Nonlinear Vibrations of Aerospace Structures

University of Liège, Belgium

L01a Introduction

Motivation

Objectives

Two keywords in the title:

Vibration & Nonlinear

Vibration in everyday life… also in the sky…

The Hippies Were Right: It's All about Vibrations, Man!

All things in our universe are constantly in motion, vibrating. Even objects that appear to be stationary are in fact vibrating, oscillating, resonating, at various frequencies. Resonance is a type of motion, characterized by oscillation between two states. And ultimately all matter is just vibrations of various underlying fields.

Vibration as a result of motion

Objective: create motion

↓

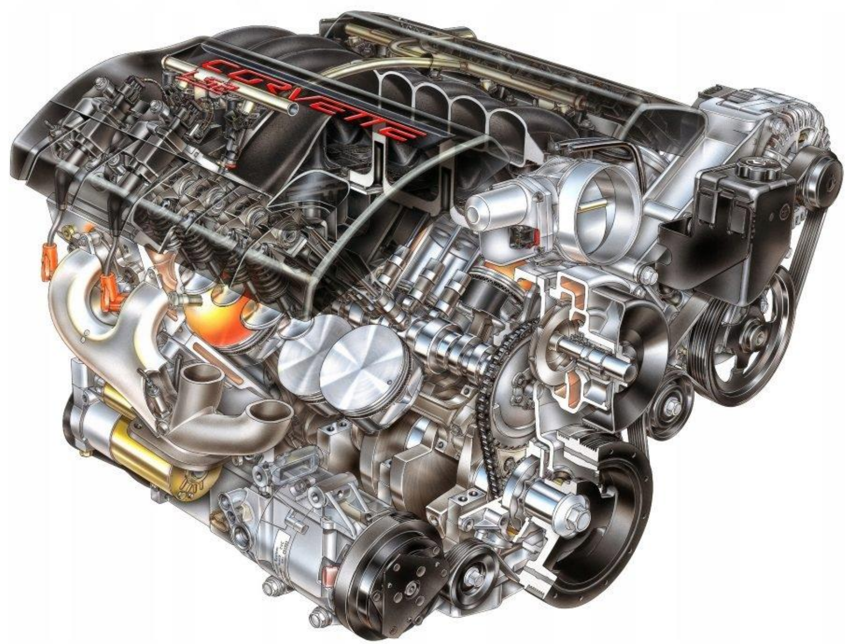
Apply force

↓

Vibration is generated

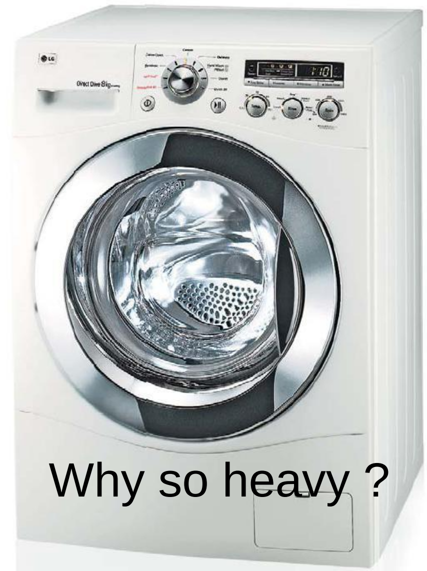
↓

Uncomfort, fatigue,
noise, damage,…



Elastomeric engine mounts are designed to absorb vibrations from the engine and isolate them from the vehicle's chassis, thereby reducing noise and vibrations felt by the driver and passengers

Vibration in everyday life ?

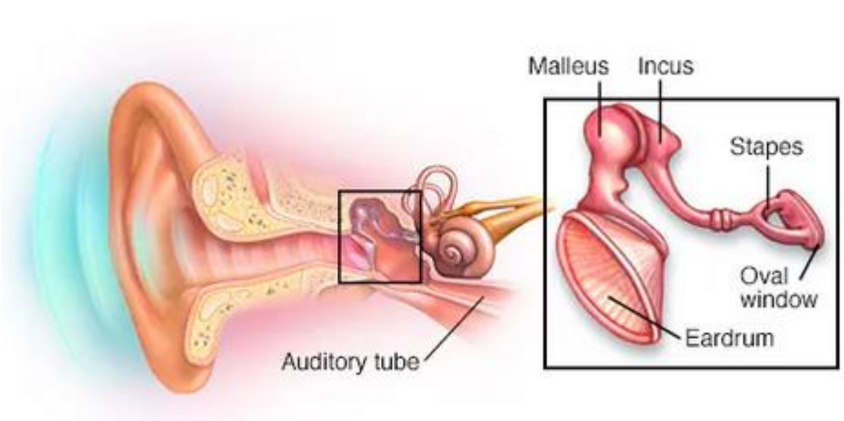


Why so heavy ?

