

# Nonlinear Vibrations of Aerospace Structures

Tutorial 01

Introduction to NI2D

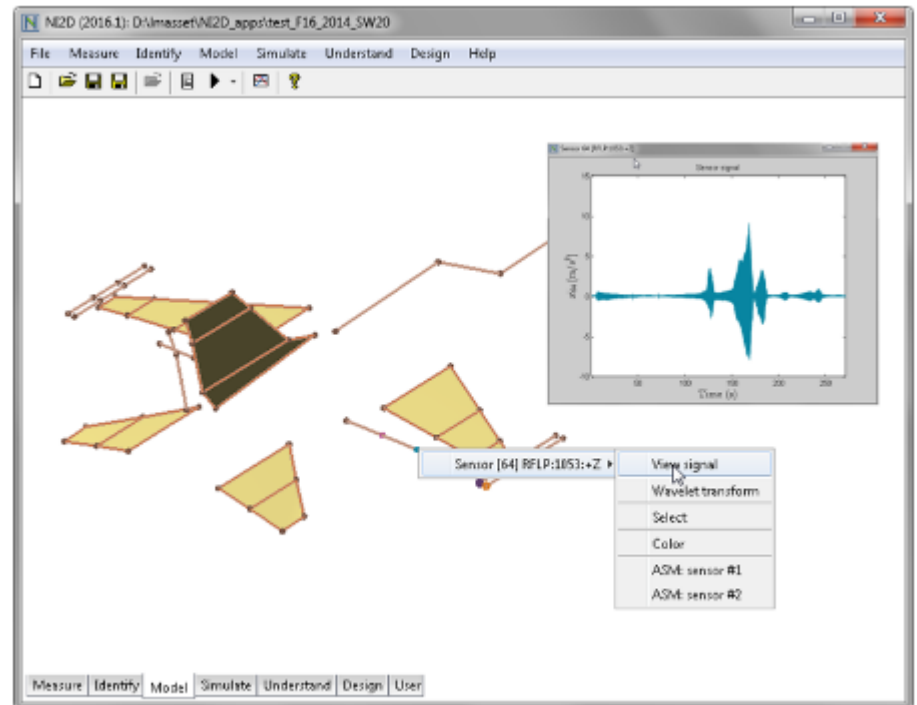


# Tutorials and Project on the NI2D Software

Check [www.nolisys.com](http://www.nolisys.com)

You'll get free access to the NI2D software.

Matlab environment; stand alone .exe.



# NI2D Philosophy

## Nonlinear Identification to Design

- *A prototype of the structure is available:*
  - Test it, identify the nonlinearities and upgrade the linear FEM.
- *A priori knowledge about the nonlinearities is available:*
  - Load the linear FEM into NI2D and implement the nonlinearities using NI2D elements library.

# How To Read the Slides

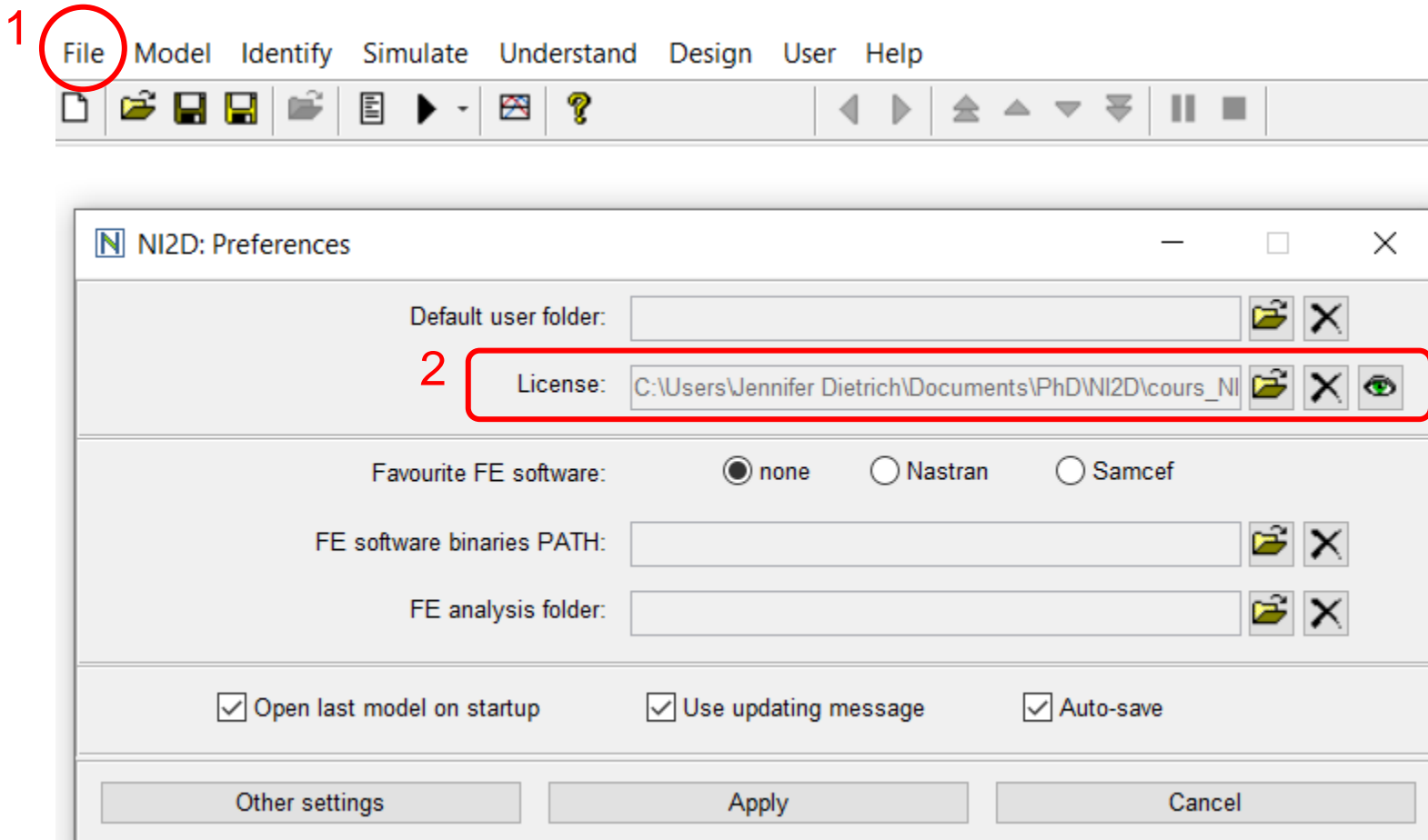
Click Here

Change  
parameters  
here

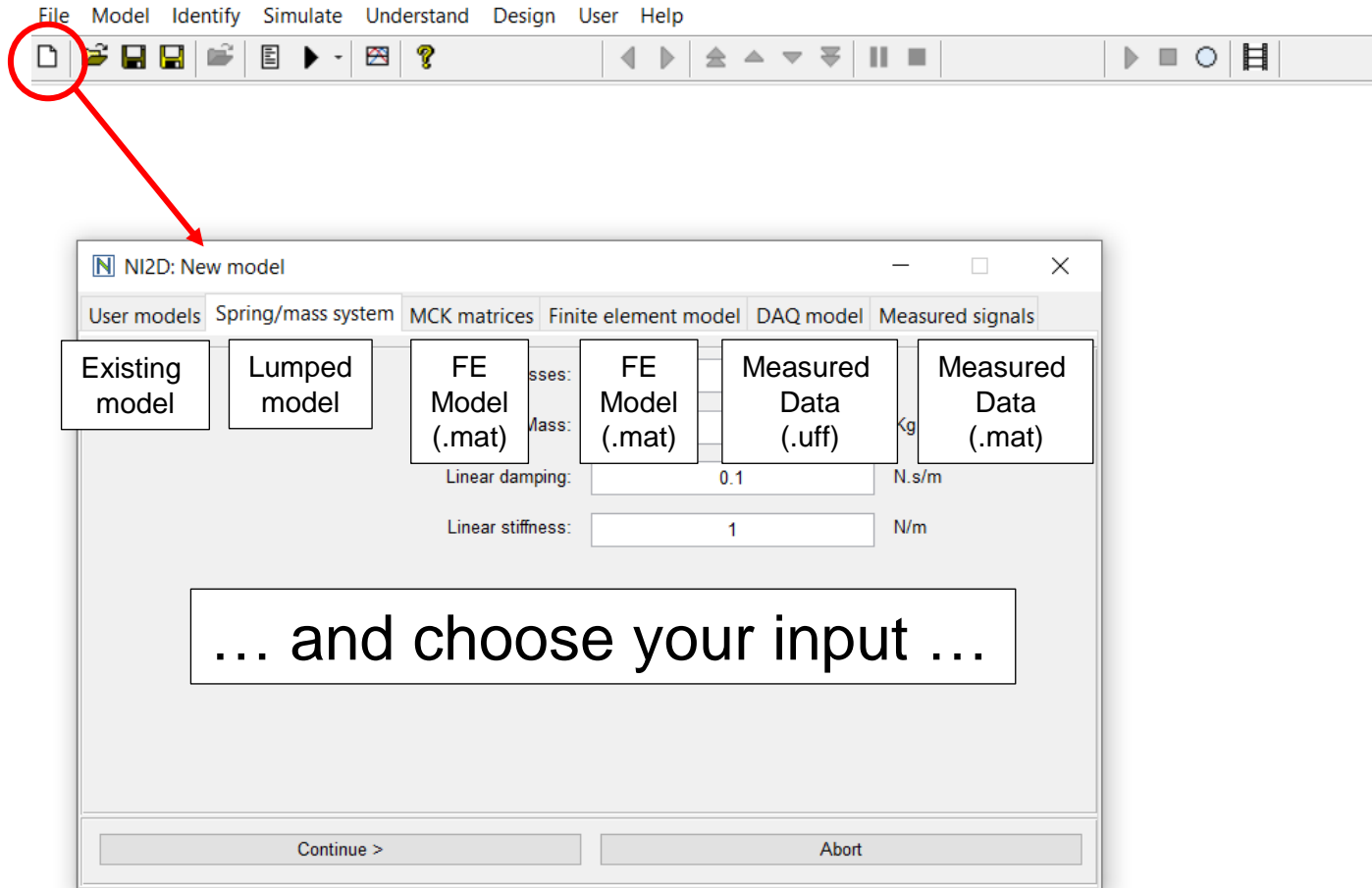
**Comments**

# Get Started with NI2D

Launch NI2D and activate your license under *Preferences* ...



# Create a New Model



# Open an existing model



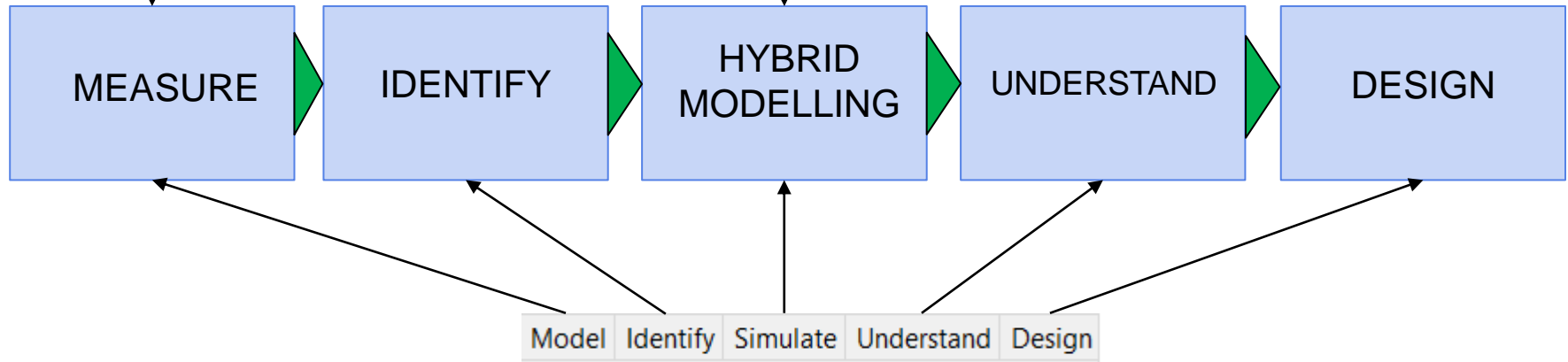
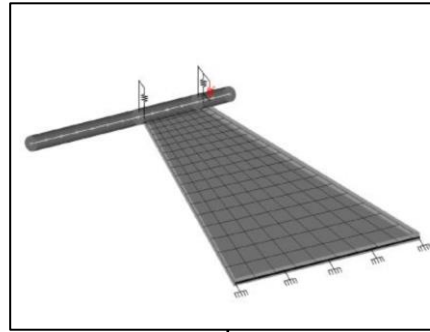
Choose here

# The Different Solvers

VIBRATION MEASUREMENTS

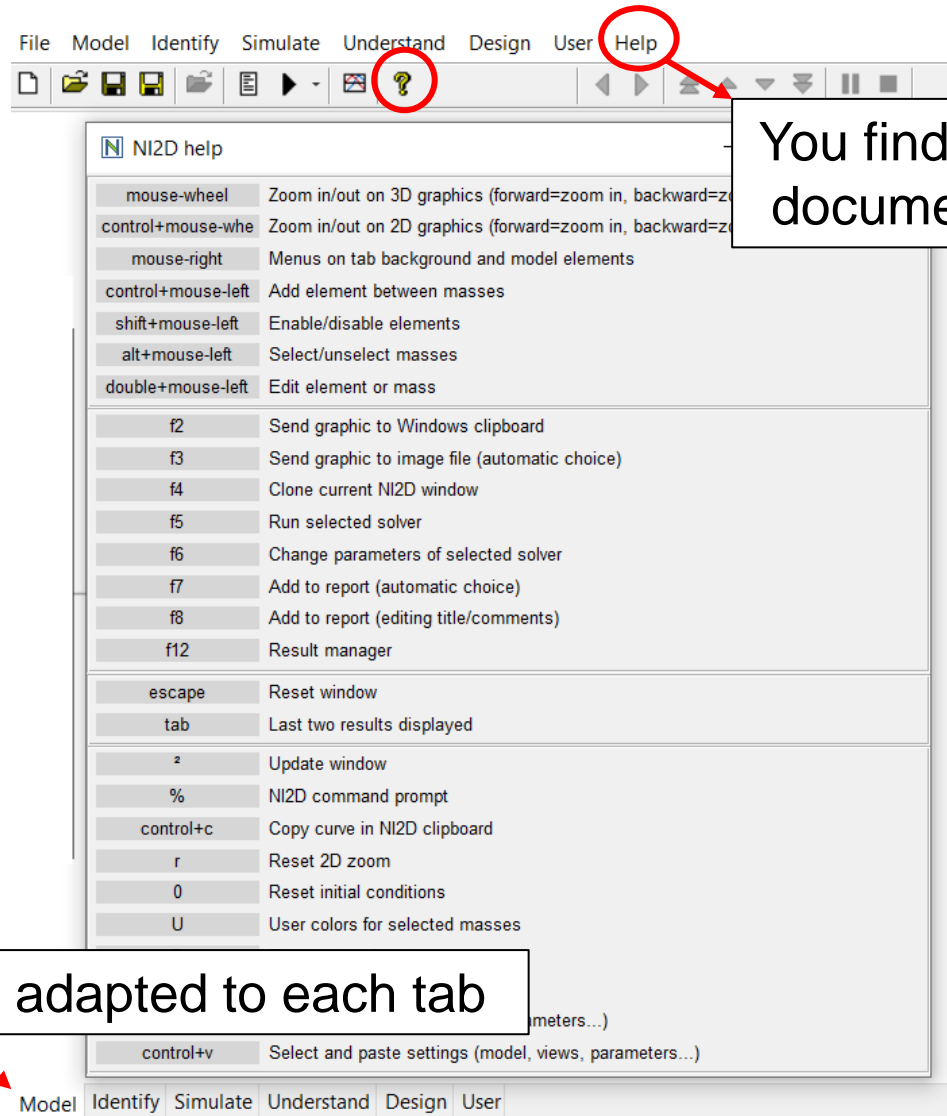


LINEAR FE-MODEL





# Need Help?



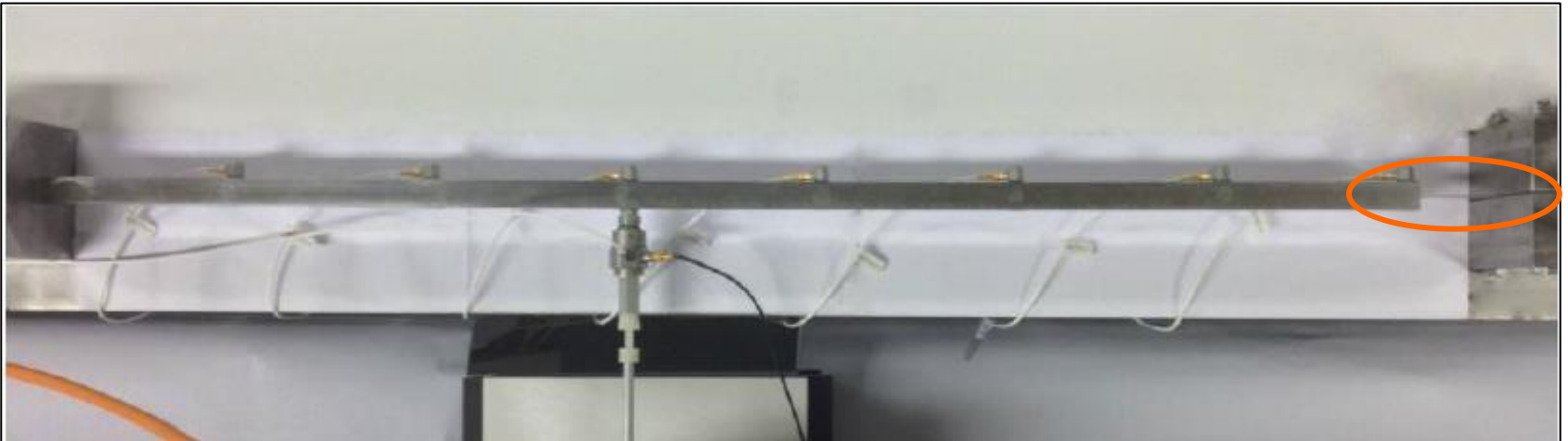
You find the software documentation here

