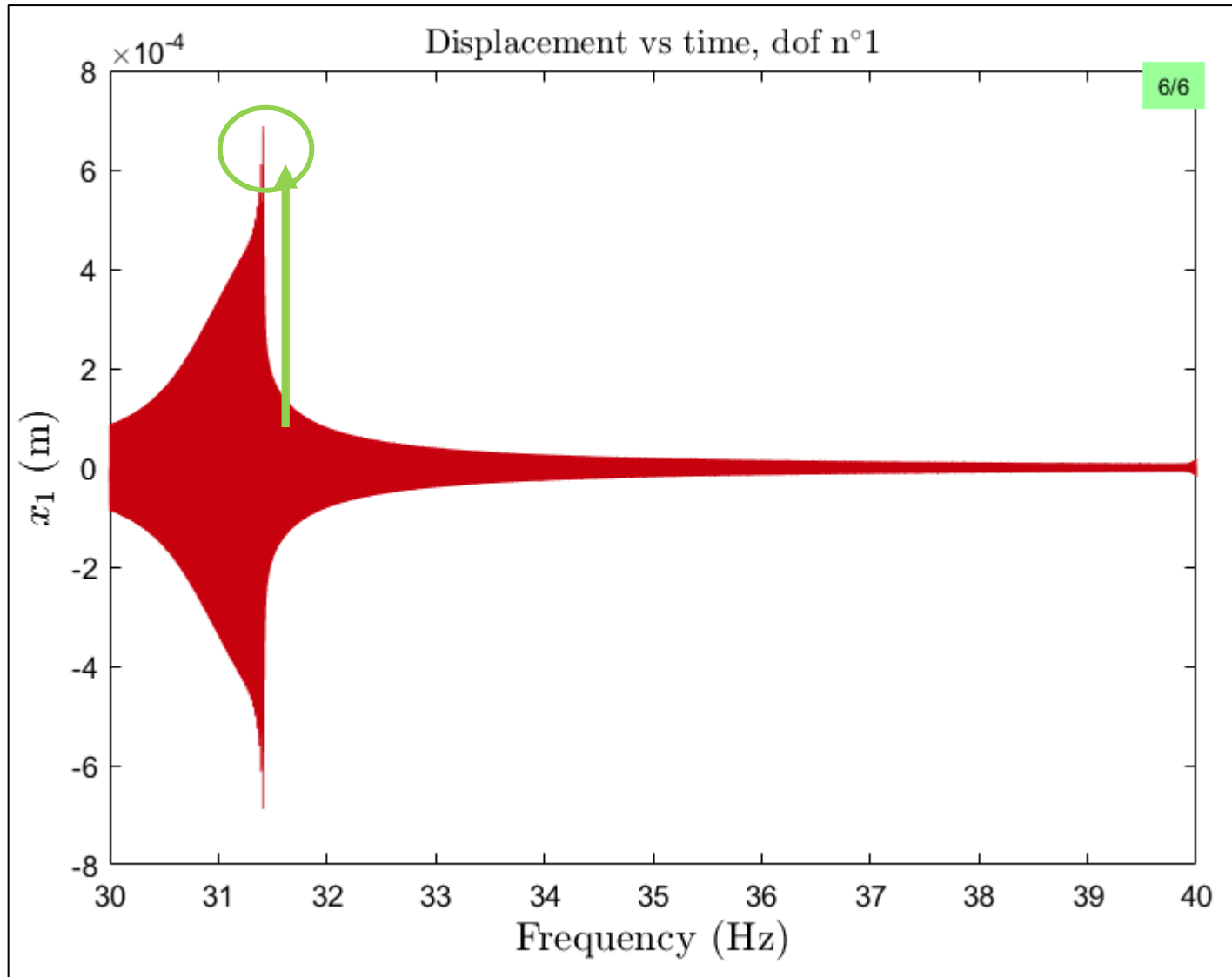
A Jump Downwards Can Be Observed



Let's Reverse the Sweep



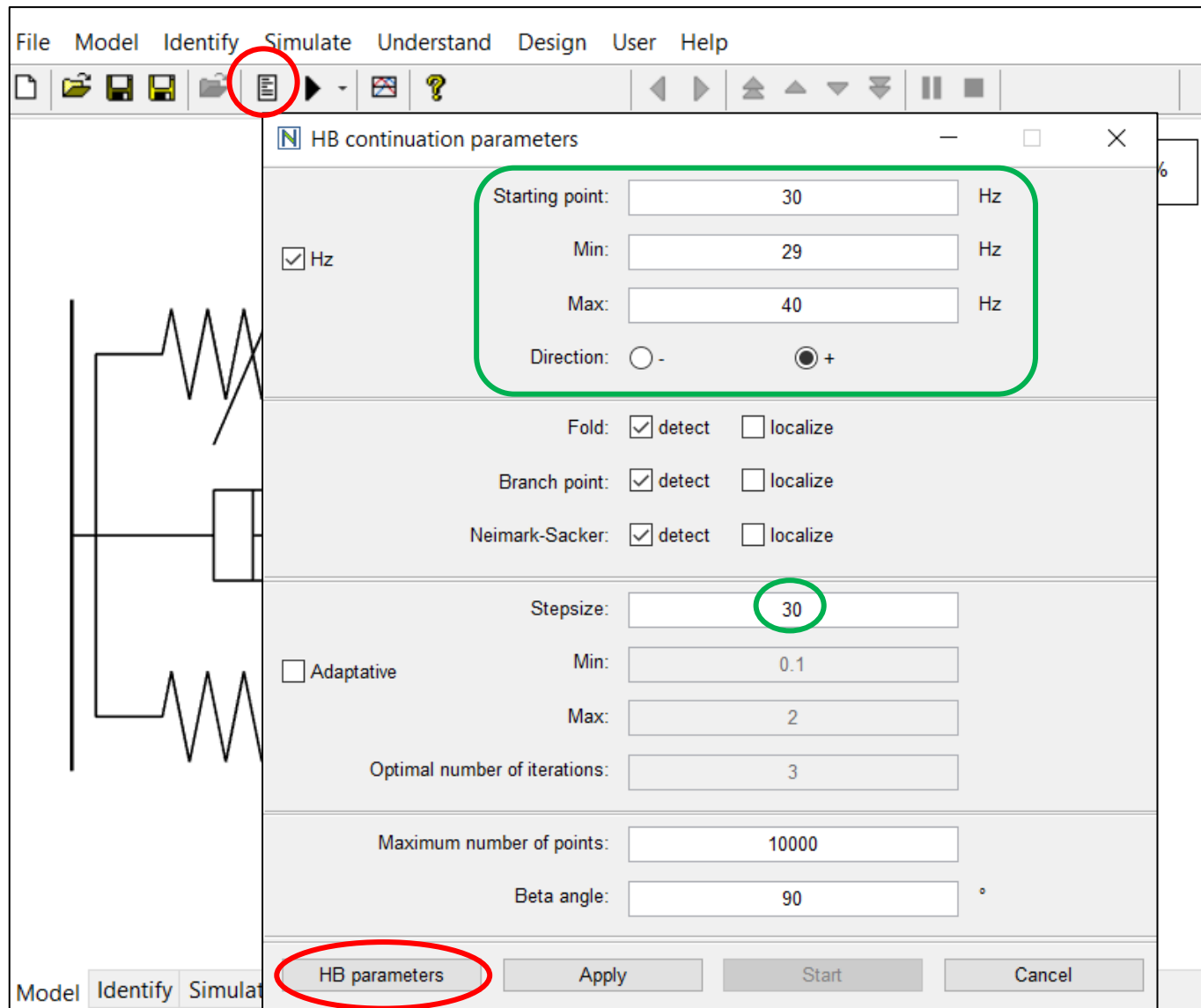
We Can Jump Up Too!