3 reasons: acoustics,
comfort, damage !

Vibrations can also be beneficial: hear sound

Sound vibrations from the bones of the middle ear transfer to the fluids of the cochlea. Tiny sensors lining the cochlea, called hair cells, change the vibrations into electrical impulses that are sent along the auditory nerve to the brain.



The vibration of the eardrum starts a chain of vibrations through the bones. Because of differences in the size, shape and position of the three bones, the force of the vibration goes up by the time it gets to the inner ear. This rise in force is needed to transfer the energy of the sound wave to the fluid of the inner ear.

Vibrations can also be beneficial: create sound

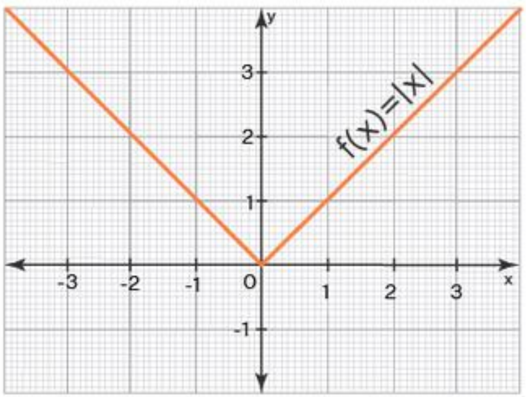
Vibrations of aerospace structures

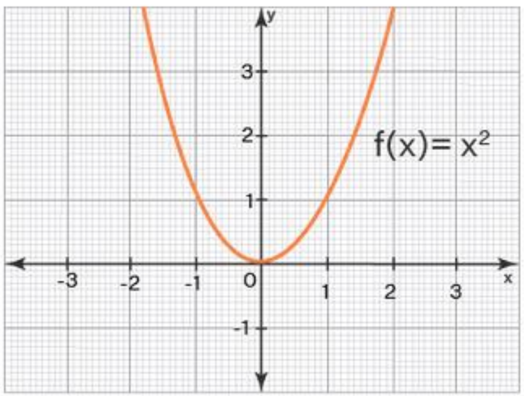
Aeroelastic flutter of
the F-16 aircraft

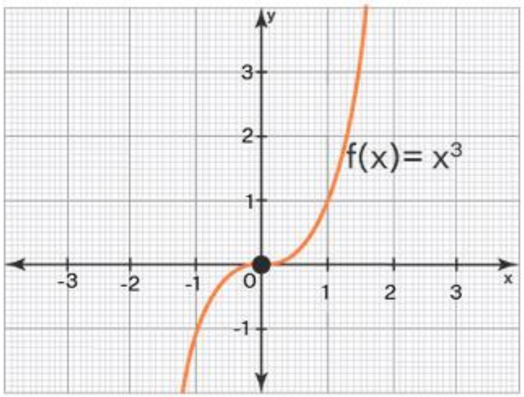
Ground resonance of
an helicopter

This course is about nonlinear vibration

A nonlinear function, as its name suggests, is a function that is NOT linear. Thus, the graph of a nonlinear function is NOT a line; its graph can be anything other than a line.