The help menu is adapted to each tab

# Launch Your First Numerical Simulation

# Consider a Thin Short Beam ...

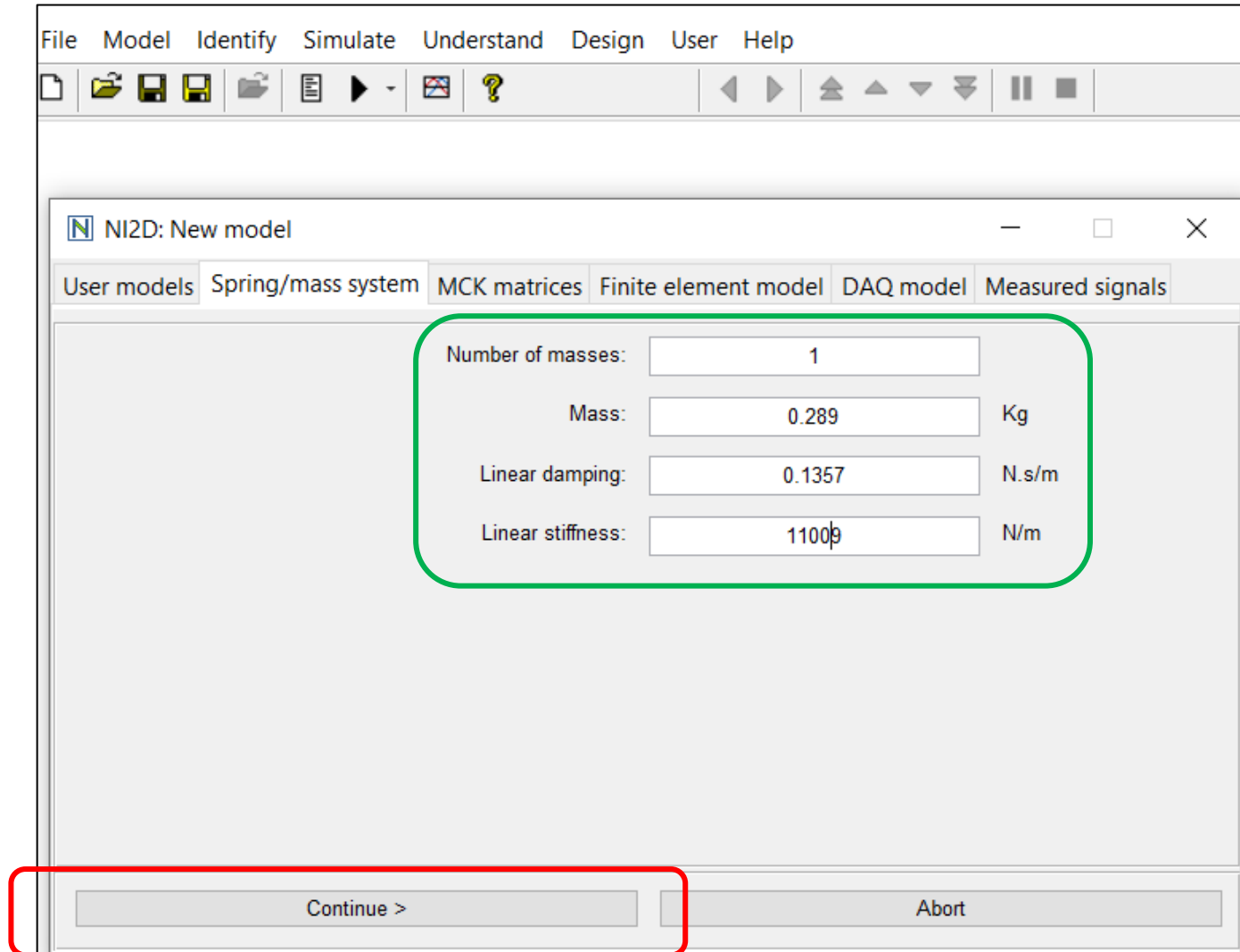
... connected to a cantilever beam (ECL benchmark)



Linear model identified at low level (31 Hz, 0.12%):

$$0.289\ddot{x} + 0.1357\dot{x} + 11009x = F \sin \omega t$$

# Create a New Model: 1 DOF Linear Oscillator



# You Can Change the Coefficients Anytime ...

The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The main workspace shows a circuit diagram with a blue rectangular component. Three numerical values are circled in red:  $0.1357 \cdot \dot{x}$ ,  $11009 \cdot x$ , and  $0.289$ . A red text label "Double click" is positioned above the first two circled values. Below the circuit, a box contains the text "mode 1: 31.0631 Hz / 0.12 %", with a red text label "Linear modal properties" to its right. The bottom status bar shows the tabs "Model", "Identify", "Simulate", "Understand", "Design", and "User".

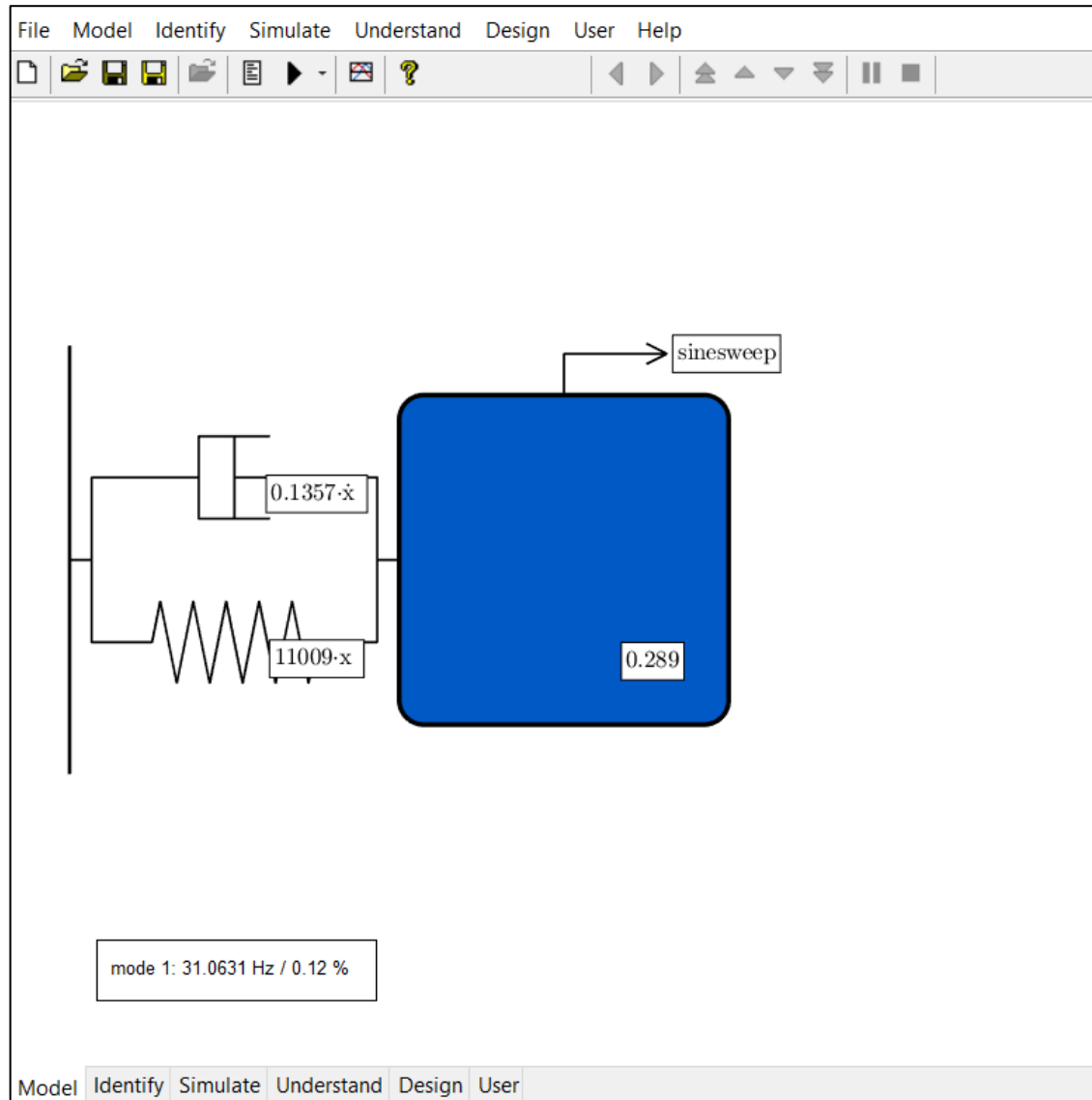
# Add an External Force

The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The main workspace shows a mechanical model consisting of a vertical bar, a spring, and a mass. A blue block labeled "Double click" is positioned over the mass, with an arrow pointing to a text box containing the mathematical expression  $\sin(2 \cdot \pi \cdot t)$ . A dialog box titled "External force on dof n°1" is open, showing tabs for "Sine", "Sine Sweep", "Random", "User", and "Measure". The "Sine Sweep" tab is selected and circled in red. The dialog box contains the following settings:

- Amplitude: 0.06 N
- Starting frequency: 30 Hz
- Ending frequency: 40 Hz
- Sweep rate: 0.5 Hz/min
- Sweep style:  linear  log

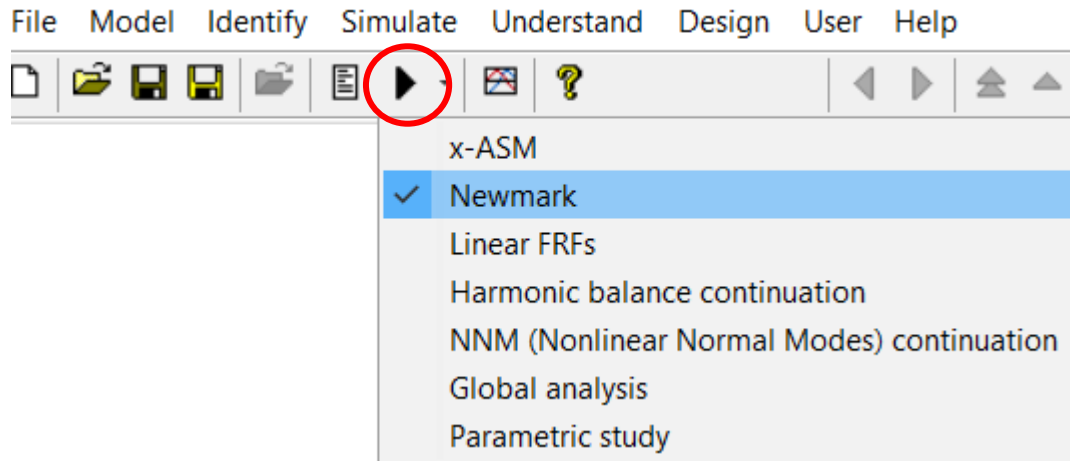
At the bottom of the dialog box, the "Apply" button is circled in red. Other buttons include "Newmark (F5)" and "Cancel". The status bar at the bottom of the window shows "mode 1: 31.0631 Hz /".

# The Final Linear Model

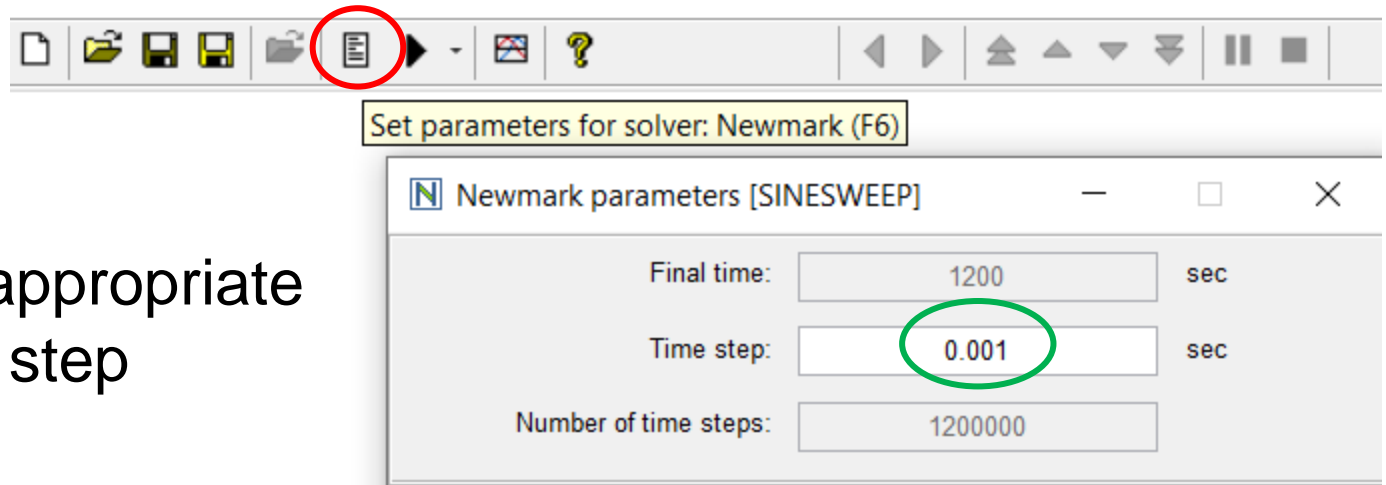


# Calculate the Time Response with Newmark

1.



2.



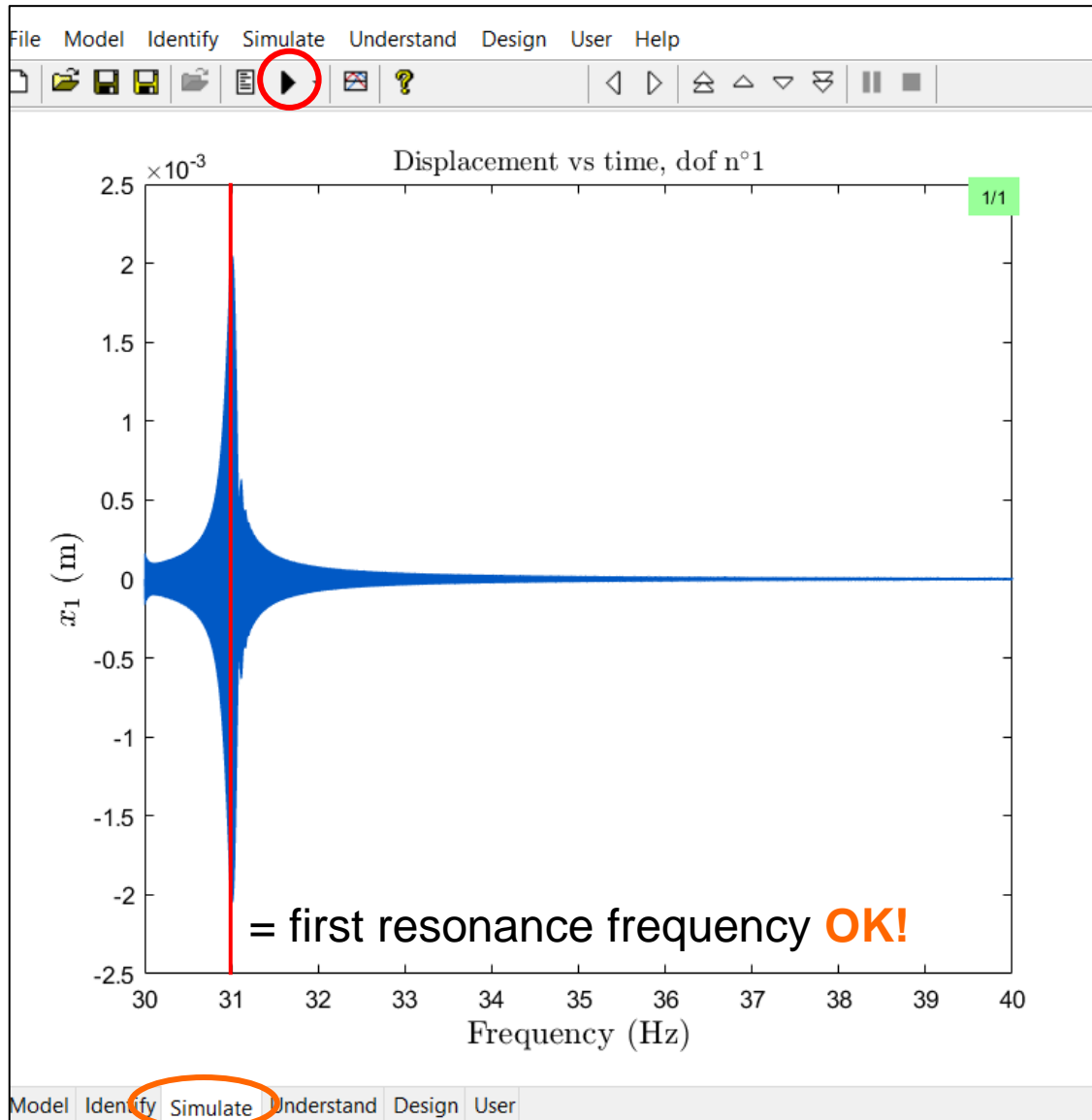
Select an appropriate time step



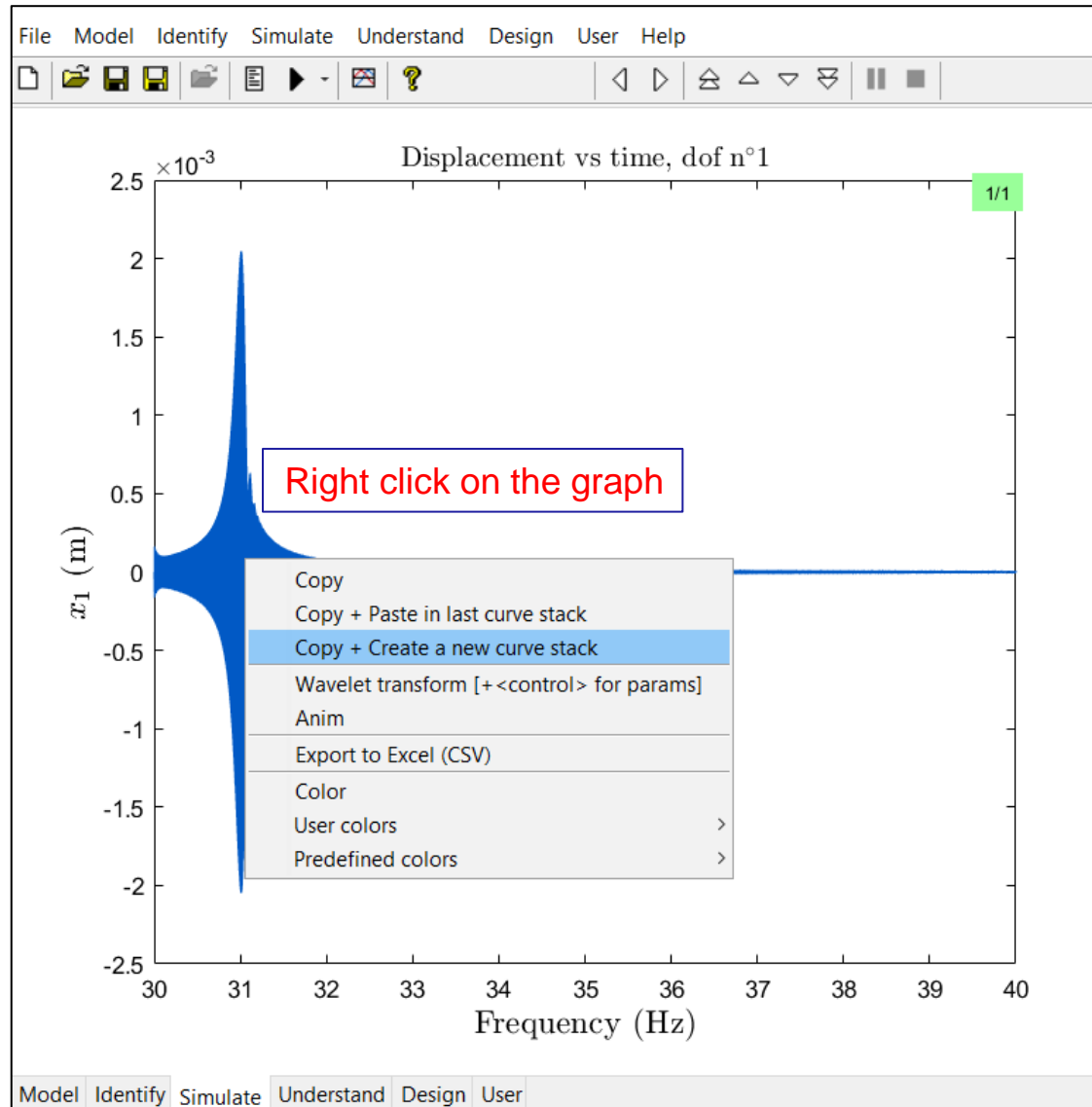
# How to Choose the Right Sampling Frequency?

