Cinématique et dynamique des machines

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L05 Isolation & rotors

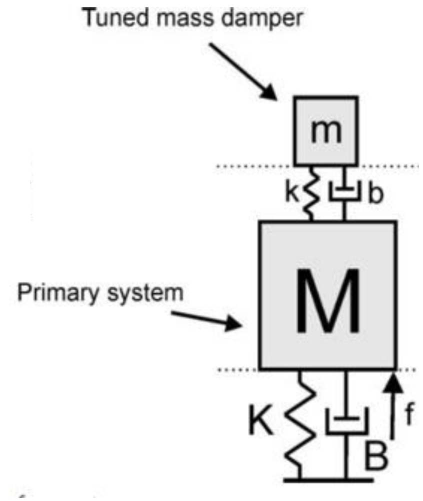
Outline

1. Vibration isolation

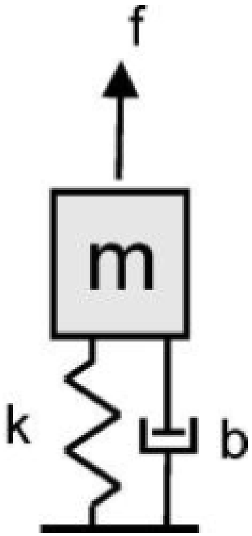
2. Rotor dynamics

Last lecture

A tuned mass damper is an additional mass aimed at protecting the structure.



Vibration isolation rather aims at designing the interface between the structure and its environment.



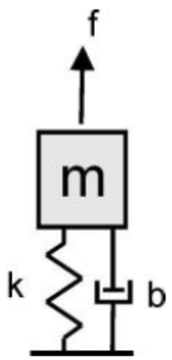
Motivation for vibration isolation

https://www.youtube.com/watch?v=SRbFxgezAX0

Real-life examples

[https://www.berleburger.com]

Vibration isolation: mathematically



$$m\ddot{x} + b\dot{x} + kx = f$$

$$x(t) = Xe^{i\omega t}$$

$$f(t) = Fe^{i\omega t}$$

$$\rightarrow \quad (k - \omega^2 m + i\omega b)X = F$$

$$|F| = \sqrt{(k - \omega^2 m)^2 + (\omega b)^2}|X|$$

Force transmitted to the ground:

$$F_T = (k + i\omega b)X$$

$$|F_T| = \sqrt{k^2 + (\omega b)^2}|X|$$

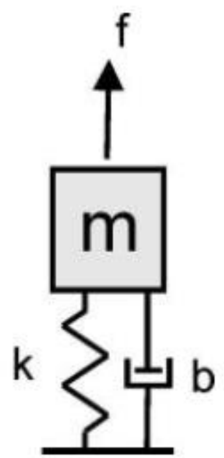
Isolation factor:

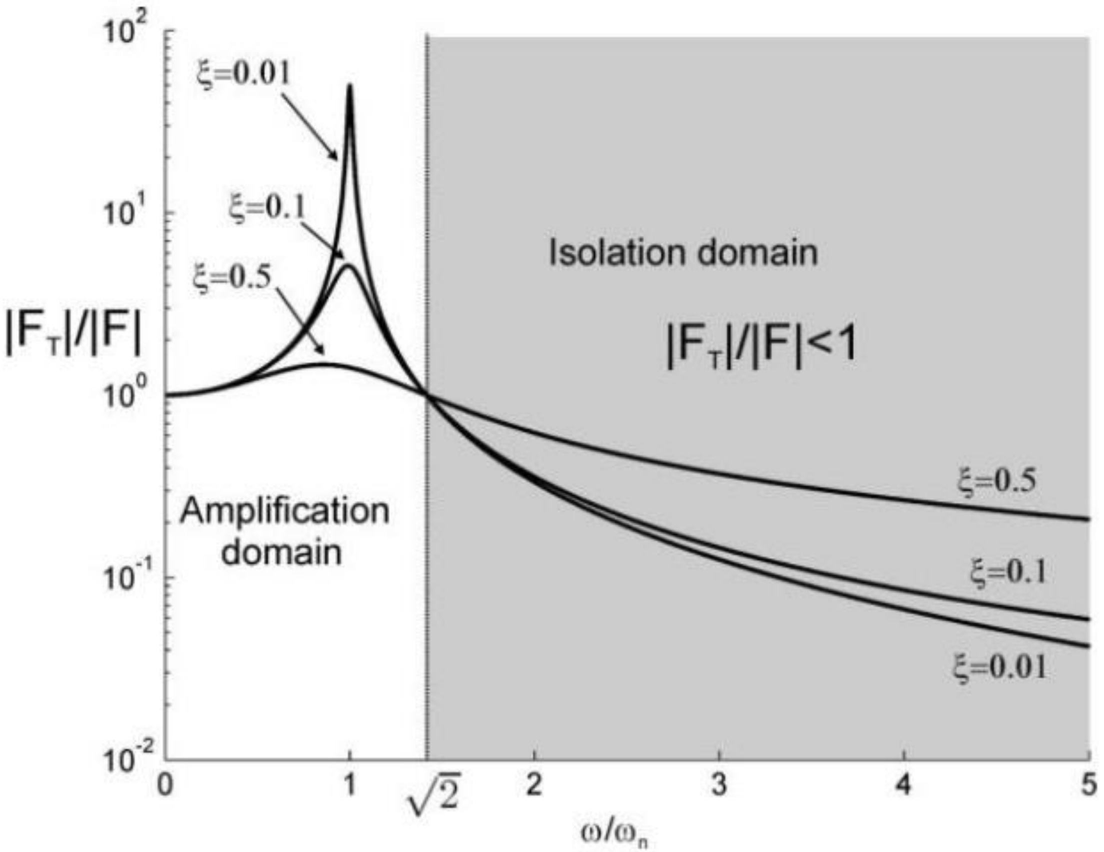
$$\boxed{\frac{|F_T|}{|F|} = \frac{\sqrt{k^2 + \omega^2 b^2}}{\sqrt{(k - \omega^2 m)^2 + \omega^2 b^2}}}$$

$$\xi = b/(2\sqrt{km})$$
$$\omega_n = \sqrt{k/m}$$

$$\boxed{\frac{|F_T|}{|F|} = \frac{\sqrt{1 + (2\xi\frac{\omega}{\omega_n})^2}}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\xi\frac{\omega}{\omega_n})^2}}}$$