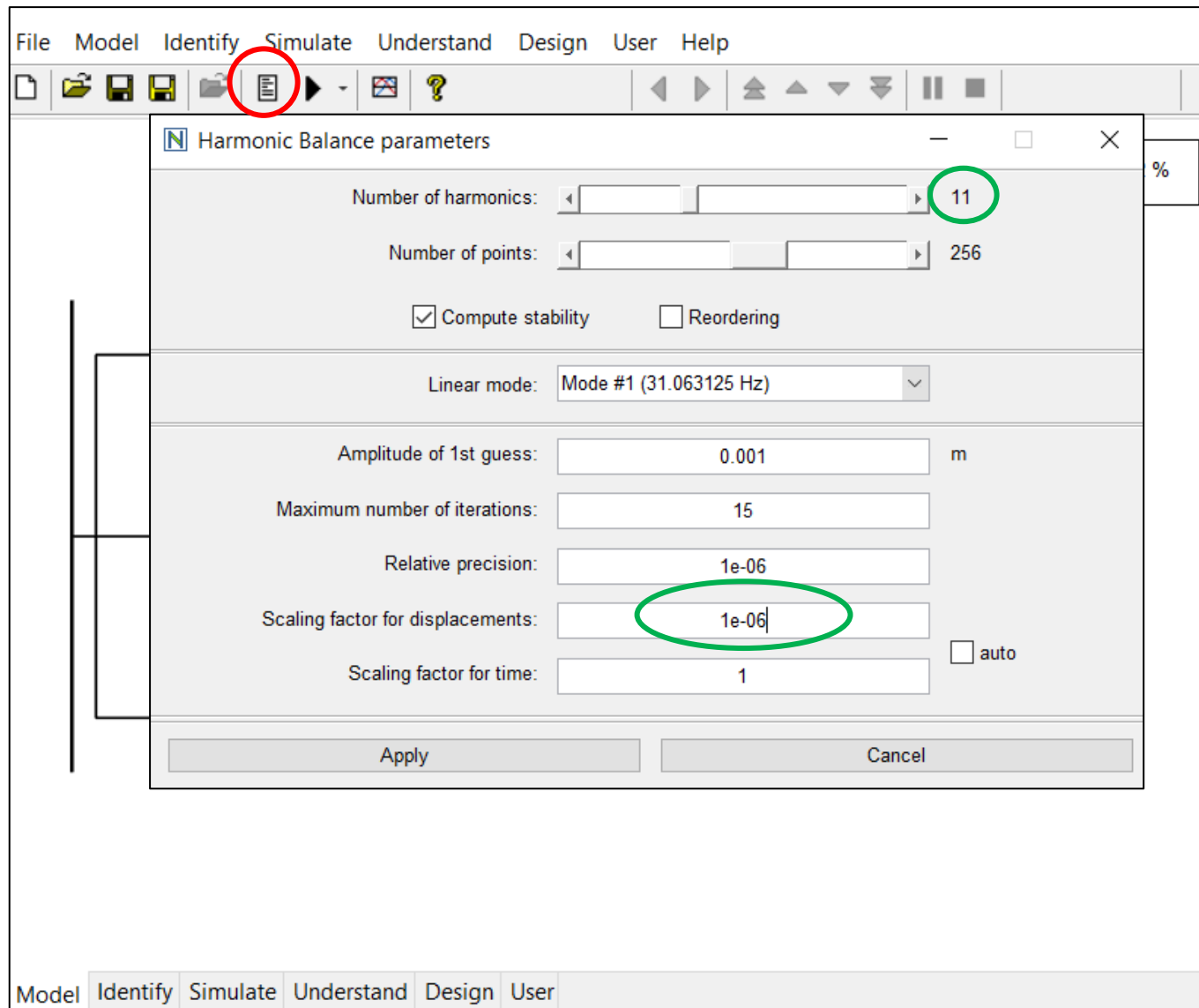
Calculate the Nonlinear FRF

The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The 'Simulate' menu is open, showing options: x-ASM, Newmark, Linear FRFs, **Harmonic balance continuation** (checked), NNM (Nonlinear Normal Modes) continuation, Global analysis, and Parametric study. A red circle highlights the play button in the toolbar. The main workspace shows a mechanical model with a blue rectangular component labeled '0.289'. A modal analysis result box indicates 'mode 1: 31.0631 Hz / 0.12 %'. The model includes a spring with stiffness $2.37e+09 \cdot x^3$, a damper with coefficient $0.1357 \cdot \dot{x}$, and another spring with stiffness $11009 \cdot x$. The bottom status bar shows 'Model Identify Simulate Understand Design User'.

But Set Appropriate Numerical Parameters First



But Set Appropriate Numerical Parameters First



And Change the Excitation Signal to a Sine

File Model Identify Simulate Understand Design User Help

mode 1: 31.0631 Hz / 0.12 %

$0.06 \cdot \sin(2 \cdot \pi \cdot t)$

External force on dof n°1

Sine Sine Sweep Random User Measure

Amplitude: 0.06 N

Frequency: 1 Hz

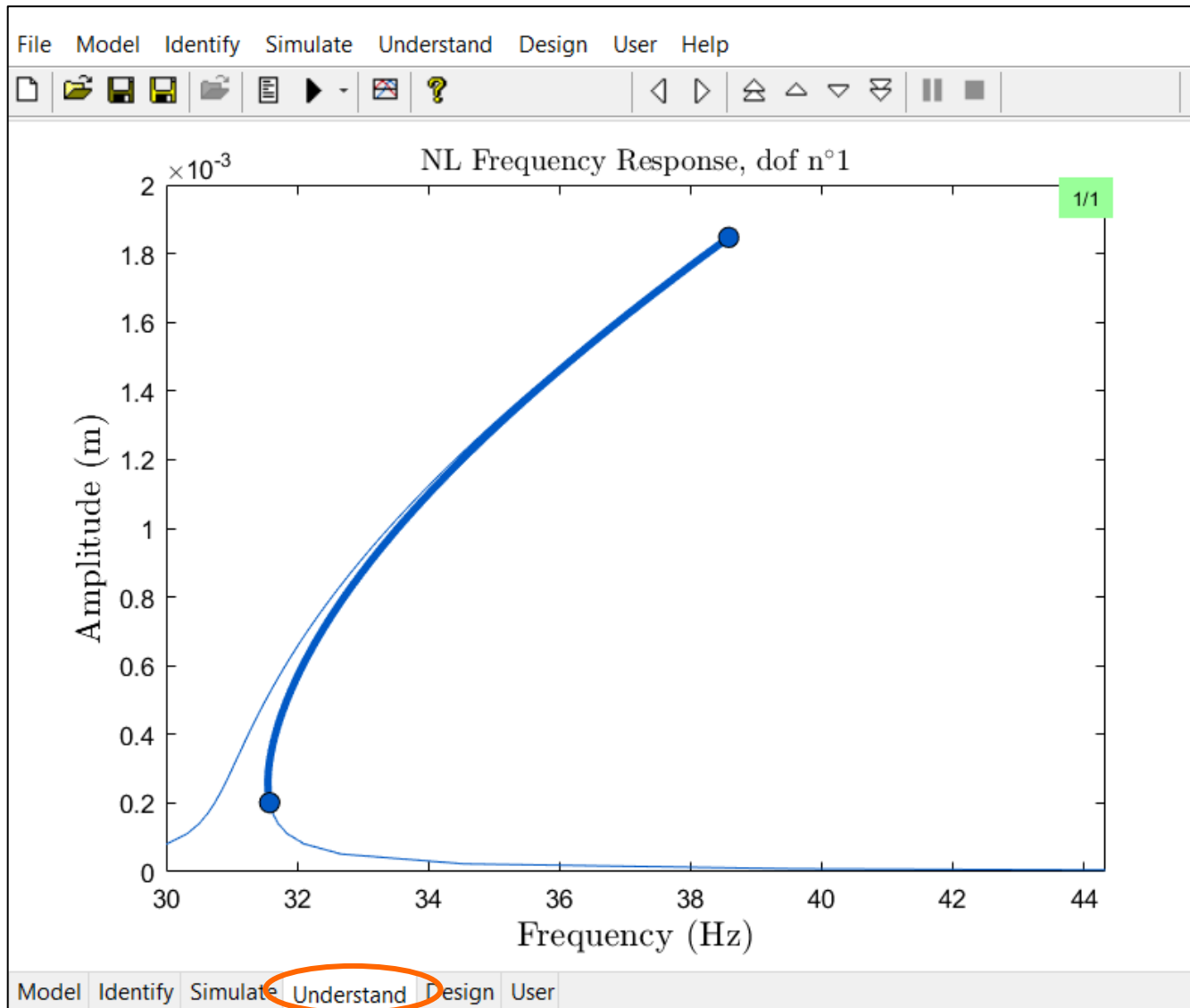
Phase: 0 °

Note that the chosen frequency is not relevant for the computation of the nonlinear FRF

