Nonlinear Vibrations of Aerospace Structures

University of Liège, Belgium

L01 Introduction

Course objectives

Review of linear theory

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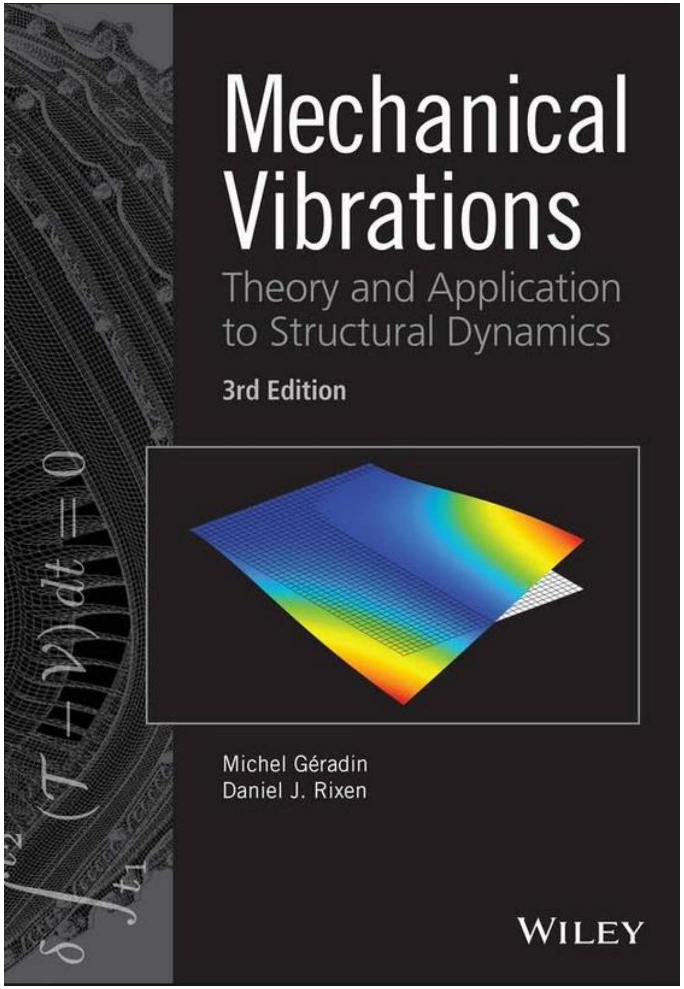
Course details:

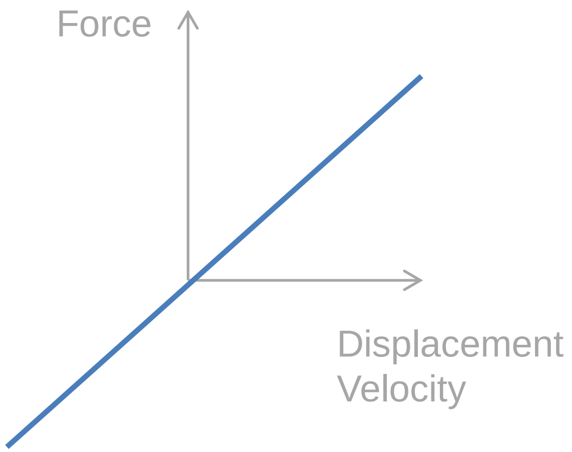
- http://www.s3l.be

The starting point: what you know

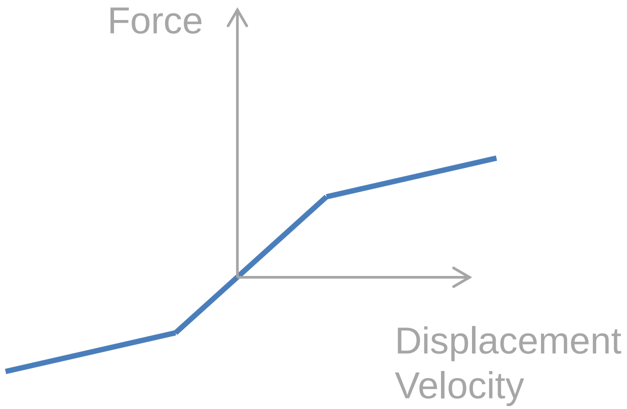


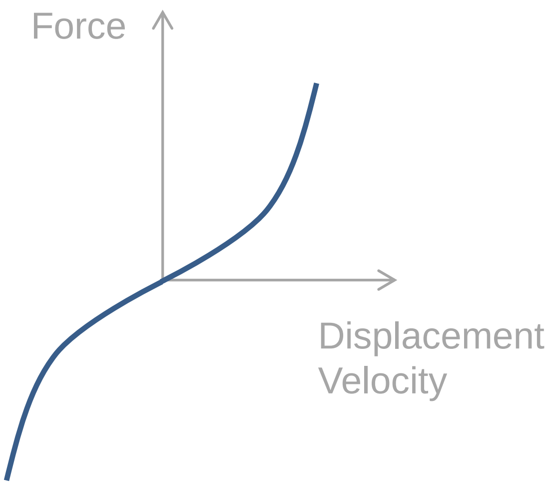
What is a nonlinearity ?

LINEAR



NONLINEARITY



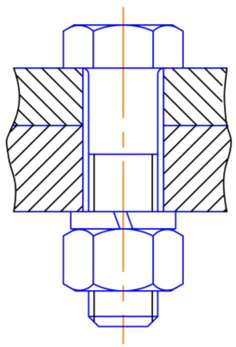


What is a nonlinear vibration ?