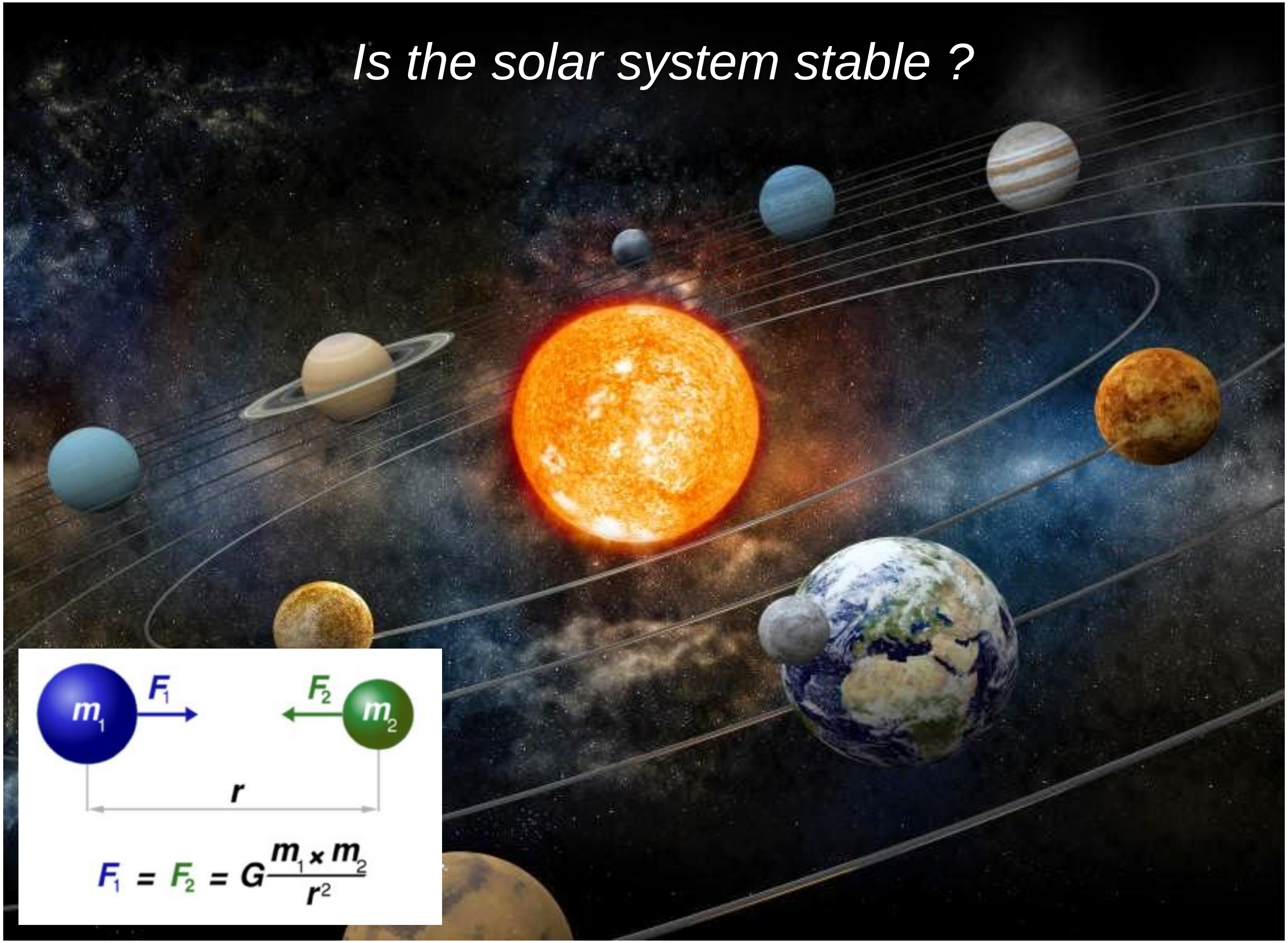






Nonlinear: why ?

Nature is nonlinear



You are nonlinear

YALE JOURNAL OF BIOLOGY AND MEDICINE 75 (2002), pp. 247-260.