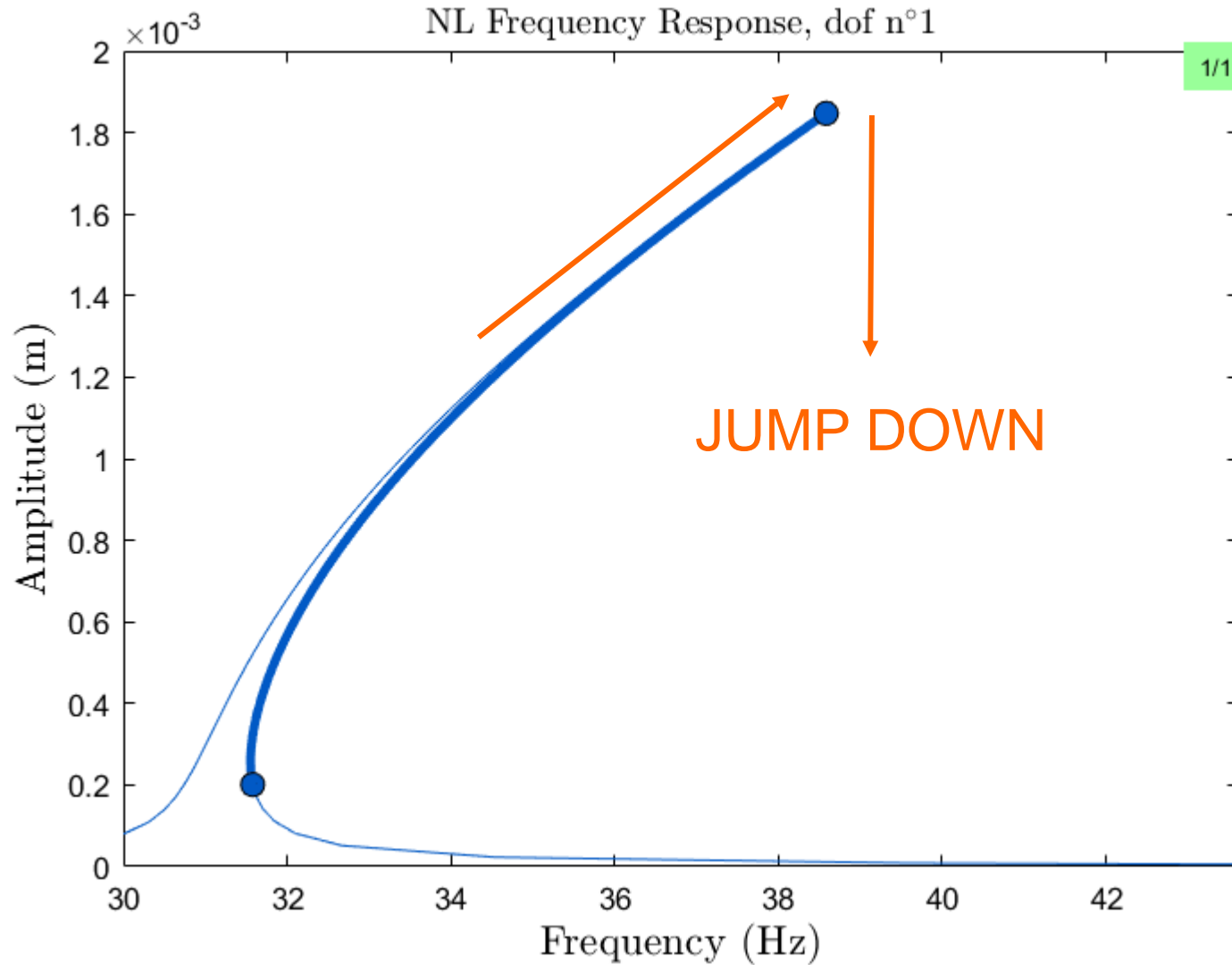
Apply HB cont (F5) Cancel

Model Identify Simulate Understand Design User

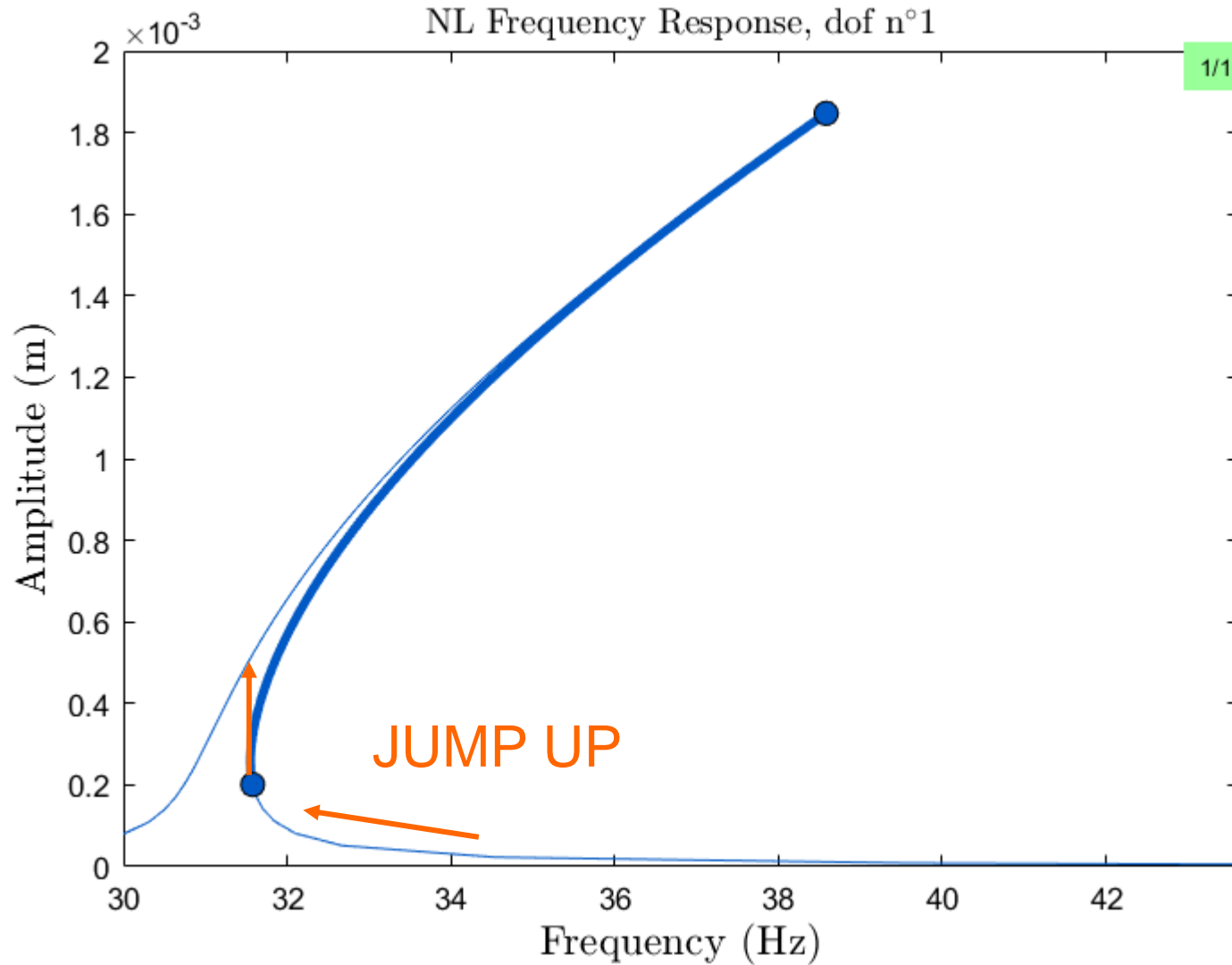
The Nonlinear FRF



The Jump Phenomenon Can Be Understood

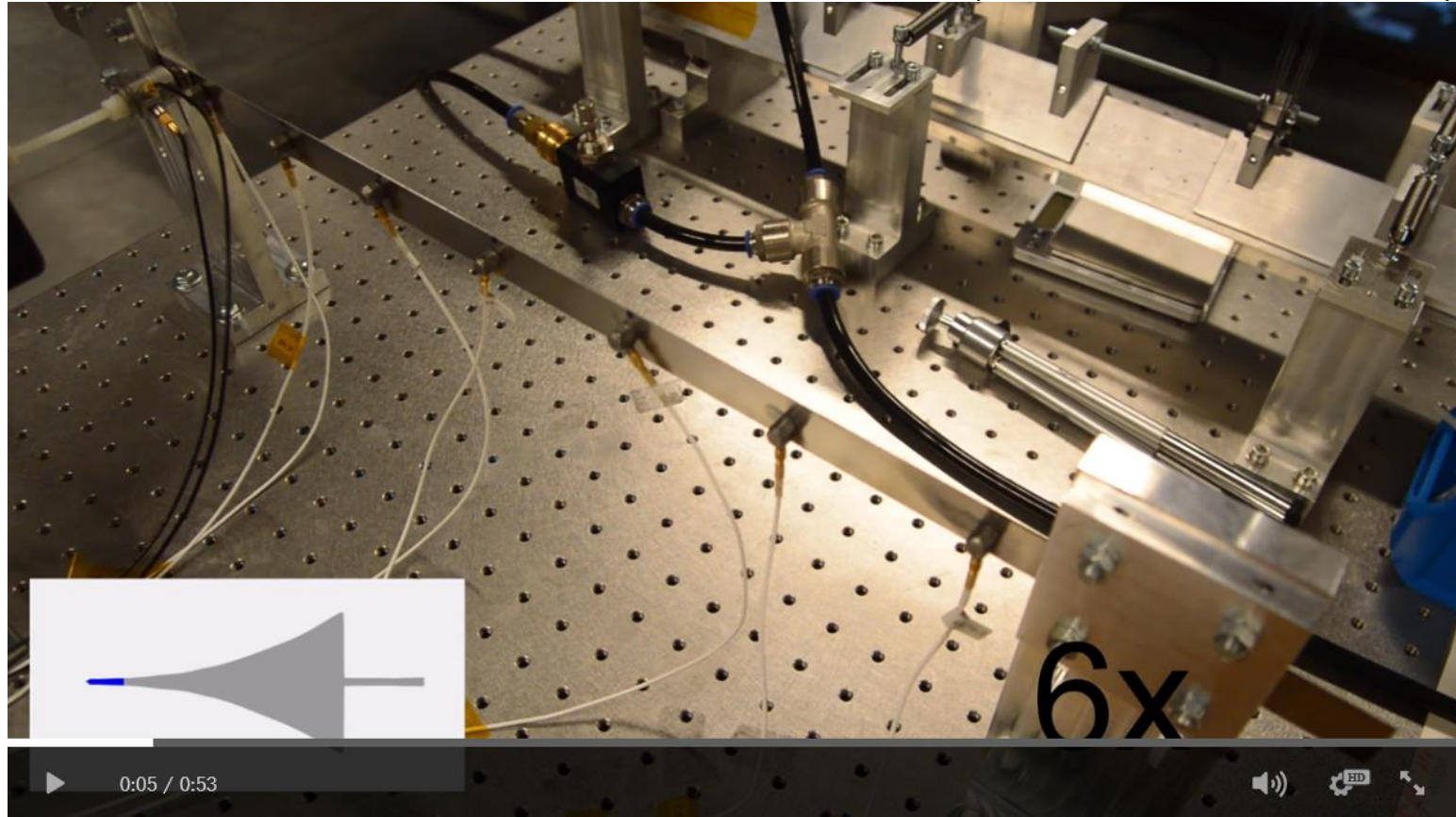


The Jump Phenomenon Can Be Understood



