Cinématique et dynamique des machines

Dynamics and introduction to vibrations

Exercise Session 1

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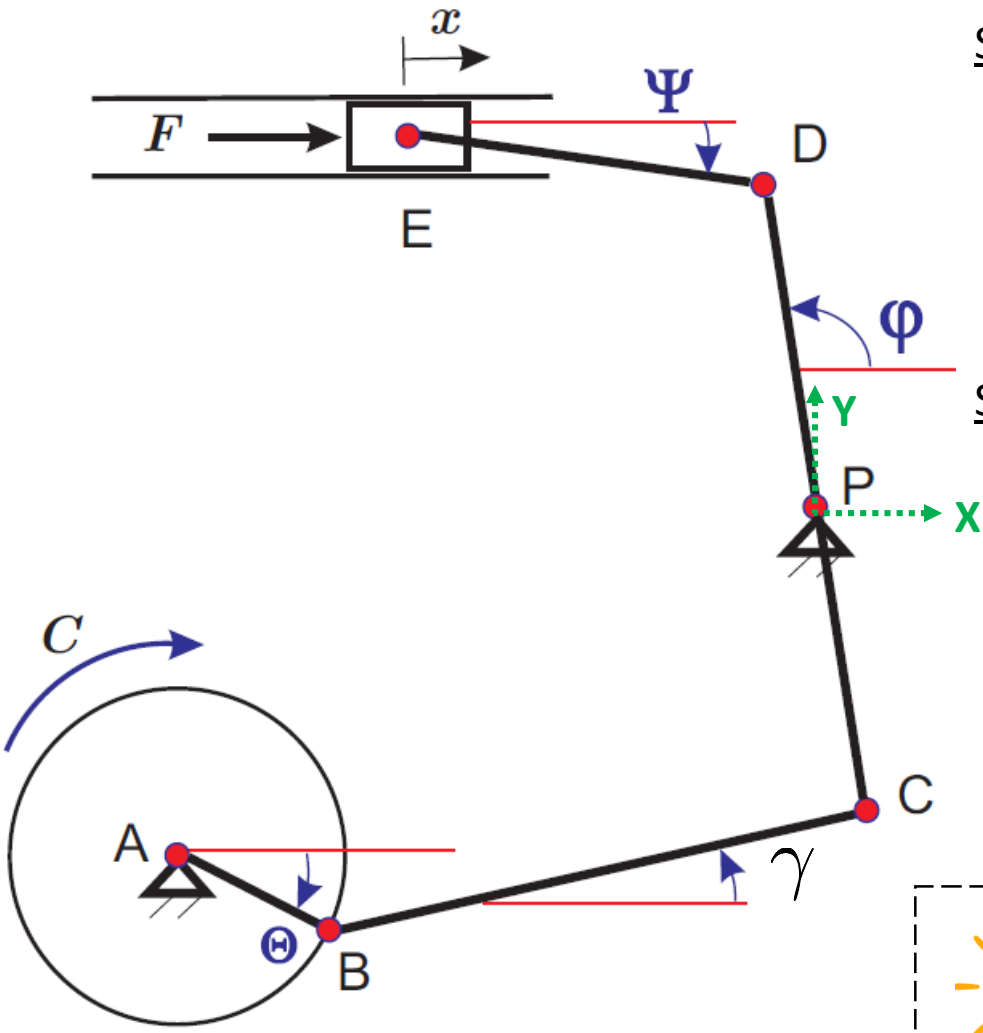
Problem 1

Kinematics

Express the ratio between the force and the torque (as a function of the system variables) when the system is in equilibrium



Step 0: Take a look at the system!

Step 1: How many degrees of freedom ?

