Cinématique et dynamique des machines

University of Brussels, Belgium

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Who am I ?

Professor of Aerospace Engineering @ ULiège.

Courses taught: satellite engineering, orbital mechanics and nonlinear vibration.

Research interests: aerospace structures, smart structures, vibration absorbers, nonlinear vibration, orbital mechanics.

Spin-off company active in nonlinear vibration.

Instructors: G. Kerschen, D. Piron

Contact details:

- Space Structures and Systems Lab (S3L)
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- BEAMS Department
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Course details:

- http://www.s3l.be

Course organization

Today/Nov. 14/Nov. 21/Nov. 28/Dec. 5/Dec. 12
from 14.00 to 16.00:

Theoretical lectures

30/11, 7/12, 14/12 from 10.00 to 12.00:

Exercice sessions

Jan. 11:

Written exam 50 % theory, list of questions provided

50% problems (similar to exercise sessions)

This course is an **introduction** to **vibration**

Objective: create motion

↓

Apply force

↓

Vibration is generated

↓

Uncomfort, fatigue,
noise, damage,…

Vibration in everyday life

The Hippies Were Right: It's All about Vibrations, Man!

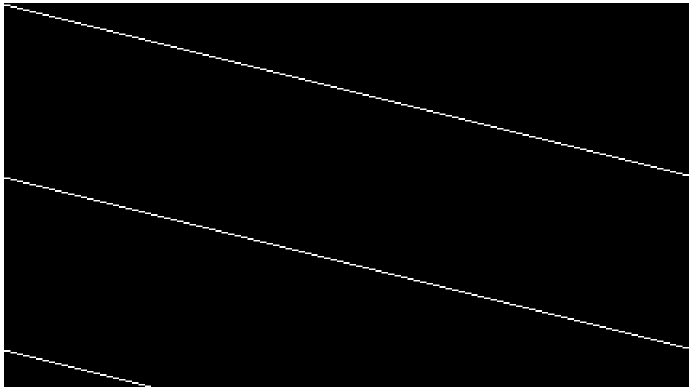
All things in our universe are constantly in motion, vibrating. Even objects that appear to be stationary are in fact vibrating, oscillating, resonating, at various frequencies. Resonance is a type of motion, characterized by oscillation between two states. And ultimately all matter is just vibrations of various underlying fields.

Vibration in everyday life ?

Why so heavy ?

3 reasons: acoustics,
comfort, damage !

Vibration in engineering structures ?

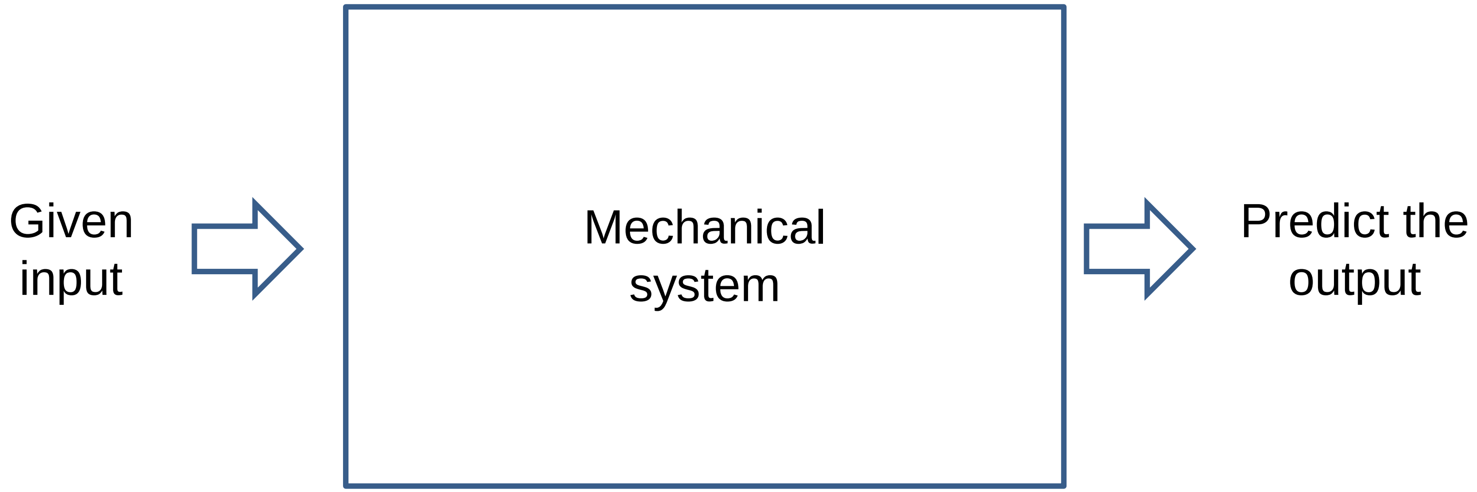


Vibrations can be beneficial !

Vibrateur … élasticité … force de rappel …
centaines de fois par seconde … résonateurs

Vocal cords

Course objectives



- Write down the equations of motion $$m\ddot{x} + c\dot{x} + kx = f(t)$$
- Calculate the response analytically or numerically $$x = 0.19 \sin 2.3t$$
- Is the structure safe ?