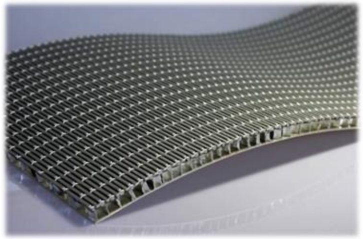
$$M\ddot{q} + C\dot{q} + Kq = 0$$

Typical nonlinearities in real-life structures

Bolts, joints and gaps



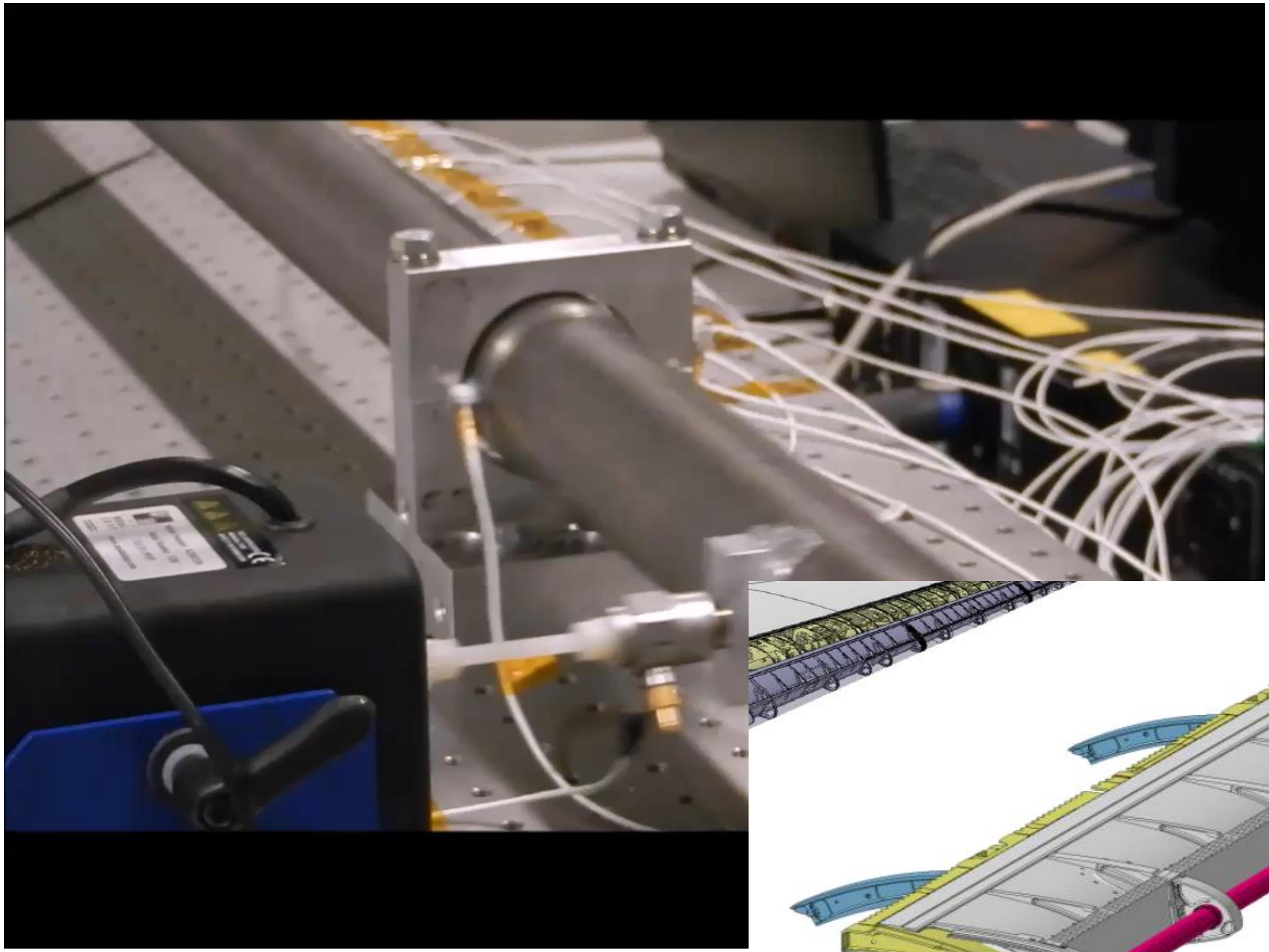
Elastomers and composites

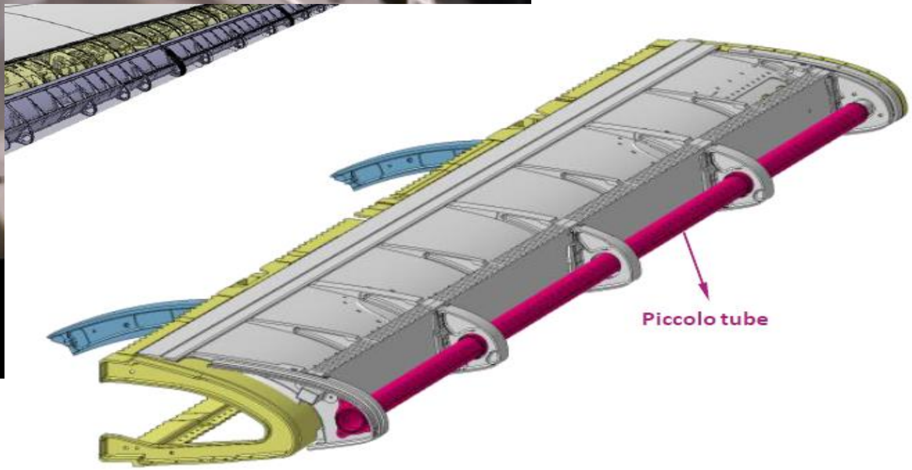


Friction and contact

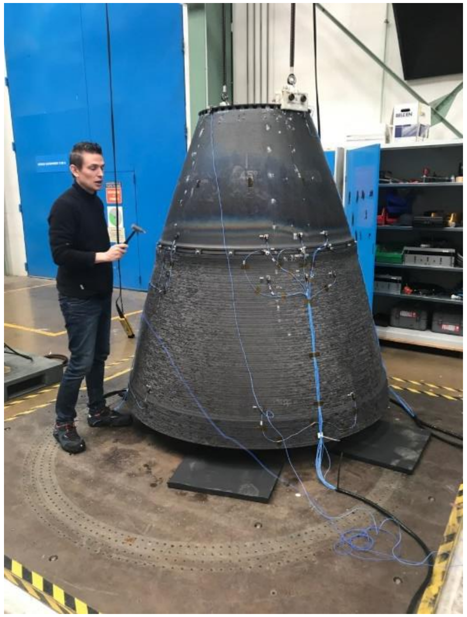
Large amplitudes

Contact phenomena in an aerospace structure





More nonlinear aerospace structures

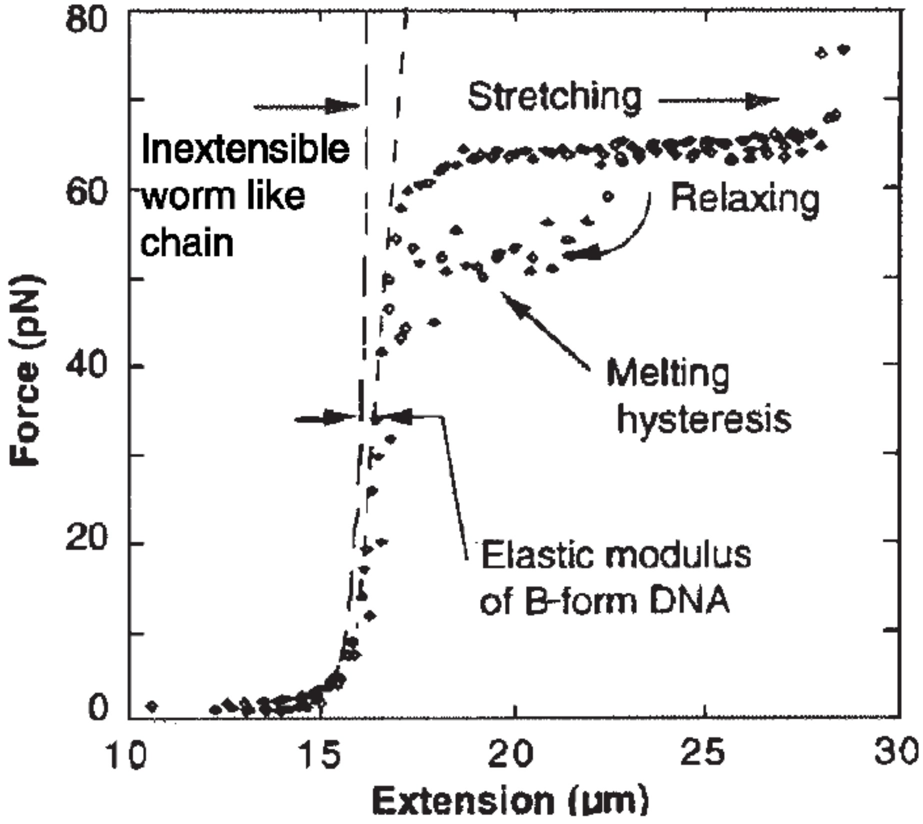


Nozzle (Ariane)

Oil tank (SAB)

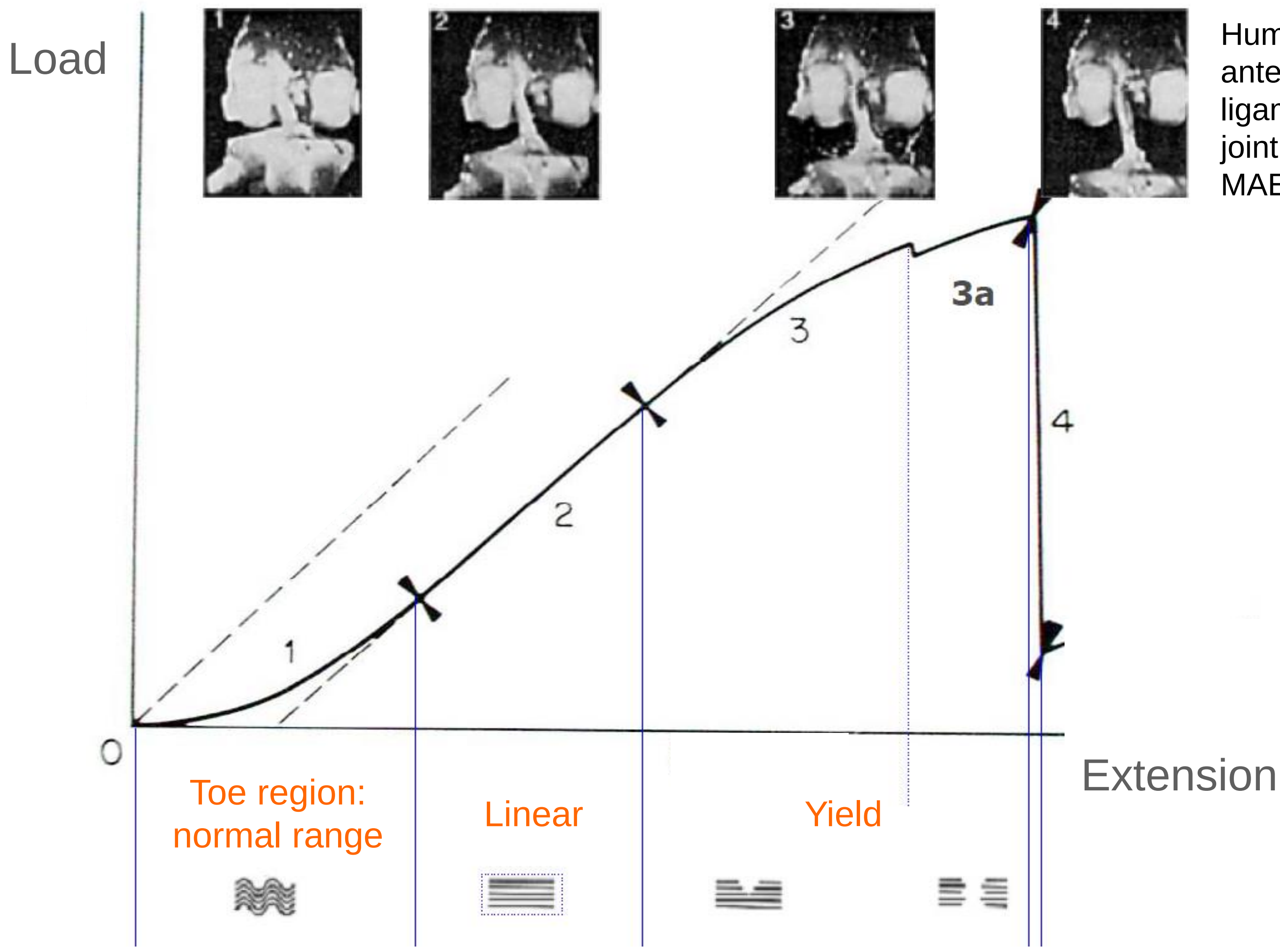
Airbus A320 (Airbus)