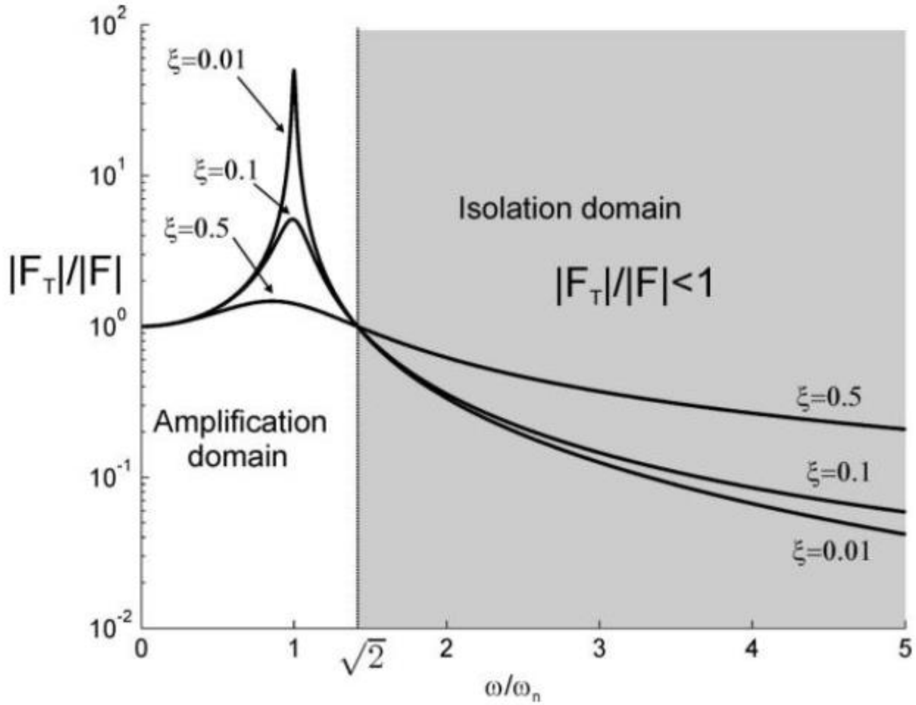
Vibration isolation: graphically





Isolation if $\omega > \sqrt{2}\omega_n$

Role of stiffness

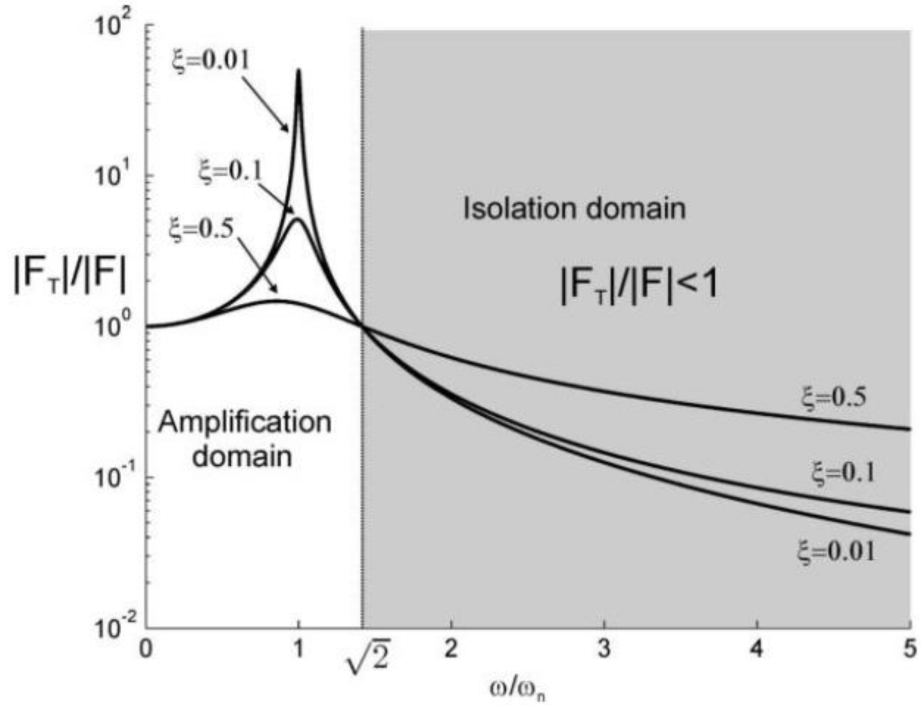


Isolation if $\omega > \sqrt{2k/m}$

Usually ω and *m* are given $\rightarrow$ choose *k* as low as possible.

However, *k* should respect the static constraints.

Role of damping



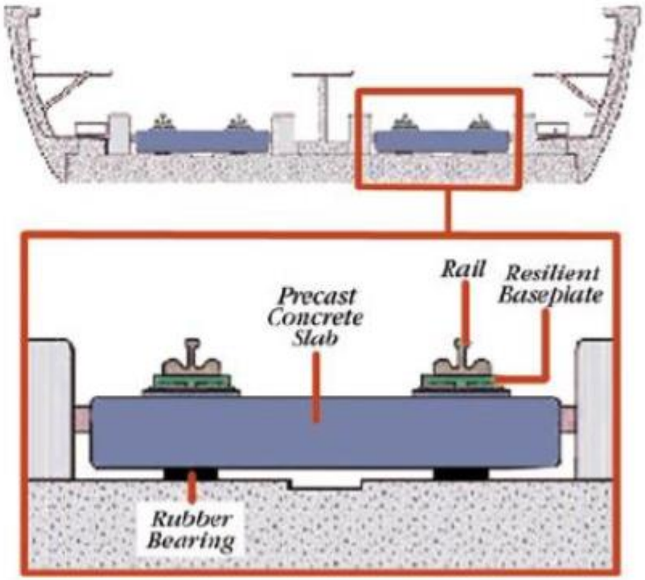
High damping reduces isolation performance but increases attenuation at resonance (there are transients !)

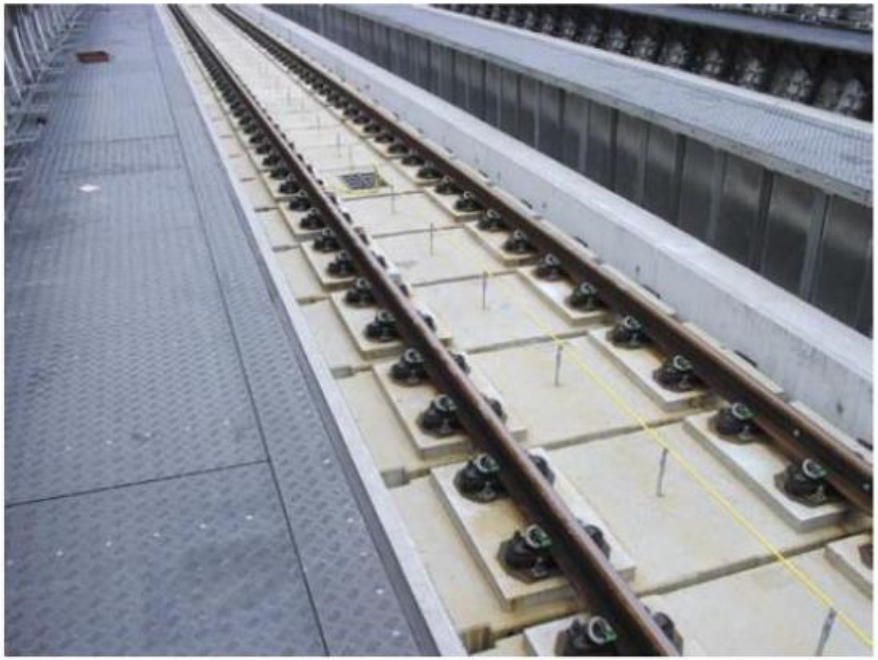
Low damping increases isolation performance but decreases attenuation at resonance.

Ideally, frequency-dependent damping (rubber and elastomers).