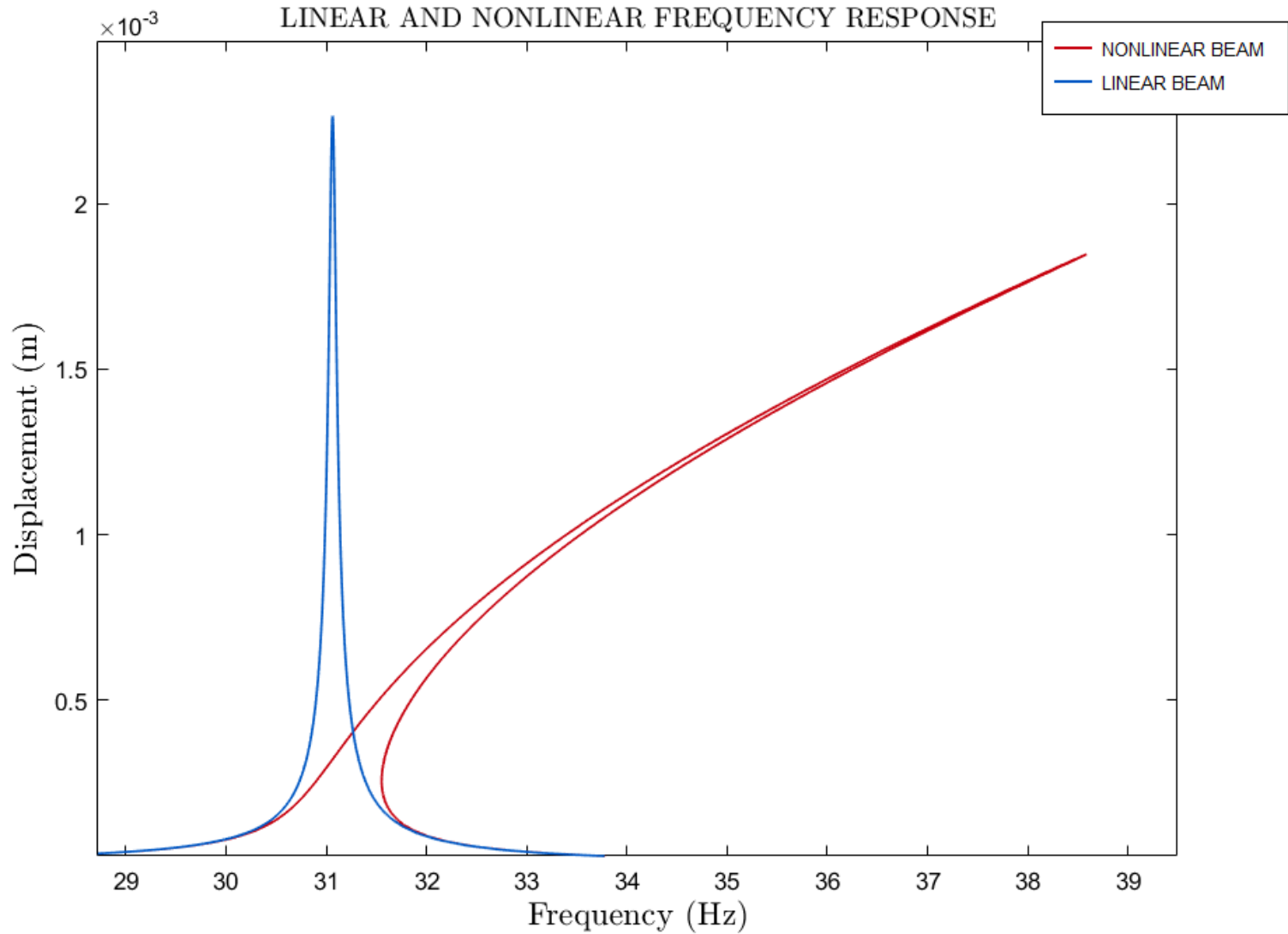
The Jump Phenomenon in Practice

(See the attached video)



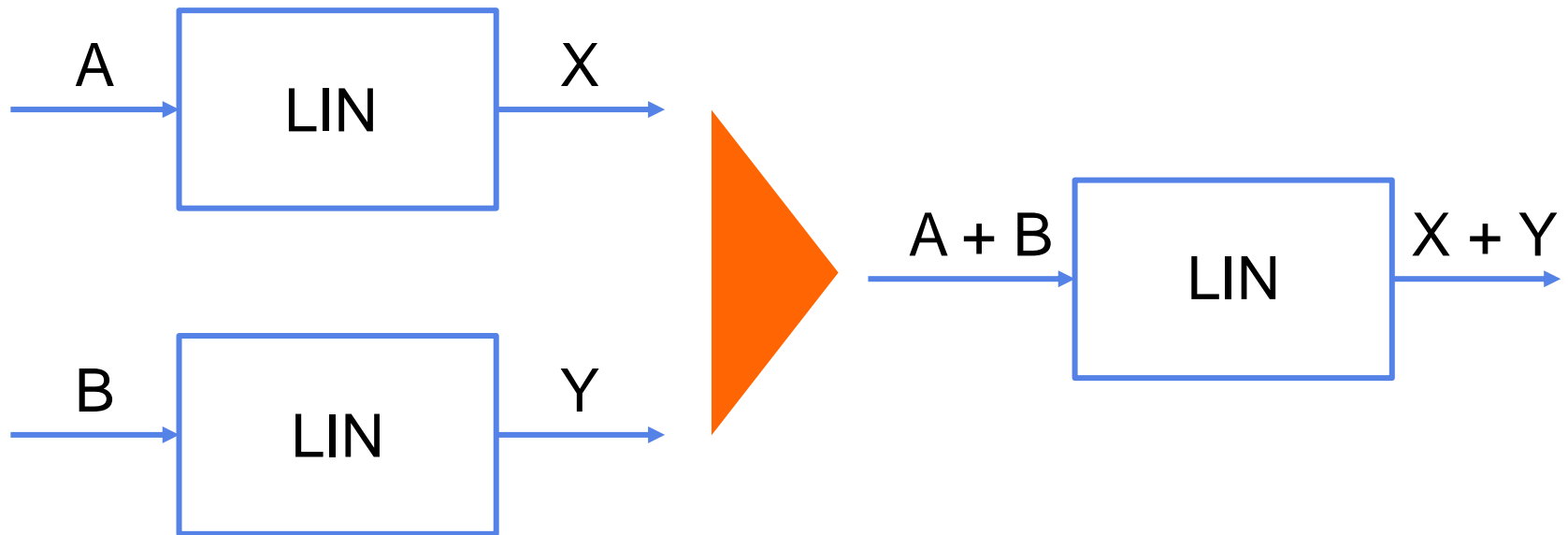
The video shows the dynamics of a clamped-clamped thin beam with geometrical nonlinearities during a sweep over the second beam mode.