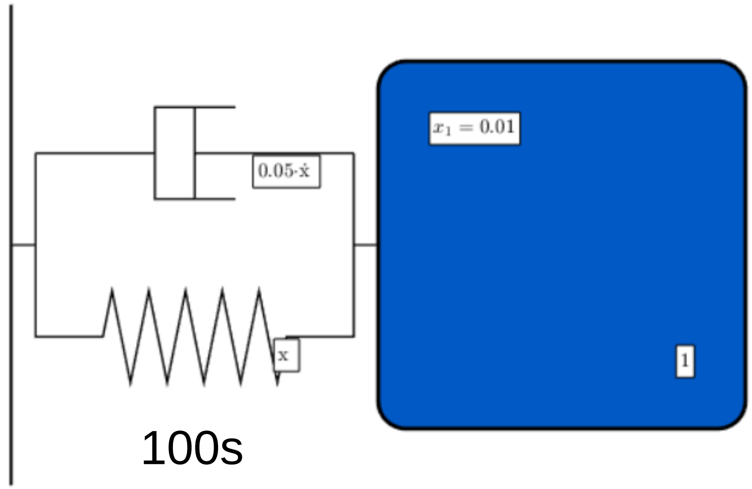
Course objectives

Given input → Mechanical system → Predict the output

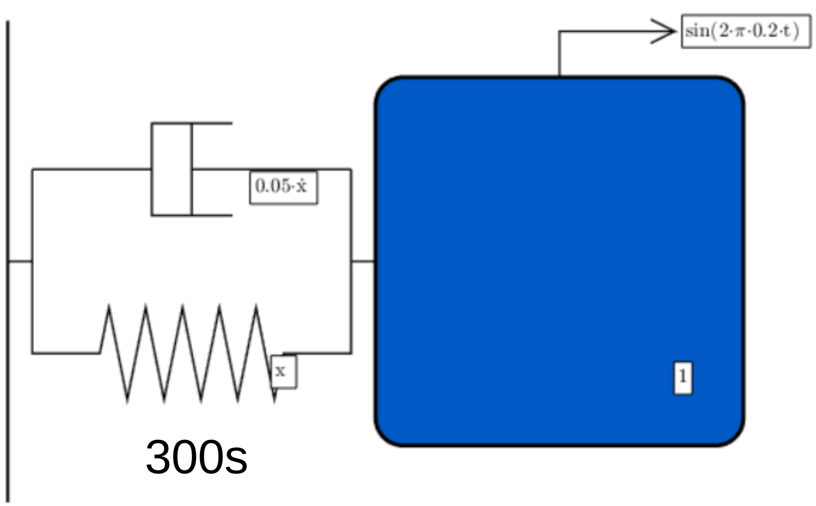
Wind gust →

→ Aircraft response

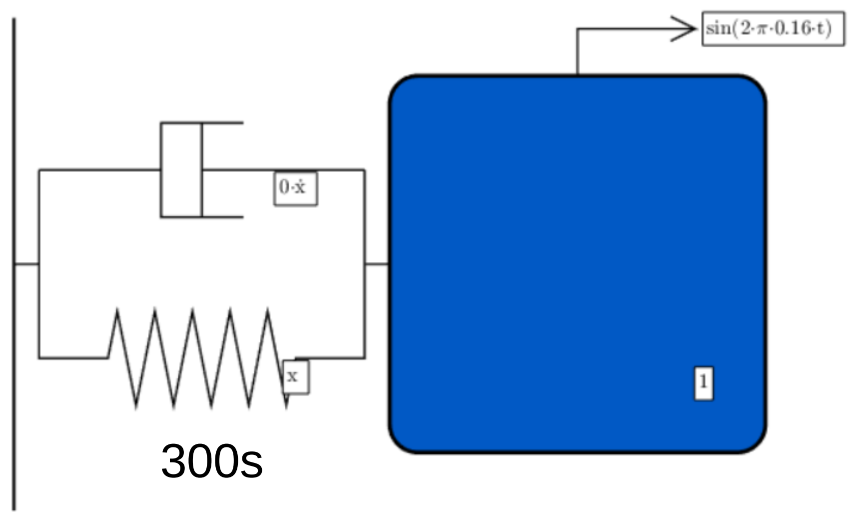
Free and forced responses with animation