Why study nonlinearity ? Nature is nonlinear



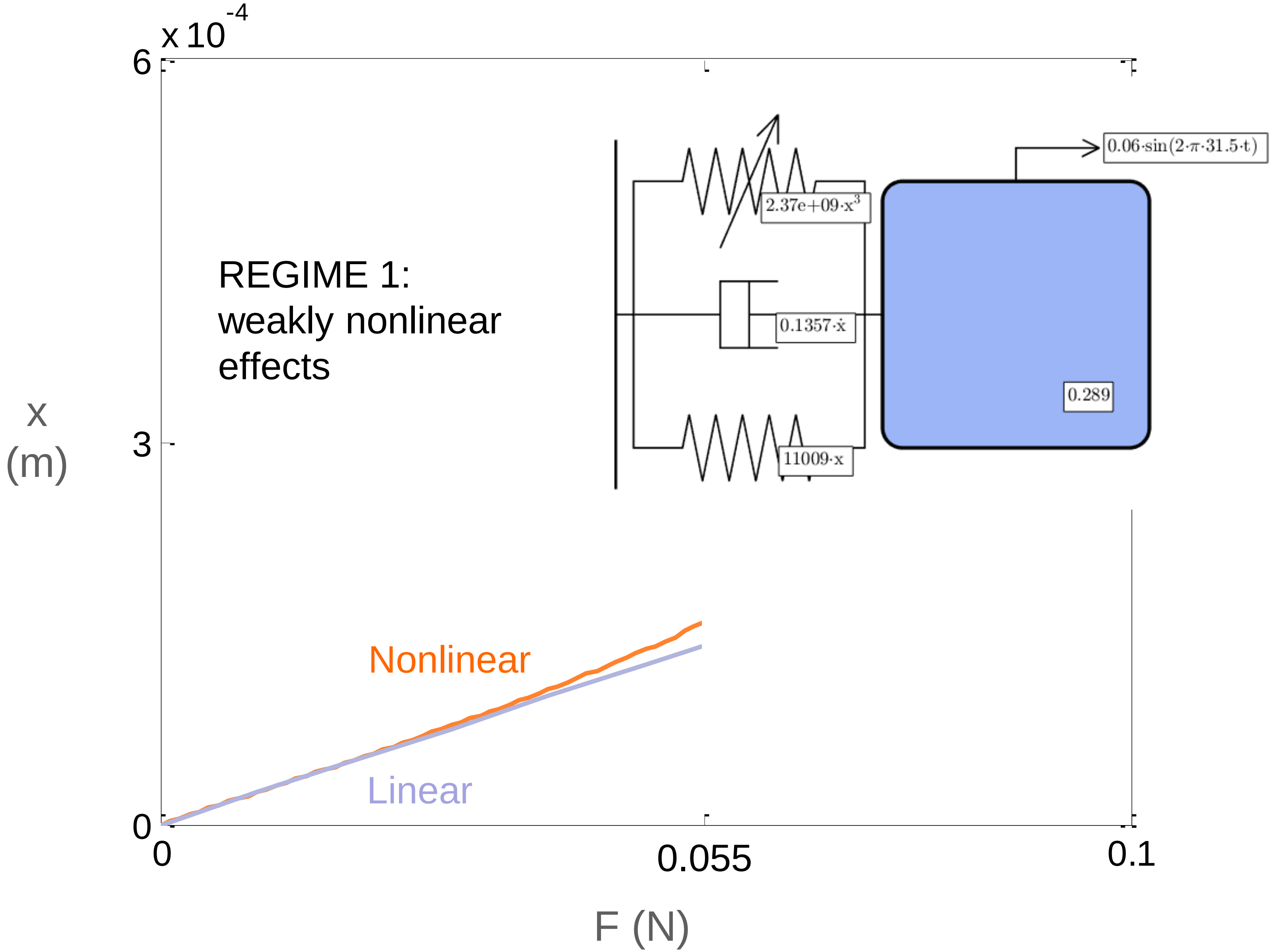
DNA molecule: Smith, Finzi, Bustamante, Science, 1992

Why study nonlinearity ? Nature is nonlinear

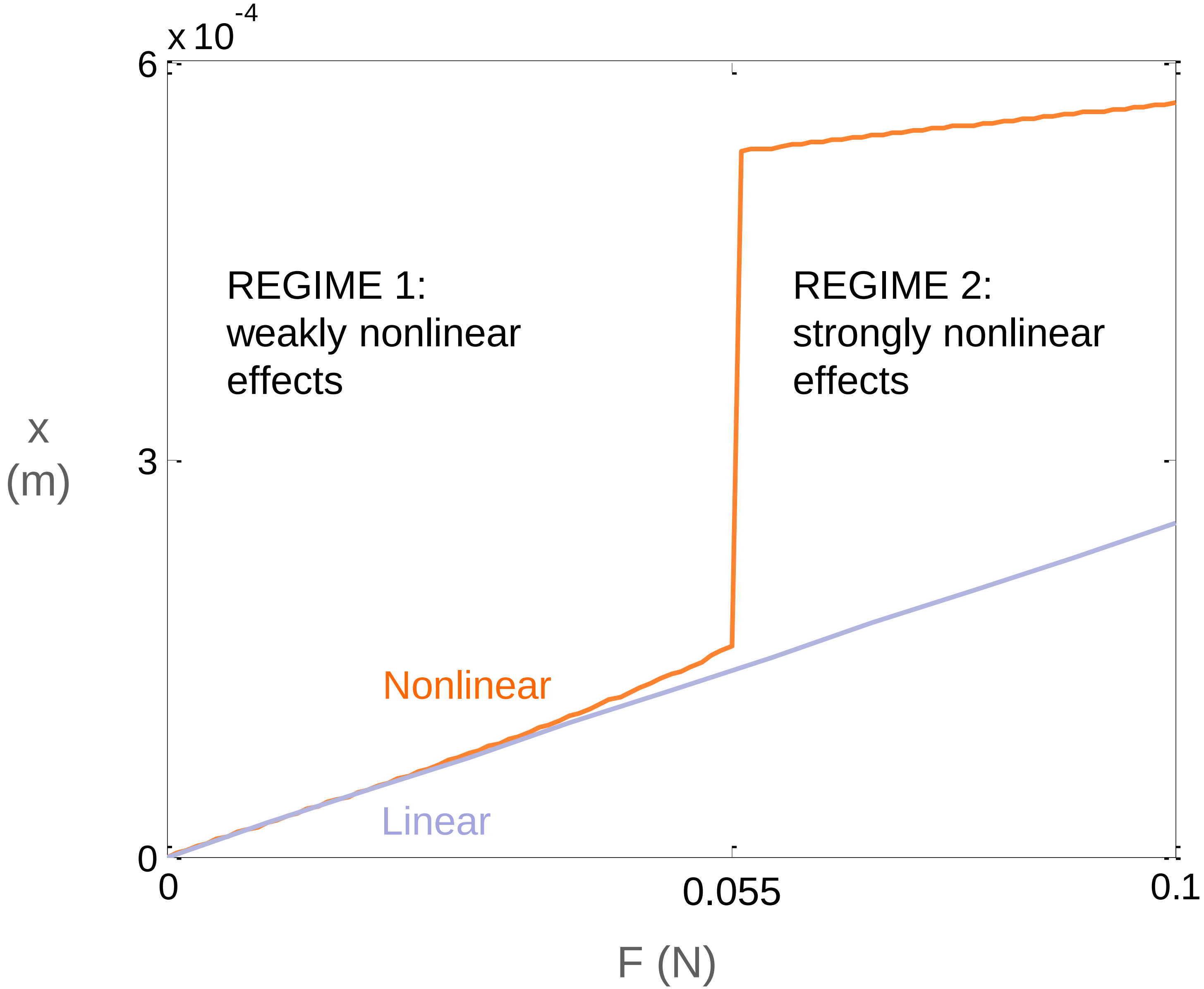


Human cadaveric anterior cruciate ligament in knee joint (Dr. Ziv, MAE, Buffalo)

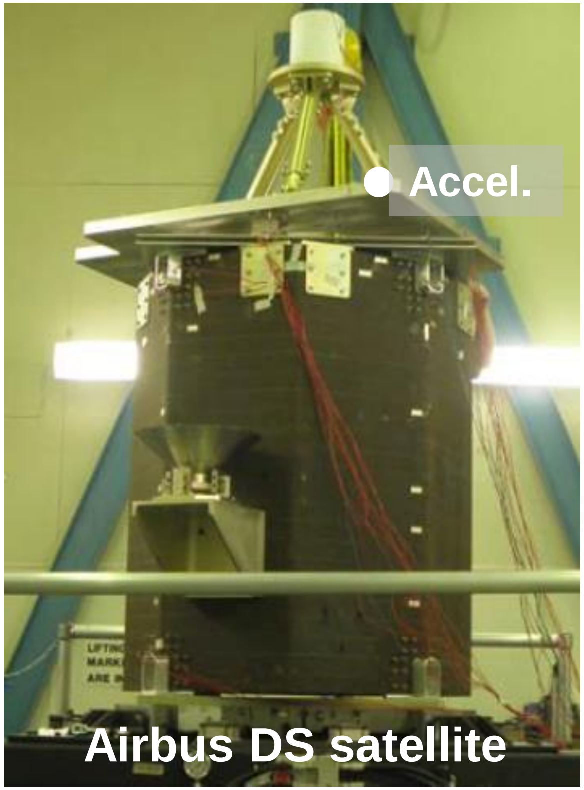
Why study nonlinearity ? No superposition principle