Algorithm	$\gamma$	$\beta$	$\omega h$	Accuracy	
				Stability limit	Amplitude error
				$\rho - 1$	$\frac{\Delta T}{T}$
Purely explicit	0	0	0	$\frac{\omega^2 h^2}{4}$	—
Central difference	$\frac{1}{2}$	0	2	0	$-\frac{\omega^2 h^2}{24}$
Fox & Goodwin	$\frac{1}{2}$	$\frac{1}{12}$	2.45	0	$O(h^3)$
Linear acceleration	$\frac{1}{2}$	$\frac{1}{6}$	3.46	0	$\frac{\omega^2 h^2}{24}$
Average constant acceleration	$\frac{1}{2}$	$\frac{1}{4}$	$\infty$	0	$\frac{\omega^2 h^2}{12}$
Average constant acceleration (modified)	$\frac{1}{2} + \alpha$	$\frac{(1 + \alpha)^2}{4}$	$\infty$	$\alpha - \frac{\omega^2 h^2}{2}$	$\frac{\omega^2 h^2}{12}$

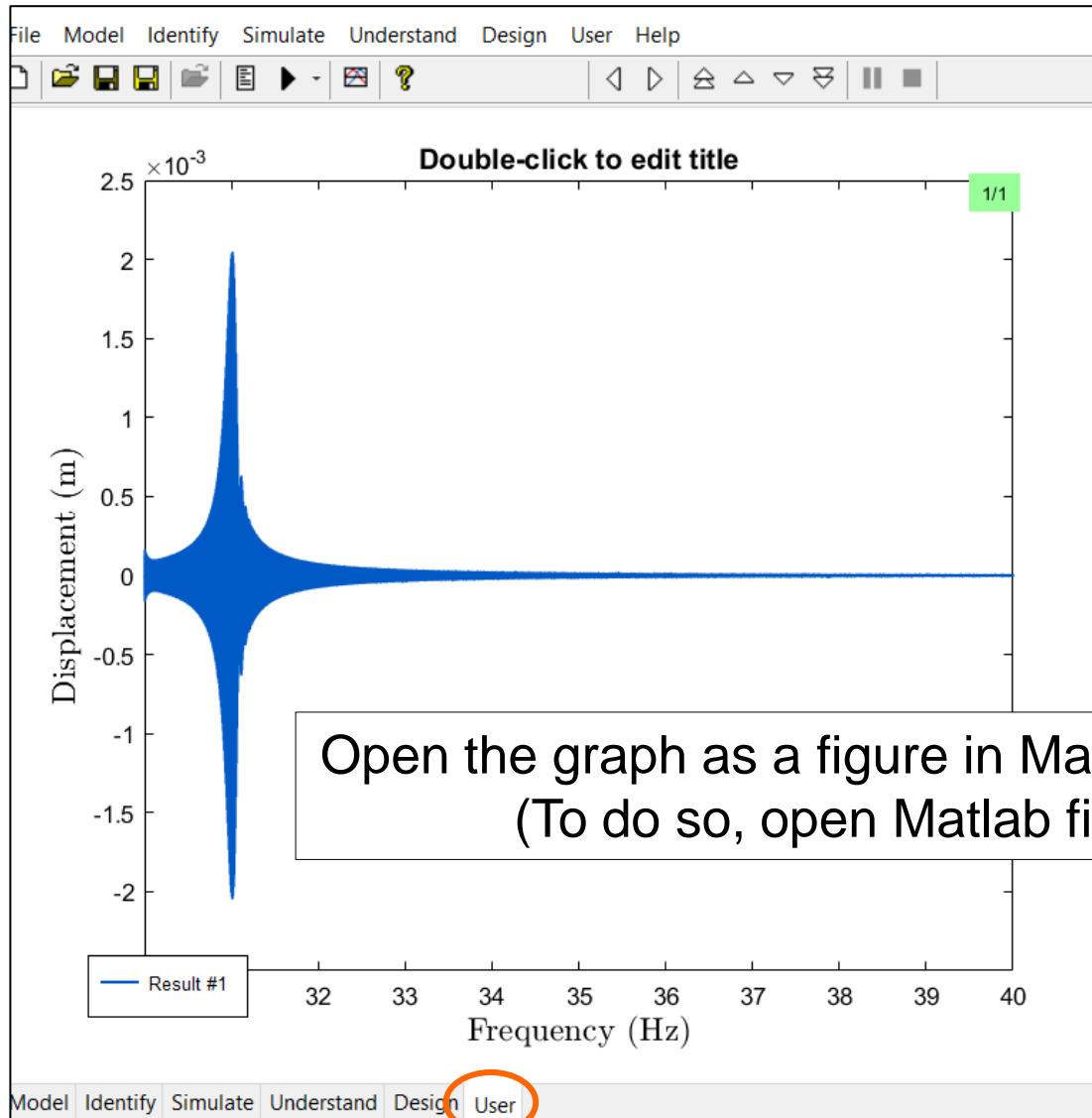
# Now: Run



# Save Your Results in a Curve Stack

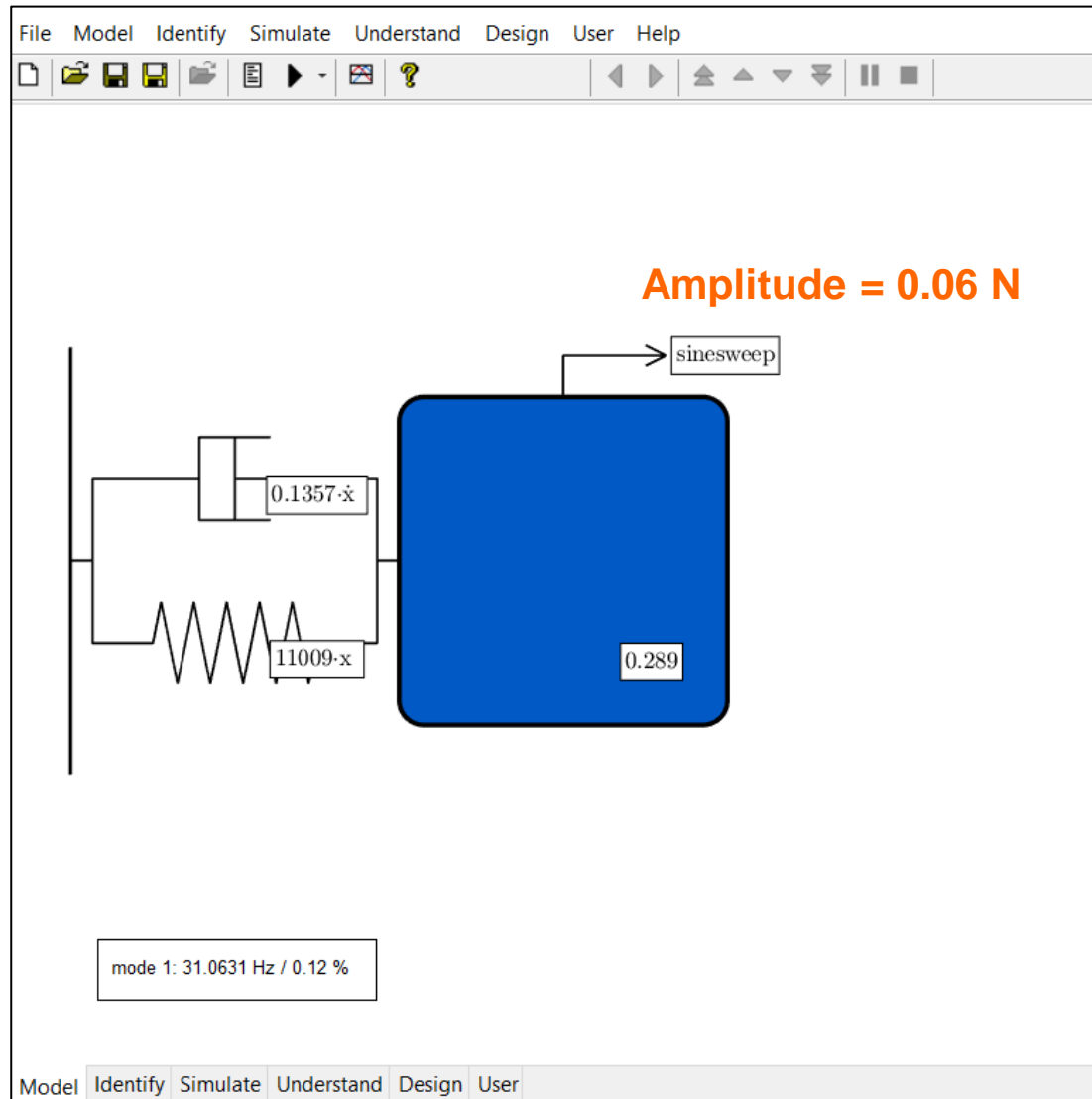


# Check Your Results Anytime

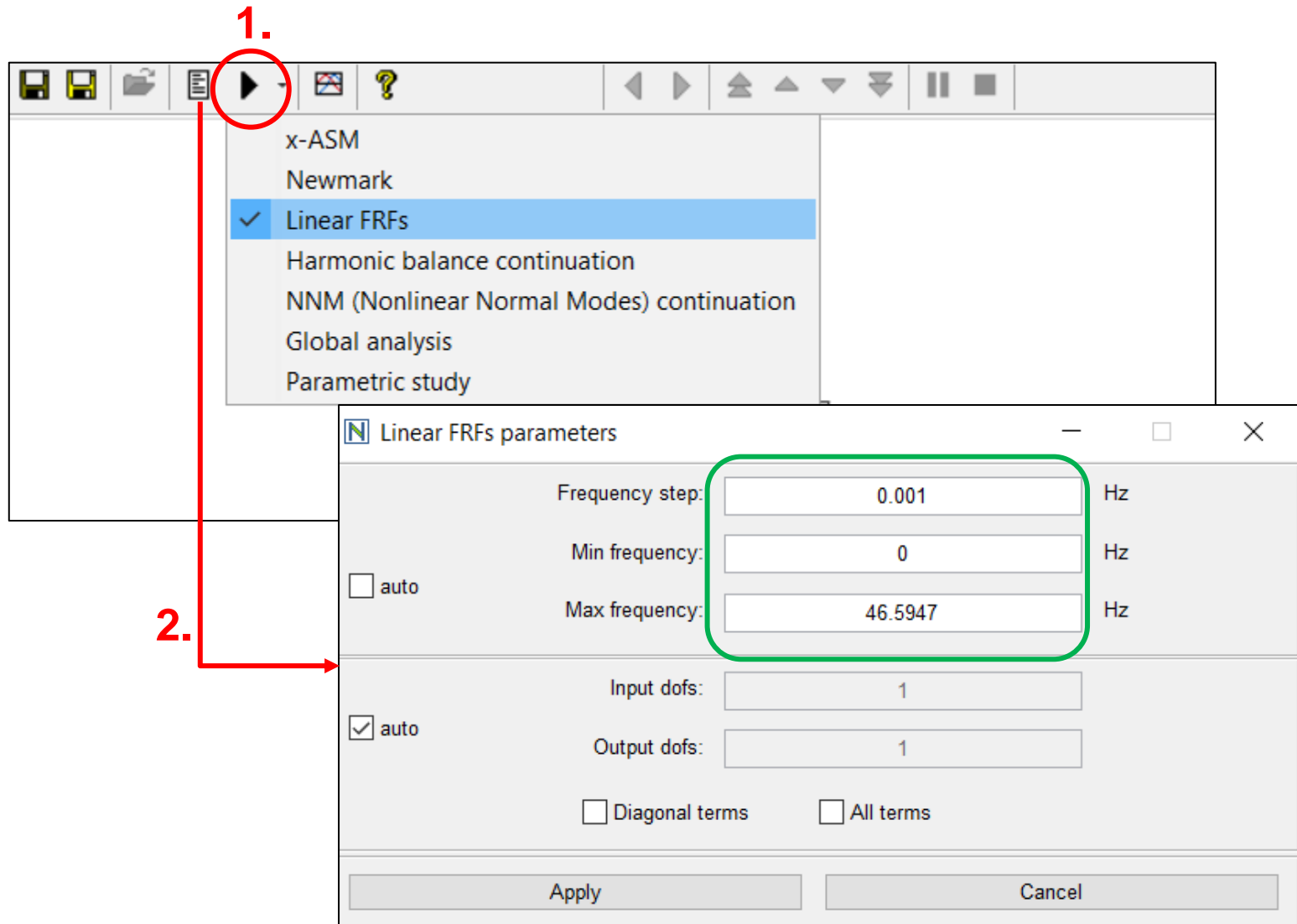


# Frequency Response Functions

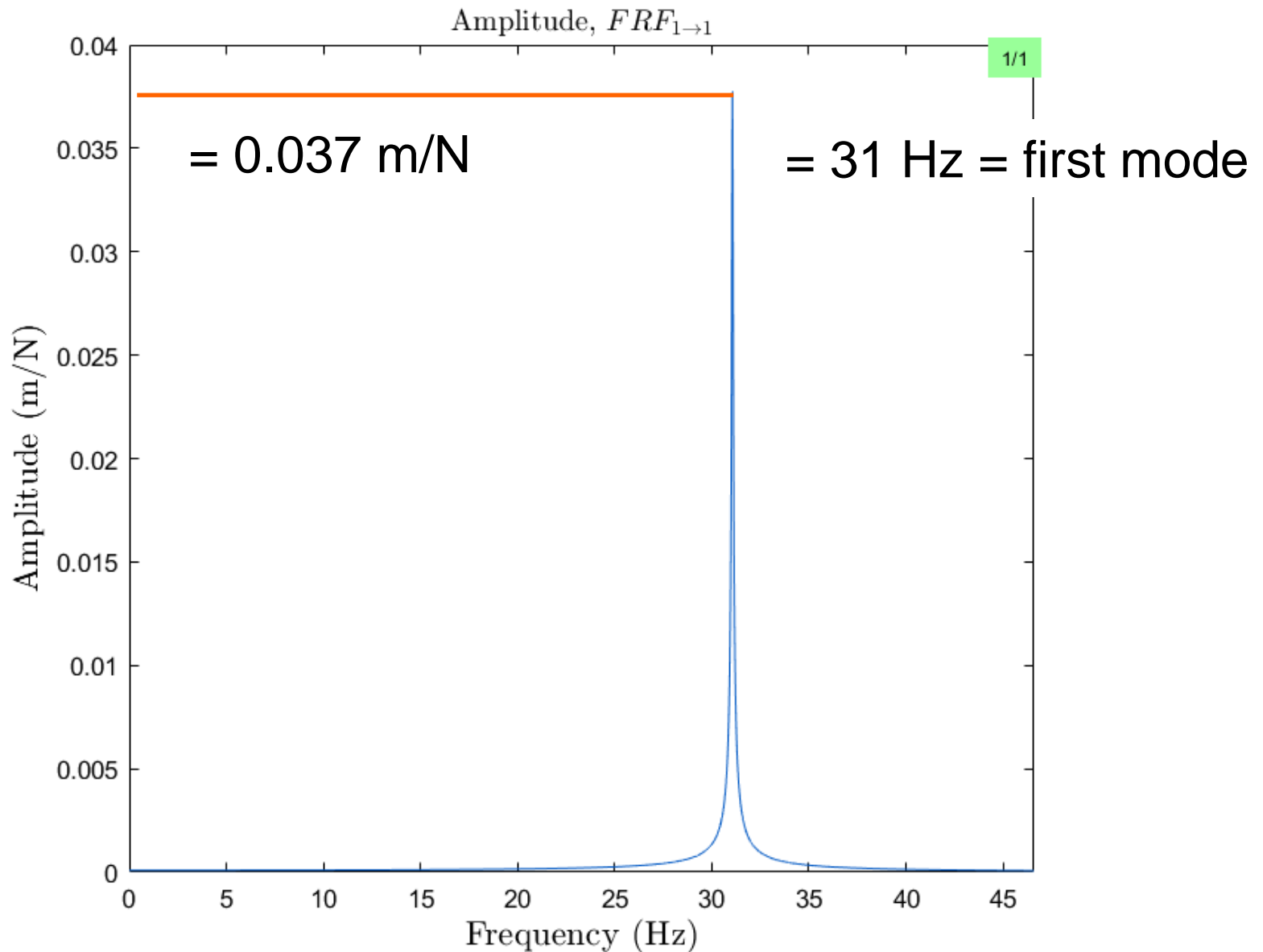
# The Linear Model



# Calculate the Linear FRF ...

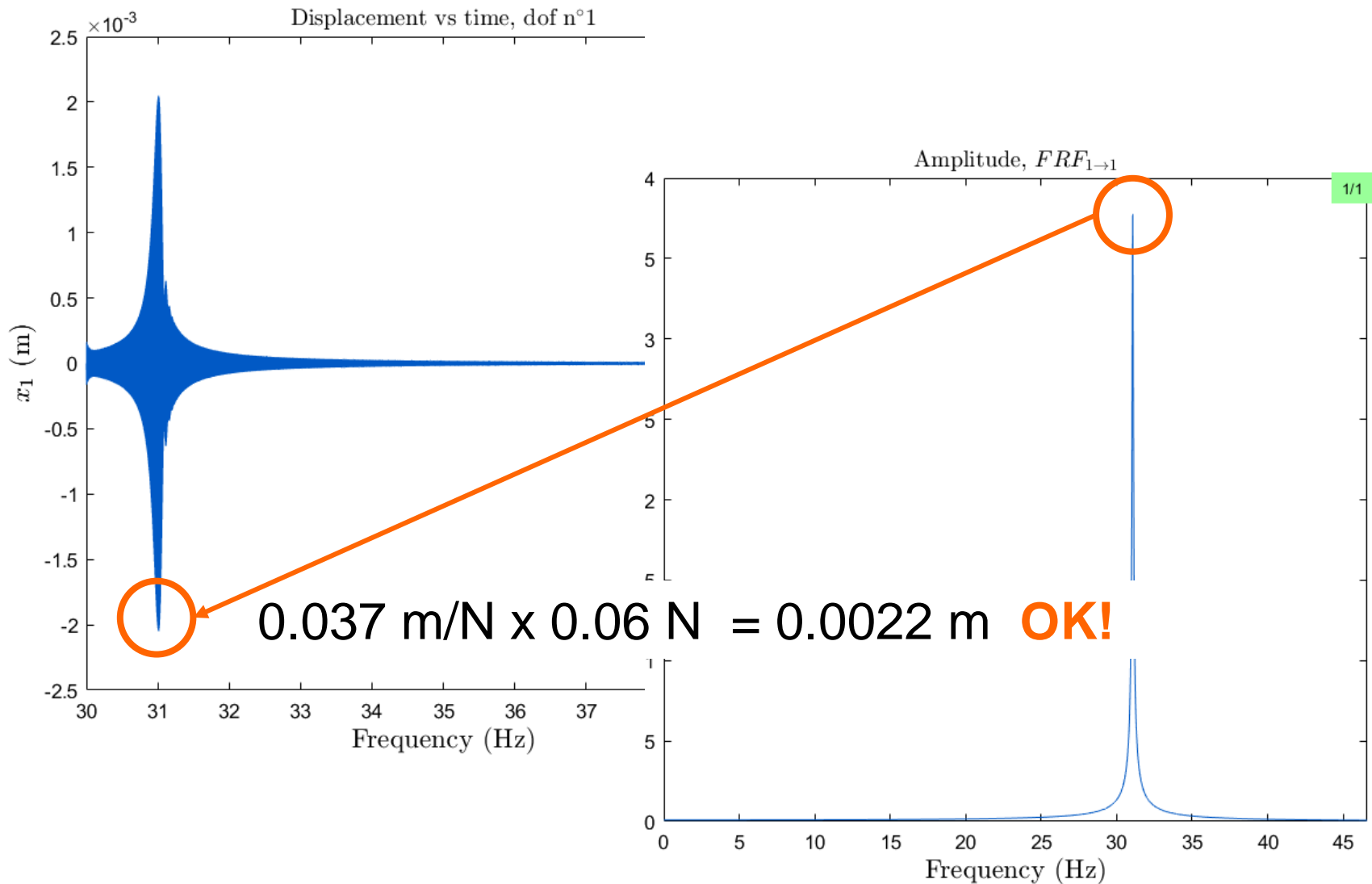


# And Check the Result

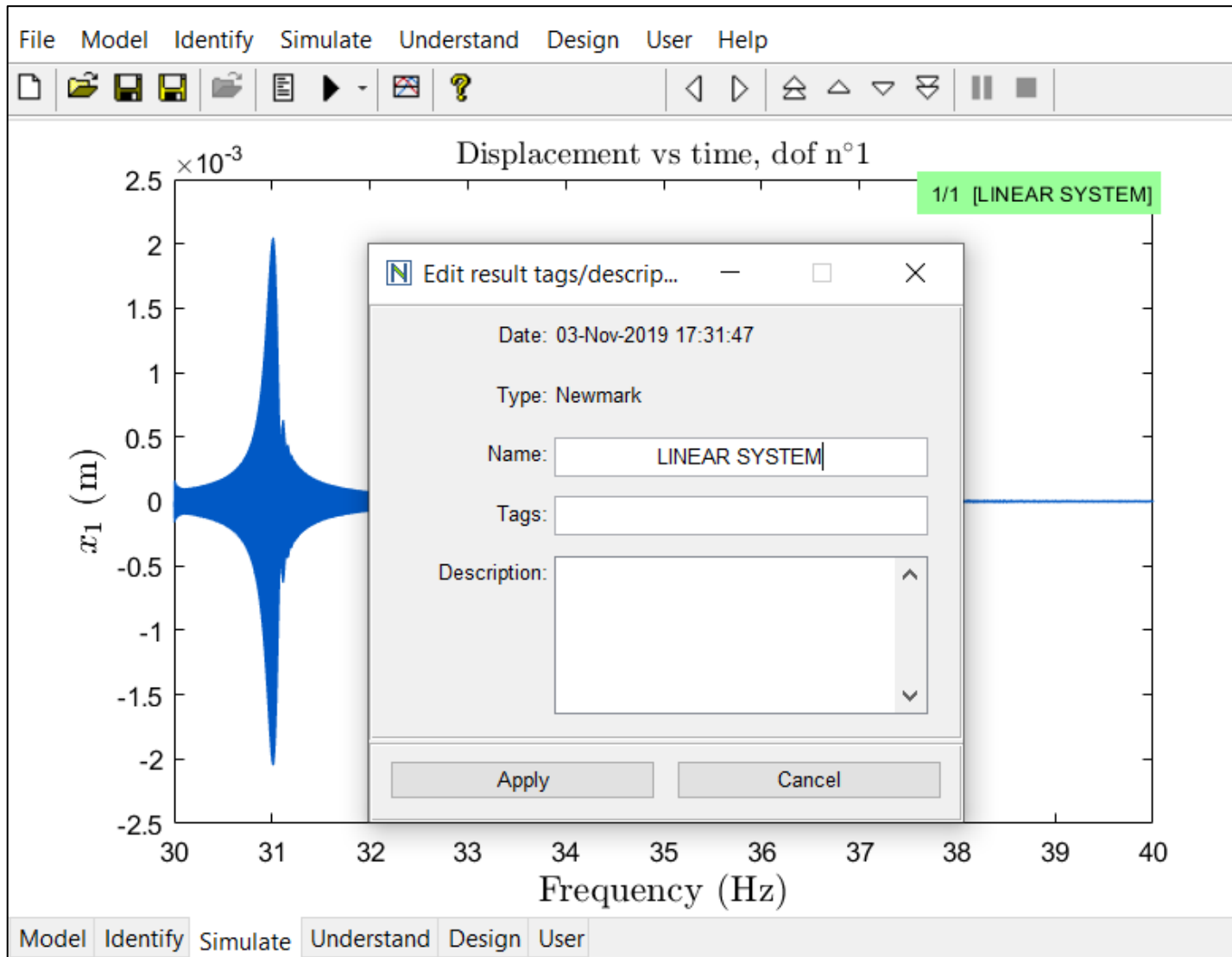




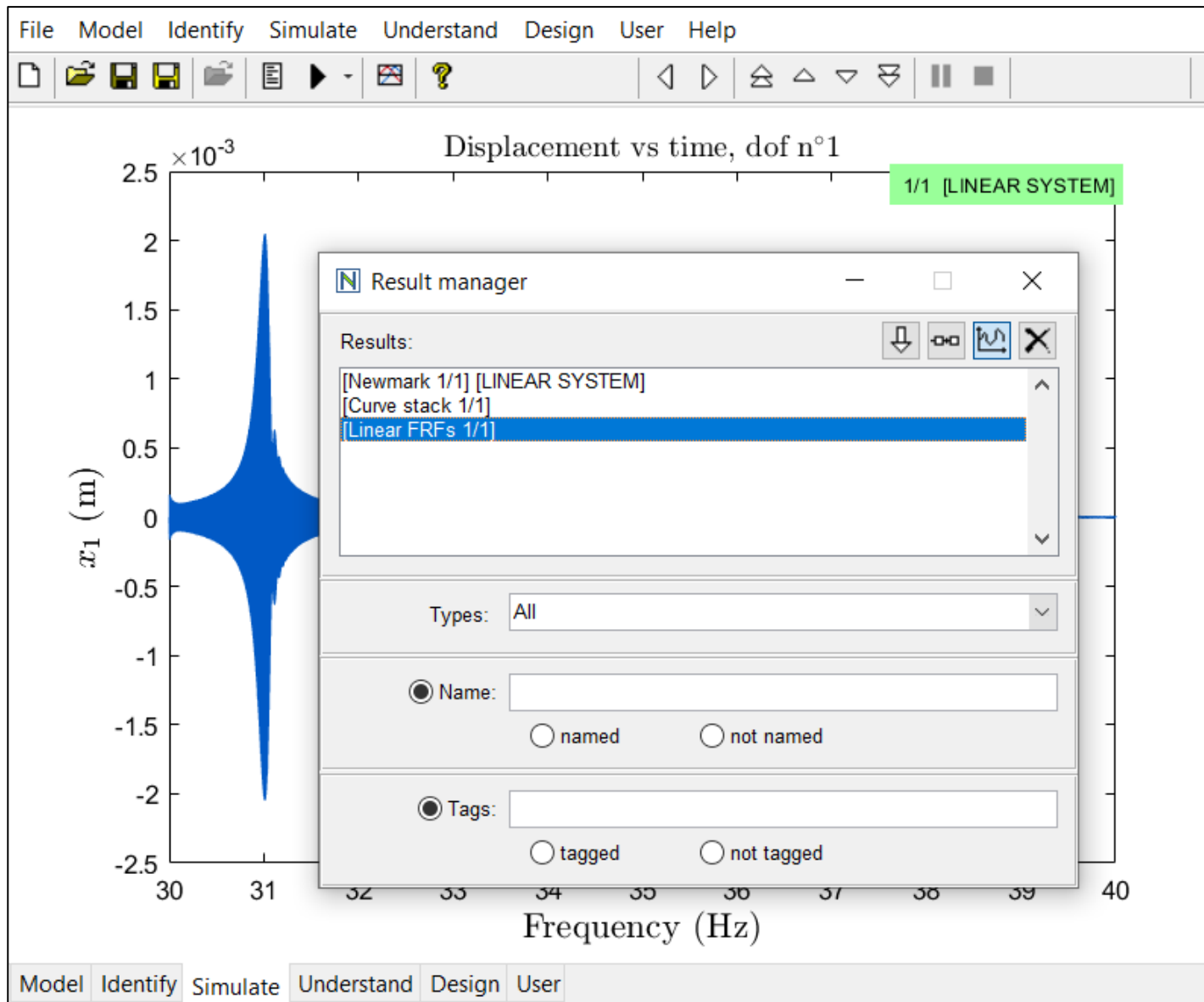
# Compare the Time and the Frequency Response



# You Can Tag Your Results (F11) ...

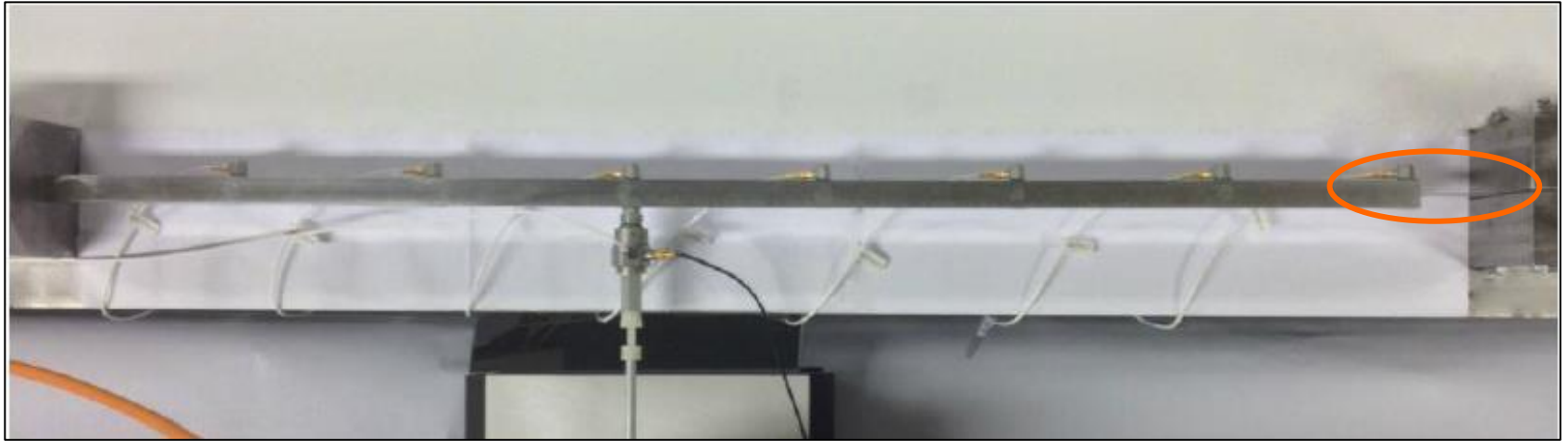


# And Manage All Your Results (F12)



# Launch Your First Nonlinear Simulation

# Nonlinear Model of the 1<sup>st</sup> Beam Mode



Linear model identified at low level (31 Hz, 0.12%):

$$0.289\ddot{x} + 0.1357\dot{x} + 11009x = F \sin \omega t$$

Nonlinearity identified at high level:  $2.37 \cdot 10^9 x^3$