100s

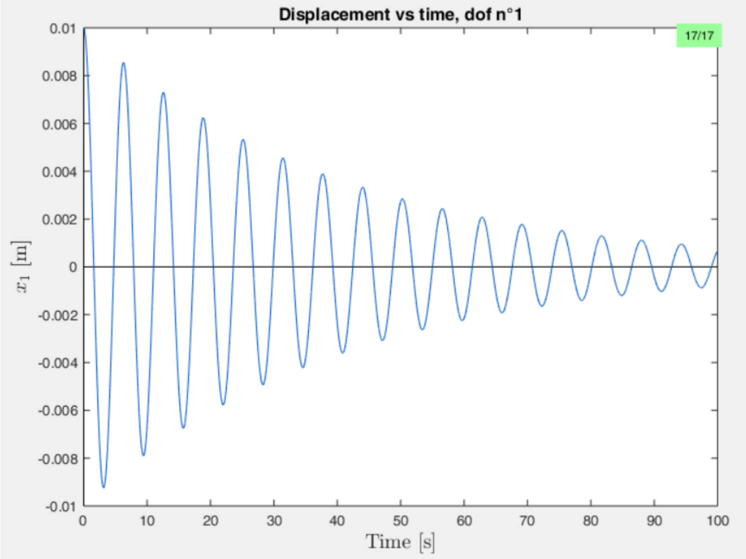


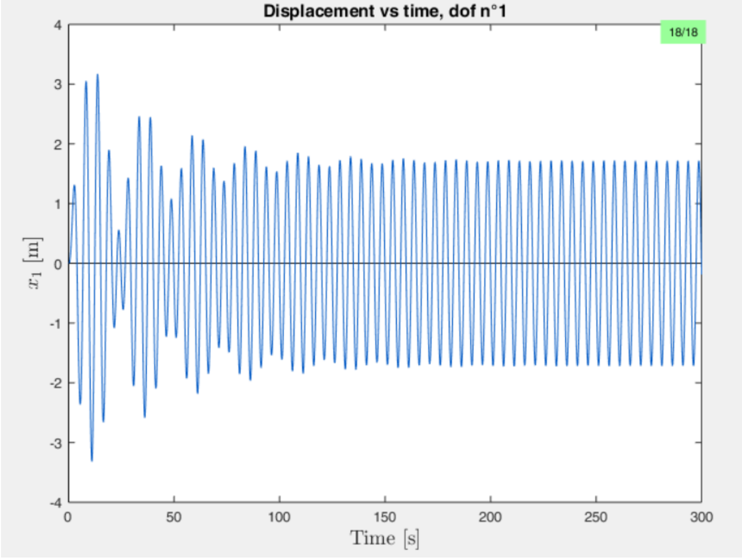
300s

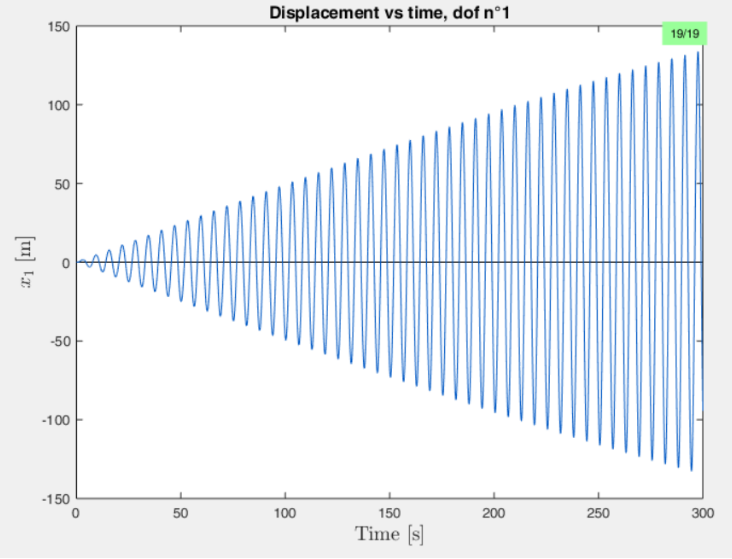


300s

Corresponding time series

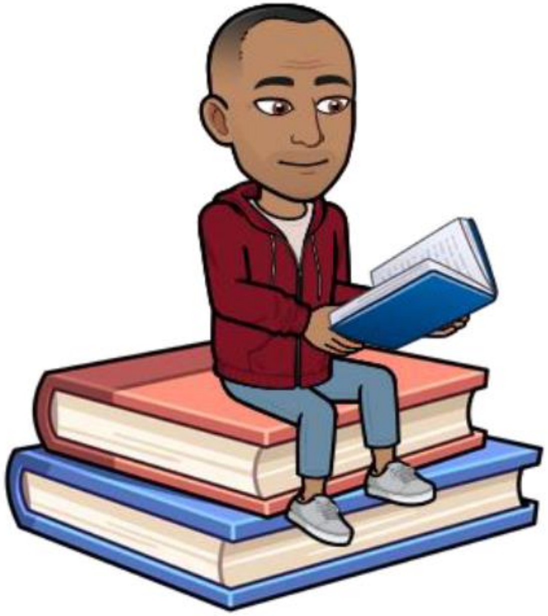




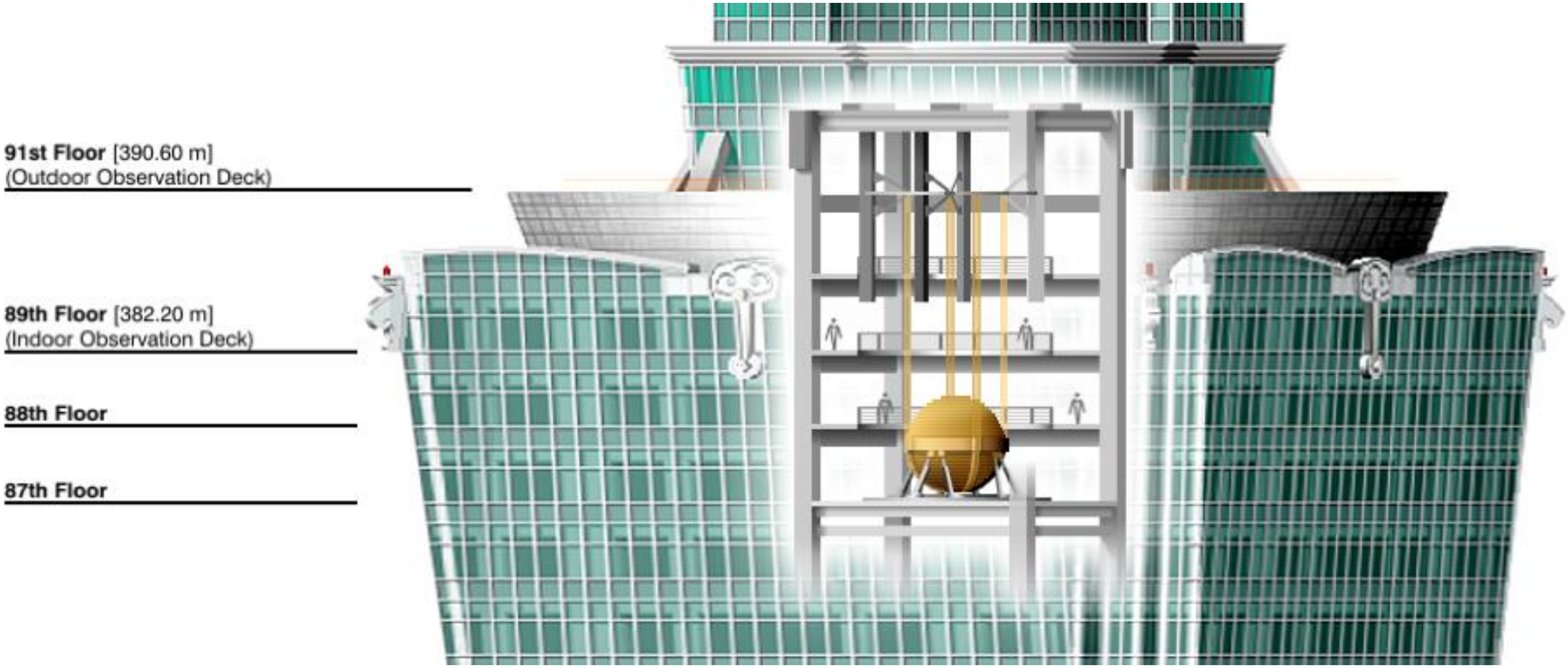


Course outline

- Basics of kinematics
- Basics of dynamics
  - Dynamic equations
  - One-degree-of-freedom system
  - Two-degree-of-freedom system
- Applications
  - Vibration isolation
  - Tuned mass damper
  - Jefcott rotor

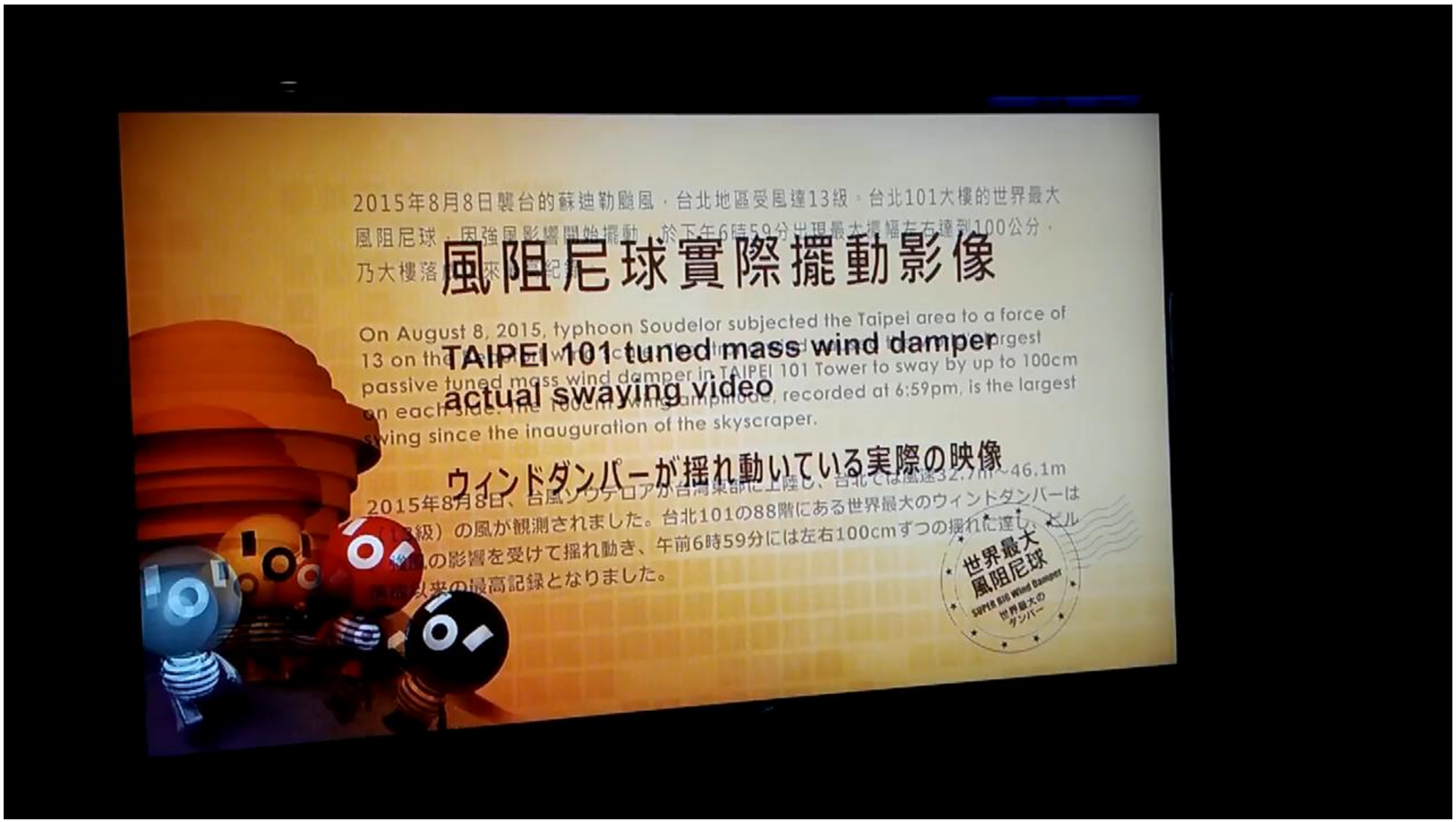


Importance of vibration isolation



730 tons - $4 Million

Importance of vibration isolation



Importance of rotor dynamics

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L01 Kinematics

Course objectives

Given input → Mechanical system → Predict the output

→ Write down the equations of motion

Degree of freedom

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

→ Calculate the response analytically or numerically

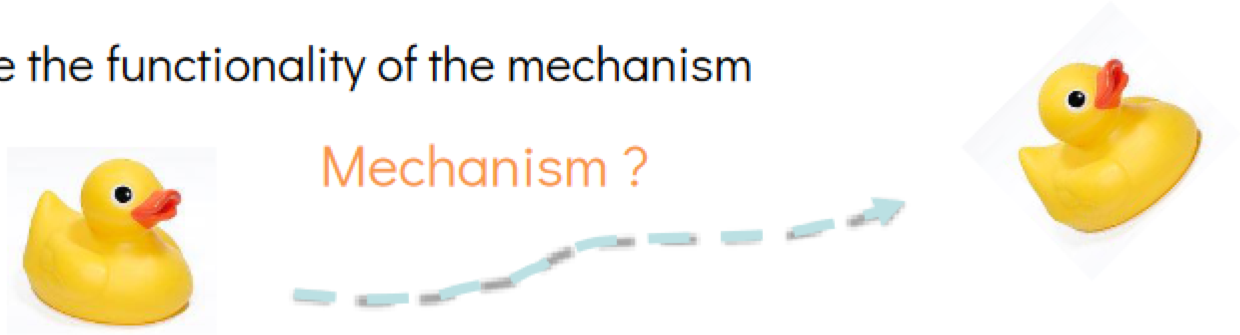
$$x = 0.19 \sin 2.3t$$

→ Is the structure safe ?