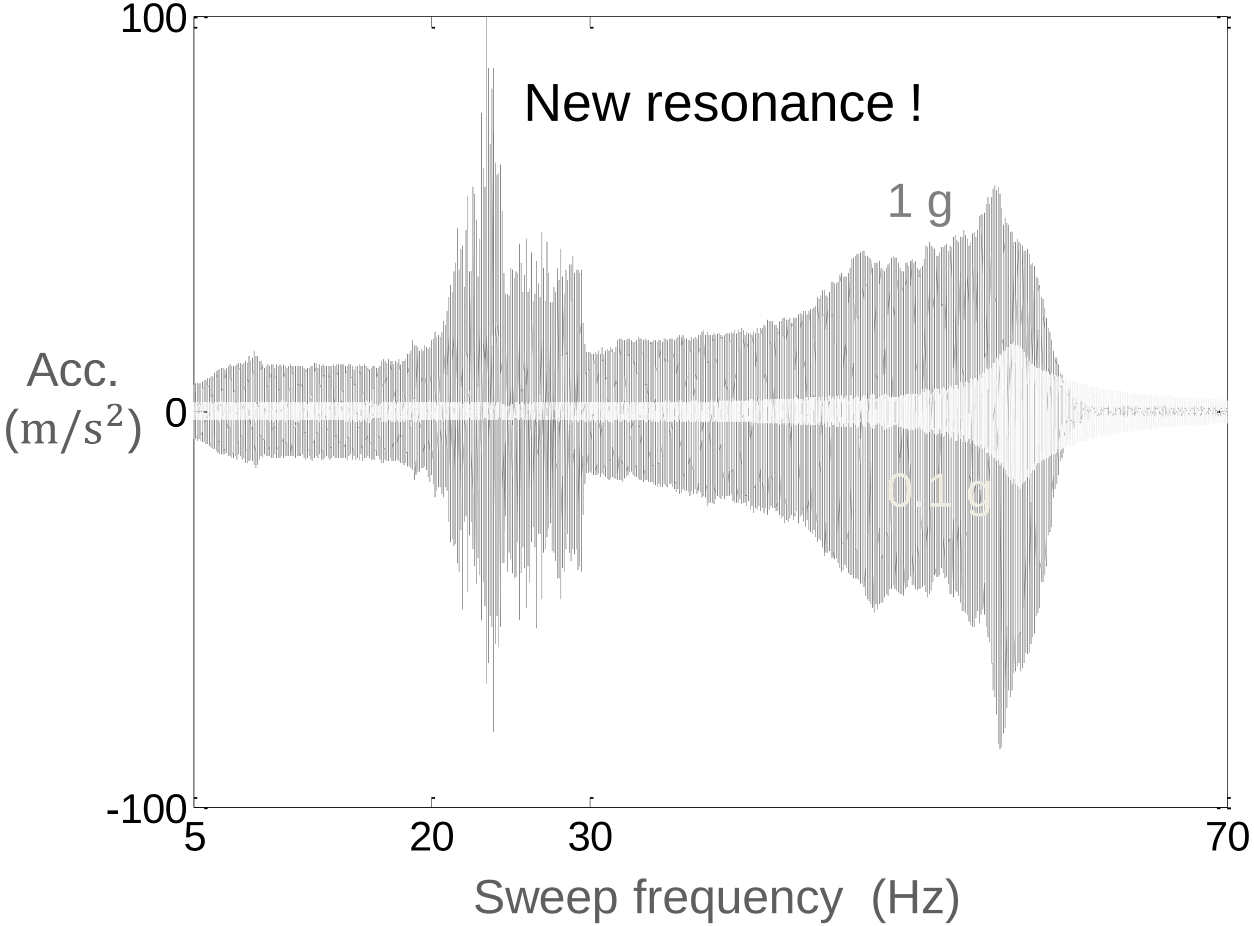


Why study nonlinearity ? Very different dynamics



Very different dynamics: a real-life example

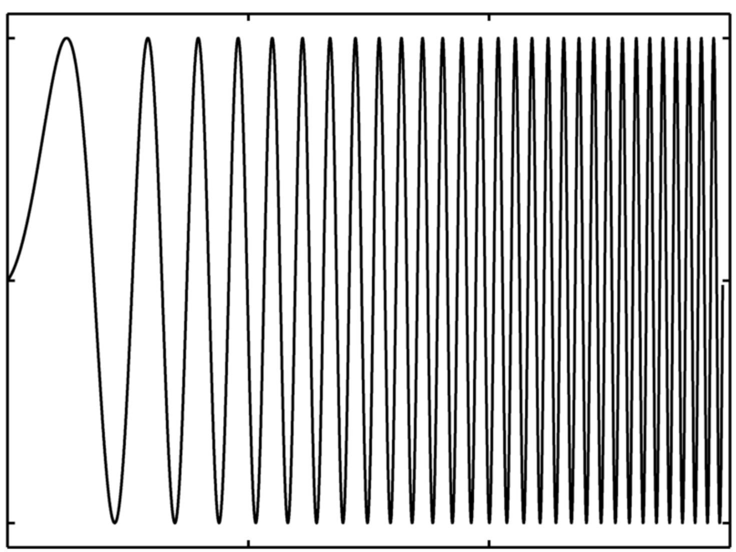




What is a sine sweep ?

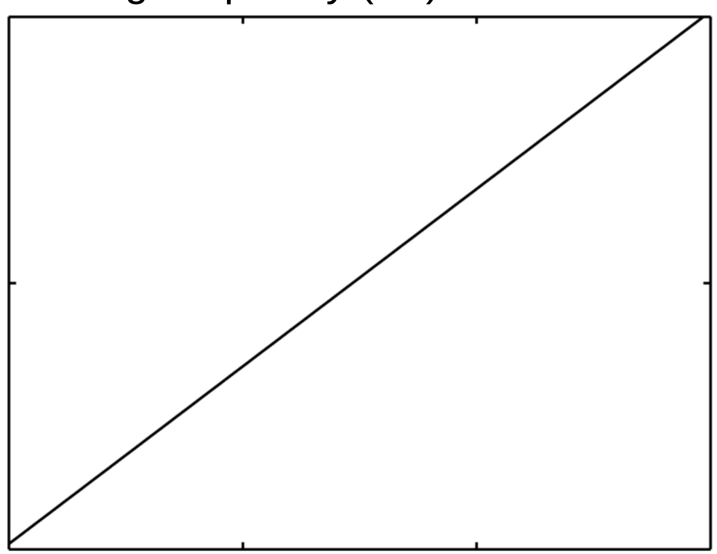
$$u(t) = A\,sin\left(2\pi f_0(t - t_0) + 2\pi \frac{r}{2}(t - t_0)^2 + \varphi_0\right)$$

Force (N)



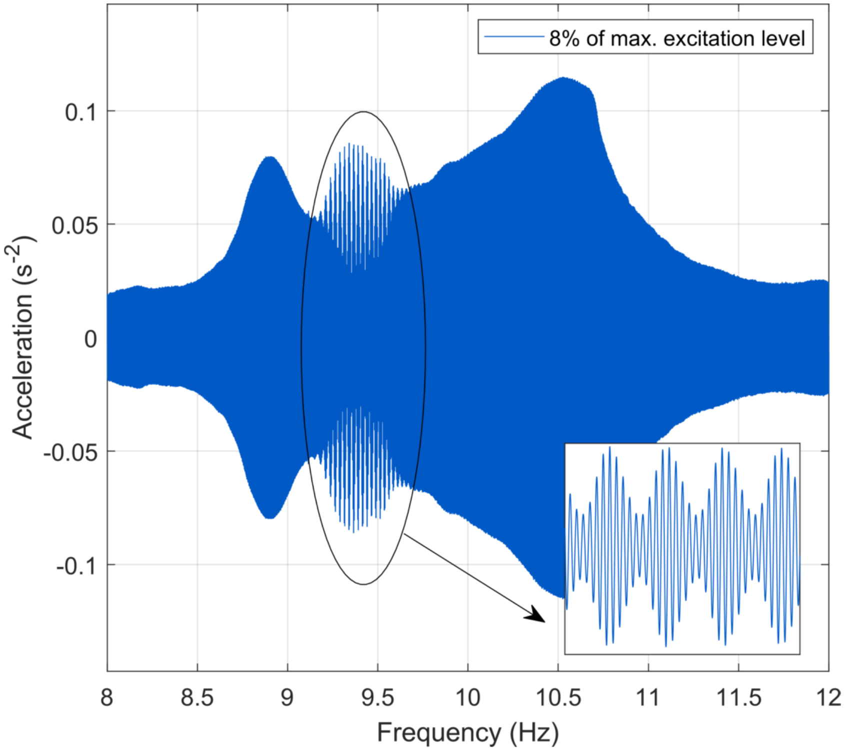
Time (s)

Forcing frequency (Hz)



Time (s)

Very different dynamics: another real-life example



Evidence of quasiperiodic motion between two resonances

Course objectives

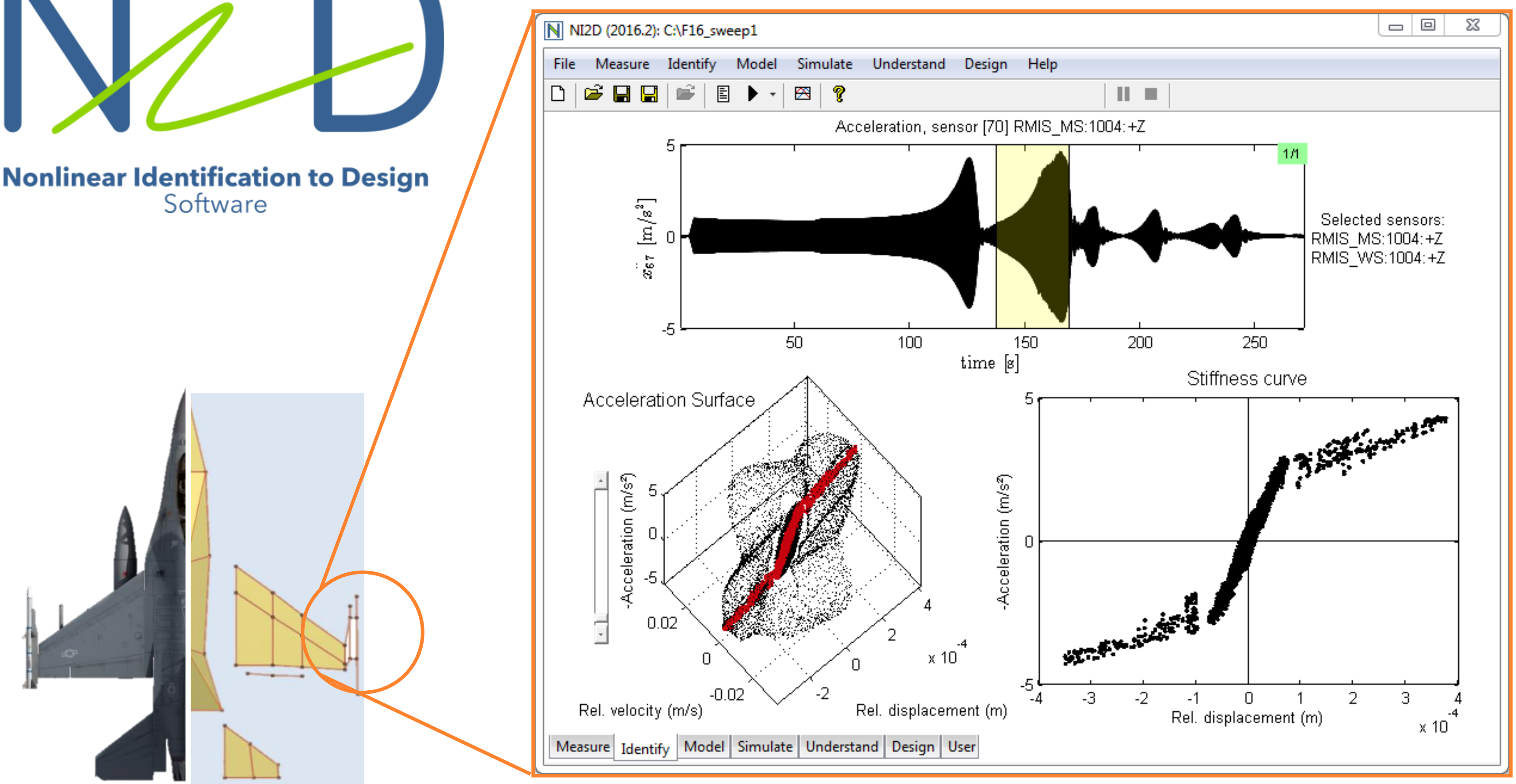
At the end of this course, you will

- Understand the impact of nonlinearity on system dynamics.
- Master the concepts of mode shape, resonance frequency and frequency response function of nonlinear systems.
- Be familiar with new nonlinear concepts incuding stability and bifurcations.
- Recognize nonlinearity in real-world (aerospace) structures.
- Know how to use the NI2D software.