# Upgrade the Linear Model from T02 ...

The screenshot shows a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The main workspace contains a block diagram with a blue rectangular block. A red arrow points from a text box labeled "Ctrl + left click" to the blue block. A dialog box titled "Add element at position 1" is open, displaying a list of elements: Nonlinear polynomial stiffness (highlighted), Nonlinear cubic spline stiffness, Nonlinear piecewise linear stiffness, Nonlinear contact, Nonlinear polynomial damping, Coulomb friction, Trilinear damping, Point-by-point damping, and Hysteretic damping (Bouc-Wen). The dialog has "OK" and "Cancel" buttons. The status bar at the bottom shows "mode 1: 31.0631 Hz / 0.12 %".

File Model Identify Simulate Understand Design User Help

Ctrl + left click

mode 1: 31.0631 Hz / 0.12 %

Nonlinear polynomial stiffness  
Nonlinear cubic spline stiffness  
Nonlinear piecewise linear stiffness  
Nonlinear contact  
Nonlinear polynomial damping  
Coulomb friction  
Trilinear damping  
Point-by-point damping  
Hysteretic damping (Bouc-Wen)

OK Cancel

Model Identify Simulate Understand Design User

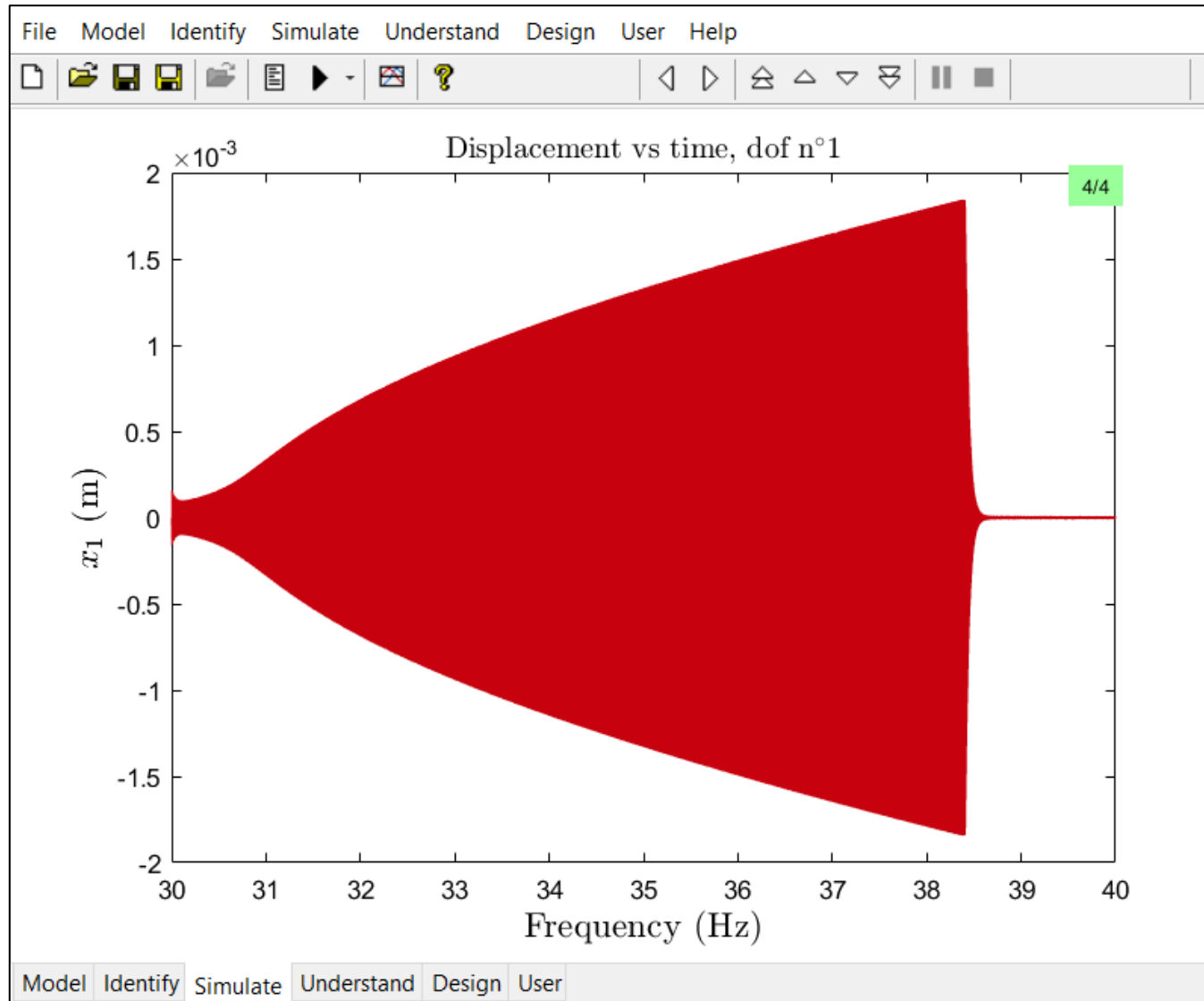
# With a Cubic Spring

The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The main workspace shows a mechanical model of a mass-spring-damper system. A blue mass is connected to a fixed wall on the left by three parallel elements: a spring with a coefficient of  $2.37e+09 \cdot x^3$ , a damper with a coefficient of  $0.1357 \cdot \dot{x}$ , and a linear spring with a coefficient of  $11009 \cdot x$ . A 'sinesweep' input is applied to the mass. A dialog box titled 'Polynomial NL spring n...' is open, showing the following settings:

- Coefficient: 2370000000 N/m^x
- Exponent: 3
- Radio buttons:  odd,  even,  unilateral

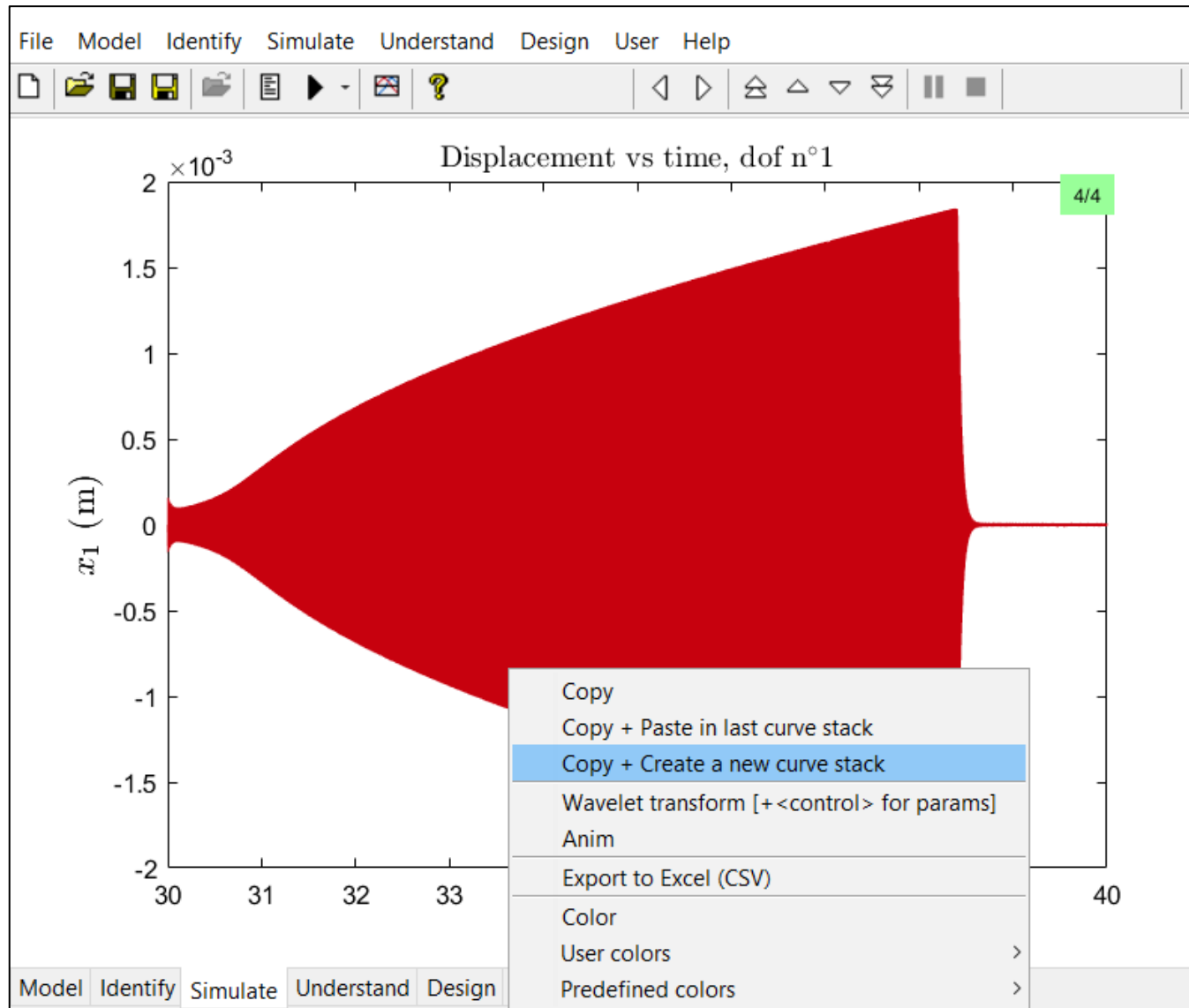
Buttons for 'Apply' and 'Cancel' are visible at the bottom of the dialog. A text box at the bottom left of the workspace indicates 'mode 1: 31.0631 Hz / 0.12 %'. The bottom status bar shows the current mode as 'Model'.