$$N = \sum_{i=1}^{l} v_i - 3b$$

Step 2: Select the coordinates q

Step 3: If q > N

Write the q-N constraint equations **Φ**

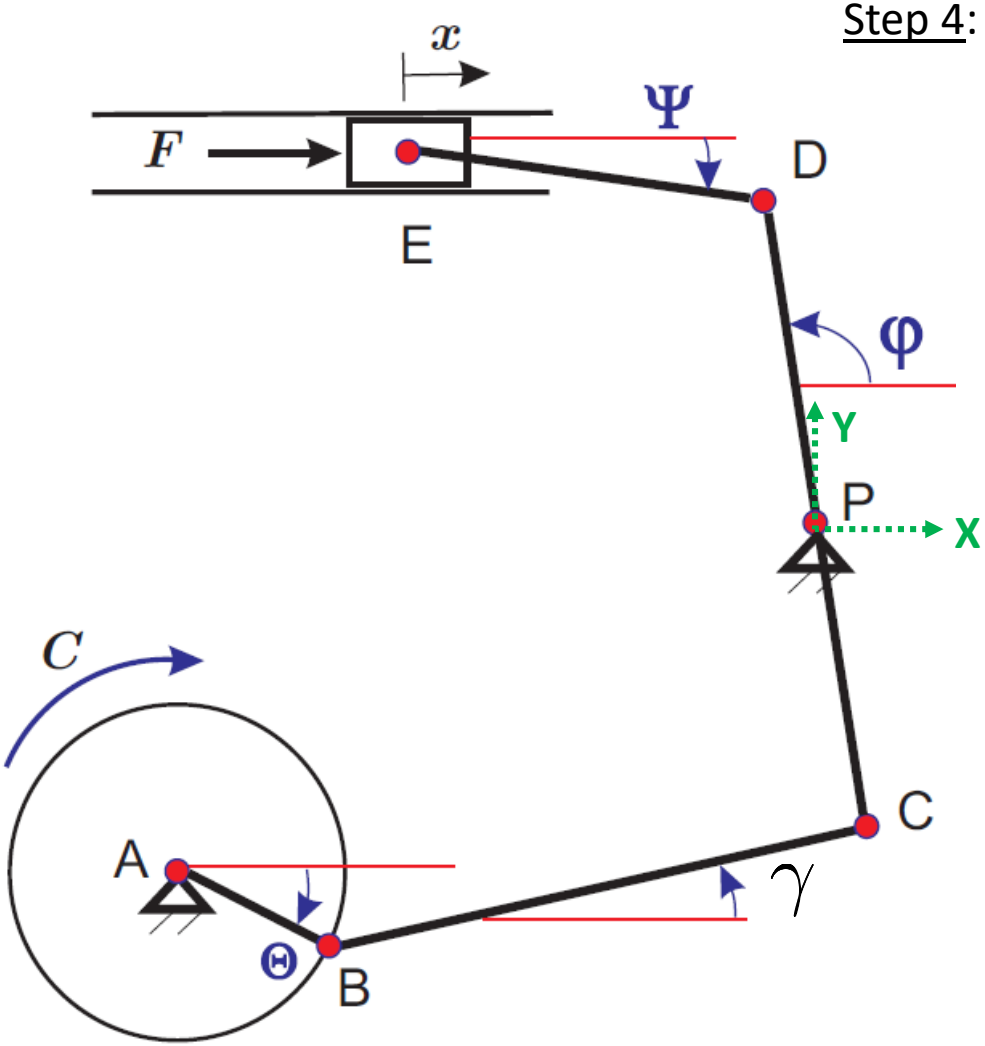
Expressing the mathematical restriction of the relative motion between the bodies

- Use of an absolute reference frame (e.g. a fixed joint…)
- Express the extremities wrt the frame

Problem 1

Kinematics

Express the ratio between the force and the torque (as a function of the system variables) when the system is in equilibrium