Definitions

- Kinematics: develop various means of transforming motion to achieve a specific task needed in applications

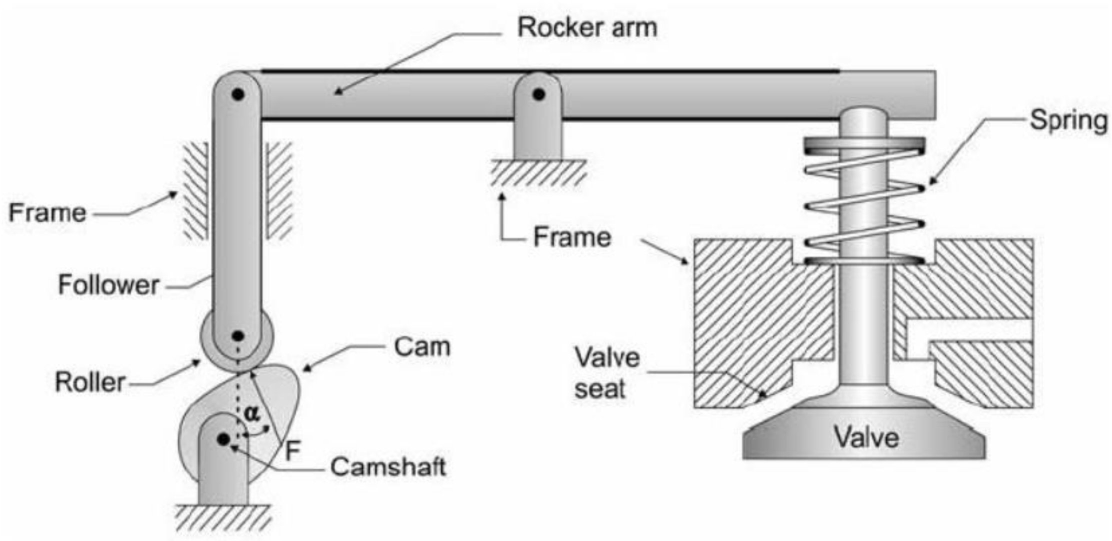
→ ensure the functionality of the mechanism



- Dynamics: behavior of a given machine or mechanism when subjected to dynamic forces

→ verify the acceptability of induced forces in parts

Example: Cam operating valve

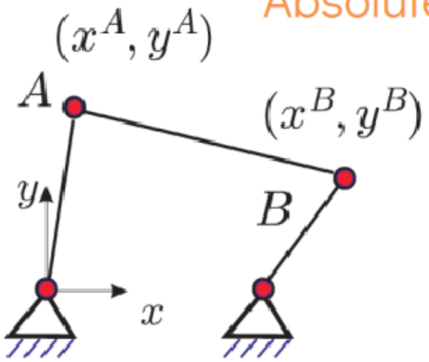


Kinematical analysis : satisfy functional requirements for valve displacements.

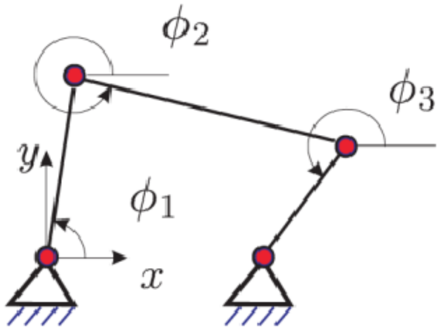
Dynamic analysis : compute forces in the system as a function of time.

Coordinates selection

Absolute coordinates

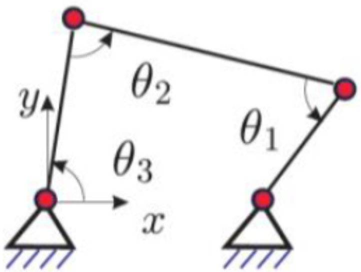


(Cartesian coordinates)



(Lagrangian coordinates)

Relative coordinates



- Absolute vs relative
- Choice is not unique
- How many coordinates ?
  (Efficiency vs simplicity)
- How many constraints ?

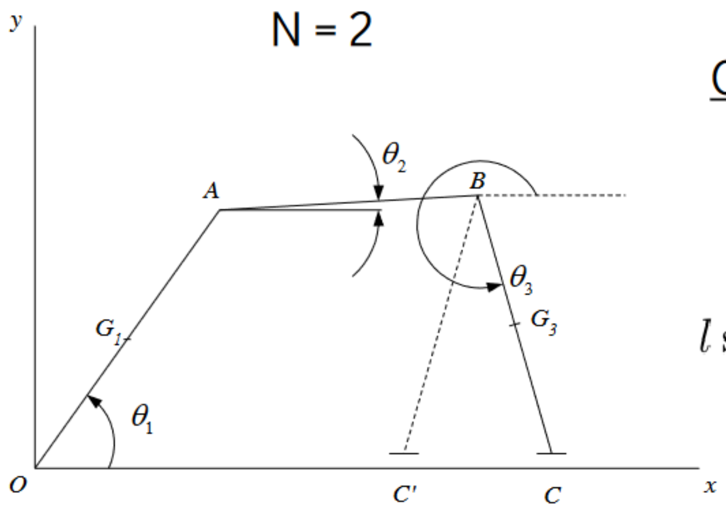
→ What is the best choice ?

Coordinates and constraints

- N = number of DOFs: minimum number of coordinates required to fully describe the configuration of a system
- Number of coordinates: q ≥ N

→ We have to find p=q-N relationships

N = 2



Option 1: q = 3 : $\theta_1, \theta_2, \theta_3$

→ p = 3 – 2 = 1

$$l \sin\theta_3 = -l \sin\theta_1 - l \sin\theta_2 \quad | \quad y_C = 0$$

Coordinates and constraints

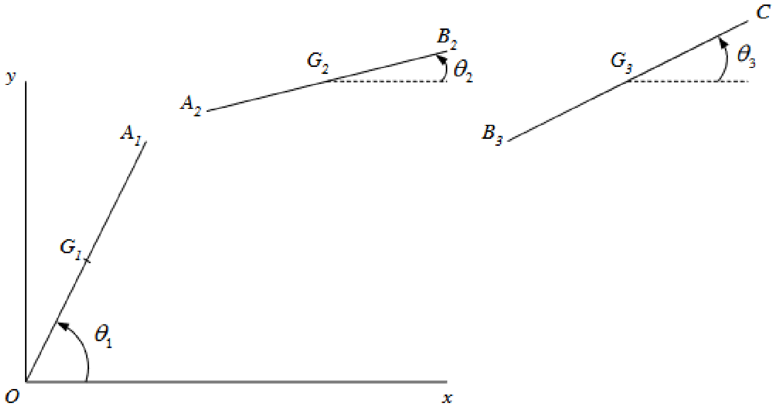
Option 2: q = 7:

$\theta_1, \theta_2, \theta_3, x_{G2}, y_{G2}, x_{G3}, y_{G3}$ → p = 7 – 2 = 5

$$1)\ l\cos\theta_1 = x_2 - \frac{l}{2}\cos\theta_2$$
$$2)\ l\sin\theta_1 = y_2 - \frac{l}{2}\sin\theta_2$$
$$A_1 = A_2$$

$$3)\ x_2 + \frac{l}{2}\cos\theta_2 = x_3 - \frac{l}{2}\cos\theta_3$$
$$4)\ y_2 + \frac{l}{2}\sin\theta_2 = y_3 - \frac{l}{2}\sin\theta_3$$
$$B_2 = B_3$$

$$5)\ y_3 + \frac{l}{2}\sin\theta_3 = 0$$
$$y_C = 0$$



Calculate the number of degrees of freedom

https://modernrobotics.northwestern.edu/nu-gm-book-resource/2-2-degrees-of-freedom-of-a-robot/#department

$$\text{dof} = m(N - 1 - J) + \sum_{i=1}^{J} f_i$$

Gruebler's formula

N = # of bodies, including ground
J = # of joints
m = 6 for spatial bodies, 3 for planar

dof f Number of DOFs at the joint

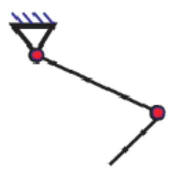
Degrees of freedom in 3D

- Number of closed loops:

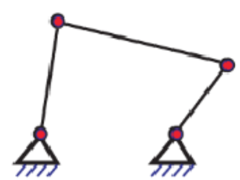
  l number of joints

  n number of bodies

$$b = l - n$$



$b = 0(l = 2, n = 2)$



$b = \mathbf{1}(l = 4, n = 3)$

- Number of DOFs (3D):

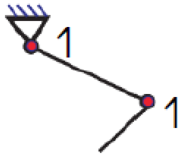
  where v_i is the number of DOFS at joint *i*

$$N = \sum_{i=\mathbf{1}}^{l} v_i - 6b$$

Degrees of freedom in 2D

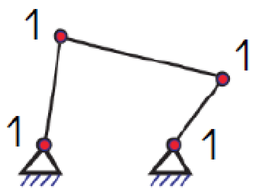
- Number of DOFs (2D):
  where v_i is the number of DOFS at joint i

$$N = \sum_{i=1}^{l} v_i - 3b$$



$$b = 0 (l = 2, n = 2)$$

$$N = 1 + 1 - 0 = 2$$



$$b = 1 (l = 4, n = 3)$$

$$N = 4 * 1 - 3 = 1$$

OK !

$$DOF = \sum_{i=1}^{l} v_i - 6b = \sum_{i=1}^{l} v_i + 6n - 6l = 6n - 6l + \sum_{i=1}^{l} v_i$$

Same notations

$$DOF = 6(N-1) - 6J + \sum_{i=1}^{J} f_i = m(N-1-J) + \sum_{i=1}^{J} f_i$$

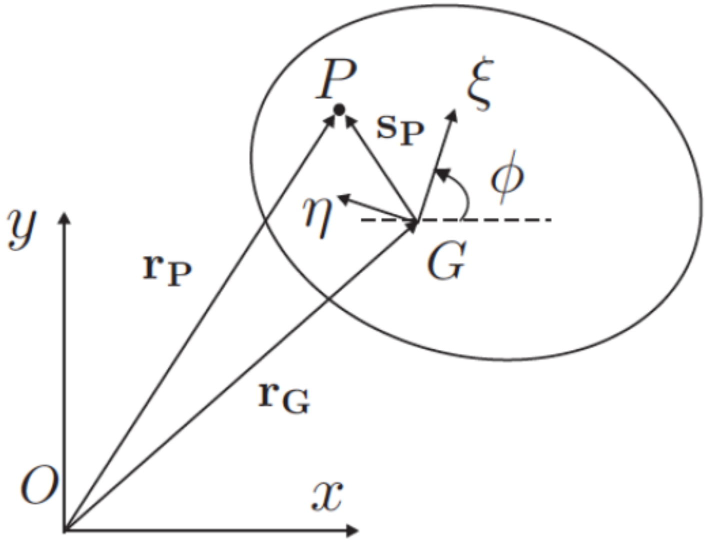
$$\text{dof} = m(N-1-J) + \sum_{i=1}^{J} f_i$$

Important remark

Our focus is on rigid bodies.

Flexible bodies may have many degrees of freedom:

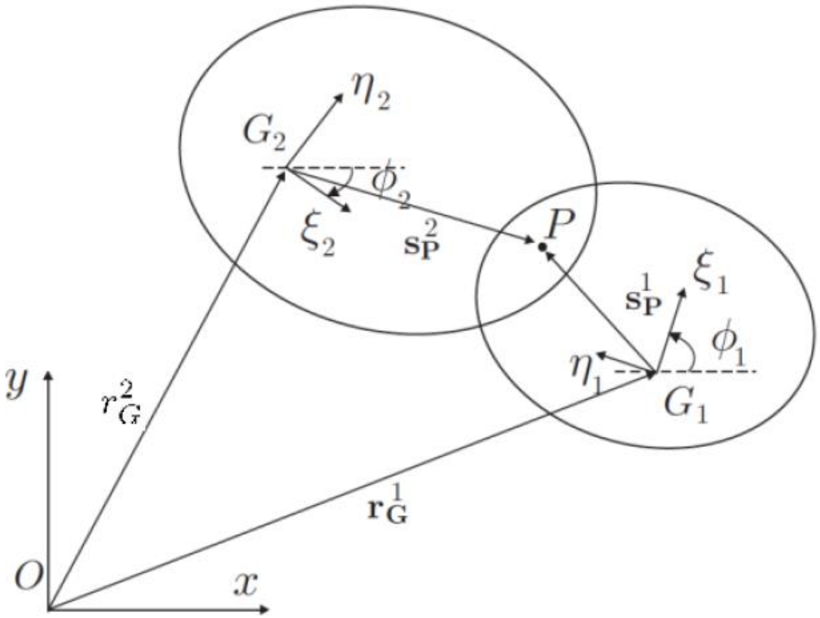
Single body revolute joint (2D): constraint equations



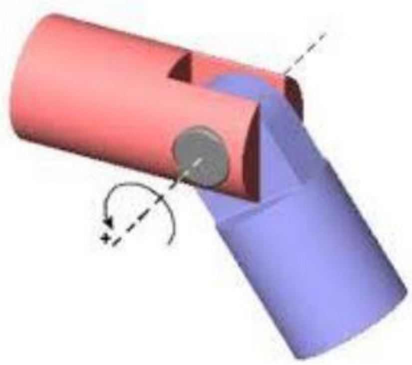
$$\overline{r}_P = \overline{r}_G + A\overline{s}_P$$

$$A = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

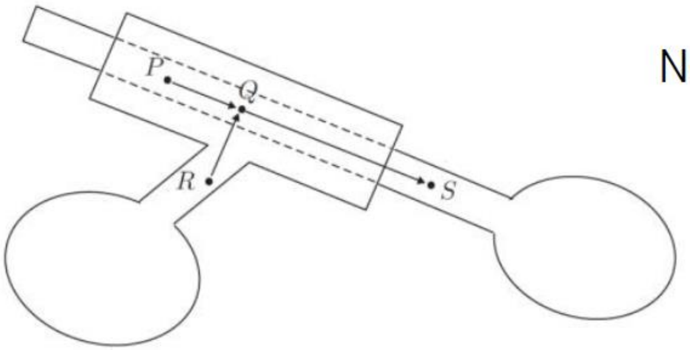
Two bodies revolute joint (2D): constraint equations