# Run the Newmark Time Integration Again ...

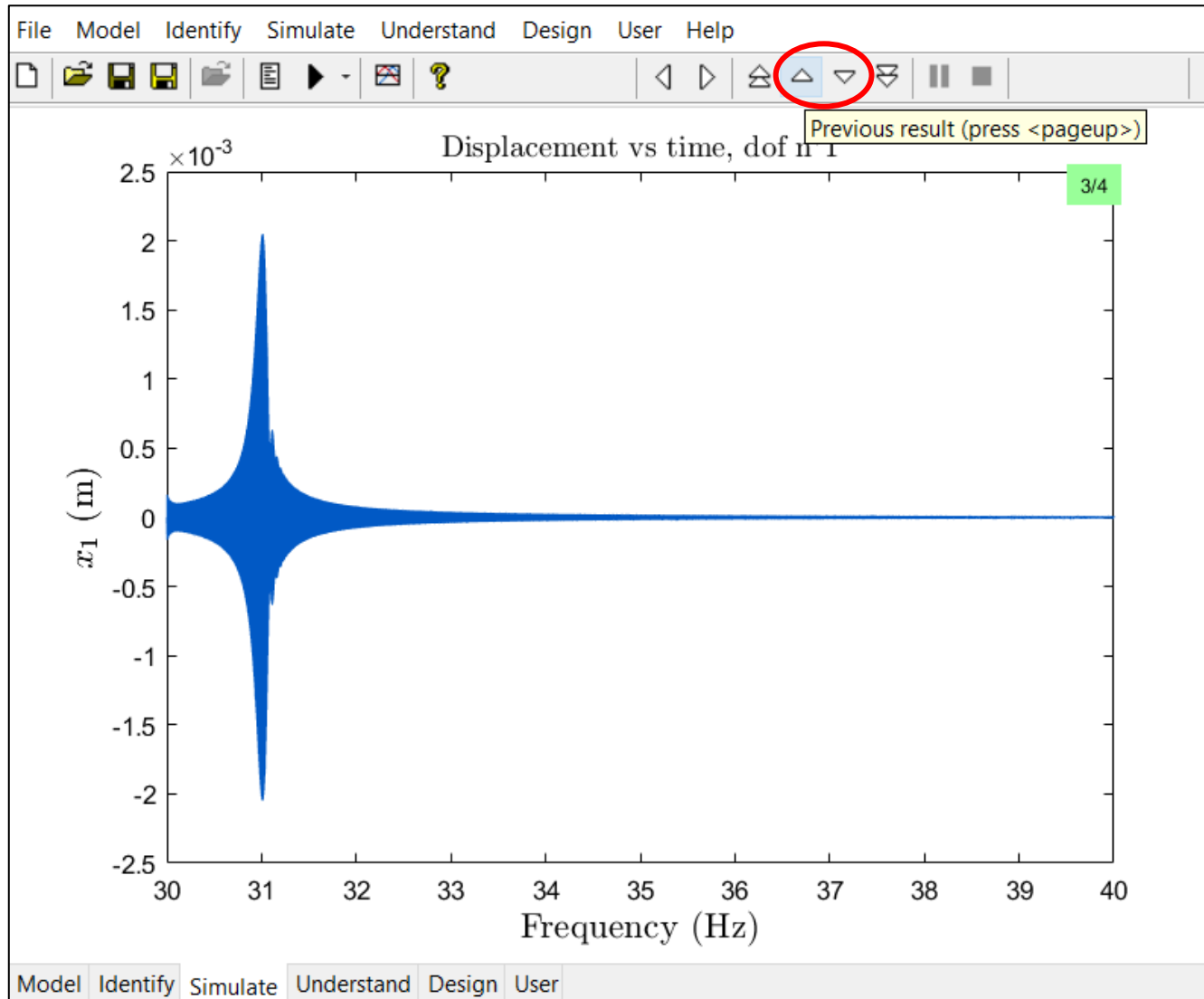




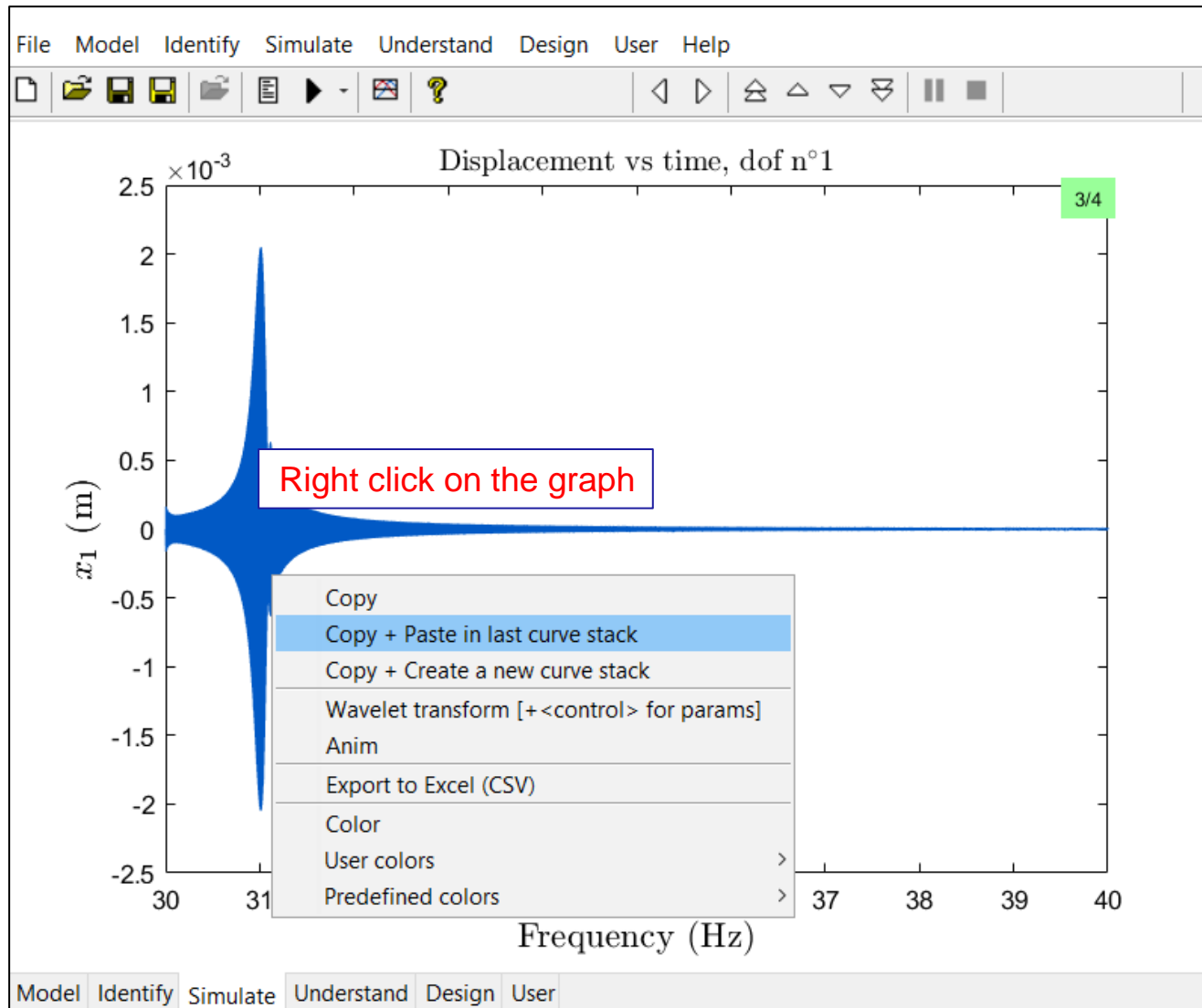
# And Save Your Results For Comparison



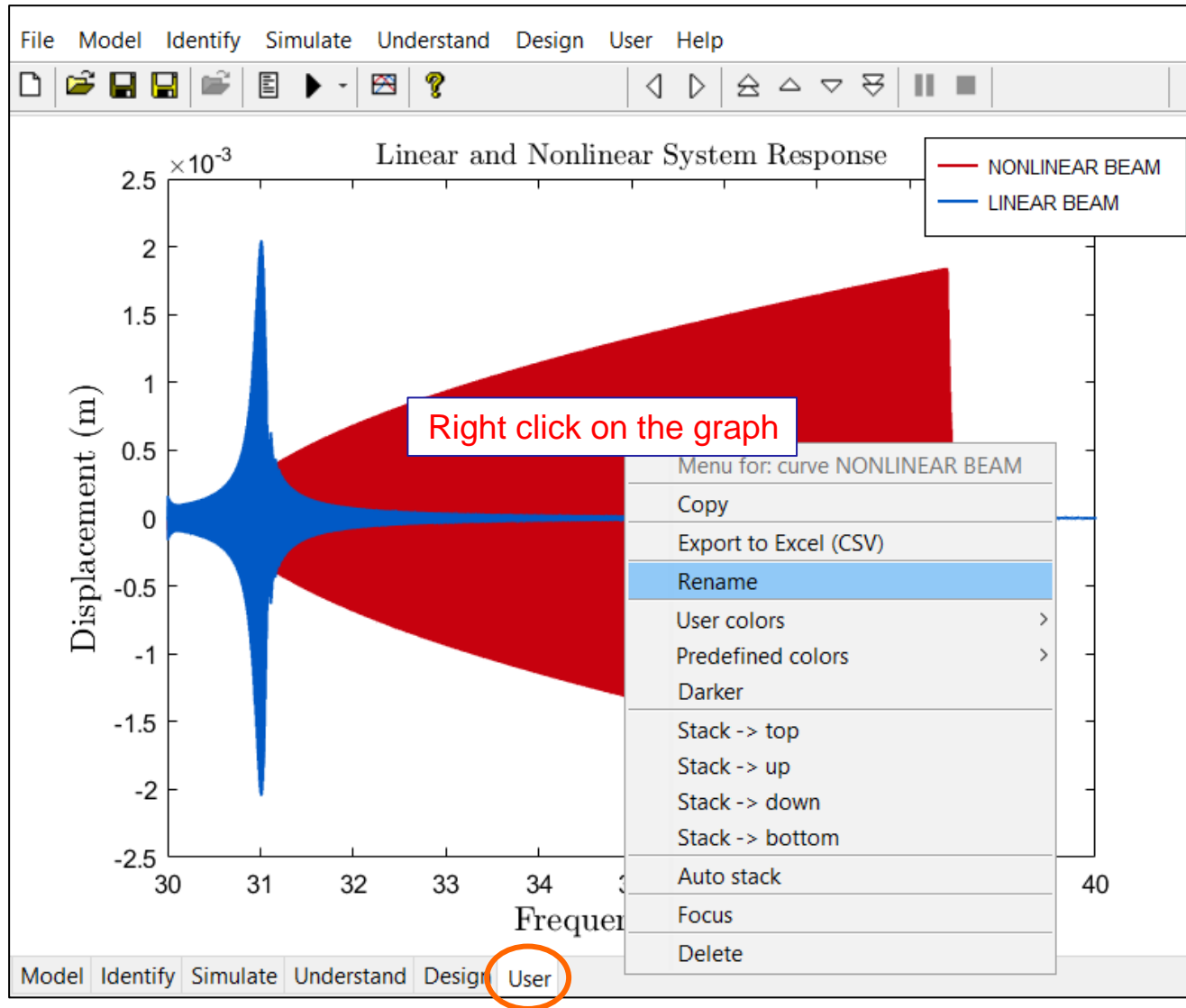
# You Can Scroll through Previous Results



# And Save Different Results in the Same Curve Stack

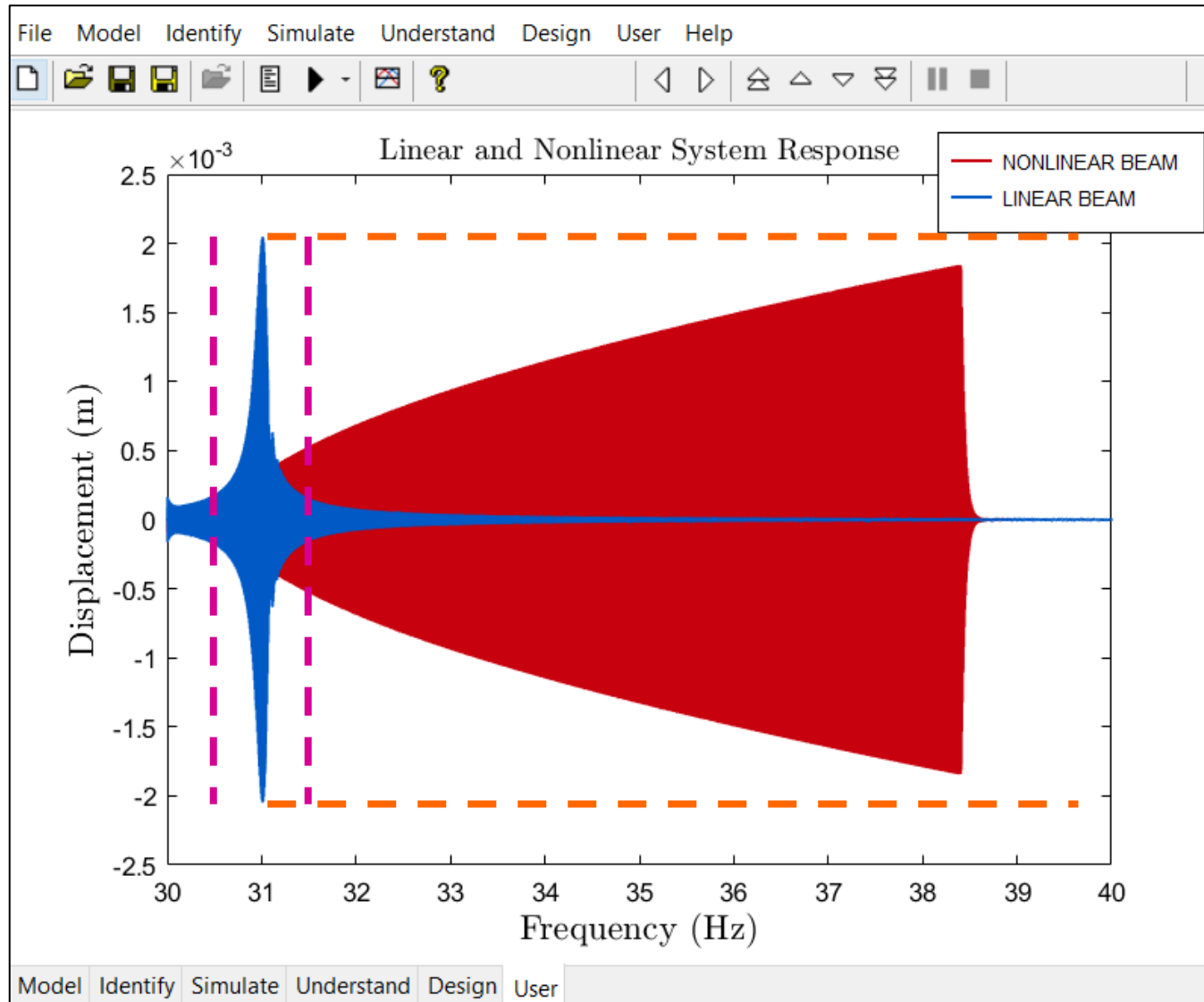


# Rename and Arrange Everything in Your Graph ...

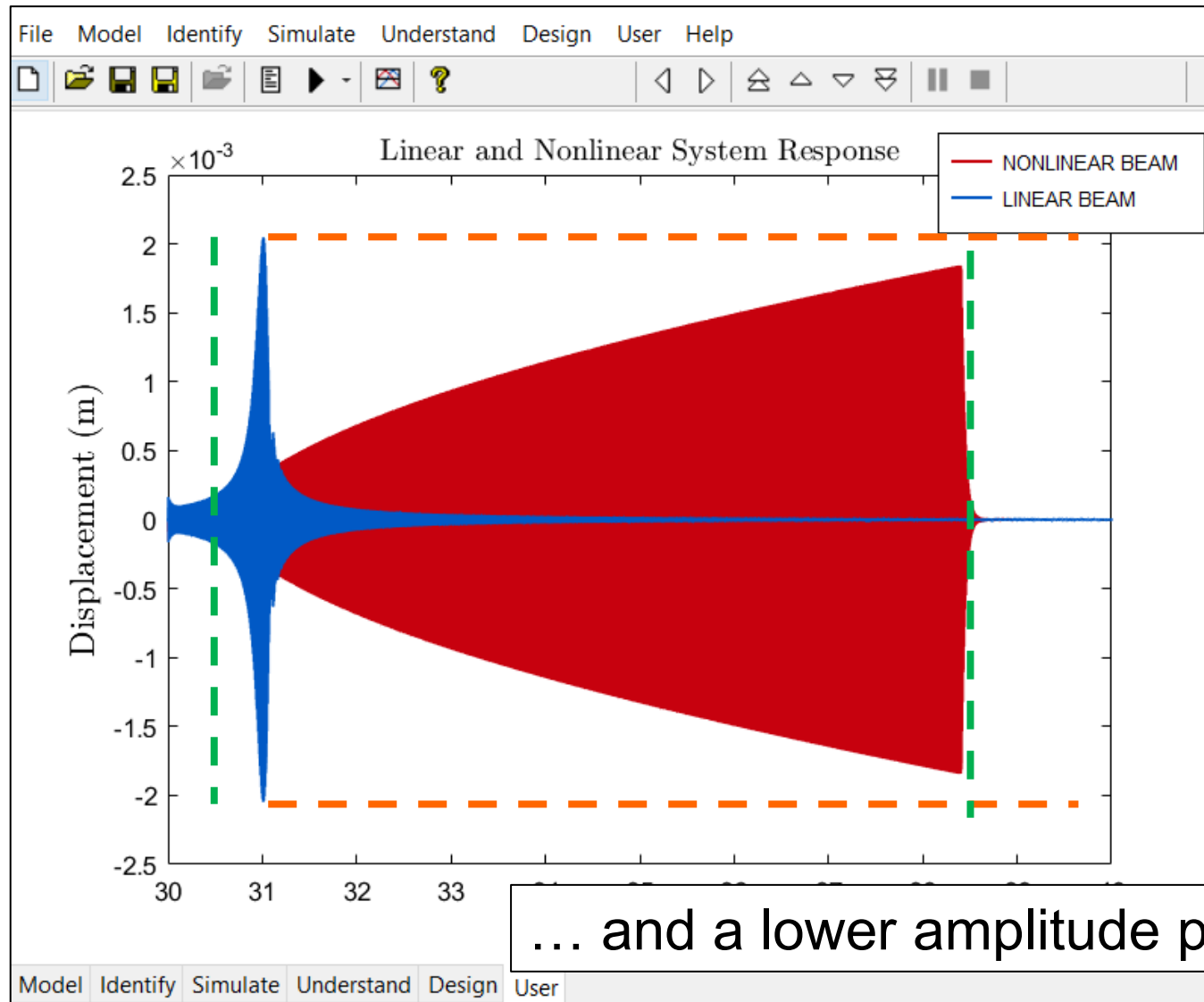


What can we observe?