Railway example

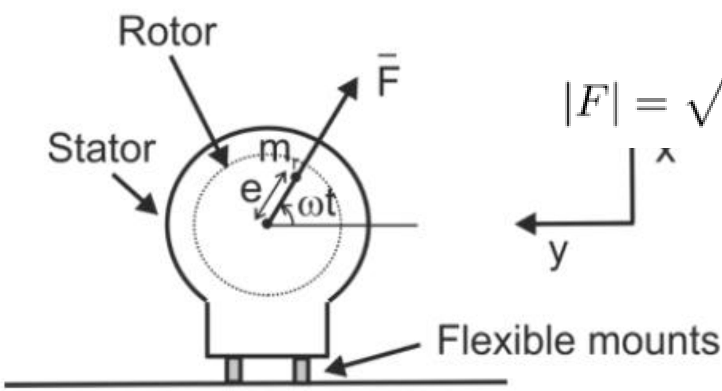
TRELLEBORG
ENGINEERED SYSTEMS

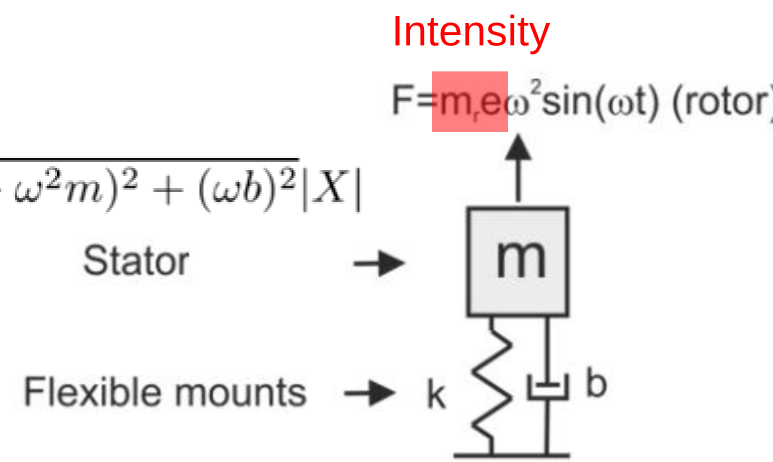




Rotating machines

$$|F| = \sqrt{(k-\omega^2 m)^2 + (\omega b)^2}\,|X|$$





$$F = m_r e \omega^2 \sin(\omega t) \longrightarrow |F| = \sqrt{(k-\omega^2 m)^2 + (\omega b)^2}\,|X| = m_r e \omega^2$$

$$\frac{|F_T|}{m_r e \omega^2} = \frac{\sqrt{k^2 + (\omega b)^2}}{\sqrt{(k-\omega^2 m)^2 + (\omega b)^2}}$$

$$\frac{|F_T|}{m_r e} = \omega^2 \frac{\sqrt{k^2 + (\omega b)^2}}{\sqrt{(k-\omega^2 m)^2 + (\omega b)^2}}$$

Rotating machines

$$\frac{|F_T|}{m_r e} = \omega^2 \frac{\sqrt{k^2 + (\omega b)^2}}{\sqrt{(k - \omega^2 m)^2 + (\omega b)^2}}$$

$$\xi = b/(2\sqrt{km})$$

$$\omega_n = \sqrt{k/m}$$

$$\boxed{\frac{|F_T|}{m_r e \omega_n^2} = \left(\frac{\omega}{\omega_n}\right)^2 \frac{\sqrt{1 + (2\xi\frac{\omega}{\omega_n})^2}}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\xi\frac{\omega}{\omega_n})^2}}}$$

$$\boxed{\frac{|F_T|}{|F|} = \frac{\sqrt{1 + (2\xi\frac{\omega}{\omega_n})^2}}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\xi\frac{\omega}{\omega_n})^2}}}$$

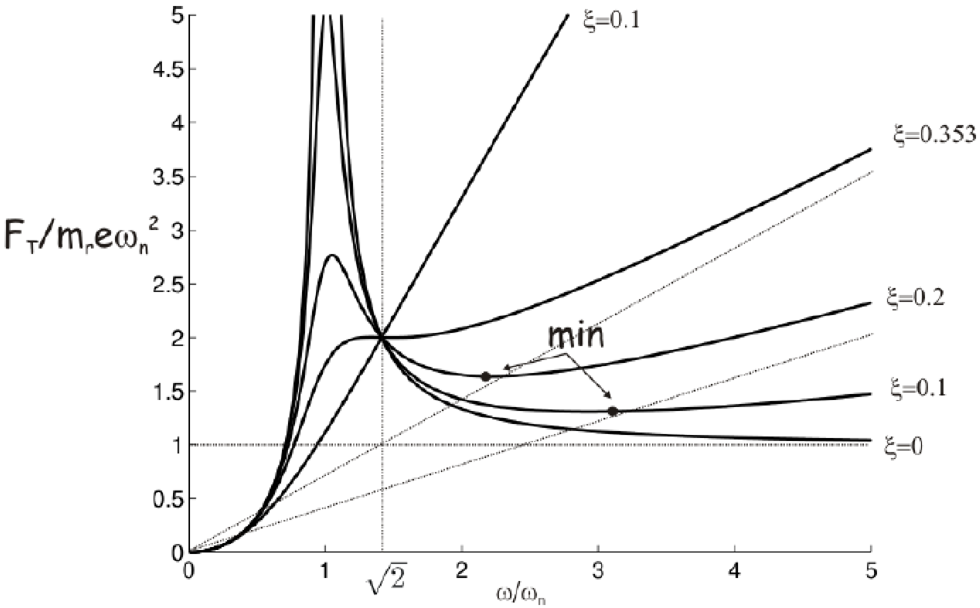
BEFORE

Rotating machines

Asymptotic behavior

$$lim_{\frac{\omega}{\omega_n}\to\infty}\left(\left(\frac{\omega}{\omega_n}\right)^2\frac{\sqrt{1+(2\xi\frac{\omega}{\omega_n})^2}}{\sqrt{(1-(\frac{\omega}{\omega_n})^2)^2+(2\xi\frac{\omega}{\omega_n})^2}}\right)=2\xi\frac{\omega}{\omega_n}$$

** Not valid when ksi=0*



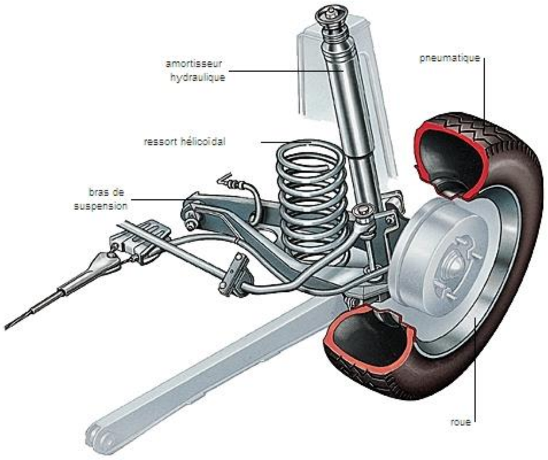
Existence of a minimum for $\xi < 0.353$ (soft isolation)

Reducing e reduces F_T

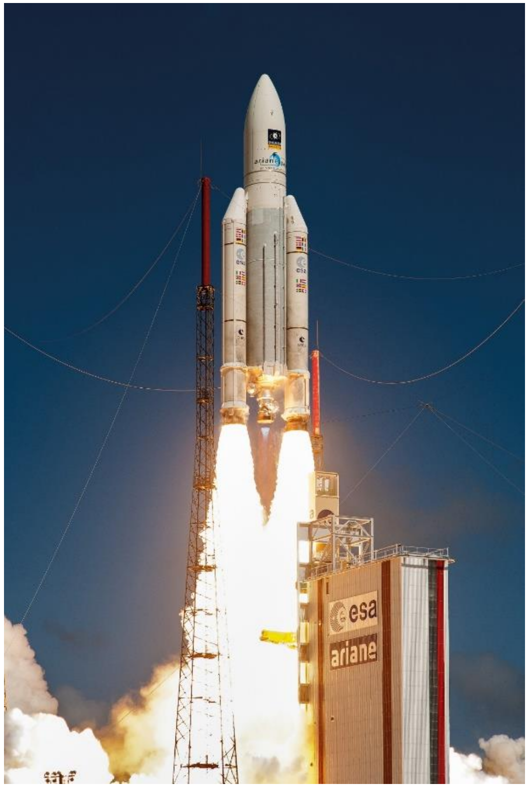
→ Rotor balancing

Vibration isolation (base excitation)