$$\bar{r}_G^1 + A_1 \bar{s}_P^1 = \bar{r}_G^2 + A_2 \bar{s}_P^2$$



Prismatic joint (2D): constraint equations

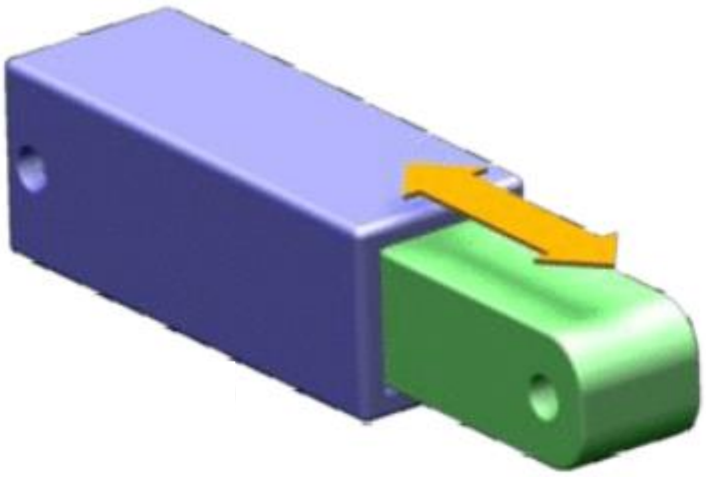


No rotation in the plane between the two bodies

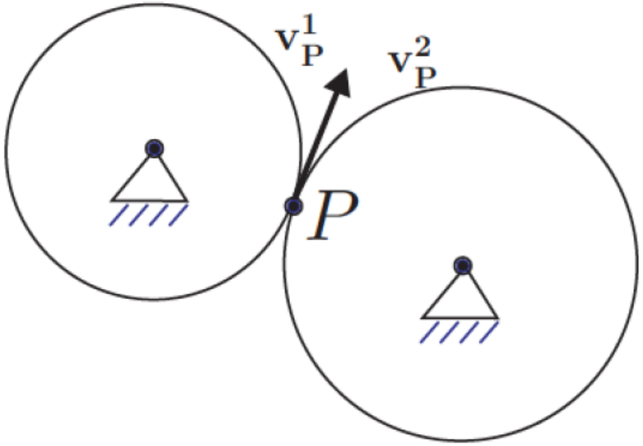
$$\overline{PQ} \times \overline{QS} = 0$$

or

$$\overline{RQ}.\overline{QS} = 0$$



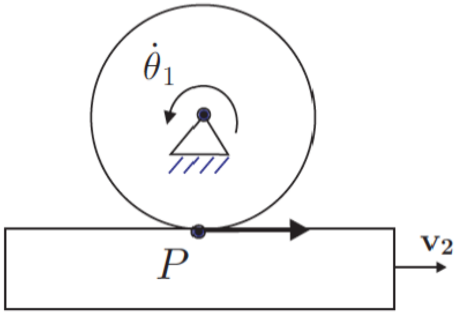
Spur gears (2D): constraint equations



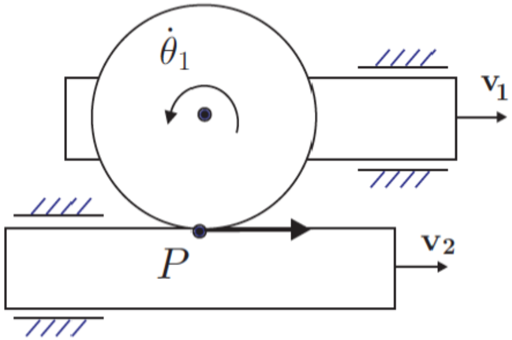
$$\overline{v}_P^1 = \overline{v}_P^2$$

$$\longrightarrow R_1\dot{\theta}_1 = R_2\dot{\theta}_2$$

Rack and pinion (2D): constraint equations

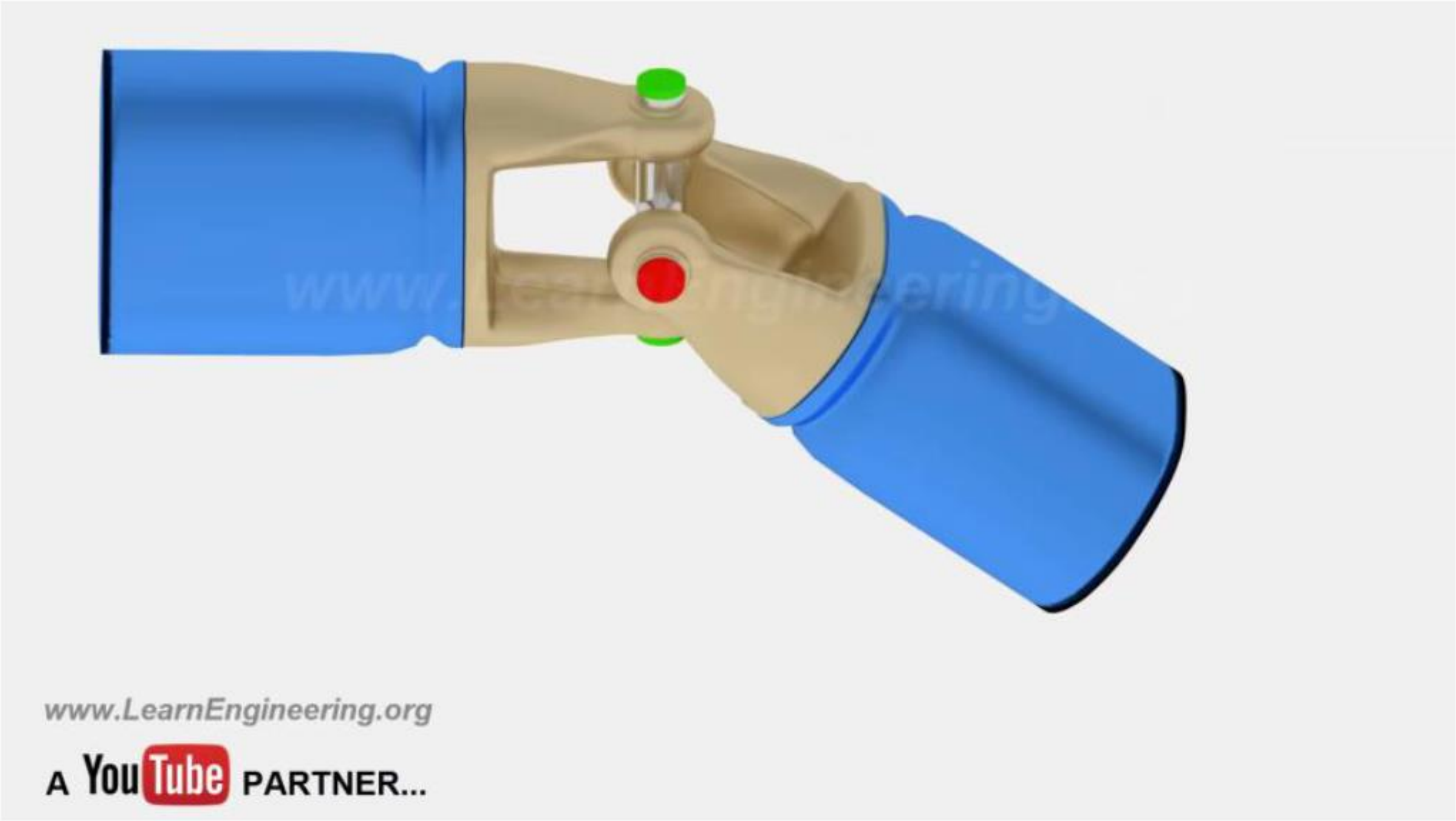


$$R_1\dot{\theta}_1 = v_2$$



$$v_1 + R_1\dot{\theta}_1 = v_2$$

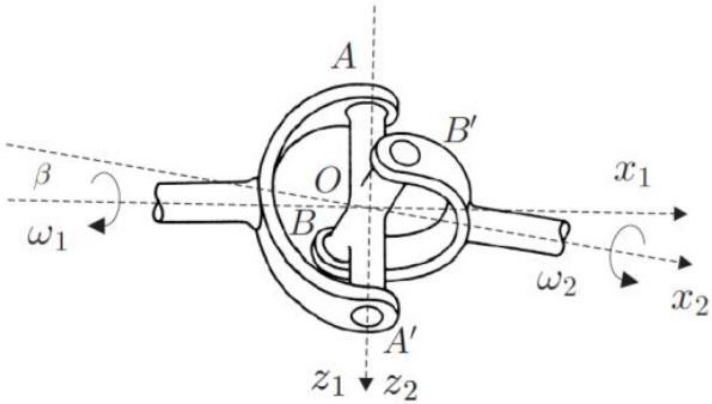
Universal joint (3D)