You will be exposed to new theoretical concepts, advanced computational methods and practical experimental techniques

The Nonlinear Identification to Design software

Nonlinear Identification to Design
Software



Course outline

1. Brief review of linear theory
2. Impact of nonlinearity, nonlinear FRFs and 4 new concepts
3. Mathematical modeling and numerical computation
4. Nonlinear modes
5. Introduction to system identification and nonlinearity detection
6. Nonlinearity characterization
7. Nonlinear parameter estimation
8. Advanced concepts: bifurcations, modal interactions, isolas.

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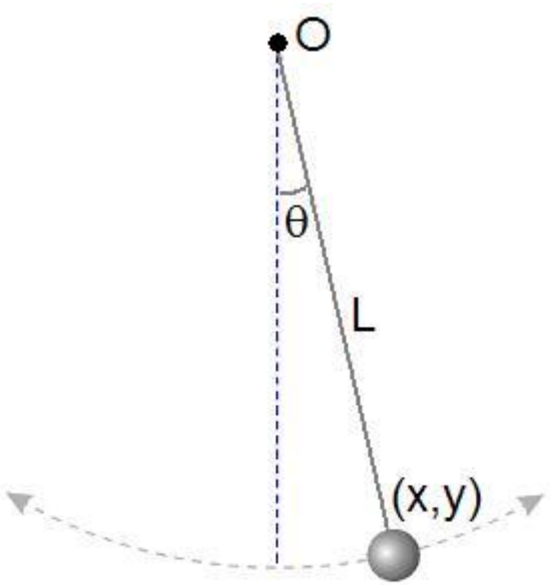
Review of linear theory

How to write the governing equations ?

$$-\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right)+\frac{\partial T}{\partial q_s}-\frac{\partial V}{\partial q_s}-\frac{\partial D}{\partial q_s}+Q_s(t)=0, s=1,\dots n.$$

Lagrange equations for n generalized coordinates



$$T=\frac{1}{2}mv^2=\frac{1}{2}mL^2\omega^2=\frac{1}{2}mL^2\dot{\theta}^2$$

$$V=mgL(1-cos\theta)$$

$$\ddot{\theta}+\frac{g}{L}\sin\theta=0$$

Let's calculate the period of the motion

$$T + V = E$$

$$\frac{1}{2}mL^2\dot{\theta}^2 + mgL(1 - cos\theta) = mgL(1 - cos\theta_0)$$

$$\dot{\theta} = \frac{d\theta}{dt} = \pm\sqrt{\frac{2g}{L}(cos\theta - cos\theta_0)}$$

$$dt = \frac{d\theta}{\dot{\theta}} = \sqrt{\frac{L}{2g(cos\theta - cos\theta_0)}}d\theta$$

$$Period = 4\sqrt{\frac{L}{2g}}\int_0^{\theta_0}\frac{d\theta}{\sqrt{cos\theta - cos\theta_0}} = 2\pi\sqrt{\frac{L}{g}}\left[1 + \frac{{\theta_0}^2}{16} + \cdots\right]$$

Linearization around an equilibrium

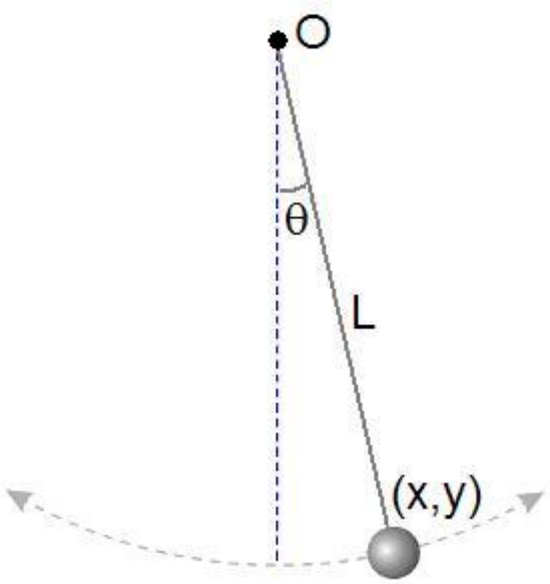
$$V(q) = \cancel{V(0)} + \sum_{s=1}^{n} \cancel{\left(\frac{\partial V}{\partial q_s}\right)_{q=0}} + \frac{1}{2}\sum_{s=1}^{n}\sum_{r=1}^{n}\left(\frac{\partial^2 V}{\partial q_s \partial q_r}\right)_{q=0} q_r q_s + \cdots$$

$$V(q) = \frac{1}{2}\sum_{s=1}^{n}\sum_{r=1}^{n} k_{rs} q_r q_s = \frac{1}{2} q^T K q$$

$$T(\dot{q}) = \frac{1}{2}\sum_{s=1}^{n}\sum_{r=1}^{n} m_{rs} \dot{q}_r \dot{q}_s = \frac{1}{2} \dot{q}^T M \dot{q}$$

$$M\ddot{q} + Kq = 0$$

Linearization of the pendulum



$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$

$$\ddot{\theta} + \frac{g}{L}\left(\theta - \frac{\theta^3}{6} + \cdots\right) = 0$$

$\theta \ll 1$

$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

$$Period = 2\pi\sqrt{\frac{L}{g}}$$

3 main assumptions in linear structural dynamics

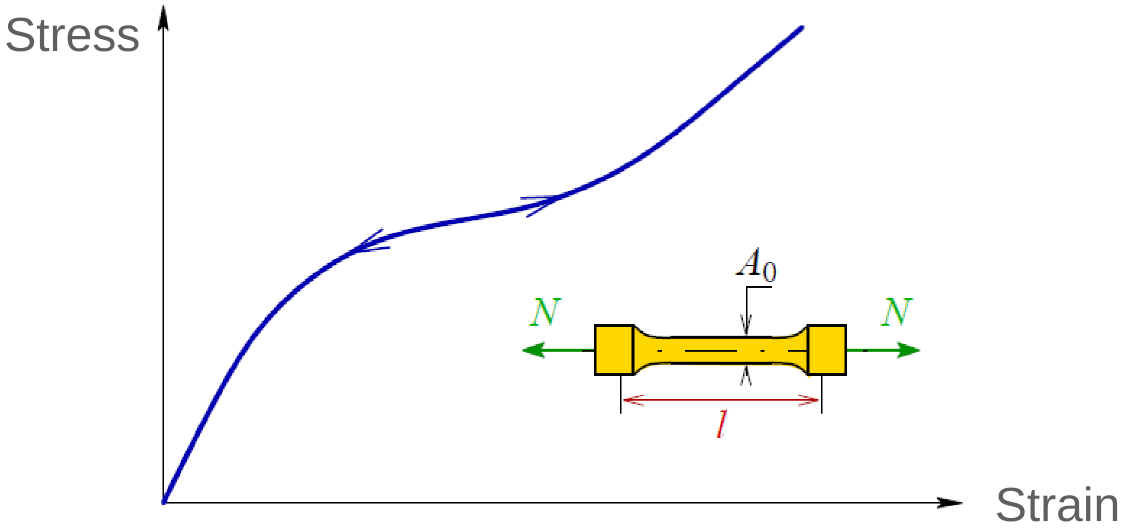
$$M\ddot{q} + C\dot{q} + Kq = 0$$

Linear elasticity → nonlinear materials

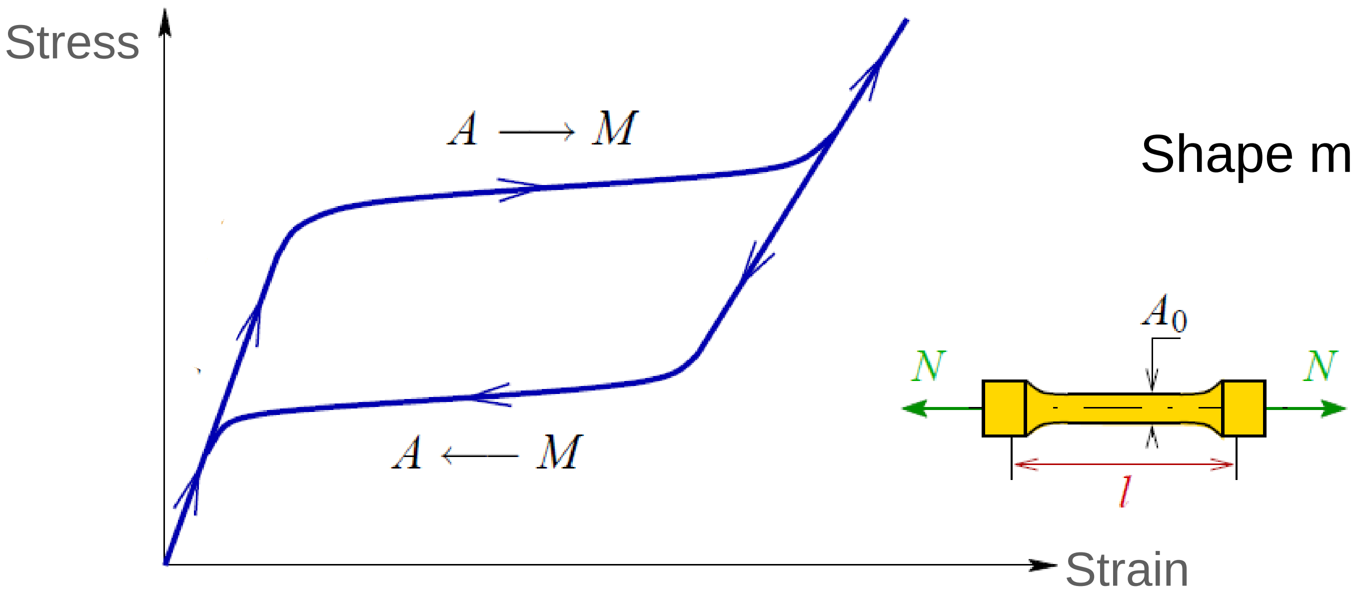
Small displ. and rotations → geometrical nonlinearity
→ nonlinear boundary conditions