Compare to the Linear Frequency Response

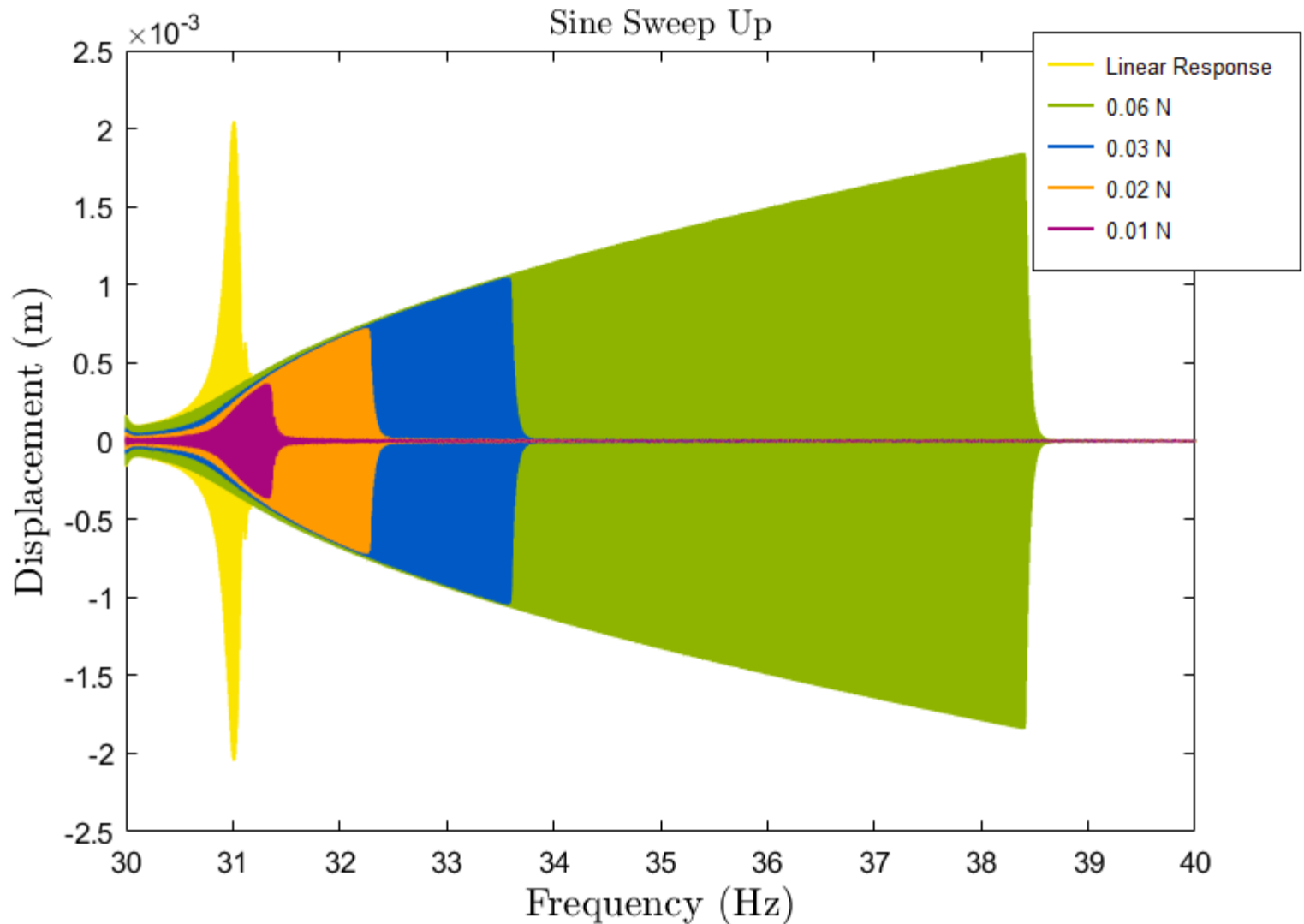


The Superposition Principle

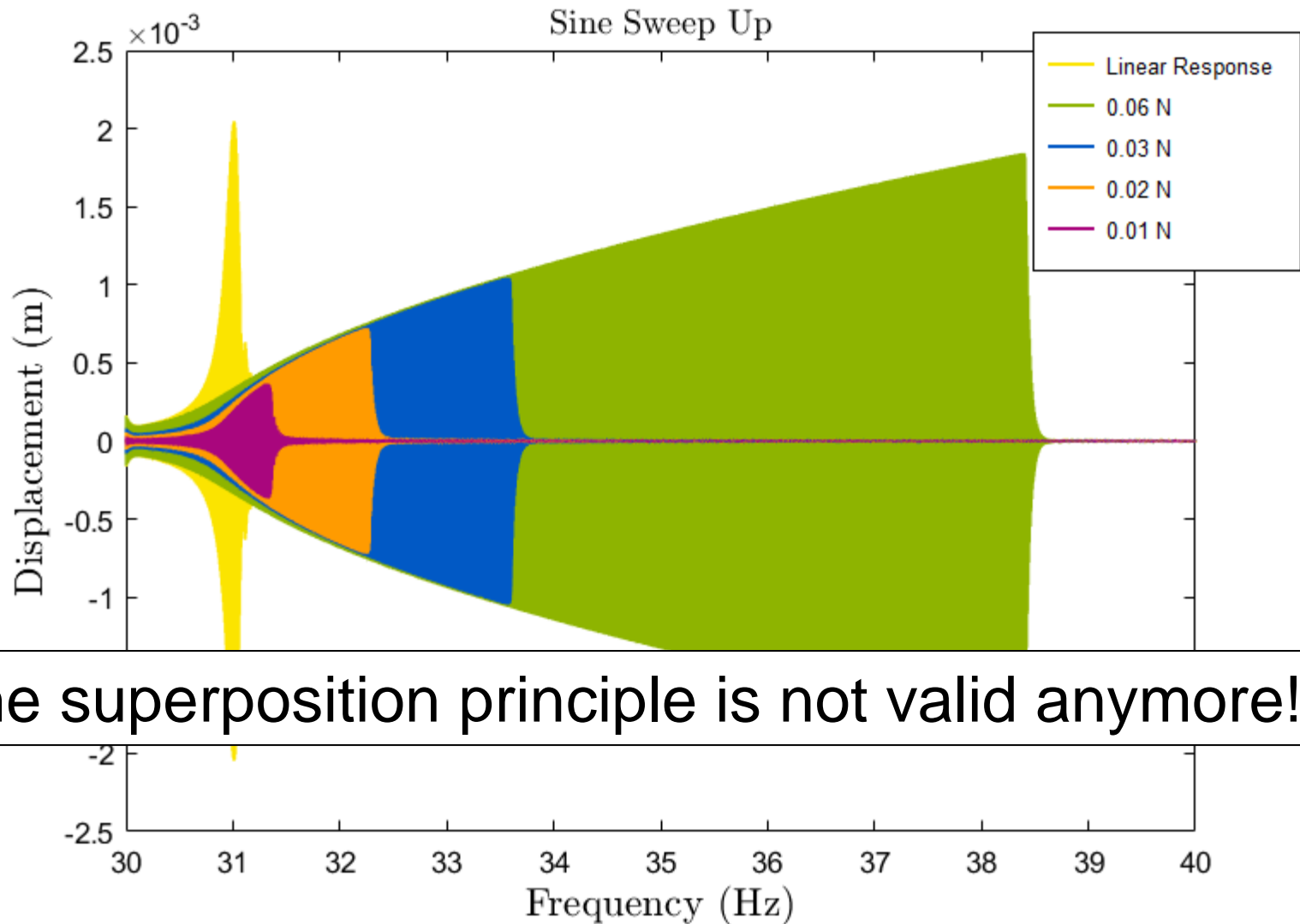
Cornerstone of linear theory:



Sweep with Different Forcing Amplitudes



Sweep with Different Forcing Amplitudes



The superposition principle is not valid anymore!

Sine with a Fixed Frequency
in the Multi-valued Region

Now: Consider an Excitation with a Fixed Frequency

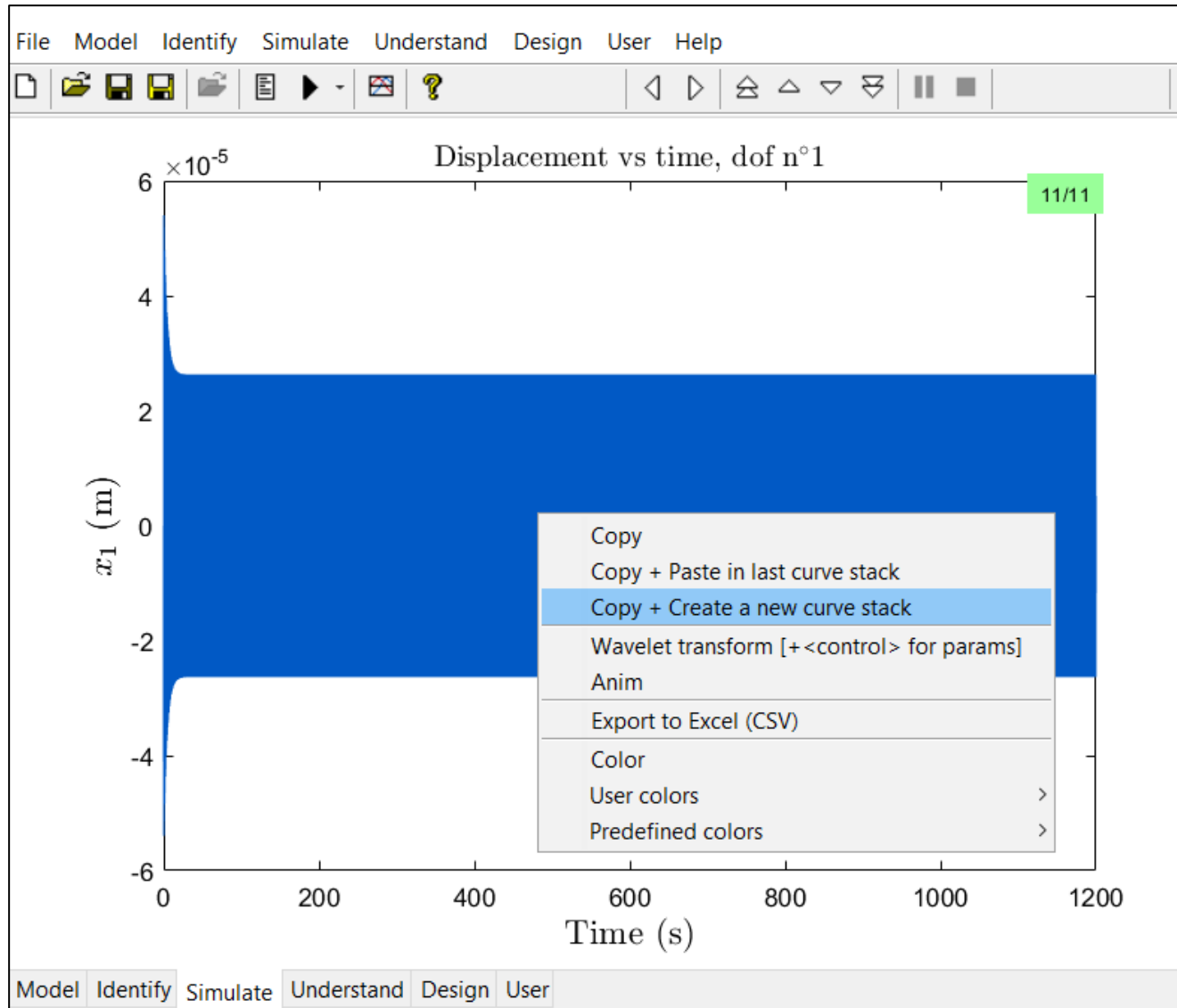
The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The main workspace shows a mechanical model with a blue component. A sine wave excitation is applied to the component, with the equation $0.06 \cdot \sin(2 \cdot \pi \cdot 34 \cdot t)$ circled in orange. A status box in the top right corner indicates "mode 1: 31.0631 Hz / 0.12 %".

An "External force on dof n°1" dialog box is open, showing the following parameters:

Parameter	Value	Unit
Amplitude	0.06	N
Frequency	34	Hz
Phase	0	°

The dialog box also includes tabs for "Sine", "Sine Sweep", "Random", "User", and "Measure", and buttons for "Apply", "Newmark (F5)", and "Cancel".