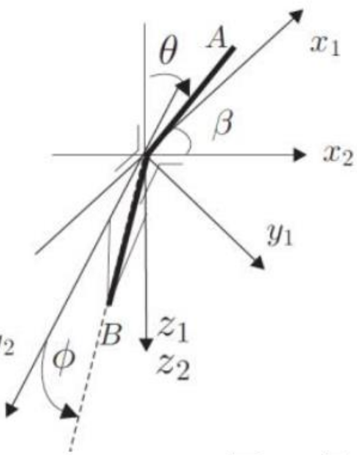


https://www.youtube.com/watch?v=LCMZz6YhbOQ

Universal joint (3D)

β is the fixed angle between the two arms
θ is the rotation angle of the first arm
ϕ is the rotation angle of the second arm





$$\overline{OA} = \cos\theta\,\overline{1}_{z_1} + \sin\theta\,\overline{1}_{y_1}$$

$$\overline{OB} = \cos\phi\,\overline{1}_{y_2} + \sin\phi\,\overline{1}_{z_2}$$

$$\overline{OA}\,.\,\overline{OB} = 0 \qquad \overline{1}_{y_1}\,.\,\overline{1}_{y_2} = \cos\beta \qquad \overline{1}_{z_1}\,.\,\overline{1}_{y_2} = 0 \qquad \overline{1}_{z_2}\,.\,\overline{1}_{y_1} = 0$$

$$-\cos\theta\sin\phi + \cos\beta\sin\theta\cos\phi = 0 \quad \longrightarrow \quad \tan\phi = \cos\beta\tan\theta$$

Universal joint (3D)

$\tan\phi = \cos\beta \tan\theta$ Differentiation: $\frac{1}{\cos^2\phi}\dot{\phi} = \frac{\cos\beta}{\cos^2\theta}\dot{\theta}$

$$\frac{1}{\cos^2\phi} = 1 + \tan^2\phi = 1 + \cos^2\beta\tan^2\theta$$

$$\omega_2 = \frac{\omega_1 \cos\beta}{1 - \sin^2\beta \sin^2\theta}$$

$\beta <<$

Transmission is not uniform unless the two axles are aligned

$$\dot{\omega}_2 = \frac{\omega_1^2 \sin^2\beta \cos\beta \sin 2\theta}{\left(1 - \sin^2\beta \sin^2\theta\right)^2}$$

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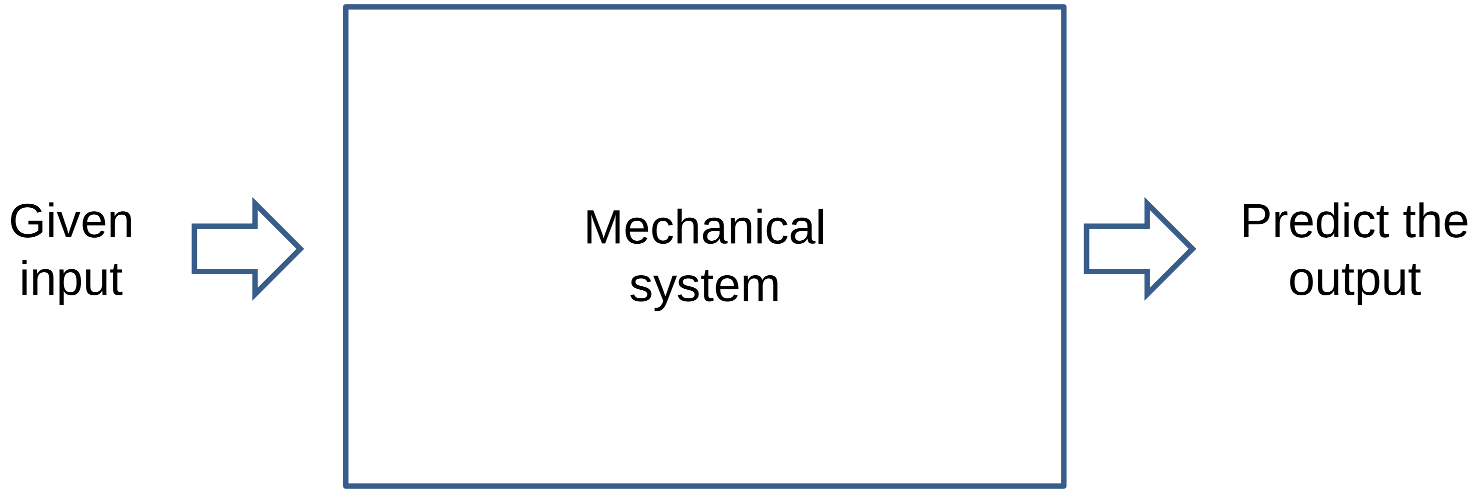
L02 Newtonian dynamics

Course objectives



Equation of motion

- Write down the equations of motion: $$m\ddot{x} + c\dot{x} + kx = f(t)$$
- Calculate the response analytically or numerically: $$x = 0.19 \sin 2.3t$$
- Is the structure safe ?

Théorème de la quantité de mouvement

When a body is acted upon by a force, the time rate of change of its momentum equals the force

Linear momentum (translation)

$$m\frac{d\overline{v}_G}{dt} = \sum_{i=1}^{N} \overline{F}_{ext,h}$$

…written in an inertial frame !...

Digression

What did Richard Feynman mean about the Second Law of Motion? Where was the error?

JANUARY 17, 2021 / FRANCES48 / 0 COMMENTS

Richard Feynman writes about Newton's Second Law of Motion in his work "Lectures on Physics" (Chapter 15):

„For over 200 years the equations of motion enunciated by Newton were believed to describe nature correctly, and the first time that an error in these laws was discovered, the way to correct it was also discovered. Both the error and its correction were discovered by Einstein in 1905.

Digression

Newton's Second Law, which we have expressed by the equation

$$F = d(mv)/dt$$

was stated with the tacit assumption that m is a constant, but we now know that this is not true, and that the mass of a body increases with velocity. In Einstein's corrected formula m has the value

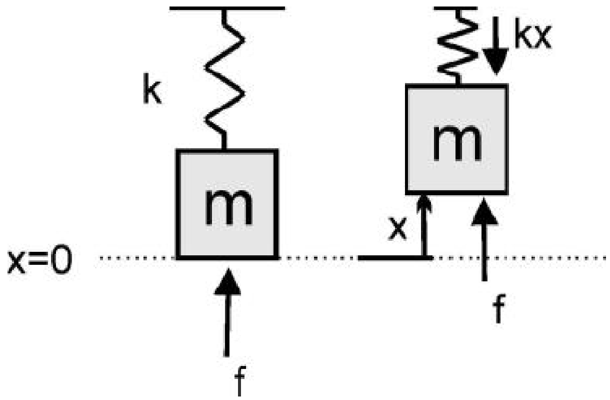
$$m = \frac{m_0}{\sqrt{1 - v^2 \,/\, c^2}}$$

where the rest mass represents the mass of a body that is not moving and c is the speed of light [...].