Earthquake protection



Car suspension

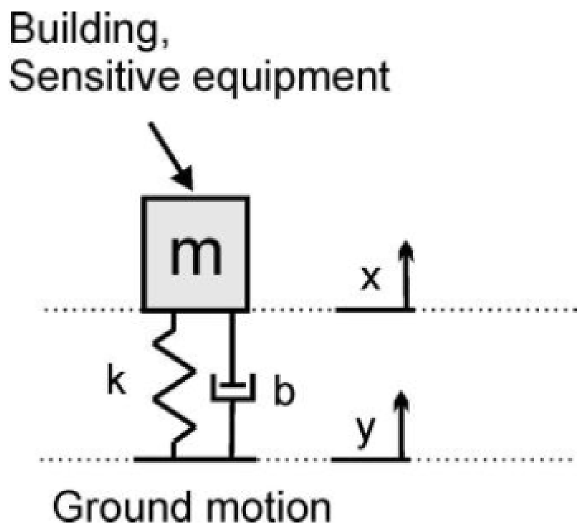


Satellite protection

Motivation

https://www.youtube.com/watch?v=ntV6LQF1GxA

Mathematically



$$m\ddot{x} + k(x-y) + b(\dot{x}-\dot{y}) = 0$$

$$x(t) = Xe^{i\omega t}$$

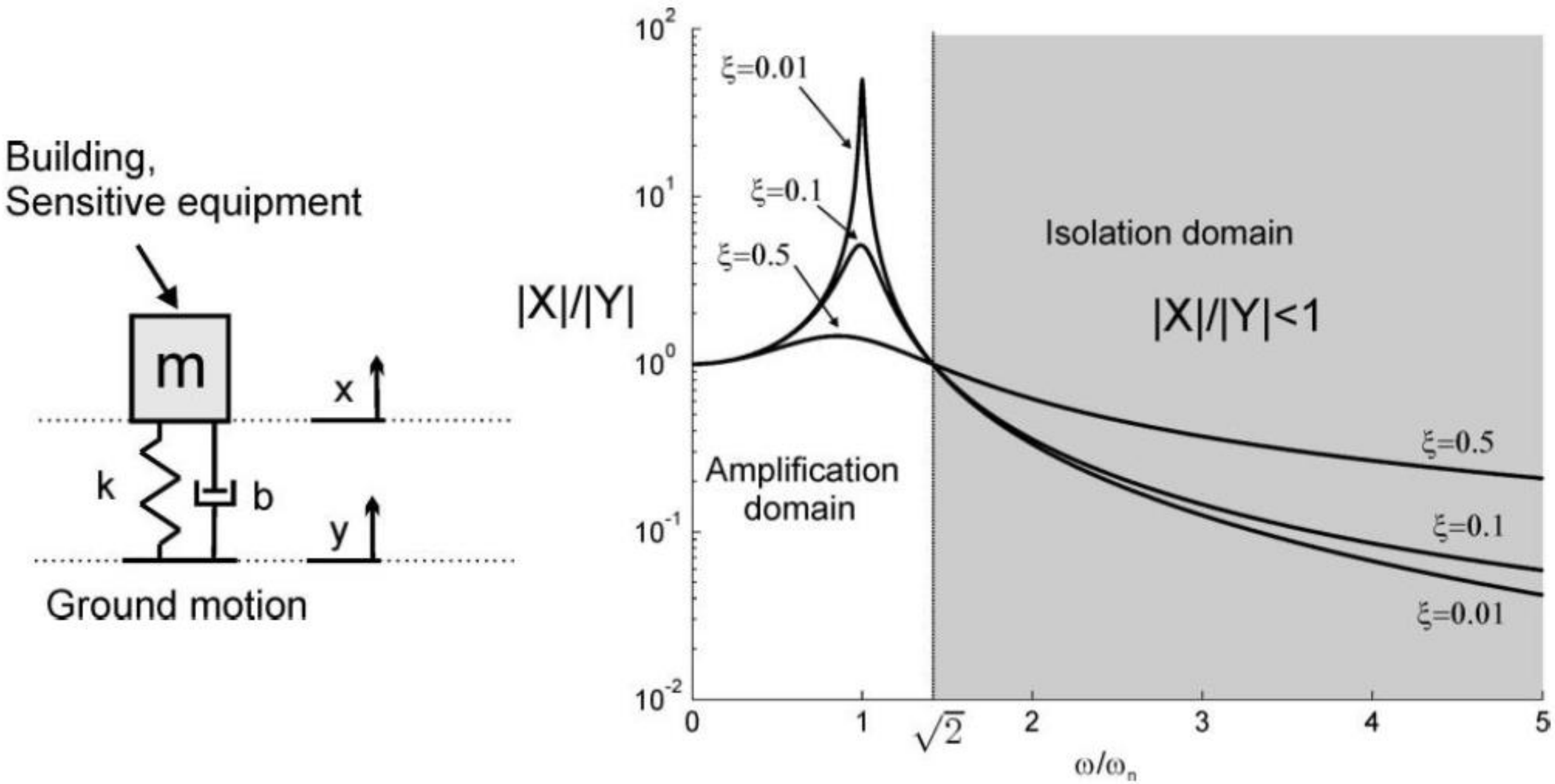
$$y(t) = Ye^{i\omega t}$$

$$\left(k - \omega^2 m + i\omega b\right) X = \left(k + i\omega b\right) Y$$

Transmissibility : $\boxed{\dfrac{|X|}{|Y|} = \dfrac{\sqrt{k^2+\omega^2 b^2}}{\sqrt{(k-\omega^2 m)^2 + \omega^2 b^2}}}$