Run the Newmark Integration and Save



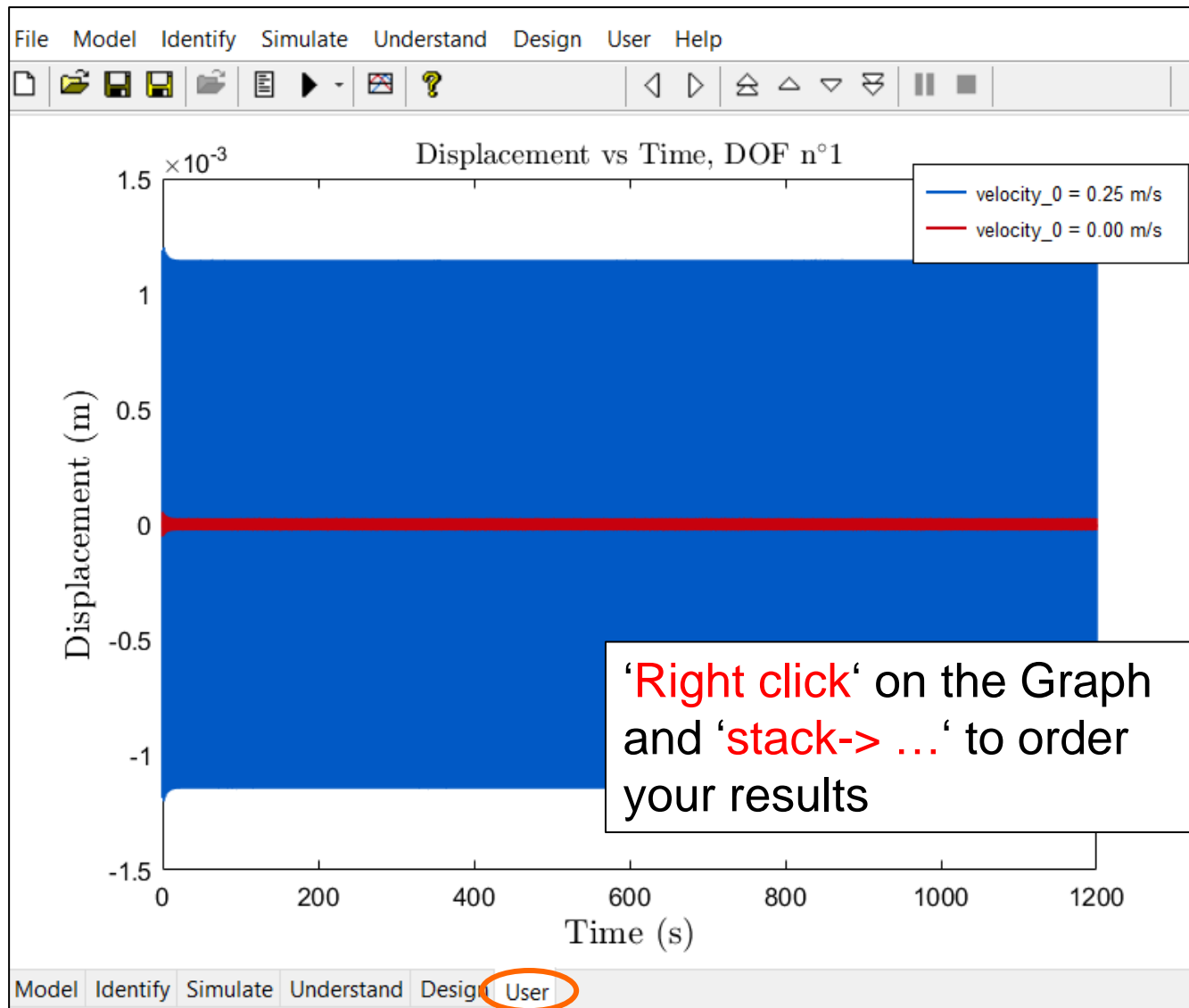
Change the Initial Conditions

The screenshot shows a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. A red circle highlights the 'Initial cond.' button in the toolbar. The main workspace displays a circuit diagram with a sine wave input labeled $0.06 \cdot \sin(2 \cdot \pi \cdot 34 \cdot t)$. A text box indicates 'mode 1: 31.0631 Hz / 0.12 %'. Two dialog boxes are open:

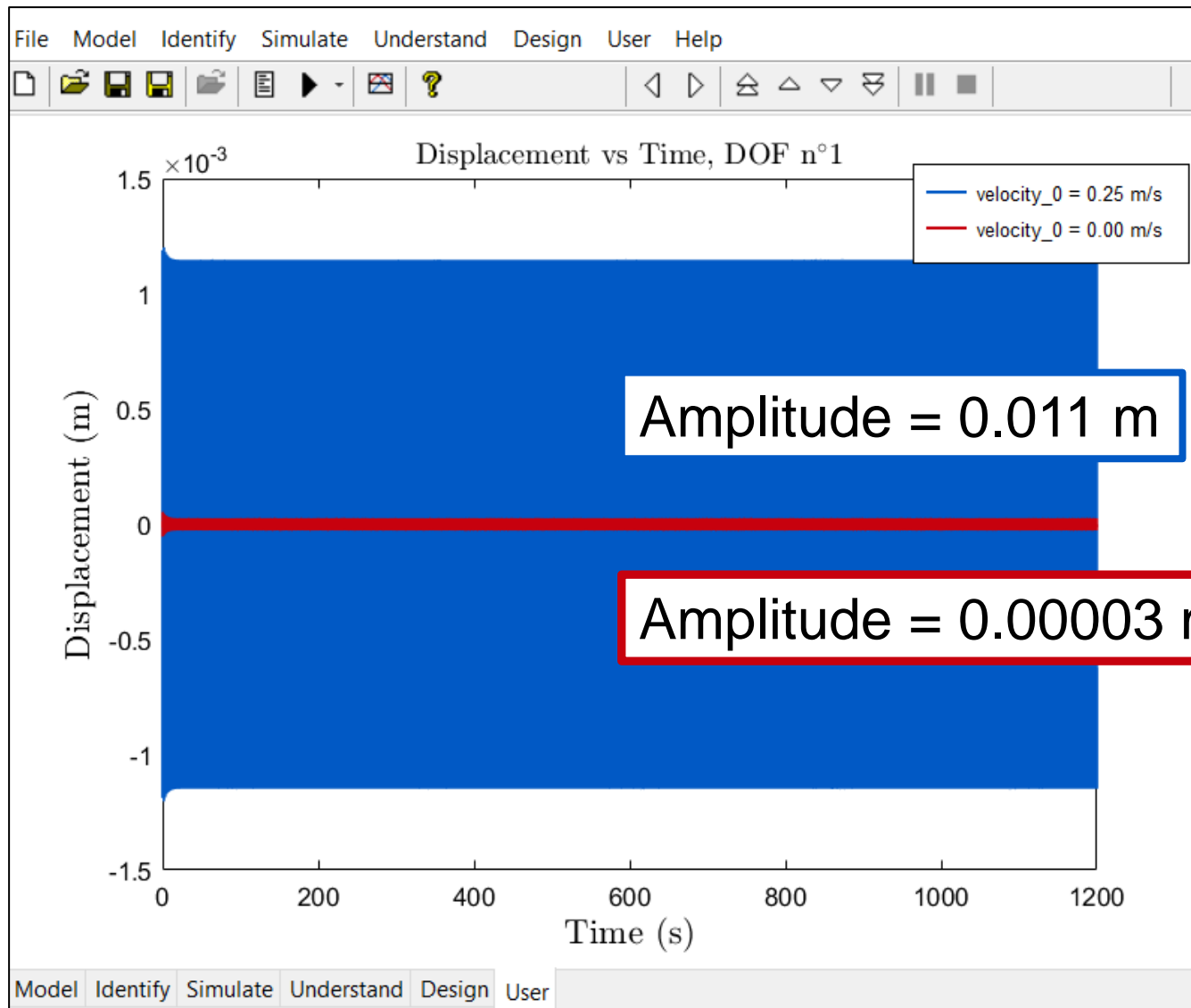
- Newmark parameters [SINE]**:
 - Final time: 1200
 - Time step: 0.001
 - Number of time steps: 1200000
 - Number of periods: 100
 - Time steps by period: 90
 - Saved dofs: all selected
- Initial conditions**:
 - Degree of freedom: all
 - Displacement: 0 m
 - Velocity: 0.25 m/sec (circled in green)

Buttons for 'Apply' and 'Cancel' are present in both dialog boxes. A red arrow points from the 'Initial cond.' button in the toolbar to the 'Initial conditions' dialog box.