Viscous damping → nonlinear damping mechanisms

Assumption 1: nonlinear materials

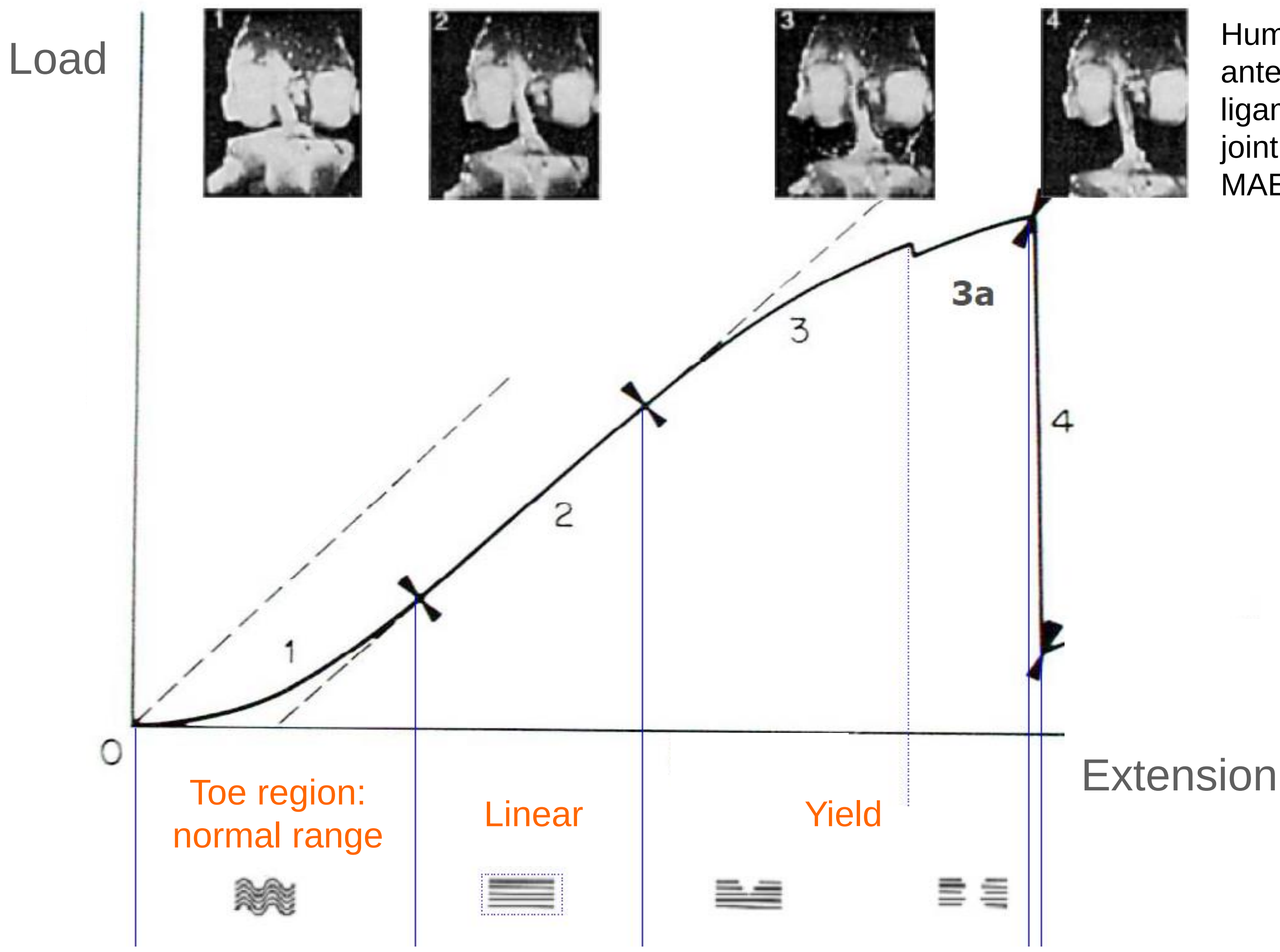


Hyperelastic material
(e.g., rubber)



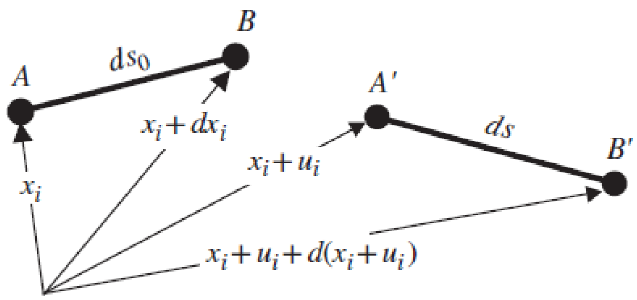
Shape memory alloy

Assumption 1: ligament in your knee joint



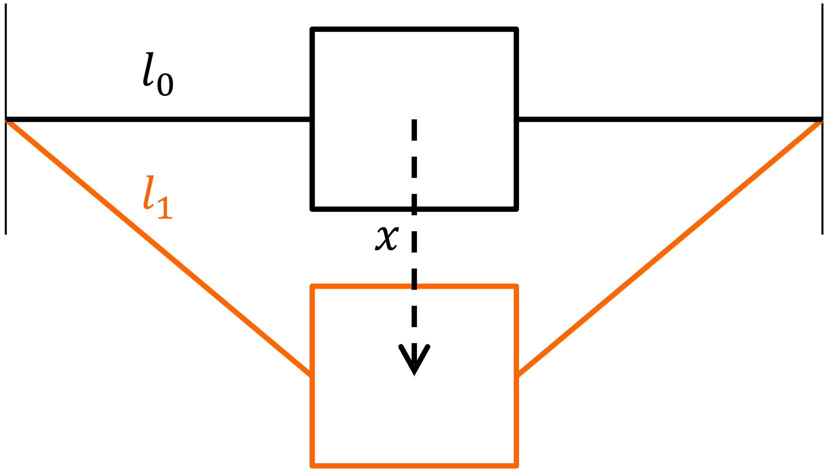
Human cadaveric anterior cruciate ligament in knee joint (Dr. Ziv, MAE, Buffalo)

Assumption 2: geometrical nonlinearities



$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_m}{\partial x_i}\frac{\partial u_m}{\partial x_j}\right)$$

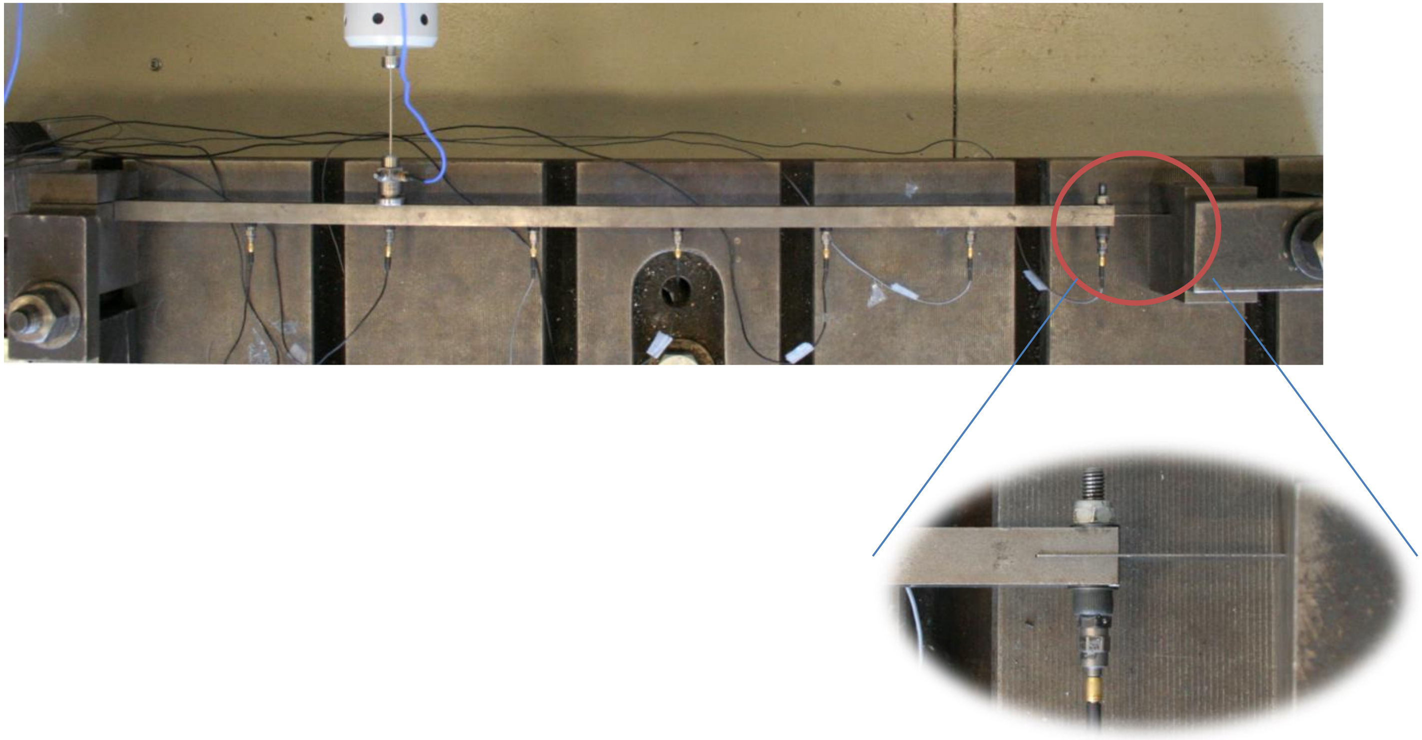
Green's strain tensor



$$F = 2k(l_1 - l_0)\frac{x}{l_1} = 2kx\left(1 - \frac{l_0}{\sqrt{x^2 + {l_0}^2}}\right)$$

$$\frac{l_0}{\sqrt{x^2 + {l_0}^2}} = 1 - \frac{x^2}{2{l_0}^2} + \frac{3x^4}{8{l_0}^4} + O(x^6)$$

