Cinématique et dynamique des machines

Dynamics and introduction to vibrations

Exercise Session 2

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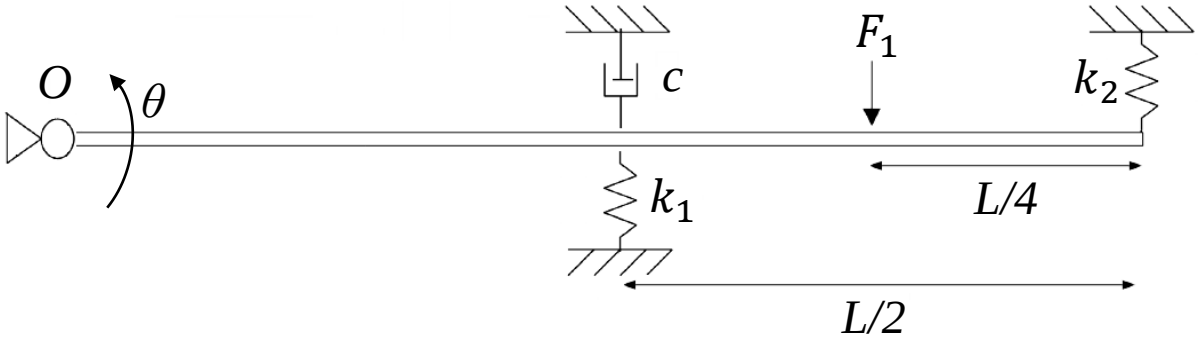
Problem 1

Dynamics

Find the resonance frequency of the following system (inertia I_o, length L). Use the small angle assumption if needed



Step 0: Take a look at the system!

Step 1: How many degrees of freedom ?

Step 2: Select the coordinates q

Step 3: Build the Lagrangian of the system

$$L = T - V$$

Step 4: Apply the Lagrangian equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k$$

≠ 0

Non-conservative forces:

$$Q_k = \sum_i F_i \frac{\partial x_i}{\partial q_k}$$

Projection of the forces on the coordinate q_k

Example:

$$\vec{F}_1 = -F_1 \, \vec{1}_{y,F}$$

$$\vec{1}_{y,F} = \frac{3L}{4} \sin\theta$$

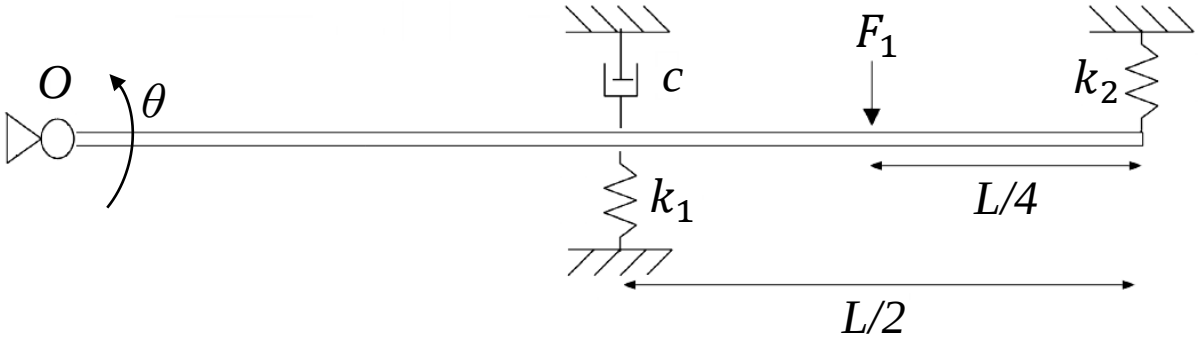
$$\frac{\partial \vec{1}_{y,F}}{\partial \theta} = \frac{3L}{4} \cos\theta$$

$$\longrightarrow Q_{\theta,F_1} = -F_1 \frac{3L}{4} \cos\theta$$

Problem 1

Dynamics

Find the resonance frequency of the following system (inertia I_o, length L). Use the small angle assumption if needed



Step 5: Use of the harmonic signal assumption

Step 5bis: Consider the free and undamped system (intrinsic property)

Step 6: Solve the equation for ω

Step 7: Critical thinking about the obtained solution

What is the major limitation of the obtained result?