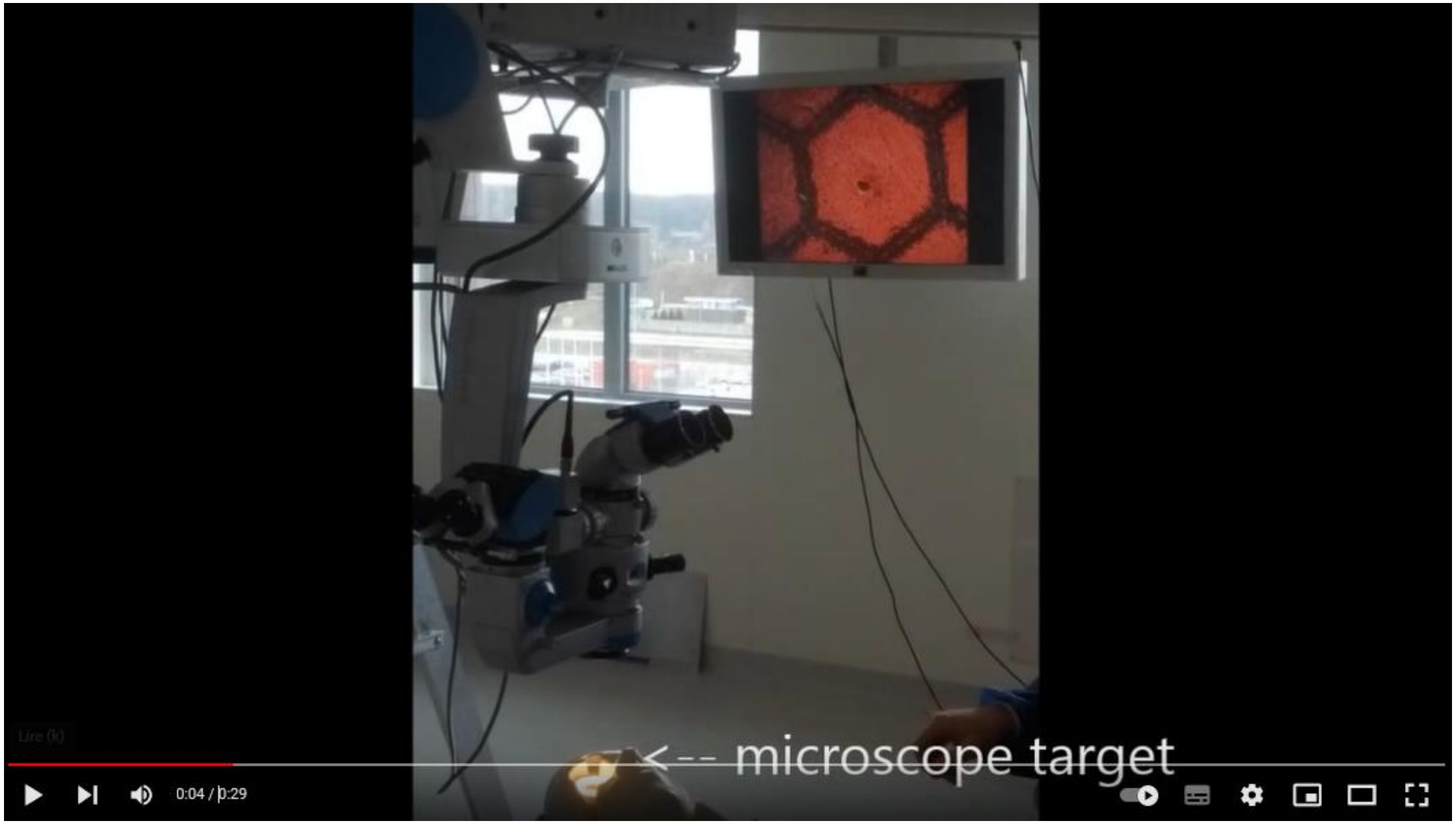
The transmissibility is equal to the isolation factor previously defined
The aim is to 'decouple' the motion of the building from the ground motion

Graphically



→ To isolate in the low frequency domain, we need k small, m high

Isolation of a microscope



https://www.youtube.com/watch?v=YPAOZXc33gE

Outline

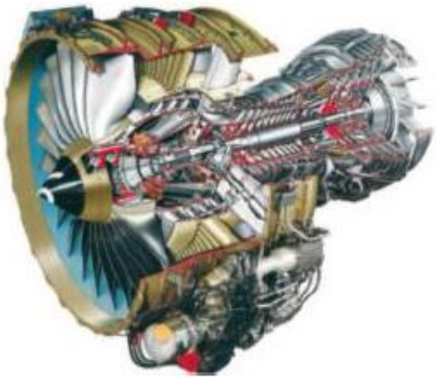
1. Vibration isolation

2. Rotor dynamics

Machines with rotor

Refrigeration compressor rotor

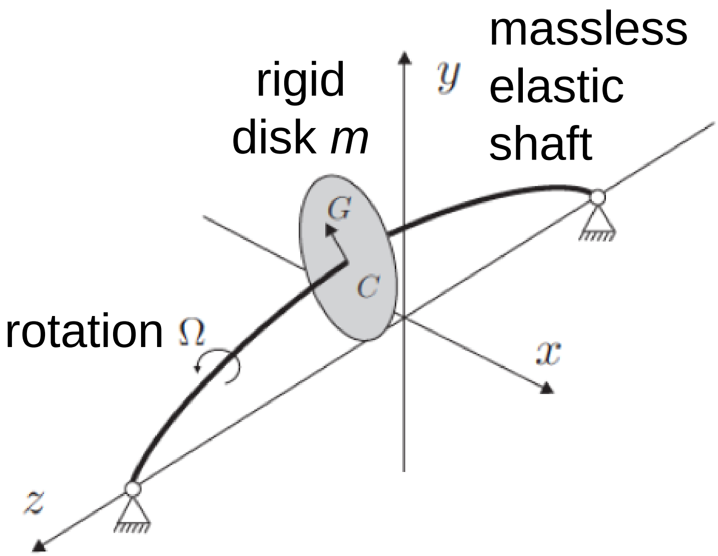
Jet engines

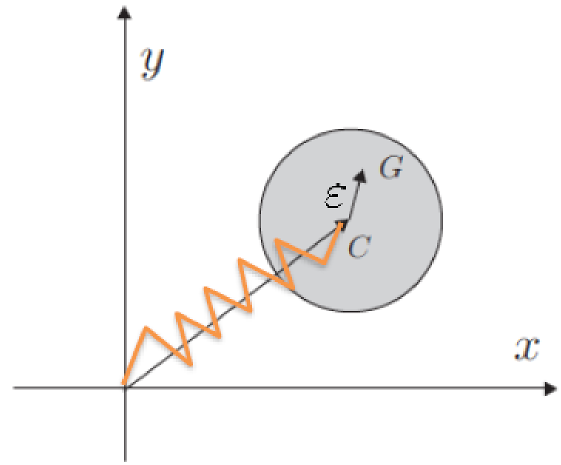


Machine tools

Electricity power plant

Simplified model: Jeffcott rotor without damping





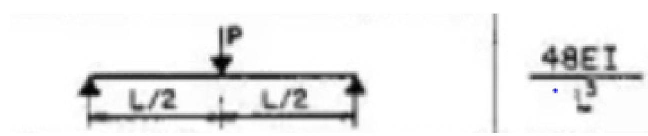
eccentricity

$$\begin{cases} x_G = x_C + \boxed{\varepsilon}\cos(\Omega t) \\ \boxed{y_G} = \boxed{y_C} + \varepsilon \sin(\Omega t) \end{cases}$$

gravity center

geometric center

Equivalent stiffness of the shaft ?



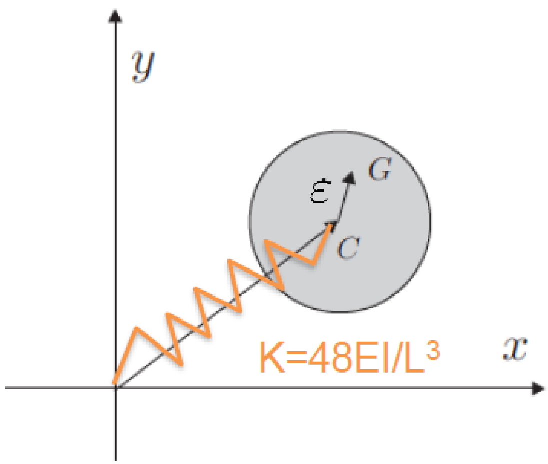
Newton's second law at gravity center

$$\begin{cases} x_G = x_C + \varepsilon\cos(\Omega t) \\ y_G = y_C + \varepsilon\sin(\Omega t) \end{cases} \longrightarrow \begin{aligned} \ddot{x}_G &= \ddot{x}_C - \varepsilon\Omega^2\cos(\Omega t) \\ \ddot{y}_G &= \ddot{y}_C - \varepsilon\Omega^2\sin(\Omega t) \end{aligned}$$

$$|F_k| = k\sqrt{x_C^2 + y_C^2}$$ spring force

$$F_{kx} = -k\sqrt{x_C^2+y_C^2}\,\frac{x_C}{\sqrt{x_C^2+y_C^2}} = -kx_C$$

$$F_{ky} = -k\sqrt{x_C^2+y_C^2}\,\frac{y_C}{\sqrt{x_C^2+y_C^2}} = -ky_C$$



$$\longrightarrow \begin{cases} m\ddot{x}_C + kx_C = m\varepsilon\Omega^2\cos\Omega t \\ m\ddot{y}_C + ky_C = m\varepsilon\Omega^2\sin\Omega t \end{cases}$$