MEDICAL REVIEW

Nonlinear Systems in Medicine

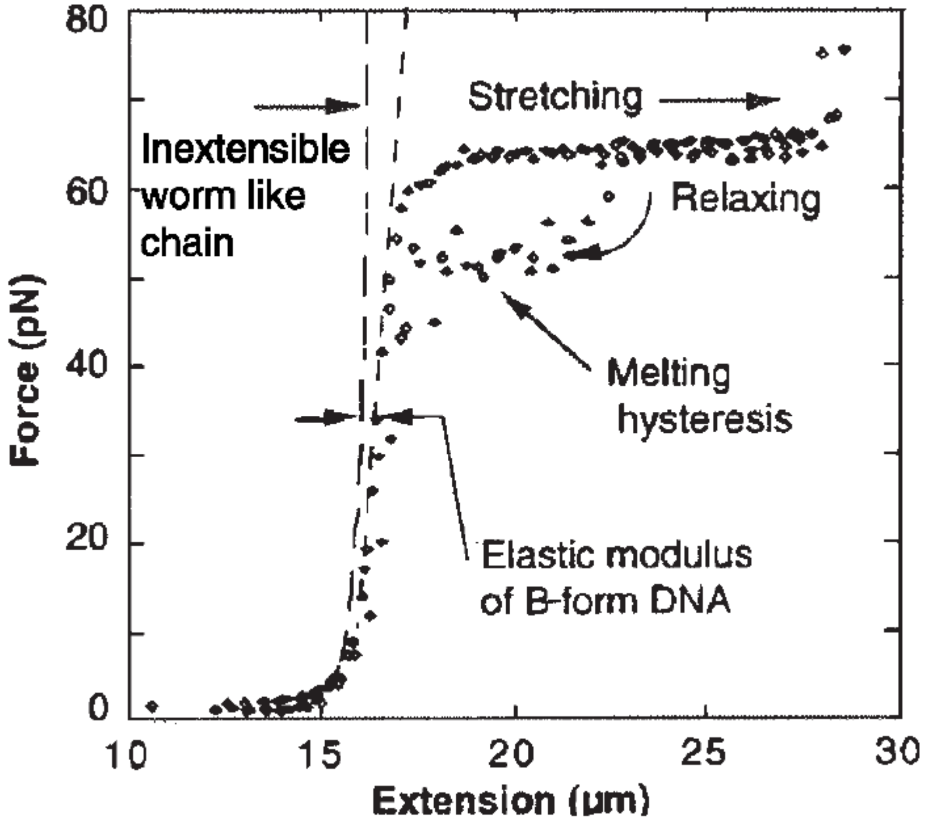
John P. Higgins[a]

Cardiology Section, Noninvasive Cardiac Laboratories, VA Boston Healthcare System, Boston, Massachusetts

Table 2. Manifestations of chaotic behavior in medicine

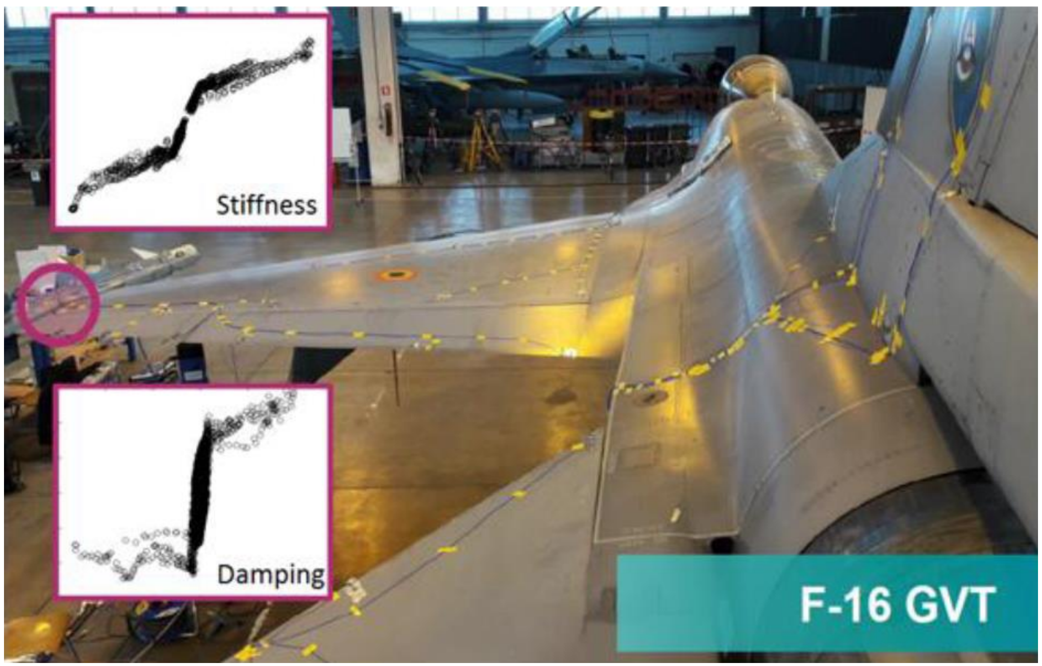
Field	Example [Reference]
Anatomy	• Fractal structure of arteries, veins, cancellous bone tissue, nerves, pulmonary alveoli, tracheobronchial tree [25-27]. • Regional myocardial blood flow heterogeneity follows a simple fractal relation [28].
Cellular biology	• Properties of ion channel proteins [29]. • Stem cell differentiation [30]. • Calcium oscillations and intracellular signaling [31].
Molecular biology	• Fractal dimension to exon structures of DNA [32].
Oncology	• Abnormal mammographic parenchymal fractal pattern to screen for breast cancer [33]. • Fractal dimension in positron emission tomography scanning to detect melanoma [34]. • Fractal analysis in detection of colon cancer [35]. • Fractal dimension analysis in evaluating response to chemotherapy [36].
Internal medicine	• Cytokine behavior exhibiting emergent patterns [37]. • Variability in respiratory impedance in Asthma [38].
Cardiology	• Atrial fibrillation can arise from a quasiperiodic stage of period and amplitude modulation, i.e., "quasiperiodic transition to chaos" [39]. • Chaos control of arrhythmias in human subjects by stabilizing an unstable target rhythm [40]. • Endothelial function behavior [7]. • Heart rate variability: - Low-dimensional chaos associated with health [41, 42]. - Reduced fractal behavior associated with increase in sudden cardiac death [43]. - Predicting mortality following myocardial infarction [44].