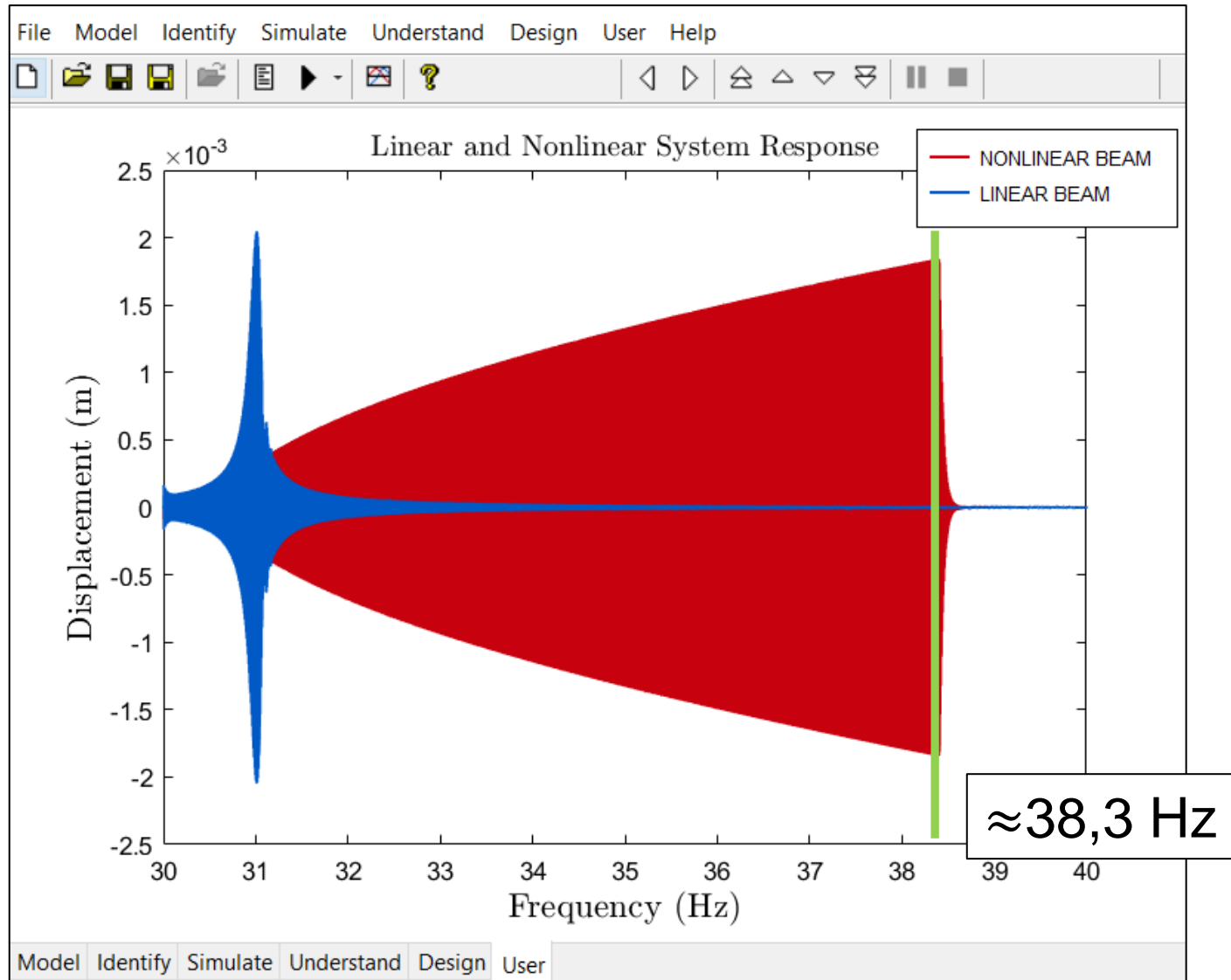
# Nonlinearity Introduces a Fundamental Change



# The Nonlinear System Response Has a Greater Bandwidth ...

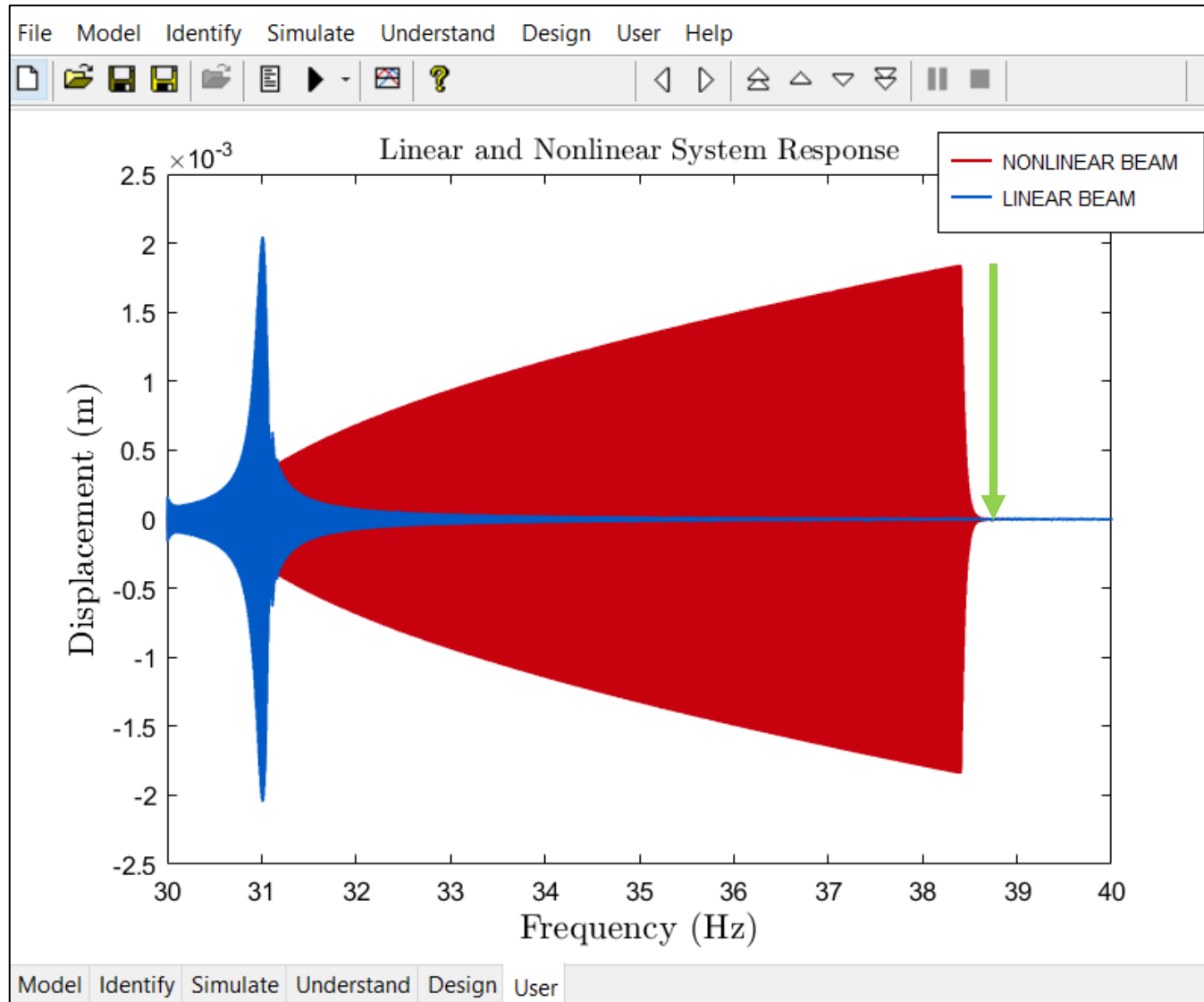


# The Resonance Frequency Shifts

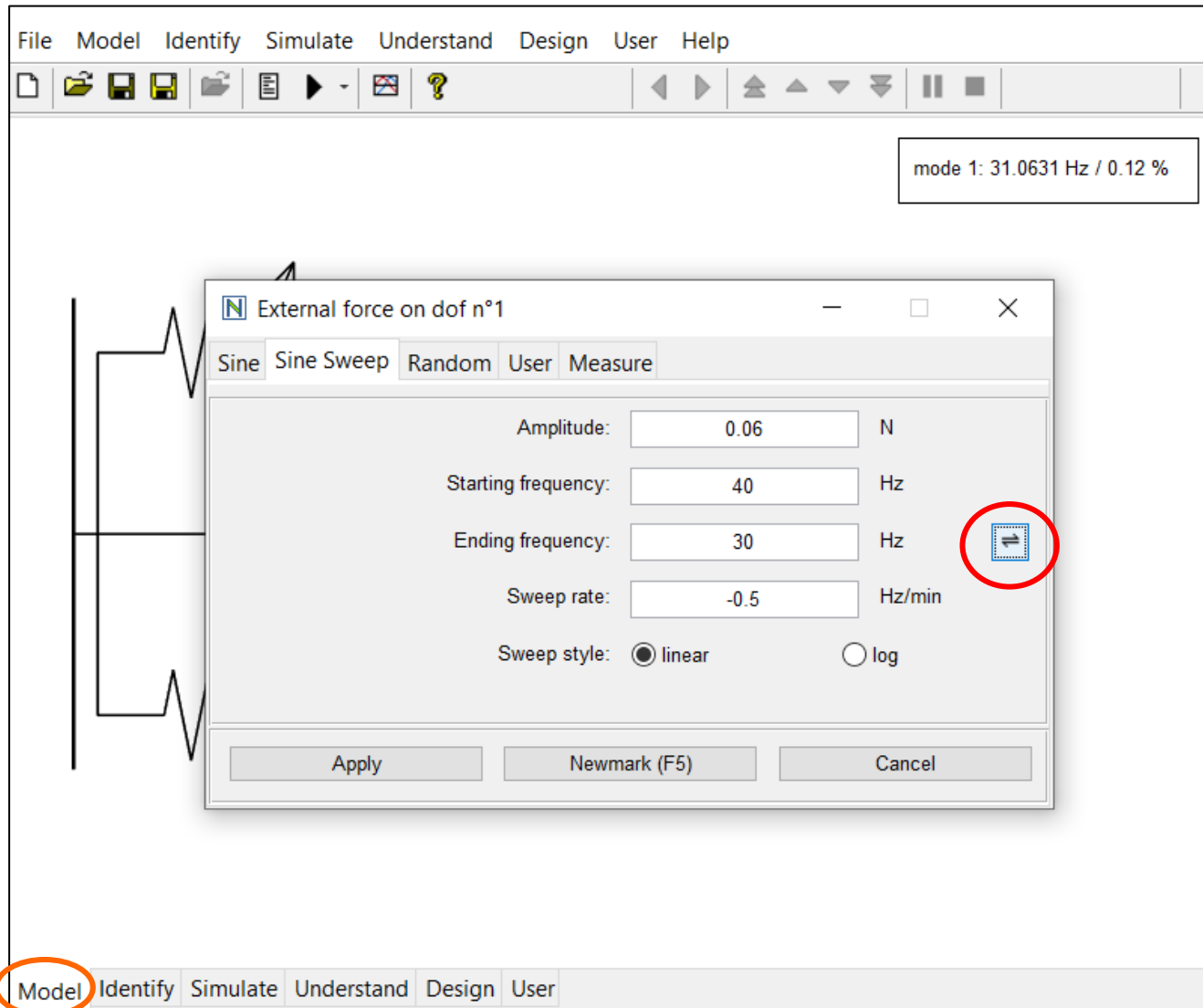




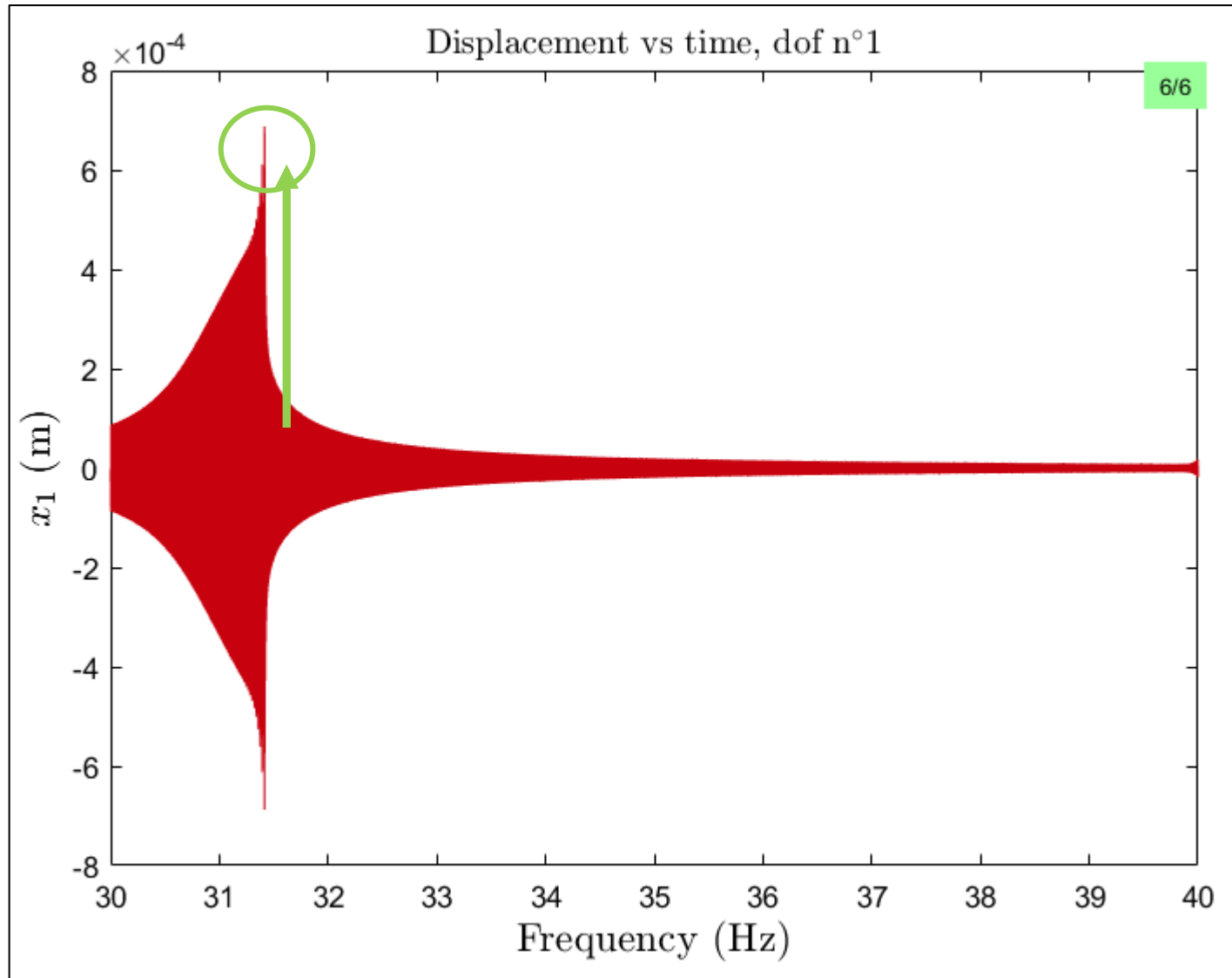
# A Jump Downwards Can Be Observed



# Let's Reverse the Sweep



# We Can Jump Up Too!



# Calculate the Nonlinear FRF

The screenshot displays a simulation software interface with a menu open over a mechanical model. The menu options are:

- x-ASM
- Newmark
- Linear FRFs
- Harmonic balance continuation
- NNM (Nonlinear Normal Modes) continuation
- Global analysis
- Parametric study

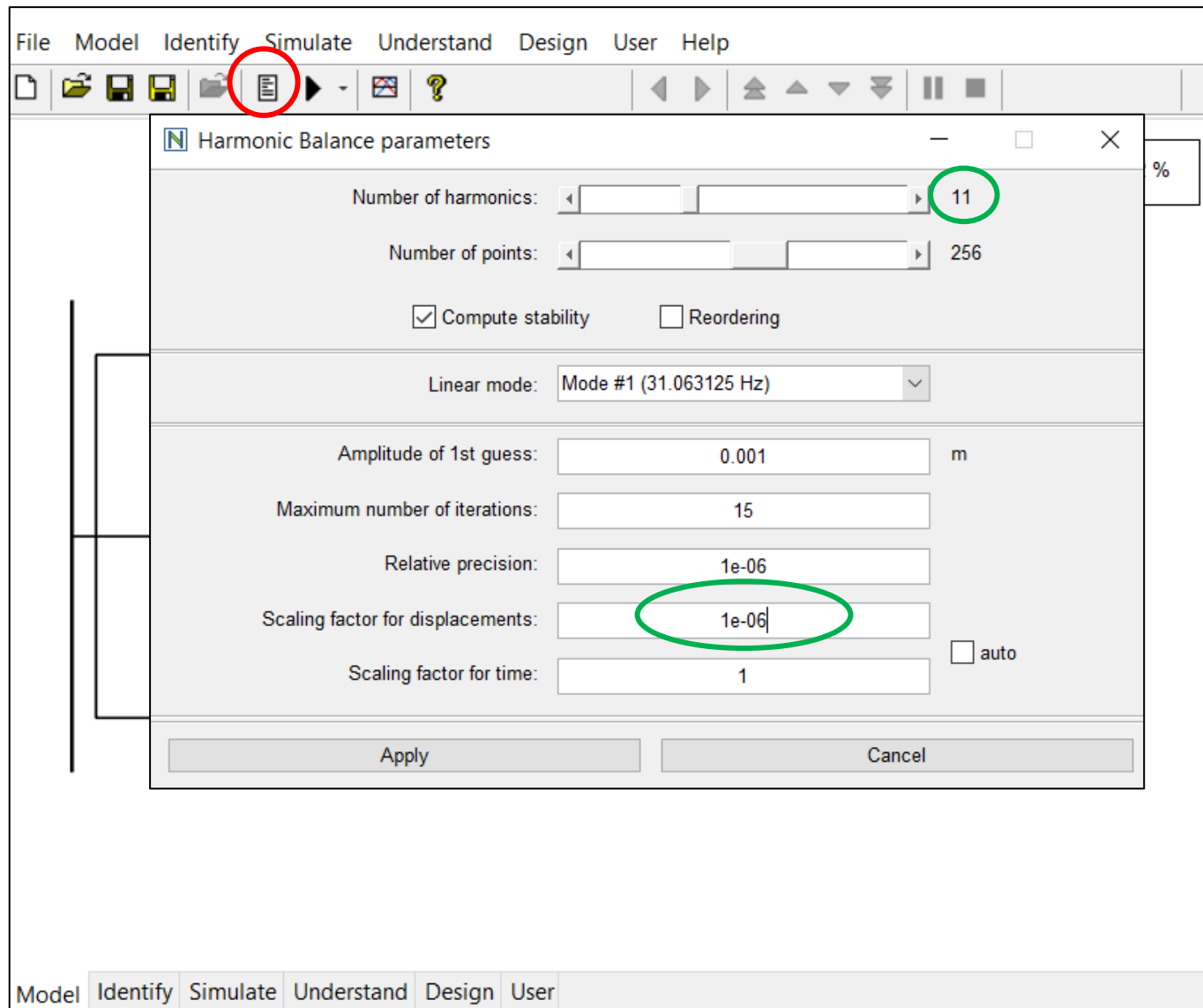
The mechanical model consists of a vertical bar on the left connected to a mass on the right. The mass is represented by a blue rounded rectangle with the value  $0.289$  inside. The bar has three components: a top spring with stiffness  $2.37e+09 \cdot x^3$ , a middle damper with coefficient  $0.1357 \cdot \dot{x}$ , and a bottom spring with stiffness  $11009 \cdot x$ . A text box in the upper right corner of the window displays "mode 1: 31.0631 Hz / 0.12 %". The software interface includes a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar with various icons. The bottom status bar shows the tabs: Model, Identify, Simulate, Understand, Design, User.