Complex coordinates

$$\begin{cases} m\ddot{x}_C + kx_C = m\varepsilon\Omega^2 \cos \Omega t \\ m\ddot{y}_C + ky_C = m\varepsilon\Omega^2 \sin \Omega t \end{cases}$$

$$\Downarrow$$

$$r_C(t) = x_C(t) + iy_C(t)$$

$$\Downarrow$$

$$\boxed{m\ddot{r}_C + kr_C = m\varepsilon\Omega^2 e^{i\Omega t}}$$

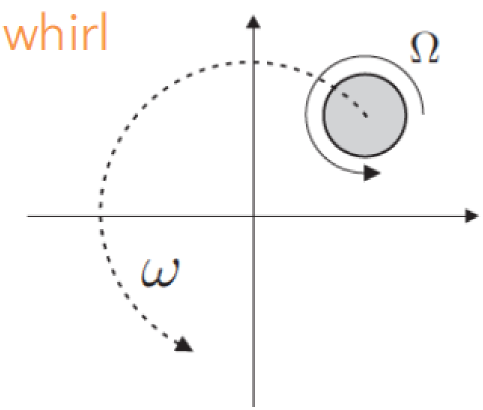
Free whirling

$$m\ddot{r}_C + kr_C = 0$$

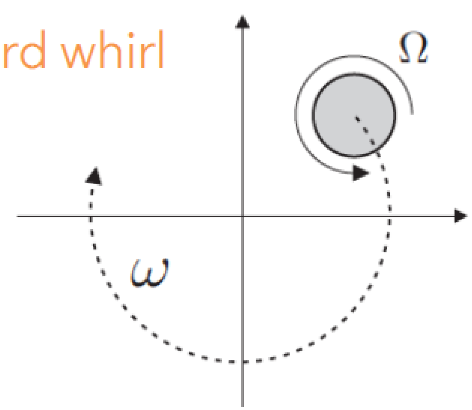
$$r_C(t) = r_0 e^{st} \longrightarrow (ms^2 + k)\, r_0\, e^{st} = 0 \qquad s = \pm i\sqrt{k/m}$$

$$\boxed{r_C(t) = R_1 e^{i\omega_0 t} + R_2 e^{-i\omega_0 t}}$$

Forward whirl



Backward whirl



Unbalance response

$$m\ddot{r}_C + kr_C = m\varepsilon\Omega^2 e^{i\Omega t}$$

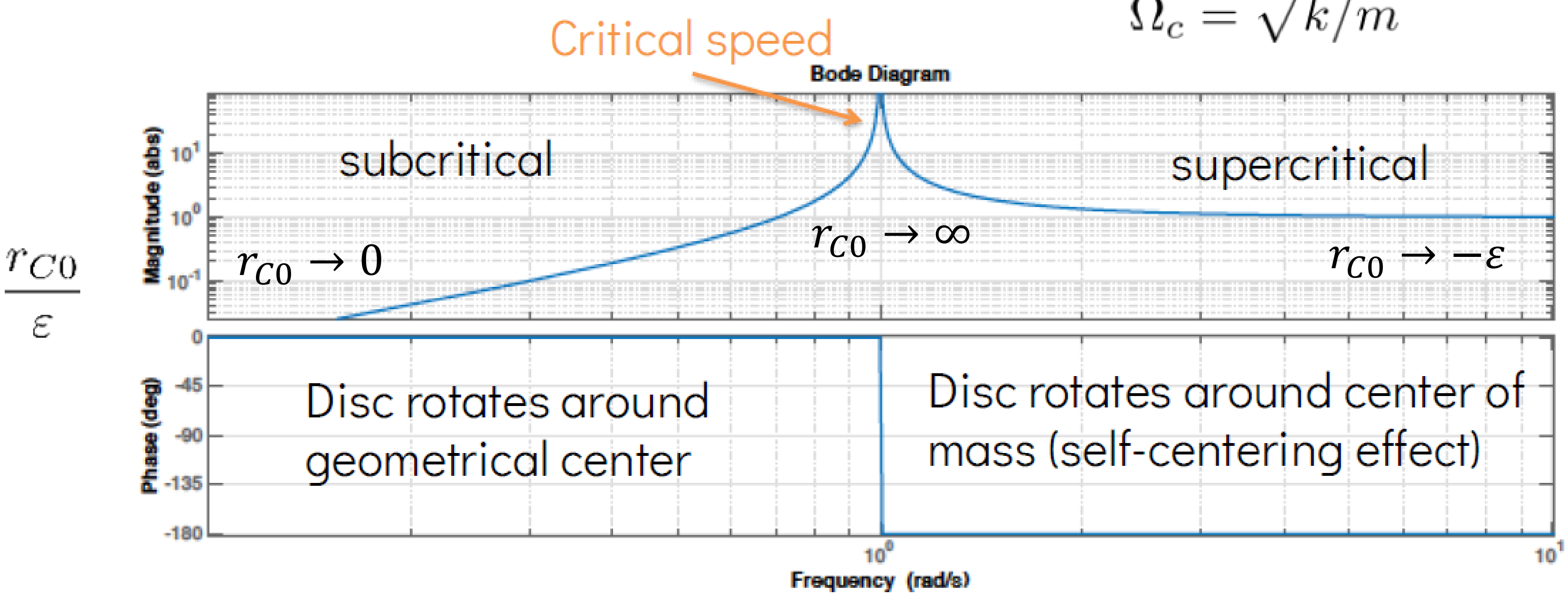
$$r_C(t) = r_0 e^{i\Omega t}$$

$$\longrightarrow \quad (-m\Omega^2 + k) r_{C_0} = m\varepsilon\Omega^2$$

Dynamic amplification

$$\boxed{r_{C_0} = \varepsilon \frac{\Omega^2}{\Omega_c^2 - \Omega^2}}$$

$$\Omega_c = \sqrt{k/m}$$

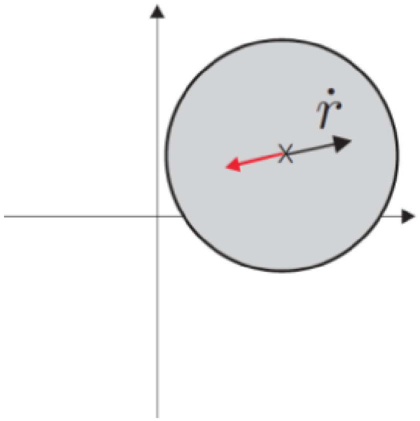


Video

Damping models

Non-rotating damping

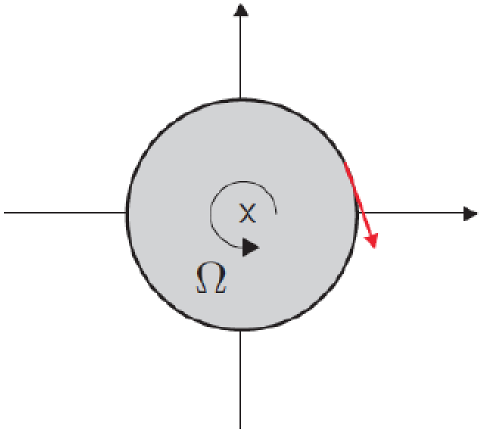
$$\mathbf{F}_n = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -c_n \begin{Bmatrix} \dot{x}_C \\ \dot{y}_C \end{Bmatrix}$$