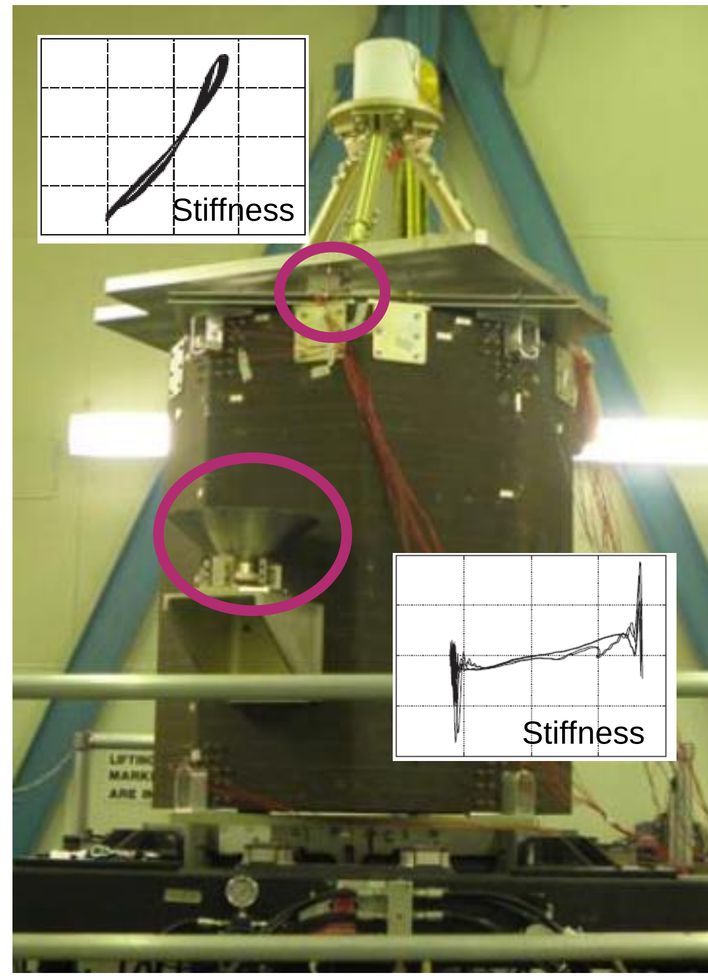
You are nonlinear



DNA molecule: Smith, Finzi, Bustamante, Science, 1992

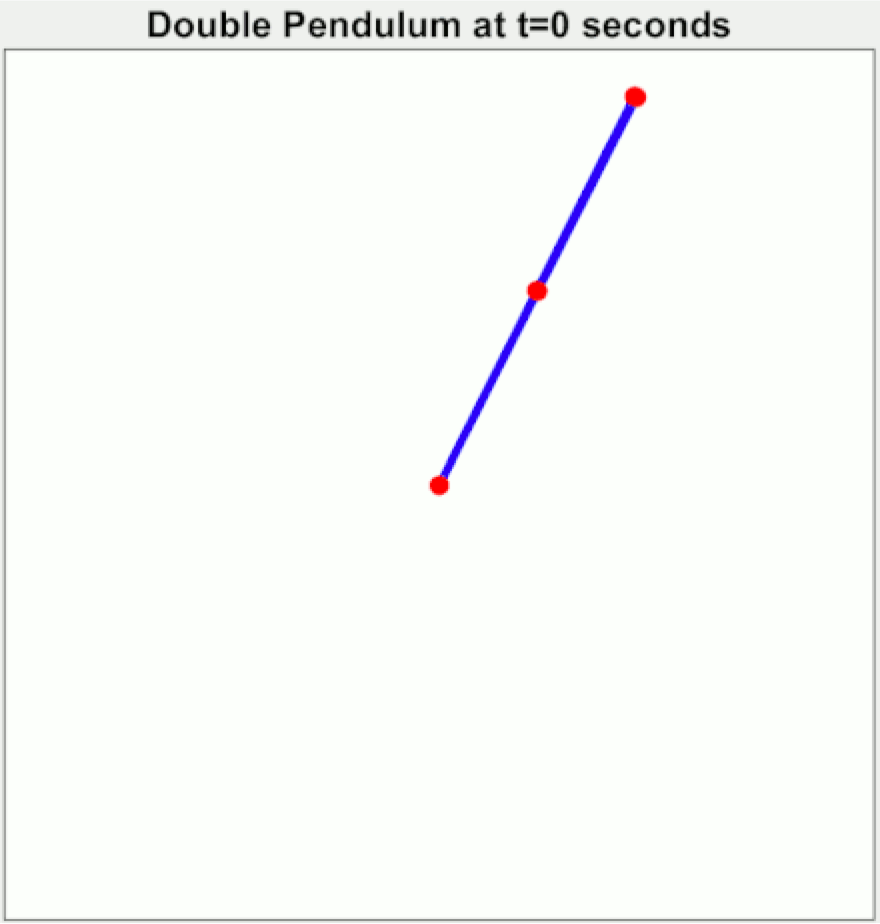
Aerospace structures are nonlinear





Nonlinear vibrations can be extremely complex

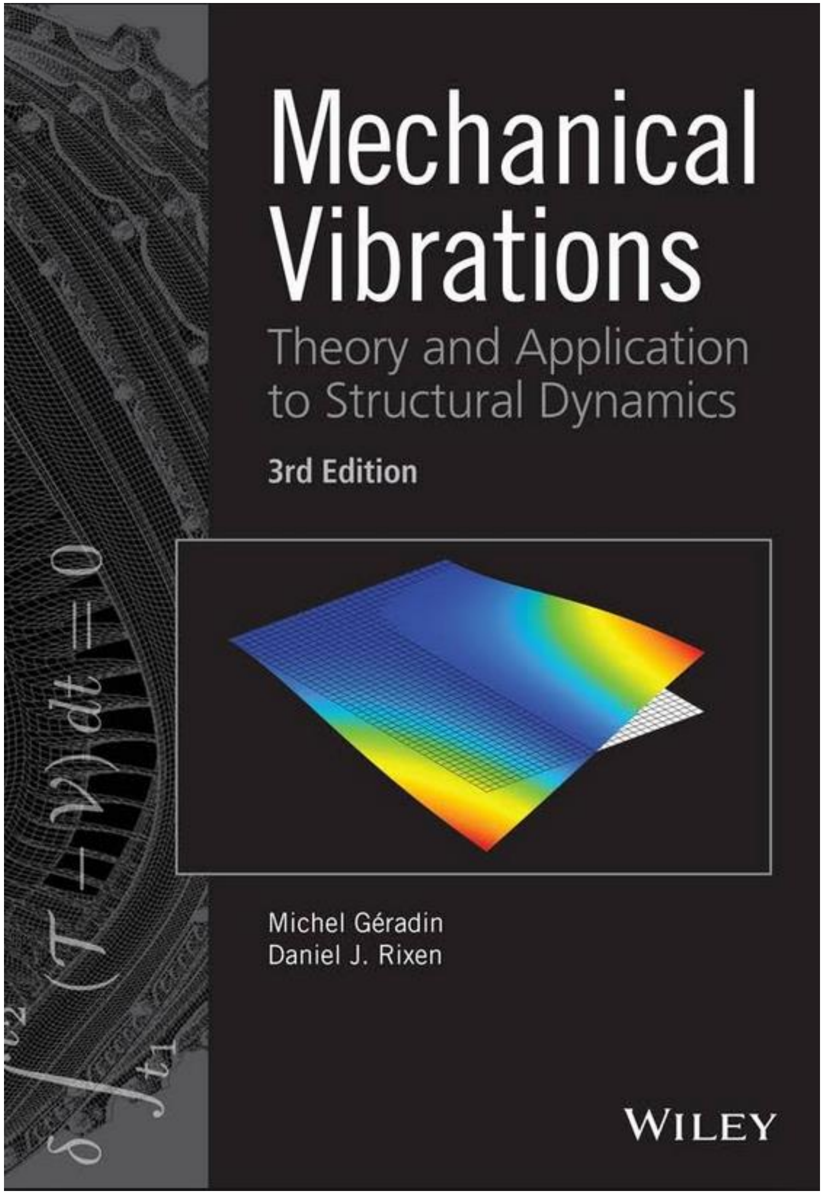
Credit: S. Troy, UWash



Three double pendulums with near identical initial conditions diverge over time, displaying the chaotic nature of the system.

What you’ve done so far

The starting point of this course



Nonlinearity is the norm

2

Undamped Vibrations of n-Degree-of-Freedom Systems

From the theory and examples discussed in Chapter 1 one should remember that the description of the dynamics of a mechanical system as obtained from Hamilton's principle, from the Lagrange equations or from the principle of virtual work generally leads to a set of nonlinear equations.

Linearization

2

Undamped Vibrations of n-Degree-of-Freedom Systems

From the theory and examples discussed in Chapter 1 one should remember that the description of the dynamics of a mechanical system as obtained from Hamilton's principle, from the Lagrange equations or from the principle of virtual work generally leads to a set of nonlinear equations. Solving such nonlinear equations usually requires applying time-integration techniques as those described later in Chapter 