Step 4: Use the virtual work principle to link F and C

$$\partial\tau = 0$$

Step 5: Express δx and $\delta\Theta$

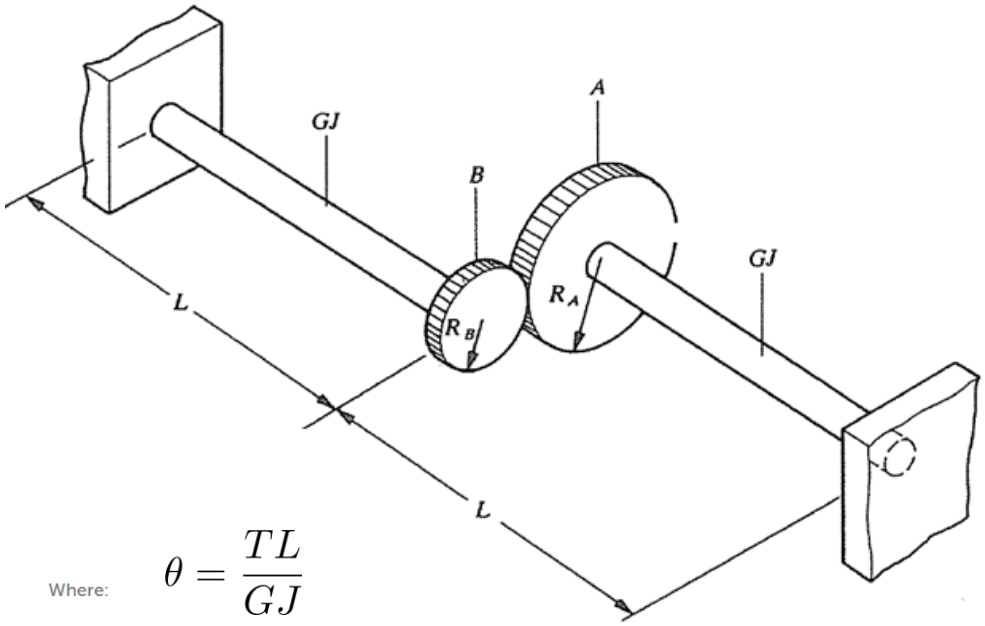
Use the constraints equations

$$d\Phi = \sum_i \frac{\partial \mathbf{\Phi}}{\partial q_i} \partial q_i = 0$$

Problem 2

Dynamics

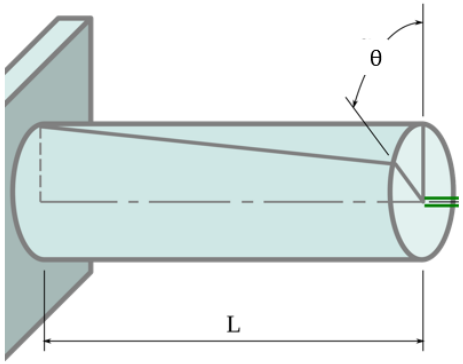
Determine the natural frequency of the system (two shafts connected with spur gears; shaft torsional rigidity GJ, shafts inertia I_A and I_B and $R_A/R_B = n$)



Where: $\theta = \frac{TL}{GJ}$

- T = Torque applied to the component
- L = Length of the component
- J = Polar Moment of Inertia (Torsion Formula Constant)
- G = Shear Modulus (Modulus of Rigidity)

The image below depicts the angle of twist:



https://www.fictiv.com/articles/design-methods-to-improve-torsional-rigidity

Step 0: Take a look at the system!

Step 1: How many degrees of freedom ?

Step 2: Select the coordinates q

Step 3: Build the Lagrangian of the system

$$L = T - V$$

$$T = \frac{1}{2}\sum_{k=1}^{N} m_k v_k^2$$

Kinematic energy

Potential energy

$$V_{spring} = \frac{1}{2}kx_q^2$$

Step 4: Apply the Lagrangian equation

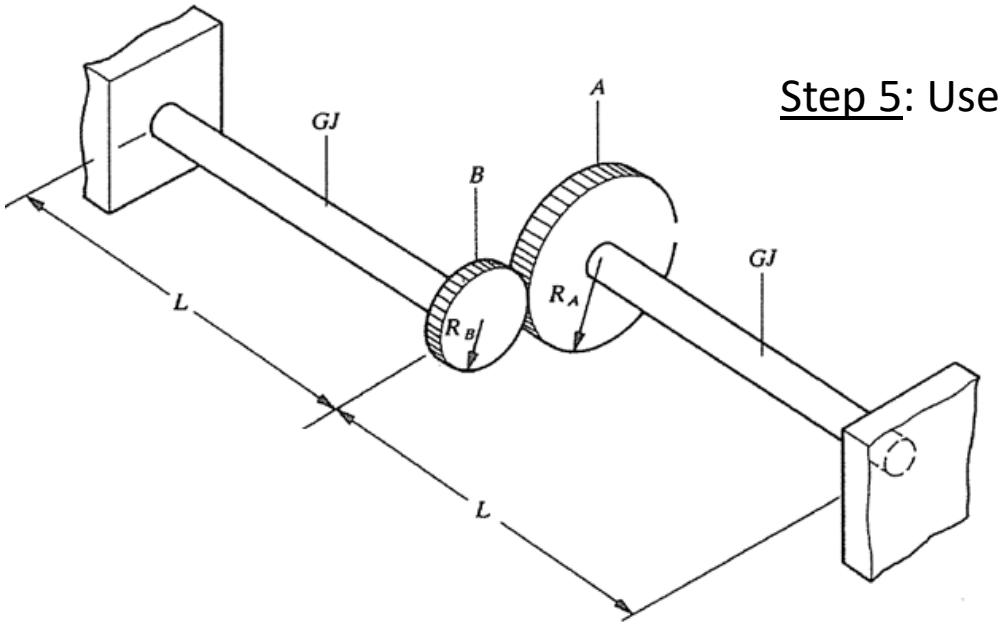
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \cancel{Q_k} = 0$$

Work from dissipative forces

Problem 2

Dynamics

Determine the natural frequency of the system (two shafts connected with spur gears; shaft torsional rigidity GJ, shafts inertia I_A and I_B and $R_A/R_B = n$)



Step 5: Use of the harmonic signal assumption

Assume harmonic motion:

$$\theta_A = Ae^{i\omega t}$$

Step 6: Solve the equation for ω

Step 7: Critical thinking about the obtained solution

Why only one frequency while we have two bodies?