# But Set Appropriate Numerical Parameters First

The screenshot shows a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The 'Simulate' menu is circled in red. A dialog box titled 'HB continuation parameters' is open, showing various settings. The 'Starting point' field is set to 30 Hz, 'Min' to 29 Hz, and 'Max' to 40 Hz. The 'Direction' is set to '+'. The 'Stepsize' field is set to 30 and is circled in green. The 'Adaptative' checkbox is unchecked. The 'Maximum number of points' is set to 10000 and the 'Beta angle' is set to 90 degrees. The 'HB parameters' button at the bottom is circled in red.

Parameter	Value	Unit
Starting point	30	Hz
Min	29	Hz
Max	40	Hz
Direction	-	
Fold	<input checked="" type="checkbox"/> detect	<input type="checkbox"/> localize
Branch point	<input checked="" type="checkbox"/> detect	<input type="checkbox"/> localize
Neimark-Sacker	<input checked="" type="checkbox"/> detect	<input type="checkbox"/> localize
Stepsize	30	
Min	0.1	
Max	2	
Optimal number of iterations	3	
Maximum number of points	10000	
Beta angle	90	°

# But Set Appropriate Numerical Parameters First



# And Change the Excitation Signal to a Sine

File Model Identify Simulate Understand Design User Help

mode 1: 31.0631 Hz / 0.12 %

$0.06 \cdot \sin(2 \cdot \pi \cdot t)$

External force on dof n°1

Sine Sine Sweep Random User Measure

Amplitude: 0.06 N

Frequency: 1 Hz

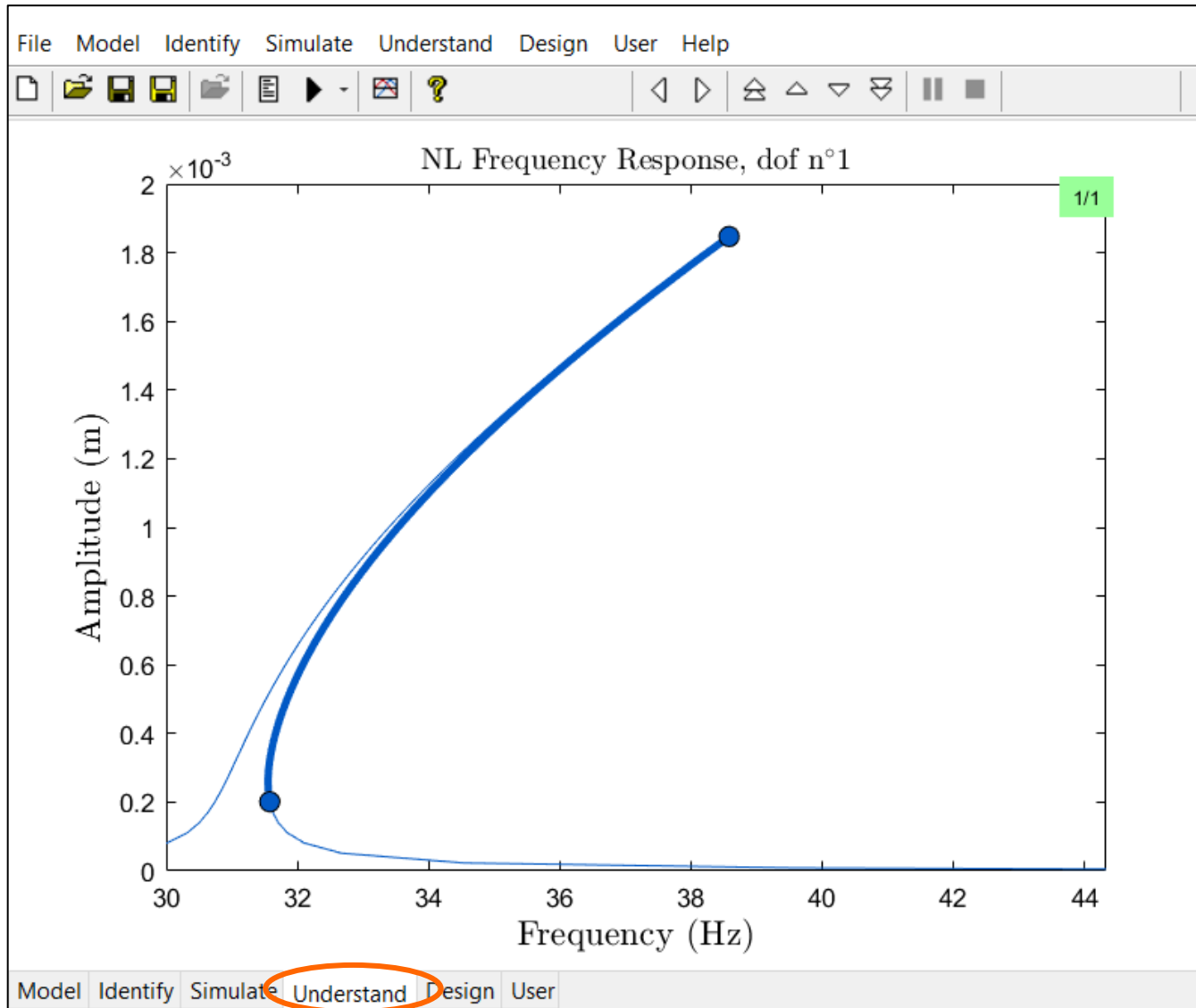
Phase: 0 °

Note that the chosen frequency is not relevant for the computation of the nonlinear FRF

Apply HB cont (F5) Cancel

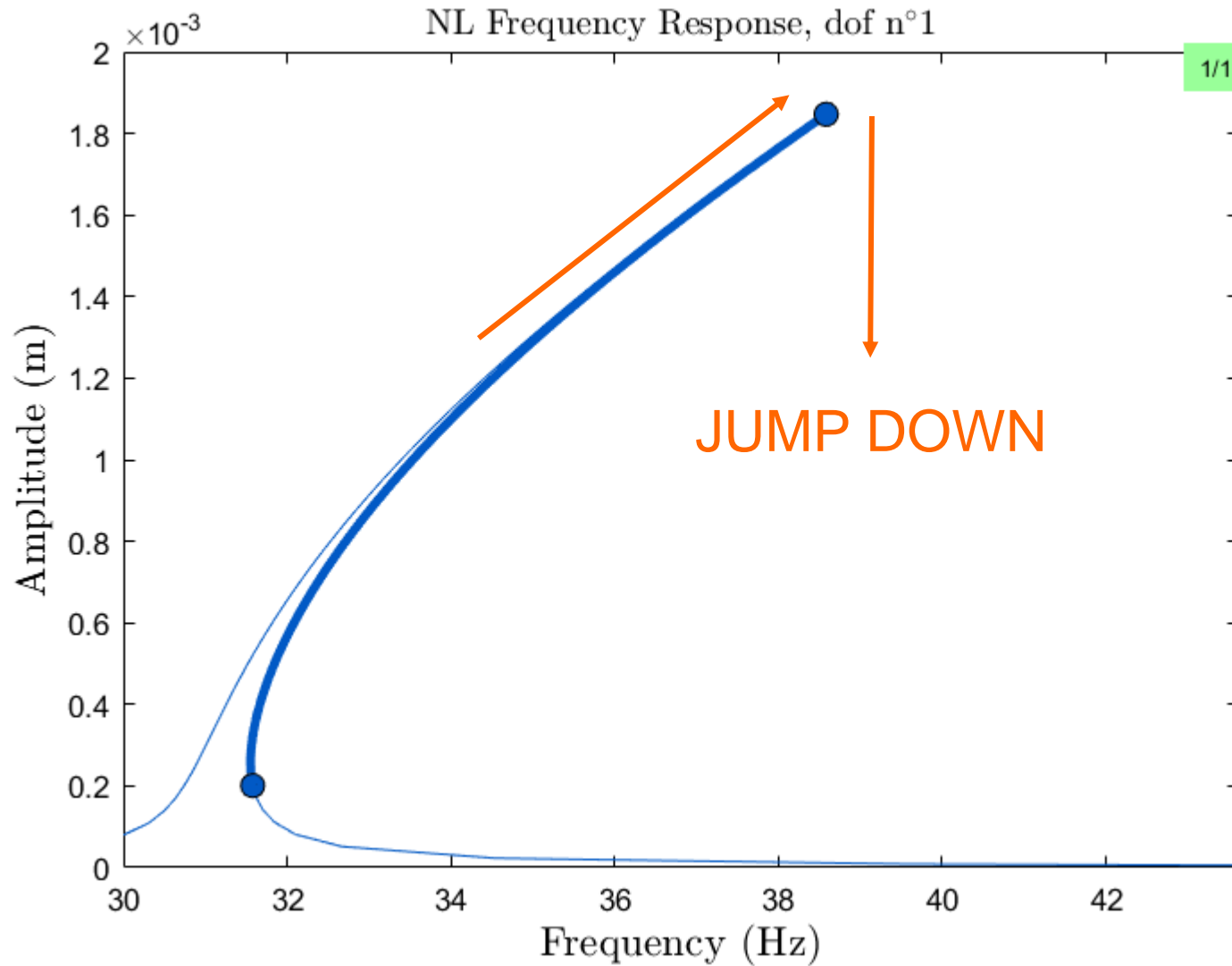
Model Identify Simulate Understand Design User

# The Nonlinear FRF

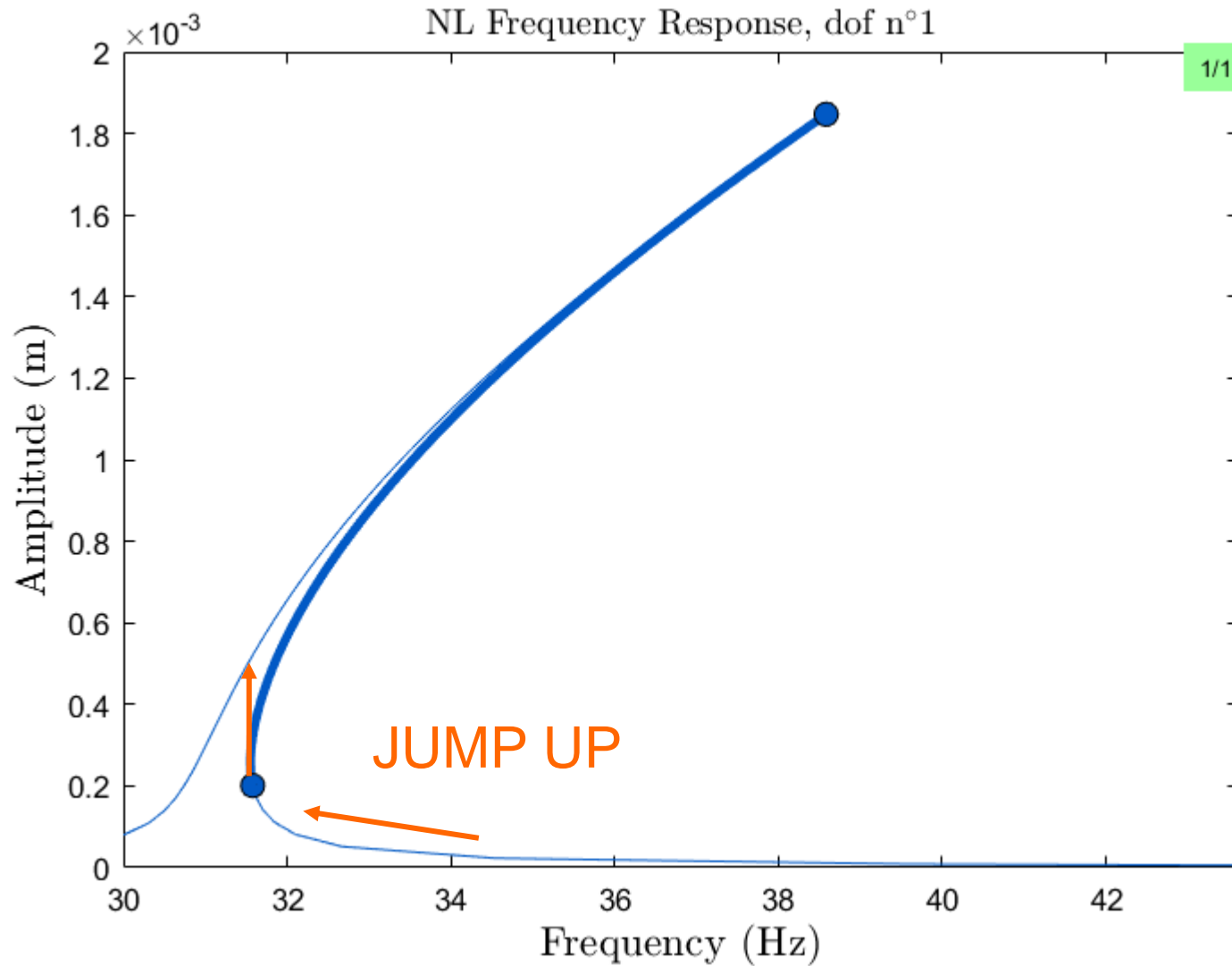




# The Jump Phenomenon Can Be Understood

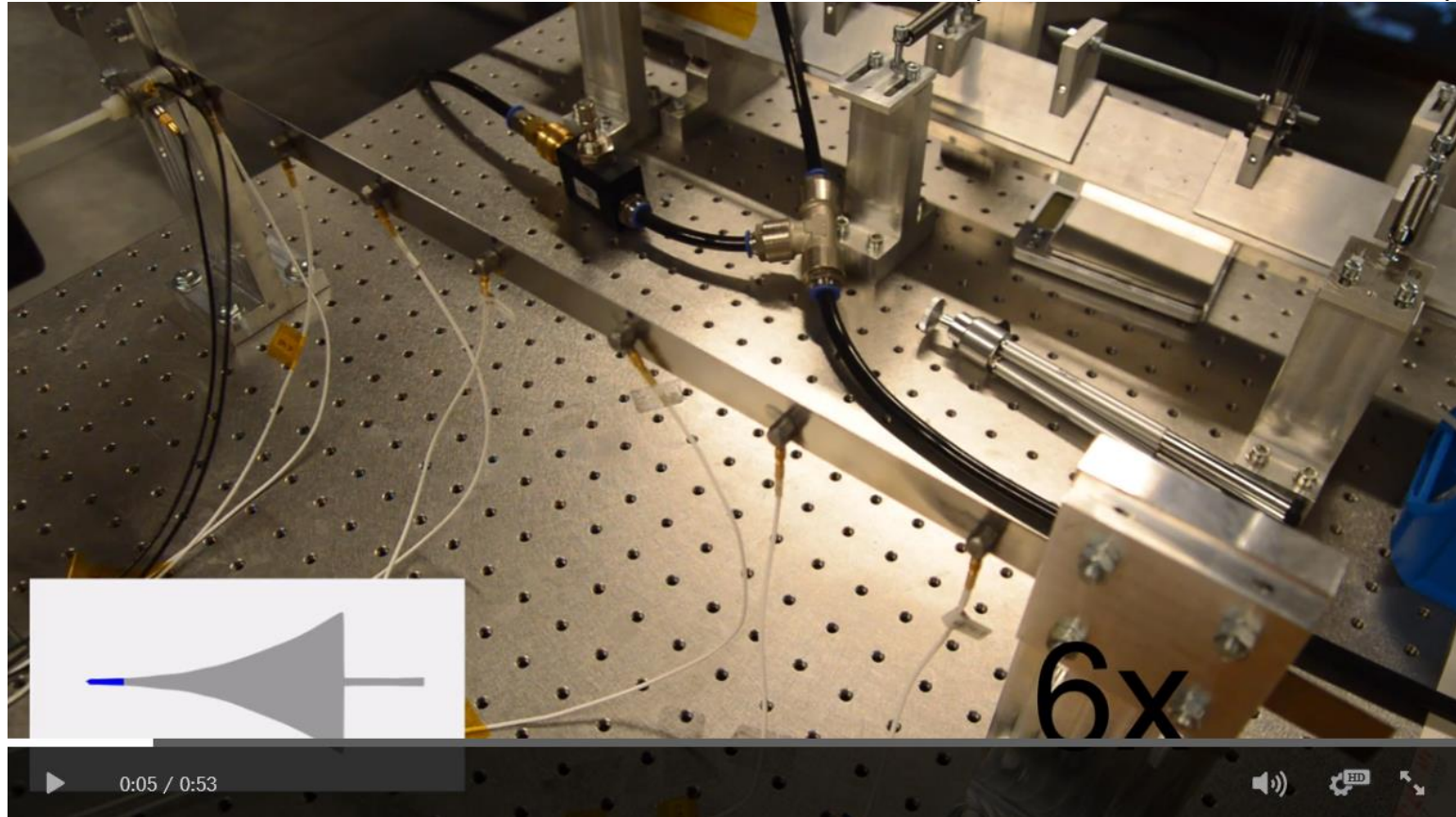


# The Jump Phenomenon Can Be Understood



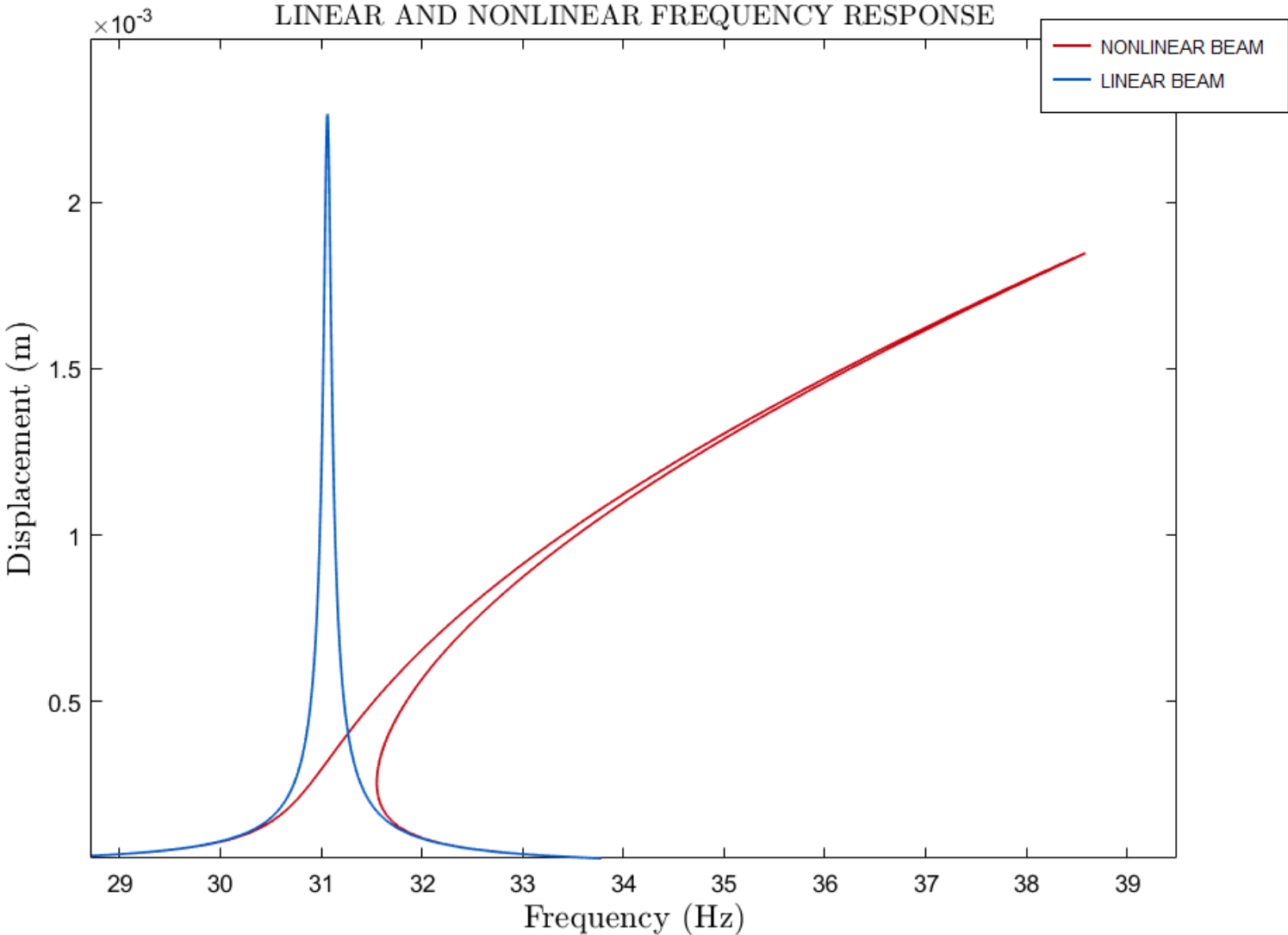
# The Jump Phenomenon in Practice

(See the attached video)