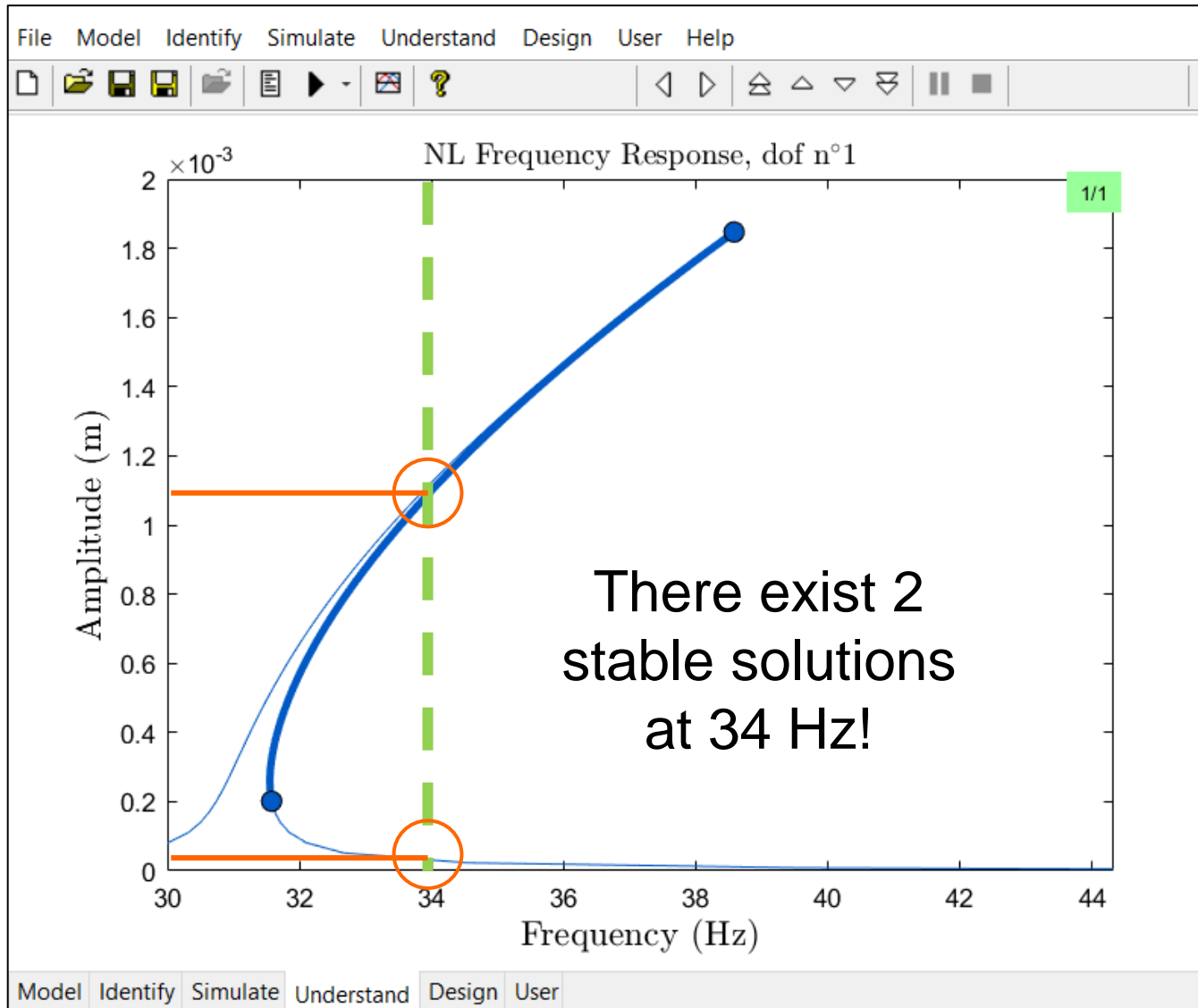
Run and Save in the Same Curve Stack



Compare

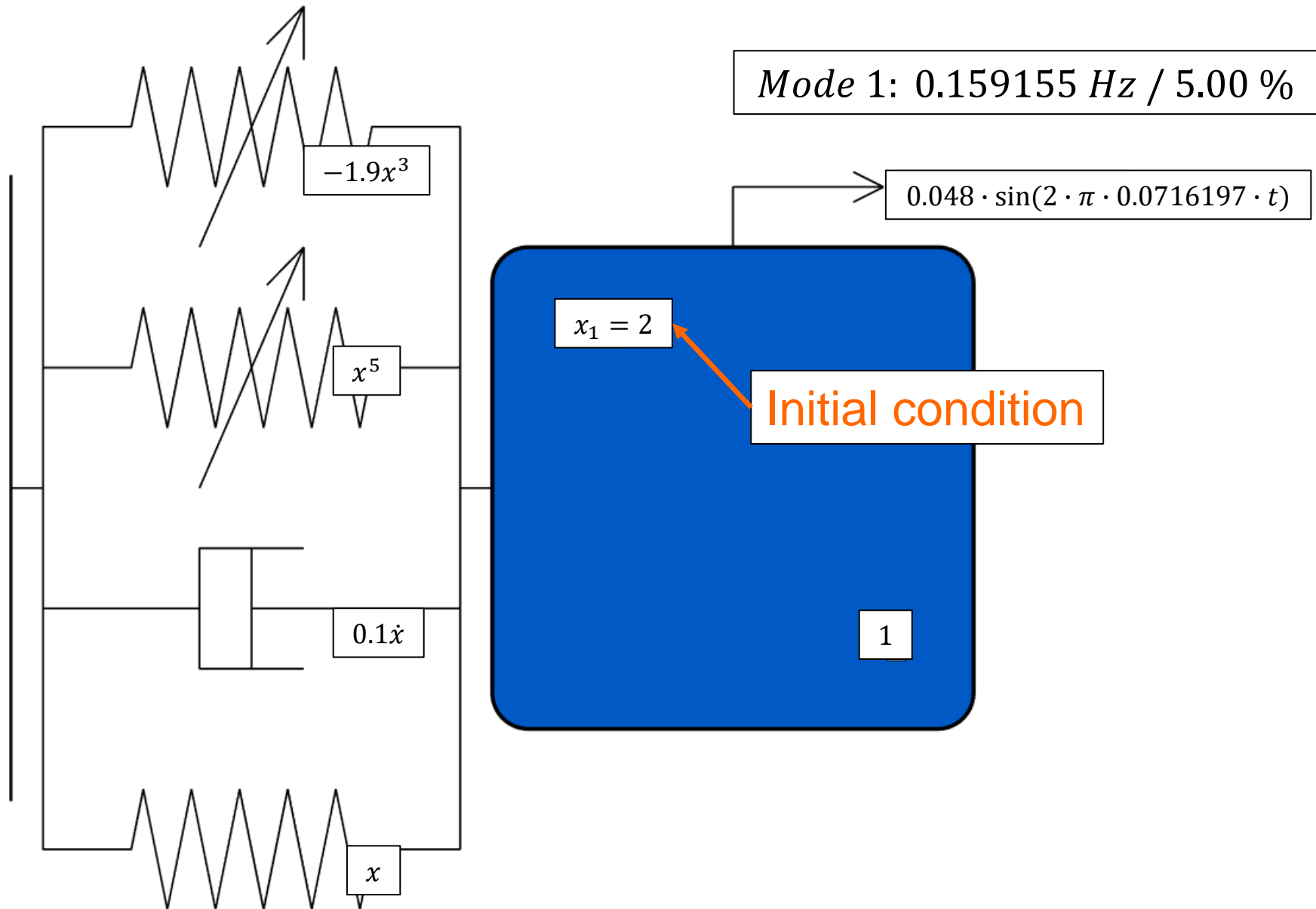


The Nonlinear FRF Helps to Understand



A New System in NI2D

The New 1 DOF-system



Parameters for Nonlinear FRF Calculation

HB continuation parameters

Hz !

Starting point: rad/s

Min: rad/s

Max: rad/s

Direction: - +

Fold: detect localize

Branch point: detect localize

Neimark-Sacker: detect localize

Stepsize:

Adaptive

Min:

Max:

Optimal number of iterations:

Maximum number of points:

Beta angle:

HB parameters Apply Start

Harmonic Balance parameters

Number of harmonics:

Number of points:

Compute stability Reordering

Linear mode:

Amplitude of 1st guess: m

Maximum number of iterations:

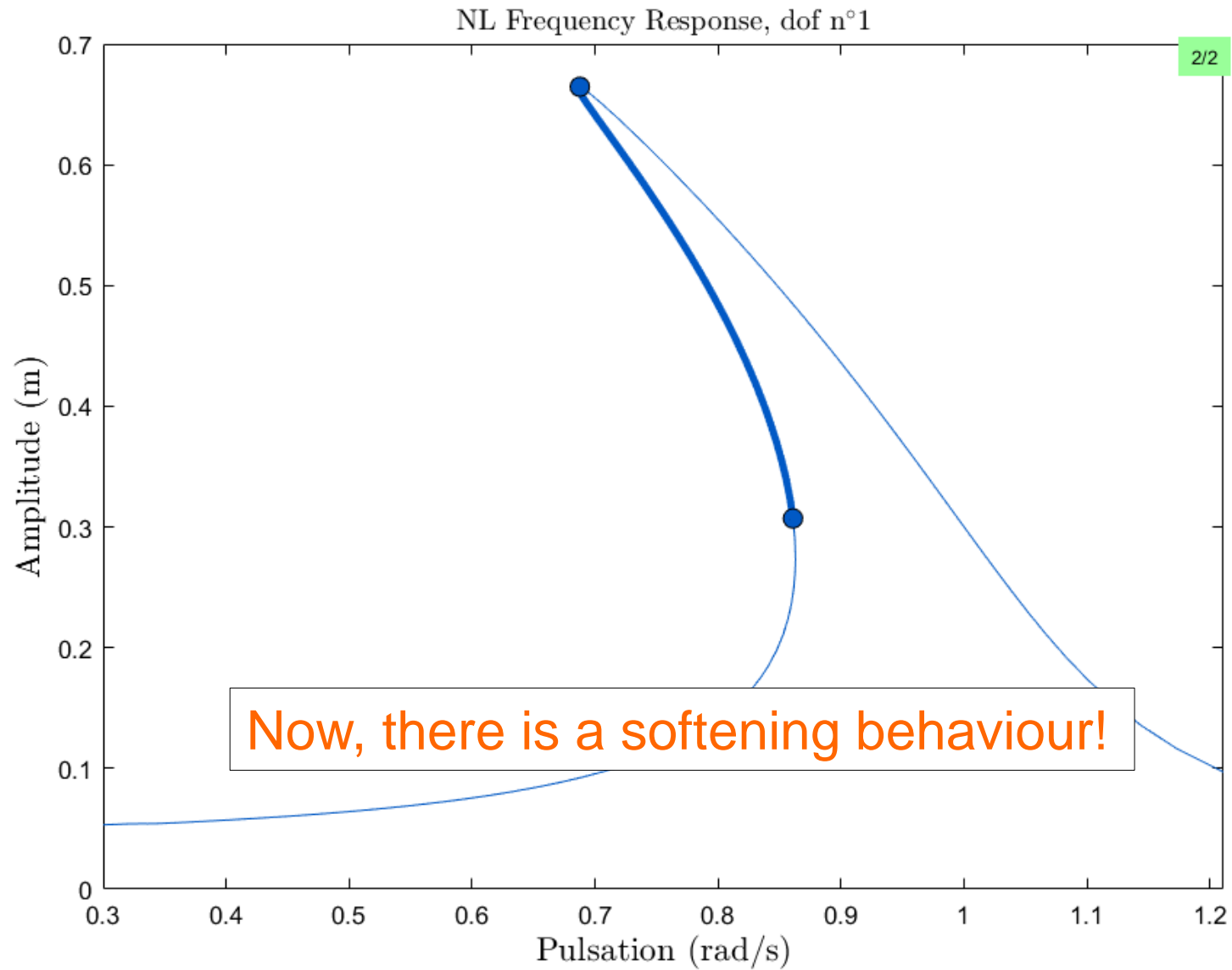
Relative precision:

Scaling factor for displacements:

Scaling factor for time: auto

Apply Cancel

The Nonlinear FRF



Nonlinear Time Integration

Newmark parameters [SINE]

Sampling frequency:	100	Hz
<input type="checkbox"/> adapt	Time step:	0.01 sec
	Time steps per period:	1397

Initial time:	0	sec
Duration:	200	sec
Number of time steps:	20000	
Number of periods:	15	

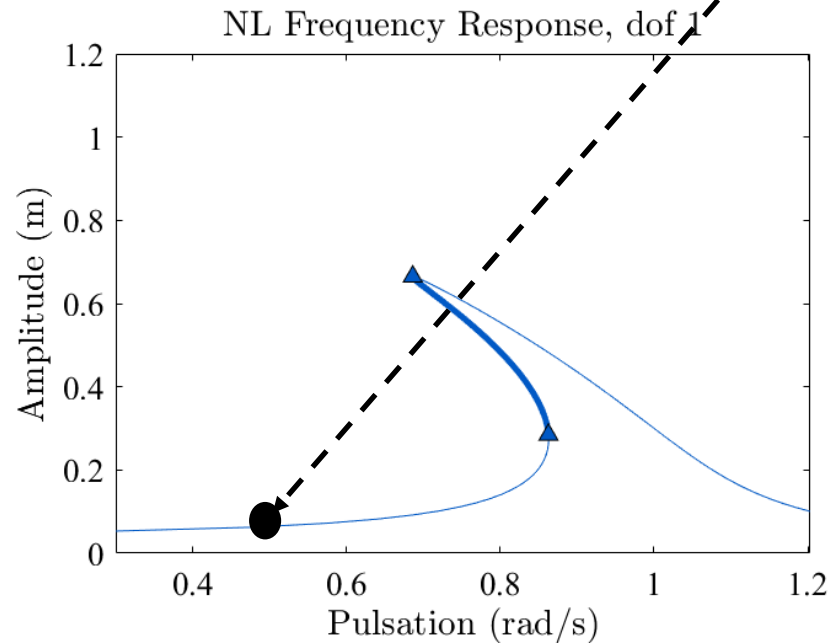
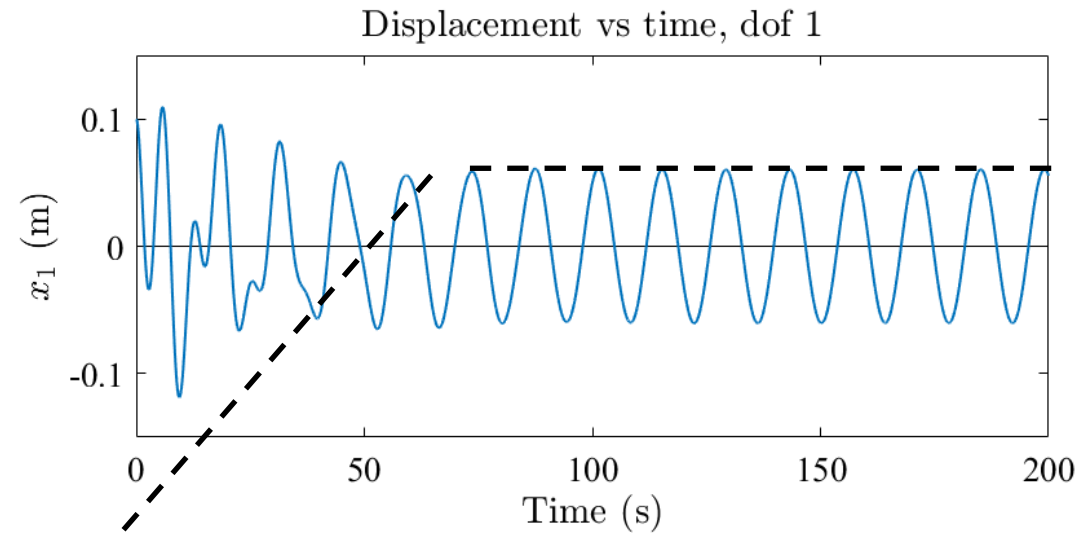
Saved dofs: all selected

disp. only overall motion

Memory: low normal auto

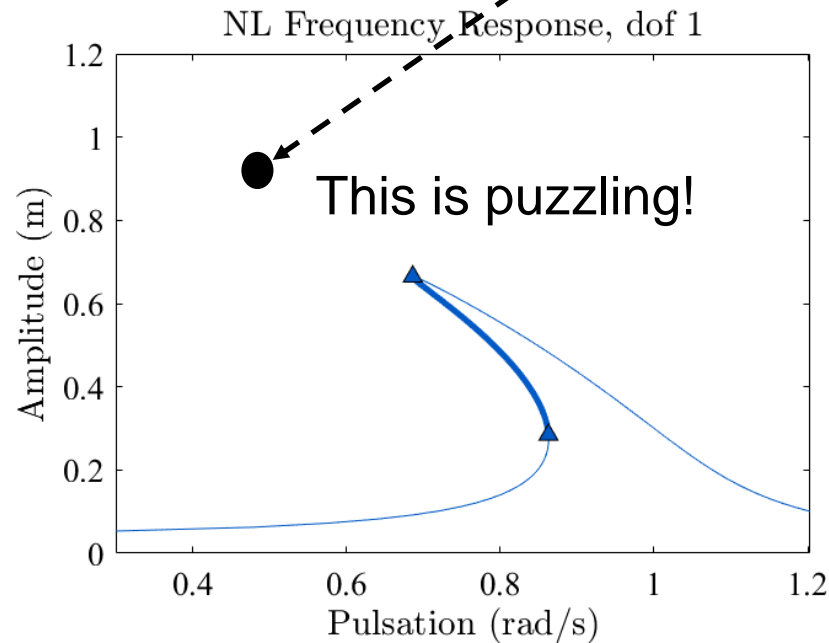
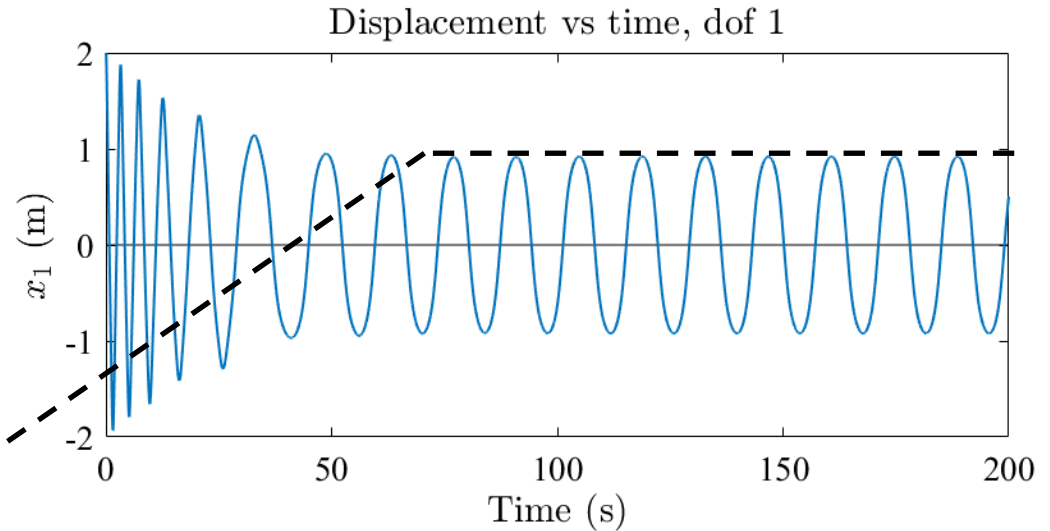
First Nonlinear Time Integration

Initial condition $x_0 = 0.1$



Second Nonlinear Time Integration

Initial condition $x_0 = 2$



Nonlinear Time Integration with Different x_0