7. Nevertheless, for many practical applications, dynamical behaviour manifests itself only as time-varying perturbation around a static solution. Indeed systems can often be described as being in an statical equilibrium configuration, around which they undergo only small dynamic motion, namely *vibrations*.

In that case, the description of the system can be significantly simplified and a linearization around an equilibrium position of the generally nonlinear dynamic equations is possible.

The linearization concept: potential energy

Linearization of potential and kinetic energies

By *linearized form of energy* we mean a quadratic expression in terms of generalized displacements or their derivatives, so that the corresponding generalized forces are linear.

Since the generalized displacements q_s represent deviations from equilibrium, the potential energy could be expanded in the form of a Taylor series in the neighbourhood of the equilibrium position:

$$\mathcal{V}(\boldsymbol{q}) = \mathcal{V}(\boldsymbol{0}) + \sum_{s=1}^{n} \left(\frac{\partial \mathcal{V}}{\partial q_s} \right)_{\boldsymbol{q}=0} q_s + \frac{1}{2} \sum_{s=1}^{n} \sum_{r=1}^{n} \left(\frac{\partial^2 \mathcal{V}}{\partial q_s \partial q_r} \right)_{\boldsymbol{q}=0} q_s q_r + O(\boldsymbol{q}^3)$$

by assuming that $\mathcal{V}$ has C_1 continuity at $q_s = 0$.