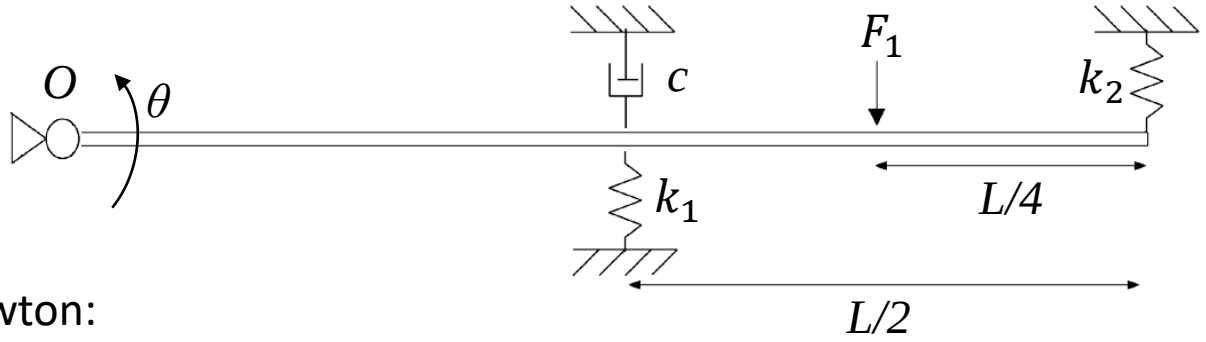
How can I compute the static deflexion?

What would happen if gravity is taken into account?

Problem 2

Dynamics

Find the resonance frequency of the following system (inertia I_o, length L). Use the small angle assumption if needed and **do not use the Lagrangian**



From Newton:

$$m\,\ddot{x} = \sum_i F_i$$

$$I\,\ddot{\theta} = \sum_i C_i$$

Similar than the previous exercise

Step 5: Use of the harmonic signal assumption

Step 5bis: Consider the free and undamped system (intrinsic property)

Step 6: Solve the equation for ω

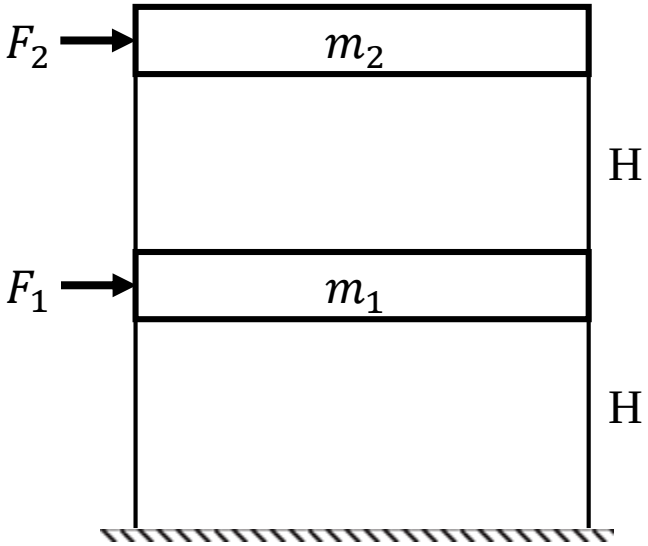
The Newton and the Lagrangian approaches provide the same EOM

(which makes sense!)

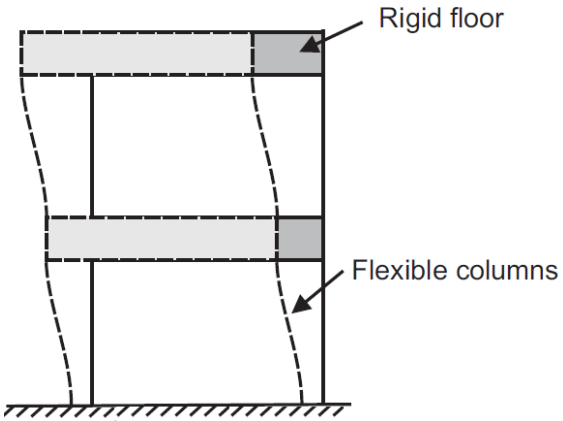
Problem 3

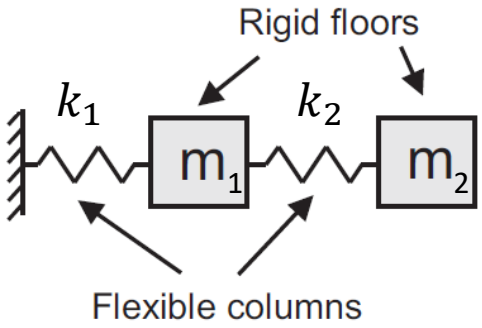
Dynamics

Calculate the natural frequencies of the simplified building where only horizontal movement of the floors are assumed.



Step 0: Take a look at the system!