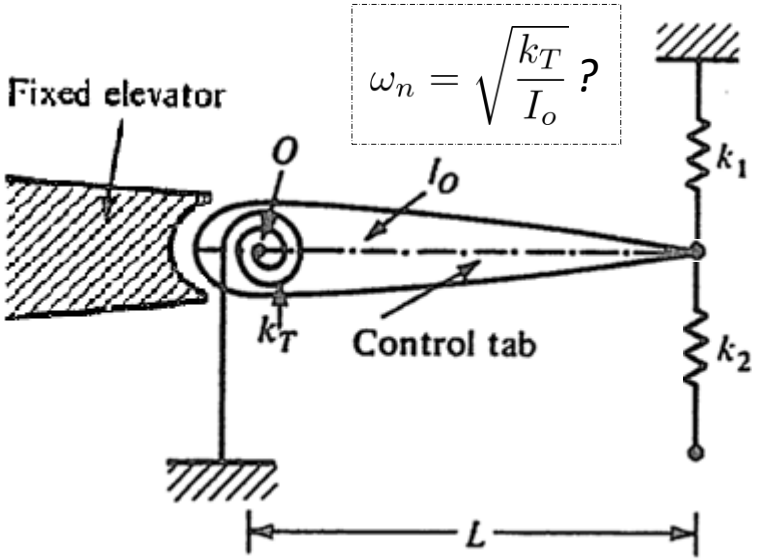
Is it coherent with the one dof natural frequency solution?

How can I modify the frequency if needed?

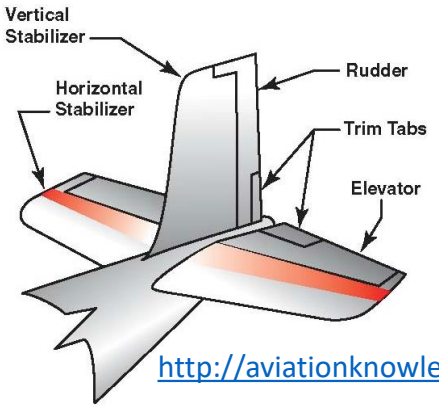
Problem 3

Dynamics

To find out the resonance frequency of a control tab (unknown internal stiffness k_t), you build a setup and you find the coupled resonance frequency ω_r. From this ω_r and the known properties of the setup, retrieve ω_n



Step 0: Take a look at the system!



http://aviationknowledge.wikidot.com/sop:stabilizers

Step 1: How many degrees of freedom ?

Step 2: Select the coordinates q

Step 3: Build the Lagrangian of the **full** system

$$L = T - V$$

Step 4: Apply the Lagrangian equation

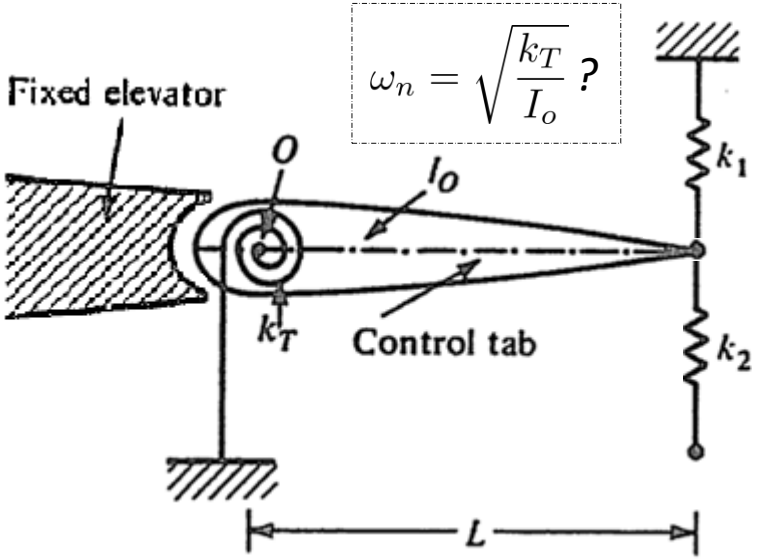
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k$$

Problem 3

Dynamics

To find out the resonance frequency of a control tab (unknown internal stiffness k_t), you build a setup and you find the coupled resonance frequency ω_r. From this ω_r and the known properties of the setup, retrieve ω_n

$$\omega_n = \sqrt{\frac{k_T}{I_o}} \; ?$$



Step 5:
Switch to harmonic assumption and solve for $\boldsymbol{\omega_r}$

Step 6:
Express ω_n as a function of ω_r

Step 7: Critical thinking about the obtained solution

Why is the natural frequency of the coupled system higher than the original one?

If my setup cannot reach the couple resonance frequency, what can I do ?

Could I used another method to find the natural frequency?