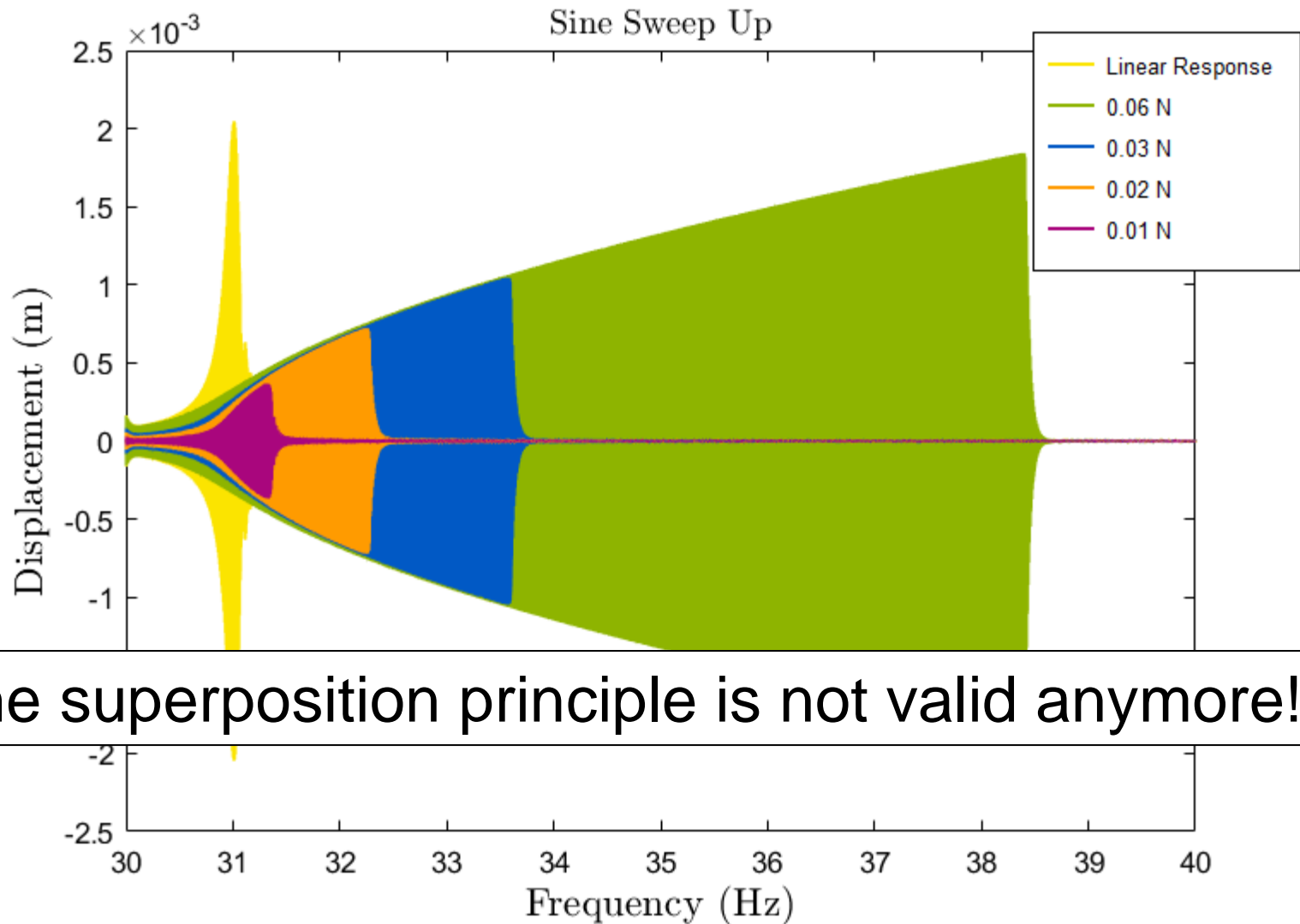


The video shows the dynamics of a clamped-clamped thin beam with geometrical nonlinearities during a sweep over the second beam mode.

# Compare to the Linear Frequency Response



# Sweep with Different Forcing Amplitudes



The superposition principle is not valid anymore!

# Sine with a Fixed Frequency in the Multi-valued Region

# Now: Consider an Excitation with a Fixed Frequency

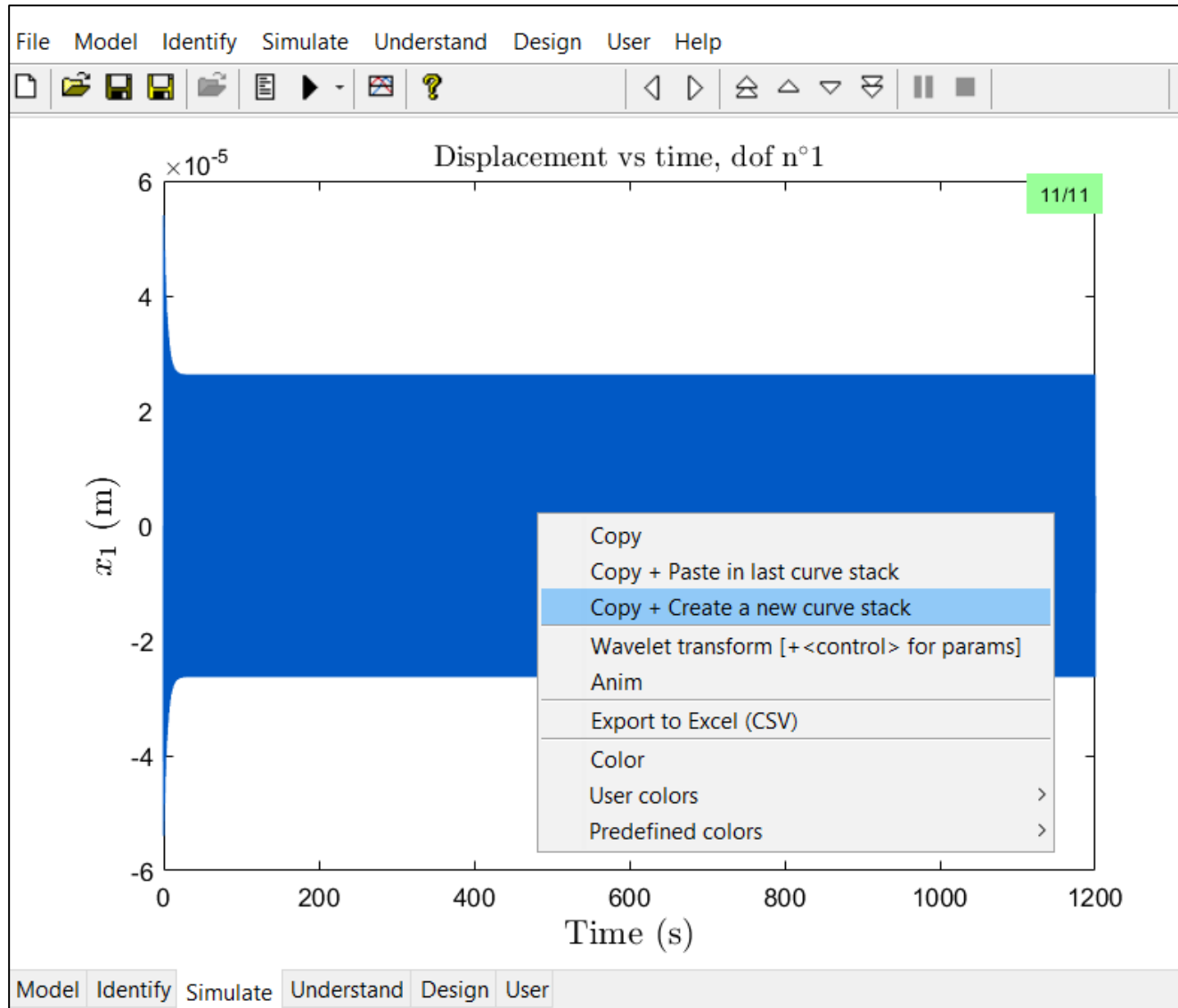
The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. The main workspace shows a mechanical model with a blue component. A sine wave excitation is applied to the component, with the equation  $0.06 \cdot \sin(2 \cdot \pi \cdot 34 \cdot t)$  circled in orange. A status box in the top right corner indicates "mode 1: 31.0631 Hz / 0.12 %".

The "External force on dof n°1" dialog box is open, showing the following parameters:

Parameter	Value	Unit
Amplitude	0.06	N
Frequency	34	Hz
Phase	0	°

The dialog box also includes tabs for "Sine", "Sine Sweep", "Random", "User", and "Measure", and buttons for "Apply", "Newmark (F5)", and "Cancel".

# Run the Newmark Integration and Save





# Change the Initial Conditions

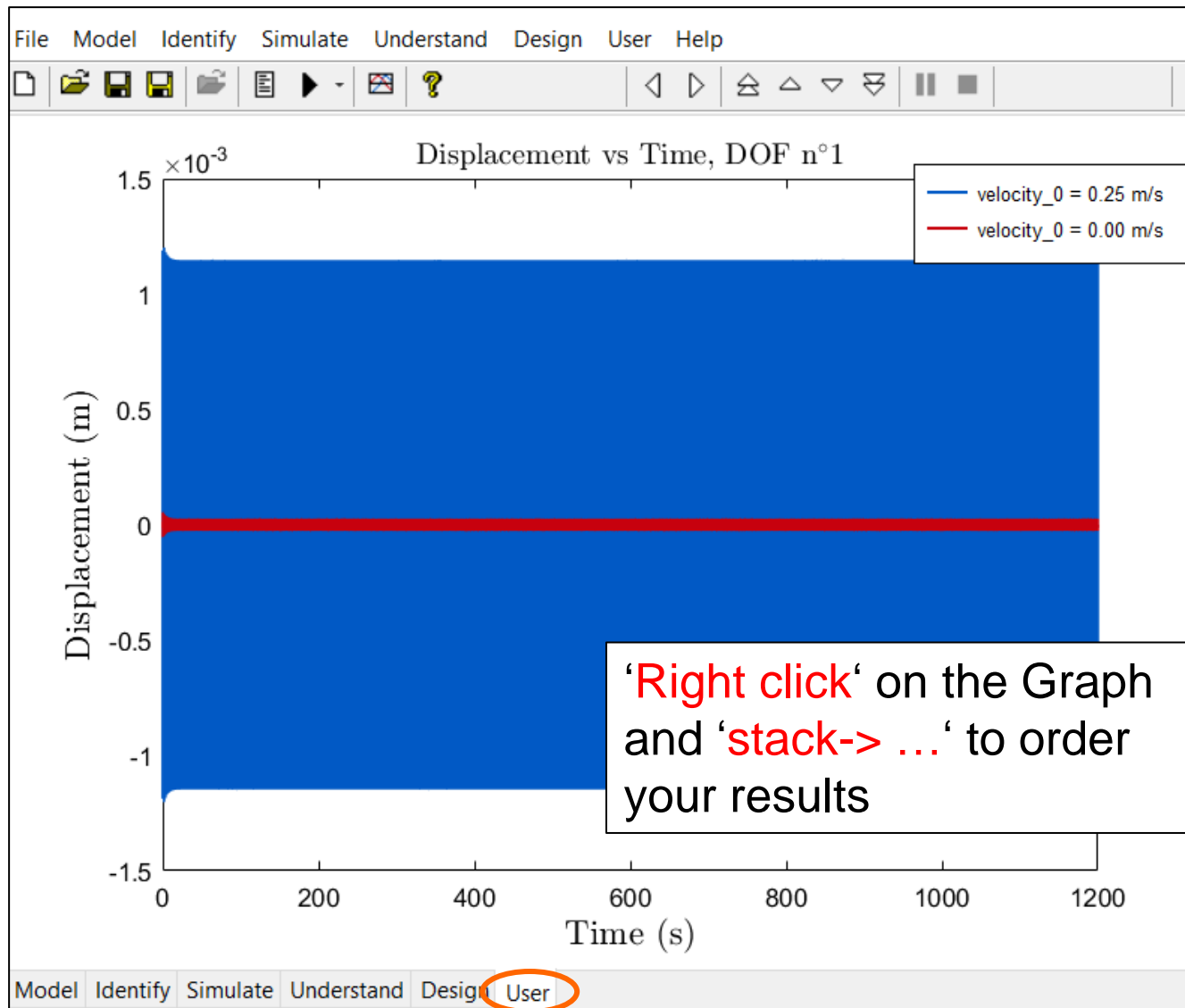
The screenshot displays a software interface with a menu bar (File, Model, Identify, Simulate, Understand, Design, User, Help) and a toolbar. A circuit diagram is visible in the background, featuring a sine wave input labeled  $0.06 \cdot \sin(2 \cdot \pi \cdot 34 \cdot t)$  and various components with numerical values like  $2.37e+$ ,  $0.135$ , and  $1100$ . A status box indicates "mode 1: 31.0631 Hz / 0.12 %".

Two dialog boxes are overlaid on the interface:

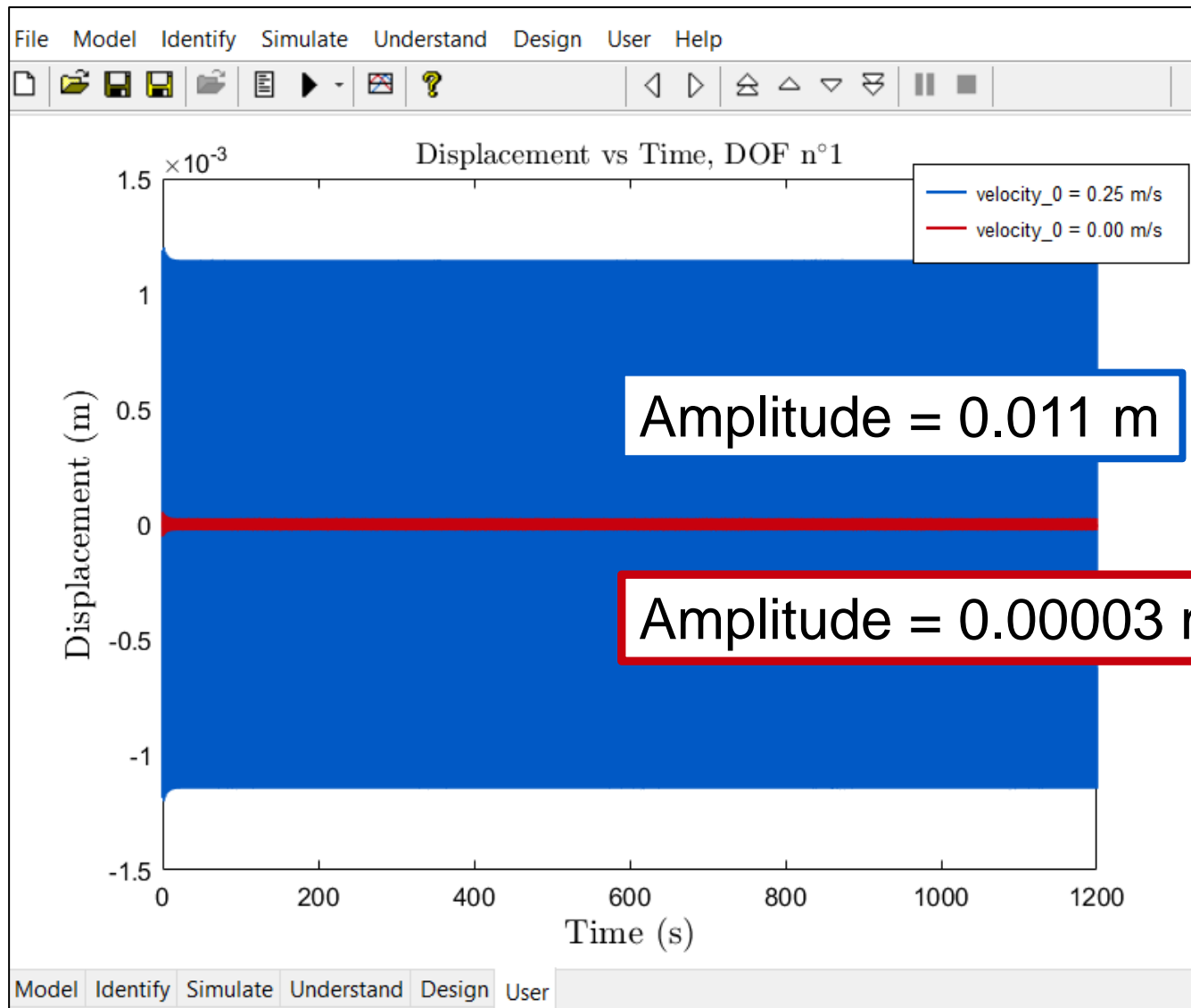
- Newmark parameters [SINE]**:
  - Final time: 1200
  - Time step: 0.001
  - Number of time steps: 1200000
  - Number of periods: 100
  - Time steps by period: 90
  - Saved dofs:  all  selected
  - Buttons: Initial cond. (circled in red), Apply, Run (F5), Cancel
- Initial conditions**:
  - Degree of freedom: all
  - Displacement: 0 m
  - Velocity: 0.25 m/sec (circled in green)
  - Buttons: Apply, Cancel

A red arrow points from the "Initial cond." button in the "Newmark parameters" dialog to the "Apply" button in the "Initial conditions" dialog.

# Run and Save in the Same Curve Stack



# Compare



# The Nonlinear FRF Helps to Understand