Spring-mass system: a 1DOF system

Linear momentum (translation)

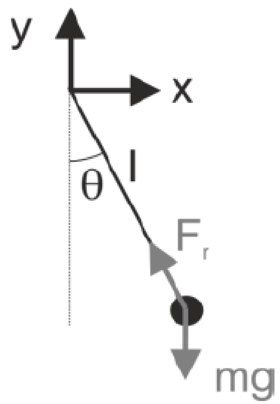
$$m\frac{d\overline{v}_G}{dt} = \sum_{i=1}^{N} \overline{F}_{ext,h}$$



- Spring force: $-kx$
- External force f acting on the mass.

$$m\ddot{x} = \sum F_x \longrightarrow m\ddot{x} + kx = f$$

Pendulum: a 1DOF system



$$m\ddot{x} = -F_r \sin\theta \quad \longrightarrow \quad F_r = \frac{-m\ddot{x}}{\sin\theta}$$

$$m\ddot{y} = F_r \cos\theta - mg \quad \longrightarrow \quad m\ddot{y} = \frac{-m\ddot{x}}{\sin\theta}\cos\theta - mg$$

$$x = l\sin\theta \qquad \dot{x} = l\dot{\theta}\cos\theta \quad \longrightarrow \quad \ddot{x} = l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta$$

$$y = -l\cos\theta \qquad \dot{y} = l\dot{\theta}\sin\theta \qquad\qquad \ddot{y} = l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta$$

$$\longrightarrow \quad ml\ddot{\theta} + mg\sin\theta = 0$$

Small displacements

$$\longrightarrow \quad \boxed{ml\ddot{\theta} + mg\theta = 0}$$

Théorème du moment cinétique

Angular momentum (rotation)

$$\frac{d\overline{M}_A}{dt} = m\overline{v}_G \times \overline{v}_A + \overline{m}_{ext,A}$$

- $G = A$
- $\overline{v}_G = 0$
- $\overline{v}_A = 0$
- $\overline{v}_G \parallel \overline{v}_A$

→

$$\frac{d\overline{M}_A}{dt} = \overline{m}_{ext,A}$$

Flexible shaft

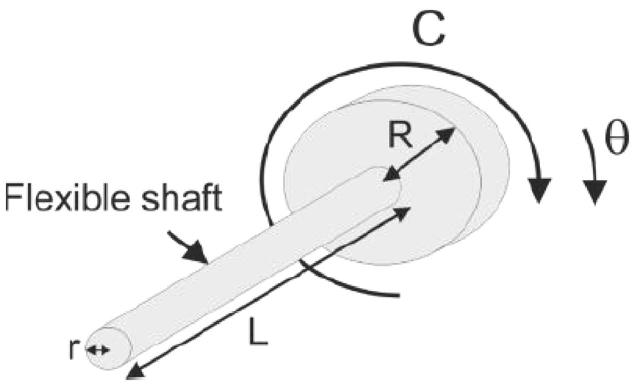
At the center of gravity

$$\frac{d\overline{M}_G}{dt} = \overline{m}_{ext,G}$$

$$\overline{M}_G = \overline{\overline{I}}_G \,.\, \overline{\omega}$$

$$\overline{M}_G = I\dot{\theta}$$

$$I\ddot{\theta} + K\theta = C$$

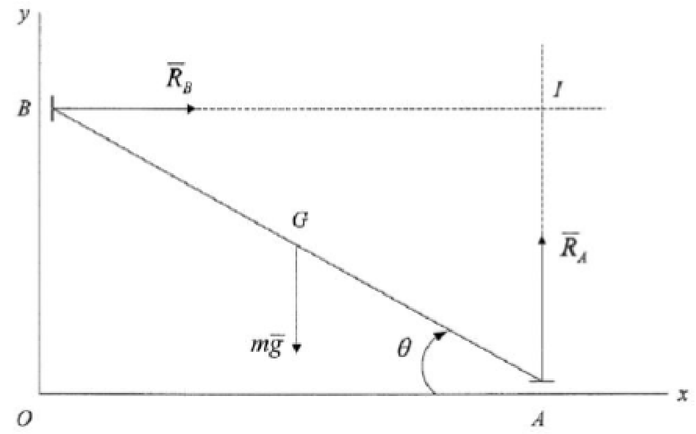


$$I = \frac{1}{4}MR^2 \qquad K = \frac{E\pi r^4}{4(1+\nu)L}$$

Rotational spring

Sliding bar

$$m\frac{d\overline{v}_G}{dt} = \sum_{i=1}^{N} \overline{F}_{ext,h}$$



$$\overline{r}_G = \frac{L}{2}(\cos\theta\overline{1}_x + \sin\theta\overline{1}_y)$$

$$\overline{v}_G = \frac{L\dot{\theta}}{2}(-\sin\theta\overline{1}_x + \cos\theta\overline{1}_y)$$

$$\overline{a}_G = -\frac{L}{2}\left((-\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta)\overline{1}_x + (\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)\overline{1}_y\right)$$

$$\frac{mL}{2}(-\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta) = \underline{R_B}$$

$$\frac{mL}{2}(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = \underline{R_A} - mg$$

Reaction forces

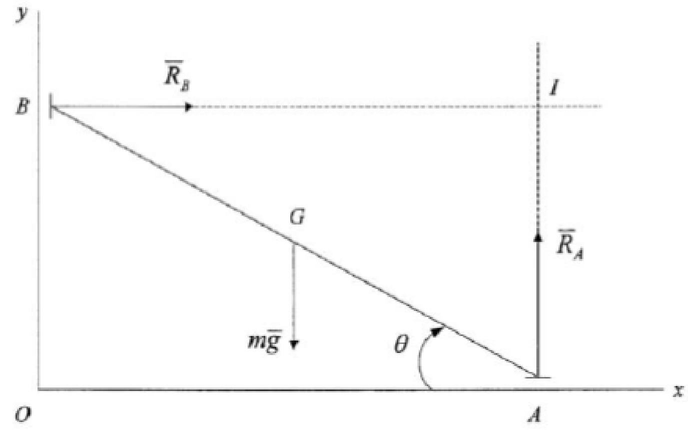
2 equations/
3 unknowns

Sliding bar

$$\frac{d\overline{M}_I}{dt} = mg\frac{L}{2}\cos\theta\overline{1}_z$$

$$\overline{v}_G \parallel \overline{v}_I$$

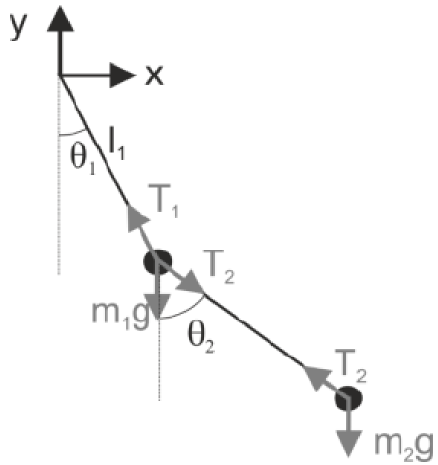
$$\overline{M}_I = \overline{M}_G + m\overline{IG} \times \overline{v}_G = -\frac{mL^2}{3}\dot{\theta}\overline{1}_z$$



Choice of I allows to get rid of reaction forces R_A and R_B

$$\rightarrow \quad -\frac{mL^2}{3}\ddot{\theta} = mg\frac{L}{2}\cos\theta$$

Double pendulum



Linear momentum (translation)

$$m_1\ddot{x}_1 = -T_1\sin\theta_1 + T_2\sin\theta_2$$
$$m_1\ddot{y}_1 = T_1\cos\theta_1 - T_2\cos\theta_2 - m_1 g$$

$$m_2\ddot{x}_2 = -T_2\sin\theta_2$$
$$m_2\ddot{y}_2 = T_2\cos\theta_2 - m_2 g$$

Accelerations

$$x_1 = l_1\sin\theta_1 \qquad y_1 = -l_1\cos\theta_1$$
$$x_2 = l_1\sin\theta_1 + l_2\sin\theta_2 \qquad y_2 = -l_1\cos\theta_1 - l_2\cos\theta_2$$

Compute $\dot{x}_1, \ddot{x}_1, \dot{x}_2, \ddot{x}_2$ And replace in equilibrium equations ....

So what ?

- Newton's second law must be applied to each solid separately
- It introduces (unknown) reaction forces
- For multiple solids, it generally leads to lengthy calculations
- The use of Lagrange equations is an alternative.

In summary

What have we achieved today ?

$\rightarrow$ Kinematics

$\rightarrow$ Newtonian dynamics

Next lecture:

$\rightarrow$ Lagrangian dynamics