Assumption 2: a nonlinear beam in our lab



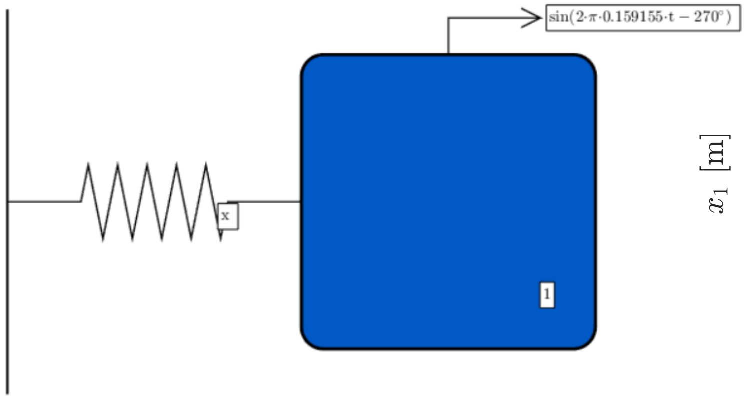
Assumption 3: nonlinear damping

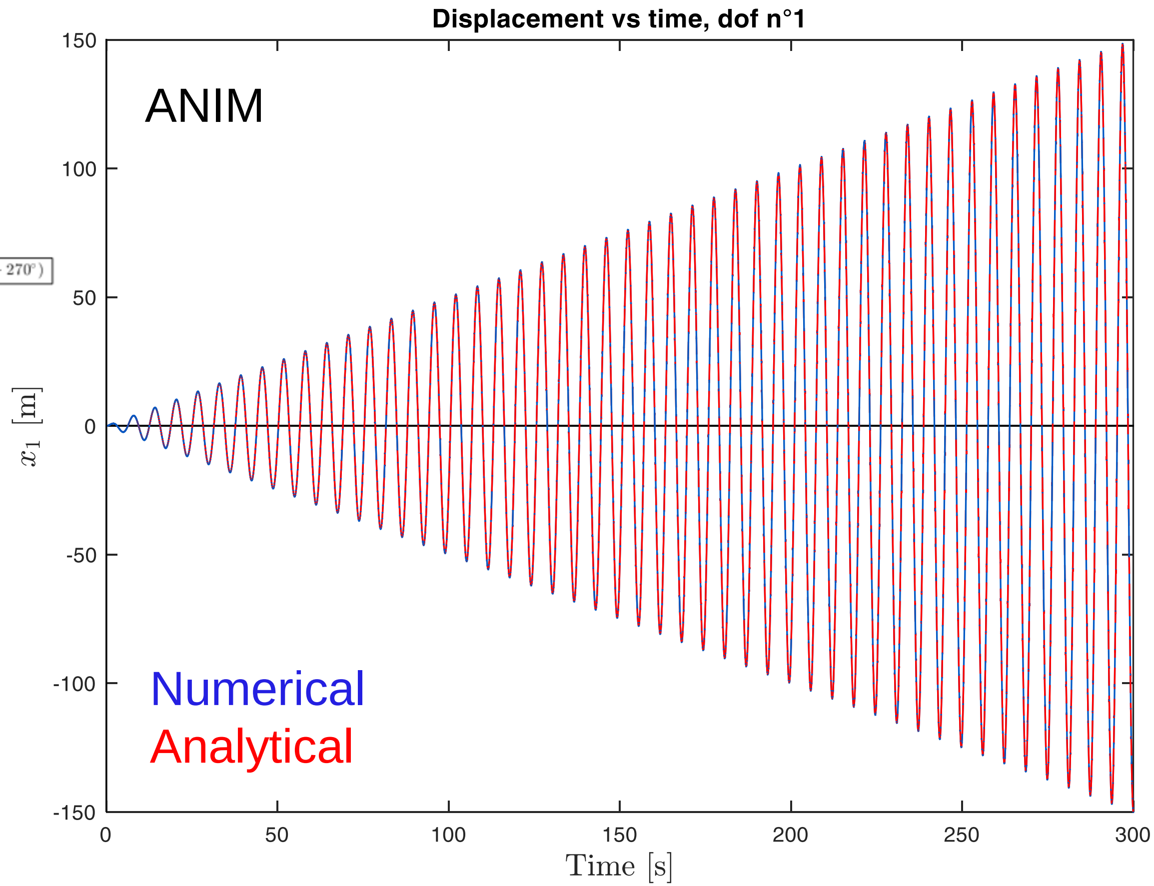
Viscous damping but also…

Coulomb friction

Aerodynamic damping

Resonance, a key concept in vibration theory

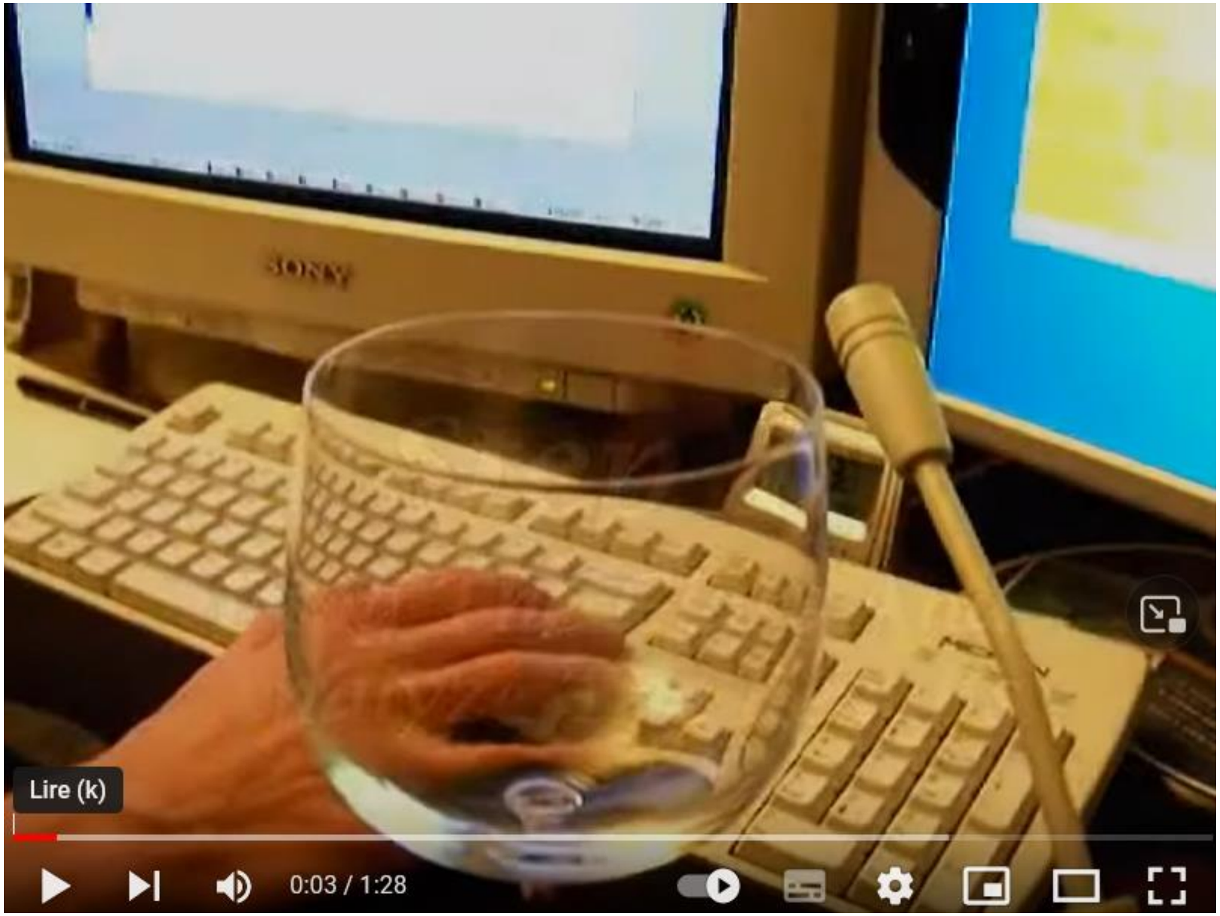




Resonance, a key concept in vibration theory

https://www.youtube.com/watch?v=10lWpHyN0Ok

Resonance, a key concept in vibration theory



https://www.youtube.com/watch?v=JiM6AtNLXX4

The concept of a mode shape

$$M\ddot{q} + Kq = 0 \qquad + \qquad q = x\,\varphi(t)$$

Synchronous vibration of the structure

$$det(K - \omega^2 M) = 0$$

n natural frequencies

$$\left(K - \omega_{(r)}{}^2 M\right) x_{(r)} = 0$$

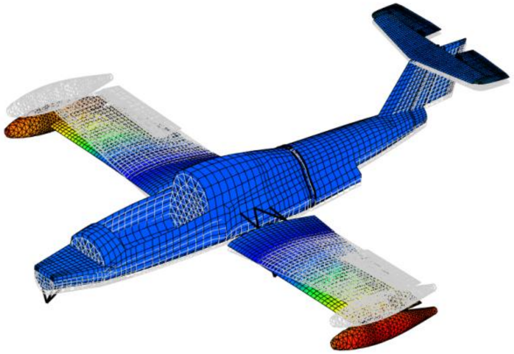
n normal modes

$$\varphi_r(t) = \alpha_r cos(\omega_r t) + \beta_r sin(\omega_r t)$$

normal coordinates

Normal nodes: important properties

Clear physical meaning:



- **Structural deformation at resonance**
- Synchronous vibration of the structure

Important mathematical properties:

- Orthogonality
- Decoupling of the equations of motion (modal superposition)
- Invariance