Translation

Rotating damping

$$\mathbf{F}_r = \begin{Bmatrix} F_u \\ F_v \end{Bmatrix} = -c_r \begin{Bmatrix} \dot{u}_C \\ \dot{v}_C \end{Bmatrix}$$



Rotation

Jeffcott rotor with damping

Express rotation damping in fixed frame :

$$\begin{Bmatrix} u_C \\ v_C \end{Bmatrix} = \mathbf{R} \begin{Bmatrix} x_C \\ y_C \end{Bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \cos(\Omega t) & \sin(\Omega t) \\ -\sin(\Omega t) & \cos(\Omega t) \end{bmatrix}$$

$$\begin{Bmatrix} \dot{u}_C \\ \dot{v}_C \end{Bmatrix} = \mathbf{R} \begin{Bmatrix} \dot{x}_C \\ \dot{y}_C \end{Bmatrix} + \dot{\mathbf{R}} \begin{Bmatrix} x_C \\ y_C \end{Bmatrix} \qquad \dot{\mathbf{R}} = \Omega \begin{bmatrix} -\sin(\Omega t) & \cos(\Omega t) \\ -\cos(\Omega t) & -\sin(\Omega t) \end{bmatrix}$$

$$\mathbf{F}_n = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -c_n \begin{Bmatrix} \dot{x}_C \\ \dot{y}_C \end{Bmatrix} \qquad \mathbf{F}_r = \begin{Bmatrix} F_u \\ F_v \end{Bmatrix} = -c_r \begin{Bmatrix} \dot{u}_C \\ \dot{v}_C \end{Bmatrix}$$

To be multiplied by $\boldsymbol{R}^{-1}$

$$\boxed{\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_C \\ \ddot{y}_C \end{Bmatrix} + \begin{bmatrix} c_n + c_r & 0 \\ 0 & c_n + c_r \end{bmatrix} \begin{Bmatrix} \dot{x}_C \\ \dot{y}_C \end{Bmatrix} + \left(\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} + \Omega \begin{bmatrix} 0 & c_r \\ -c_r & 0 \end{bmatrix} \right) \begin{Bmatrix} x_C \\ y_C \end{Bmatrix} = m\varepsilon\Omega^2 \begin{Bmatrix} \cos(\Omega t) \\ \sin(\Omega t) \end{Bmatrix}}$$

Jeffcott rotor with damping: complex notations

$$r_C = x_C + i y_C$$

$$m\ddot{r}_C + (c_r + c_n)\dot{x}_C + \boxed{(k - i c_r \Omega)} r_C = m\varepsilon\Omega^2 e^{i\Omega t}$$

Complex stiffness
= negative damping

Rotating damping performs two tasks:

1. dissipating energy
2. transferring energy from the rotation of the system to its vibration

→ Possibility of instability

Jeffcott rotor with damping: free whirling

$$r_C = r_0 e^{st} \longrightarrow ms^2 + (c_r + c_n)s + k - i\Omega c_r = 0$$

$$s = \sigma + i\omega = -\frac{c_r + c_n}{2m} \pm \sqrt{\frac{(c_r + c_n)^2 - 4m(k - i\Omega c_r)}{4m^2}}$$

$$\boxed{r = R_1 e^{(\sigma_1 + i\omega_1)t} + R_2 e^{(\sigma_2 + i\omega_2)t}}$$

Unlike the undamped case, 2 different backward anf forward frequencies

$$\begin{cases} \sigma_{1,2} = -\frac{c_r+c_n}{2m} \pm \frac{1}{\sqrt{2}}\sqrt{\sqrt{\Gamma^2 + (\frac{\Omega c_r}{m})^2} - \Gamma} \\ \omega_{1,2} = \pm\frac{\mathrm{sgn}(\Omega)}{2m} \pm \frac{1}{\sqrt{2}}\sqrt{\sqrt{\Gamma^2 + (\frac{\Omega c_r}{m})^2} + \Gamma} \end{cases}$$

$$\Gamma = \frac{k}{m} - \frac{(c_r + c_n)^2}{4m^2}$$

Jeffcott rotor with damping: free whirling

$$\boxed{r = R_1 e^{(\sigma_1 + i\omega_1)t} + R_2 e^{(\sigma_2 + i\omega_2)t}}$$

$$\begin{cases} \sigma_{1,2} = -\frac{c_r + c_n}{2m} \pm \frac{1}{\sqrt{2}}\sqrt{\sqrt{\Gamma^2 + (\frac{\Omega c_r}{m})^2} - \Gamma} \\ \omega_{1,2} = \pm\frac{\mathrm{sgn}(\Omega)}{2m} \pm \frac{1}{\sqrt{2}}\sqrt{\sqrt{\Gamma^2 + (\frac{\Omega c_r}{m})^2} + \Gamma} \end{cases} \qquad \Gamma = \frac{k}{m} - \frac{(c_r + c_n)^2}{4m^2}$$

σ_2 is always negative: OK !

ω_2 is always negative (backward).

σ_1 can be positive: KO ! Stable if

ω_1 is always positive (forward).

Jeffcott rotor with damping: free whirling

Non-dimensional values

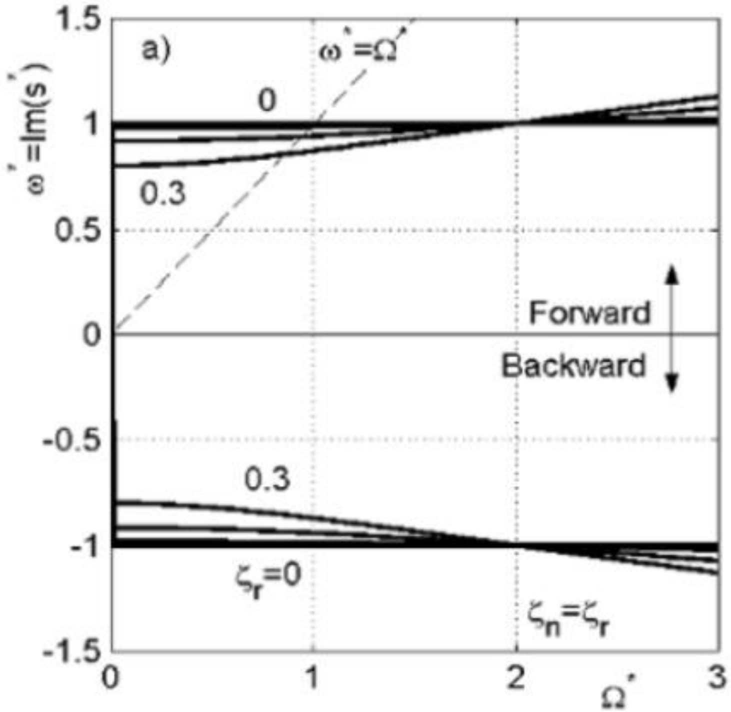
$$\begin{cases} \sigma^* = -(\zeta_r + \zeta_n) \pm \sqrt{\sqrt{\Gamma^{*2} + \Omega^{*2}\zeta_r^2} - \Gamma^*} \ , \\ \omega^* = \pm \mathrm{sgn}(\Omega^*)\sqrt{\sqrt{\Gamma^{*2} + \Omega^{*2}\zeta_r^2} + \Gamma^*} \ , \end{cases}$$

$$\begin{array}{ll} \omega^* = \omega/\sqrt{k/m} \ , & \sigma^* = \sigma/\sqrt{k/m} \ , \\ \Omega^* = \Omega/\sqrt{k/m} \ , & \Gamma^* = [1 - (\zeta_n + \zeta_r)^2]/2 \ , \\ \zeta_r = c_r/2\sqrt{km} \ , & \zeta_n = c_n/2\sqrt{km} \ . \end{array}$$