Step 1: How many degrees of freedom ?

Step 2: Select the coordinates q

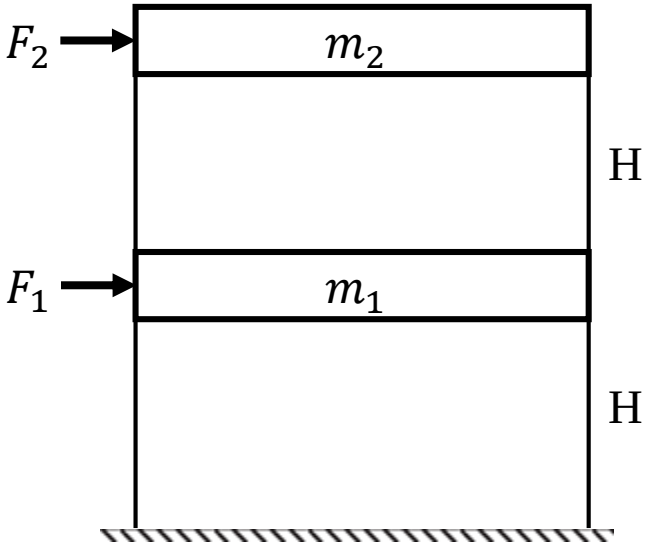
Step 3: Build the equations of motion

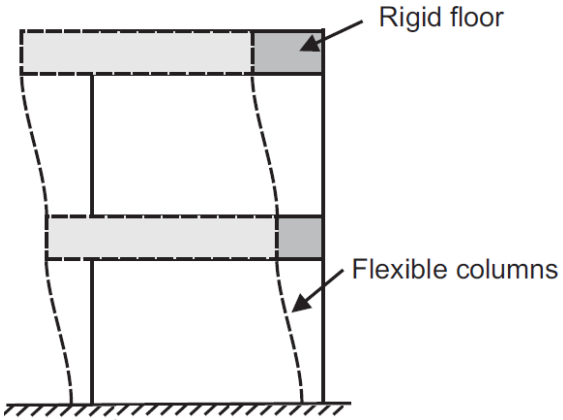
Step 4: Use of the harmonic signal assumption

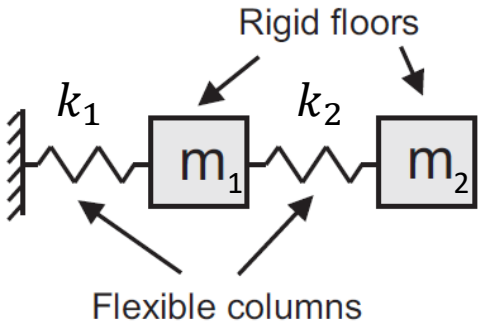
Problem 3

Dynamics

Calculate the natural frequencies of the simplified building where only horizontal movement of the floors are assumed.







Step 5: Solve for ω

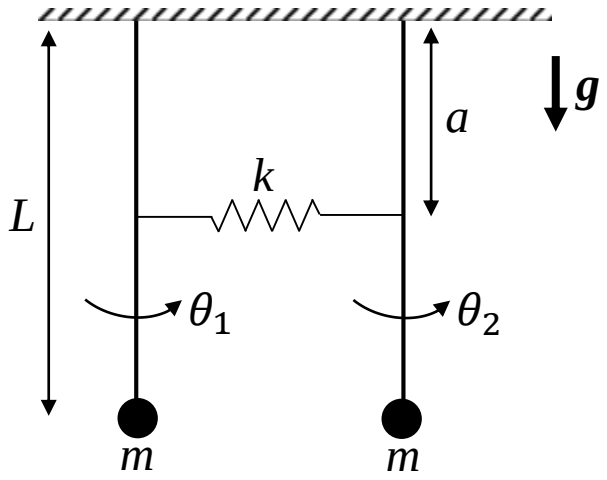
$$\left[K - M\omega^2\right]\underline{X} = \underline{0}$$

Non-trivial solution

$$\det\left(K - M\omega^2\right) = 0$$

Problem 4 Calculate the natural frequencies and the mode shapes of this pendulum

Dynamics



Step 0: Take a look at the system!

Step 1: How many degrees of freedom ?

Step 2: Select the coordinates q

Step 3: Build the equations of motion

Step 4: Use of the harmonic signal assumption

Step 5: Solve for ω

Step 6: Inject ω_1 and ω_2 in the reduced equation

Step 7: Critical thinking about the obtained solution

How can we physically interpret the two frequencies?

What is the physical representation of the mode shapes?