The concept of a FRF

$$M\ddot{q} + Kq = s\,cos(\omega t) \quad + \quad q = x\,cos(\omega t)$$

FRF

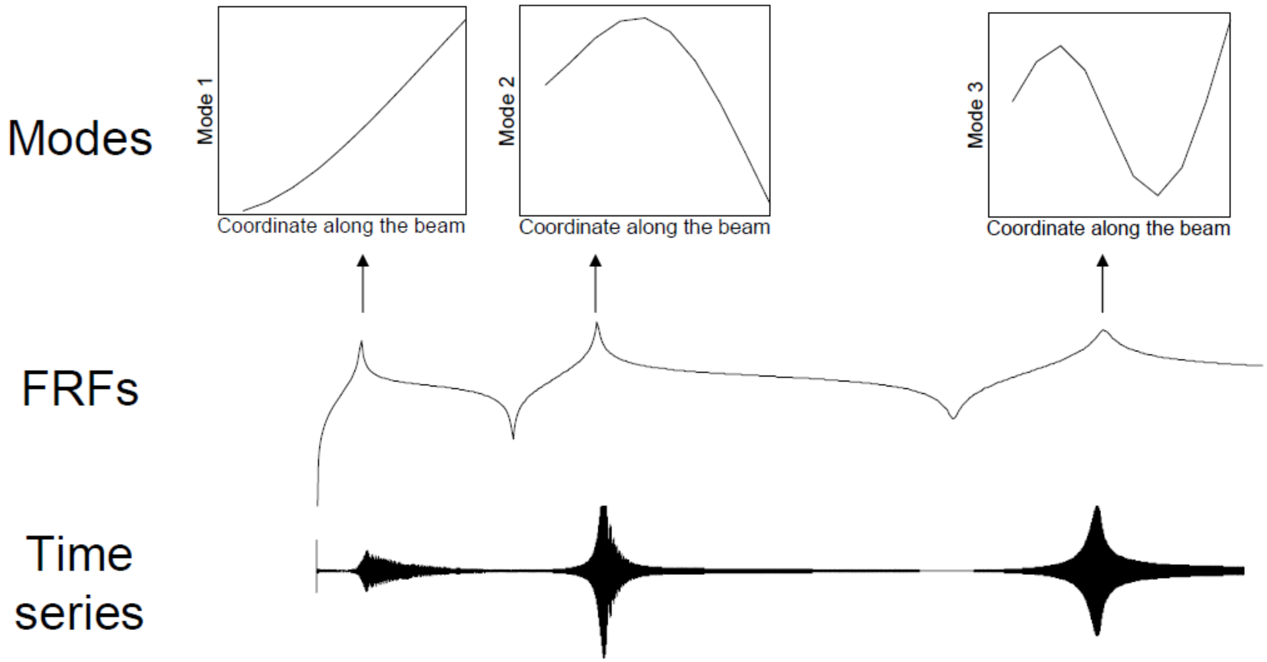
$$x = (K - \omega^2 M)^{-1} s = \boxed{H(\omega)}\, s$$

$$H(\omega) = \sum \frac{x_{(s)} {x_{(s)}}^T}{({\omega_s}^2 - \omega^2)\mu_s}$$

Clear link between the FRF
and the modal parameter

FRF: important properties

- The FRF is a constant system properties for a linear system
- FRF can be easily estimated from measured data
- Very convenient way of locating resonance frequencies



Time response: mode displacement method

$$M\ddot{q} + Kq = p(t) = g\,\varphi(t)$$

$$q(t) = \sum_{s=1}^{n} \frac{x_{(s)} {x_{(s)}}^T g}{\mu_s \omega_s} \int_0^t sin\omega_s(t-\tau)\,\varphi(t) d\tau$$ Exact

$$q(t) = \sum_{s=1}^{k<n} \frac{x_{(s)} {x_{(s)}}^T g}{\mu_s \omega_s} \int_0^t sin\omega_s(t-\tau)\,\varphi(t) d\tau$$ Approximate

Time response: numerical integration

Newmark integration scheme for linear systems

$\mathbf{M}, \mathbf{C}, \mathbf{K} \quad \mathbf{q}_0, \dot{\mathbf{q}}_0$

Compute $\ddot{\mathbf{q}}_0$

Time incrementation

$$t_{n+1} = t_n + h$$

Prediction

$$\dot{\mathbf{q}}^*_{n+1} = \dot{\mathbf{q}}_n + (1-\gamma)\, h\, \ddot{\mathbf{q}}_n$$

$$\mathbf{q}^*_{n+1} = \mathbf{q}_n + h\, \dot{\mathbf{q}}_n + (0.5-\beta)\, h^2\, \ddot{\mathbf{q}}_n$$

Computation of accelerations

$$\mathbf{S} = \mathbf{M} + h\,\gamma\,\mathbf{C} + h^2\,\beta\,\mathbf{K}$$

$$\mathbf{S}\,\ddot{\mathbf{q}}_{n+1} = \mathbf{p}_{n+1} - \mathbf{C}\,\dot{\mathbf{q}}^*_{n+1} - \mathbf{K}\,\mathbf{q}^*_{n+1}$$

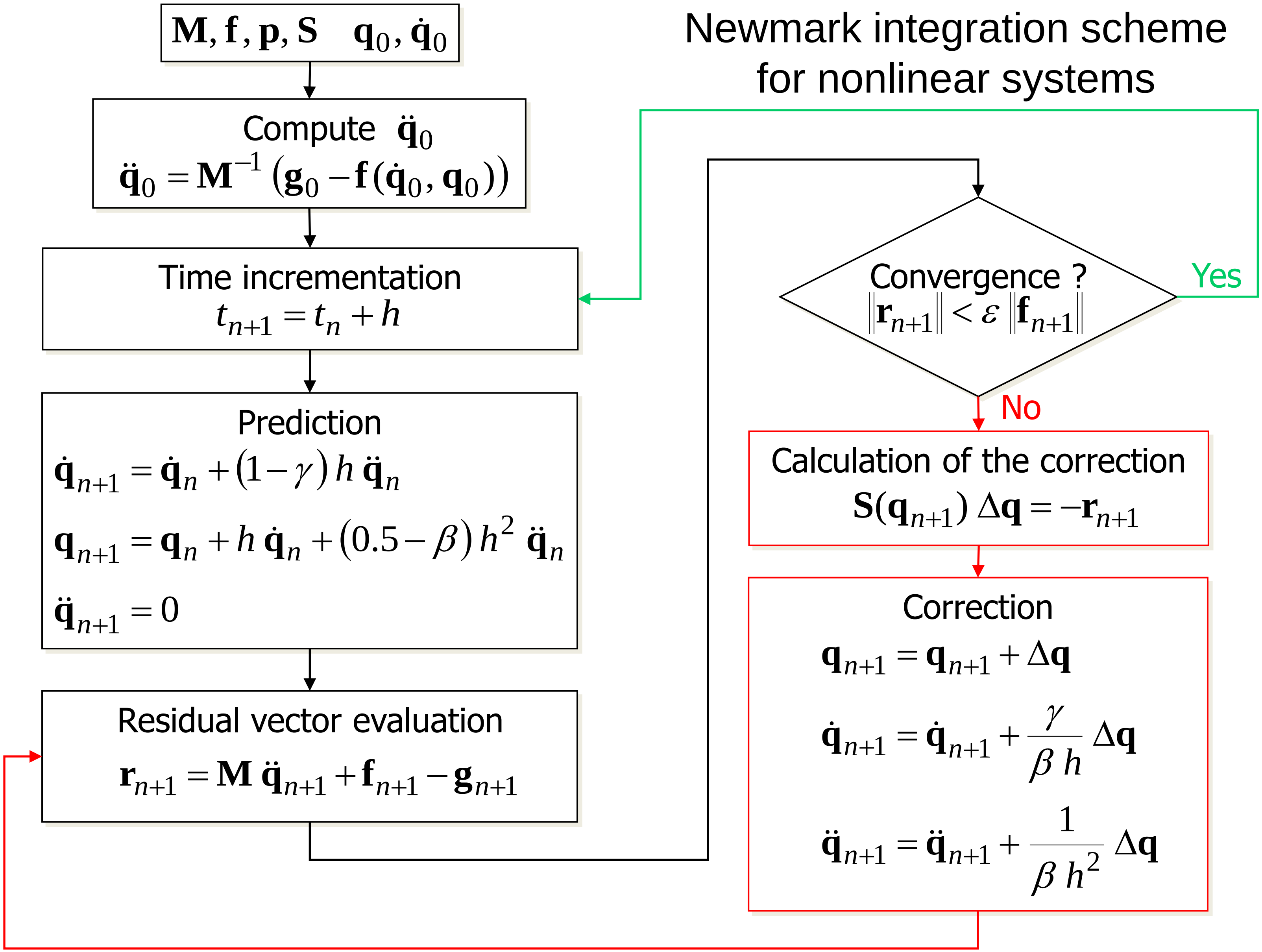
Correction

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}^*_{n+1} + h\,\gamma\,\ddot{\mathbf{q}}_{n+1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}^*_{n+1} + h^2\,\beta\,\ddot{\mathbf{q}}_{n+1}$$

Time response: numerical integration

Newmark integration scheme for nonlinear systems



Very important: sampling frequency

Algorithm	γ	β	**Stability** limit ωh	**Accuracy**: Amplitude error $\rho - 1$	**Accuracy**: Periodicity error $\frac{\Delta T}{T}$
Purely explicit	0	0	0	$\frac{\omega^2 h^2}{4}$	$-$
Central difference	$\frac{1}{2}$	0	2	0	$-\frac{\omega^2 h^2}{24}$
Fox & Goodwin	$\frac{1}{2}$	$\frac{1}{12}$	2.45	0	$O\left(h^3\right)$
Linear acceleration	$\frac{1}{2}$	$\frac{1}{6}$	3.46	0	$\frac{\omega^2 h^2}{24}$
Average constant acceleration	$\frac{1}{2}$	$\frac{1}{4}$	∞	0	$\frac{\omega^2 h^2}{12}$
Average constant acceleration (modified)	$\frac{1}{2}+\alpha$	$\frac{(1+\alpha)^2}{4}$	∞	$-\alpha \frac{\omega^2 h^2}{2}$	$\frac{\omega^2 h^2}{12}$

In Summary: important linear concepts/methods

Mode shapes, resonance frequencies, damping ratios

Frequency response functions (FRFs)

Modal superposition/numerical integration

OPEN QUESTION:

Will they remain valid/useful for nonlinear systems ?