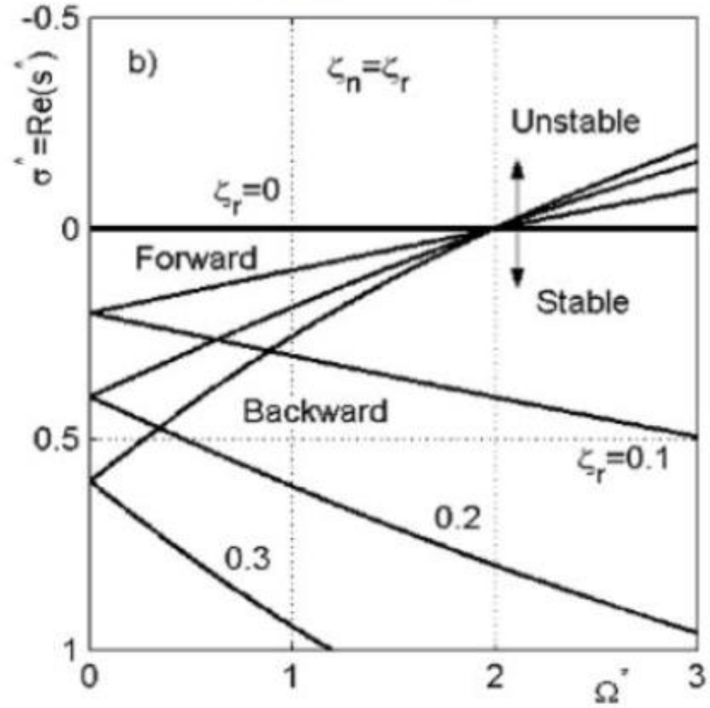
The nondimensional values of σ^* and ω^* are then functions of the spin speed Ω^* and of only two parameters ζ_n and ζ_r.

Jeffcott rotor with damping: free whirling

Campbell diagram



Decay rate plot



For increasing rotation speed, the imaginary part remains unchanged, but the forward whirl becomes unstable !

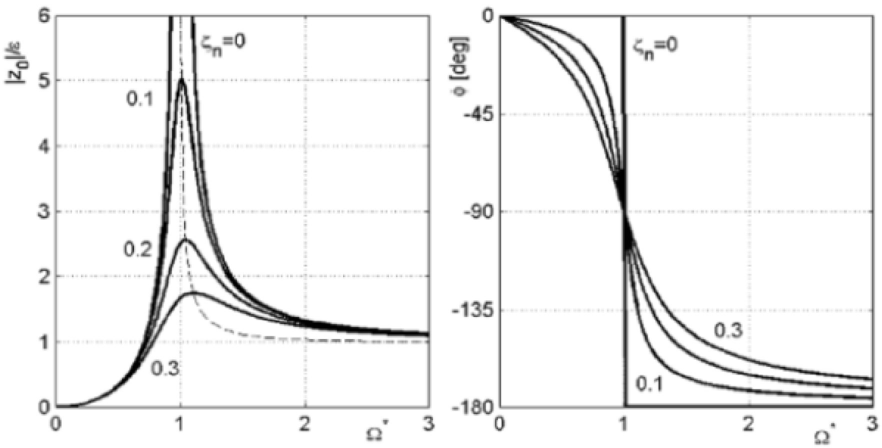
Jeffcott rotor with damping: unbalance response

$$r_C = r_{C_0} e^{i\Omega t} \qquad m\ddot{r}_C + (c_r + c_n)\dot{r}_C + (k - ic_r\Omega) r_C = m\epsilon\Omega^2 e^{i\Omega t}$$

$$\boxed{r_{C_0}(-m\Omega^2 + i\Omega c_n + k) = m\epsilon\Omega^2}$$

c_r disappears

Only translation damping



$$|r_{C_0}| = \epsilon \frac{{\Omega^*}^2}{\sqrt{(1-{\Omega^*}^2)^2 + 4\zeta_n^2 {\Omega^*}^2}}$$

$$\Phi = \arctan\left(\frac{-2\Omega^*\zeta_n}{1-{\Omega^*}^2}\right).$$

Questions for the exam (L2)

Dynamique newtonienne et Lagrange

1. Etablir les équations du mouvement d'un pendule par la méthode newtonienne et via le formalisme de Lagrange. Quelle est la méthode la plus simple ?
2. Dériver le principe des travaux virtuels et l'appliquer à un moteur mono-piston afin de calculer la force nécessaire à appliquer au piston pour obtenir l'équilibre statique.
3. Dériver le principe d'Alembert et le principe d'Hamilton et définir la notion de Lagrangien. Calculer le Lagrangien pour un pendule.
4. Dériver les équations de Lagrange sans contraintes à partir du principe d'Hamilton et établir à l'aide de celles-ci les équations du mouvement pour un pendule avec masse attachée à l'extrémité d'un ressort.
5. Dériver les équations de Lagrange avec contraintes à partir du principe d'Hamilton.

Questions for the exam (L3)

Systèmes à 1 degré de liberté

6. Etablir l'équation du mouvement d'un système à 1 degré de liberté non amorti soumis à une excitation harmonique. Dériver la solution générale et le facteur d'amplification dynamique. Représenter le module et la phase de la réponse fréquentielle (Diagramme de Bode).
7. Etablir l'équation du mouvement d'un système à 1 degré de liberté amorti soumis à une excitation harmonique. Calculer le module et la phase de la réponse de la fonction de transfert (Diagramme de Bode) et représenter graphiquement pour différentes valeurs de l'amortissement.
8. Etablir la fonction de réponse impulsionnelle d'un système à 1 degré de liberté amorti et tracer celles-ci pour différentes valeurs de l'amortissement. Expliquer comment la valeur de l'amortissement influence cette réponse.
9. Ecrire les équations du mouvement d'un système à 1 degré de liberté excité par sa base. Quel est le lien avec un système excité par une force ?

Questions for the exam (L4)

Systèmes à plusieurs degrés de liberté

10. Pour un système à n degrés de liberté non amorti, démontrez que la vibration est harmonique et établissez l'équation qui permet de calculer les fréquences propres.
11. Etablir les équations du mouvement d'un système à 2 degrés de liberté sans amortissement possédant 2 masses et 3 ressorts identiques (tous les coefficients sont unitaires). Calculer les fréquences propres et les modes propres correspondants.
12. Démontrer l'orthogonalité des modes propres et citer les propriétés mathématiques (2) et physiques (2) de ceux-ci.
13. Démontrer qu'un système à deux degrés de liberté peut posséder une antirésonance.
14. Etablir les équations du mouvement d'un système à 2 degrés de liberté avec amortissement (2 masses, 3 ressorts et trois amortisseurs identiques). Etablir l'expression de la réponse forcée (force appliquée sur la masse 2), et tracer le diagramme de Bode pour chaque degré de liberté.
15. Démontrer qu'un absorbeur de vibration peut remplacer la résonance de la structure primaire par une antirésonance. Représenter graphiquement la fonction de transfert de la structure primaire seule et de la structure primaire équipée d'un absorbeur.
16. Démontrer qu'un absorbeur de vibration pendulaire peut remplacer la résonance de la structure primaire par une antirésonance. Quelle doit être la fréquence du pendule ?