Since the potential energy is defined only to a constant, we have made earlier the assumption that $\mathcal{V}(\boldsymbol{0}) = 0$. Furthermore, all forces being in equilibrium at $\boldsymbol{q} = \boldsymbol{0}$, (2.2) holds and one obtains the second-order approximation:

$$\mathcal{V}(\boldsymbol{q}) = \frac{1}{2} \sum_{s=1}^{n} \sum_{r=1}^{n} k_{sr} q_s q_r > 0 \quad \text{for } \boldsymbol{q} \neq \boldsymbol{0} \tag{2.6}$$

with the *stiffness coefficients*:

$$k_{sr} = k_{rs} = \left(\frac{\partial^2 \mathcal{V}}{\partial q_s \partial q_r} \right)_{\boldsymbol{q}=0} \tag{2.7}$$

Expression (2.6) may also be put in the matrix positive definite quadratic form:

$$\mathcal{V}(\boldsymbol{q}) = \frac{1}{2} \boldsymbol{q}^T \boldsymbol{K} \boldsymbol{q} > 0 \quad \text{for } \boldsymbol{q} \neq \boldsymbol{0} \tag{2.8}$$

where $\boldsymbol{K}$, the *linear stiffness matrix* of the system, of general term (2.7), is *symmetric and positive definite*.

The linearization concept: kinetic energy

The kinetic energy reduces here to the sole term $\mathcal{T}_2$. Since $\mathcal{T}_2$ is homogeneous and quadratic in velocities, the linearized expression is obtained by expanding the coefficients of the quadratic form in the neighbourhood of $q_s = 0$:

$$\begin{aligned}
\mathcal{T}_2(\dot{q}, q) &= \frac{1}{2} \sum_{s=1}^{n} \sum_{r=1}^{n} \left(\frac{\partial^2 \mathcal{T}_2}{\partial \dot{q}_s \partial \dot{q}_r} \right) \dot{q}_s \dot{q}_r \\
&= \frac{1}{2} \sum_{s=1}^{n} \sum_{r=1}^{n} \left(\frac{\partial^2 \mathcal{T}_2}{\partial \dot{q}_s \partial \dot{q}_r} \right)_{q=0} \dot{q}_s \dot{q}_r + O(\dot{q}^2, q)
\end{aligned}$$

We thus adopt the second-order approximation:

$$\mathcal{T}_2(\dot{q}) = \frac{1}{2} \sum_{s=1}^{n} \sum_{r=1}^{n} m_{sr} \dot{q}_s \dot{q}_r \tag{2.9}$$

The fundamental equation of linear vibration

Equations for free vibrations about a stable equilibrium position

Making use of the definitions (2.6) and (2.9) for potential and kinetic energies, the Lagrange equations provide the system of equations of motion governing the free vibrations about a stable equilibrium position:

$$\sum_{r=1}^{n}(m_{sr}\ddot{q}_r + k_{sr}q_r) = 0 \qquad s = 1, \ldots, n$$

or, in matrix form:

$$\boxed{\boldsymbol{M\ddot{q}} + \boldsymbol{Kq} = \boldsymbol{0}} \tag{2.12}$$

This equation is the fundamental equation describing the free vibration of a conservative system around an equilibrium position.

$$\boxed{M\ddot{y} + C\dot{y} + Ky = 0}$$

with viscous damping

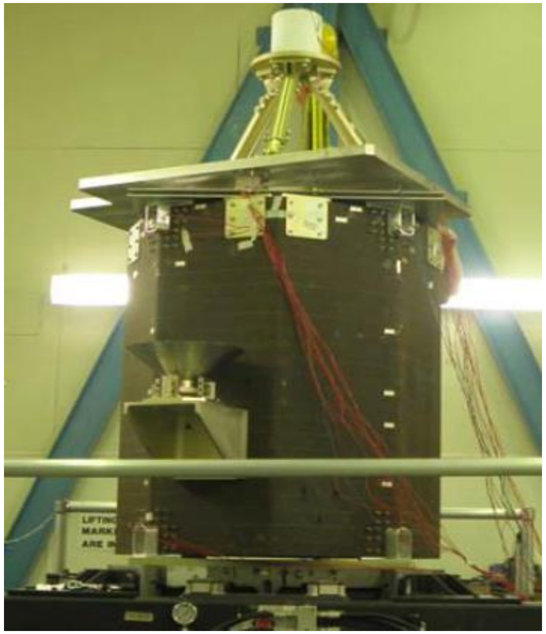
3 main assumptions in linear structural dynamics

$$\boxed{M\ddot{y} + C\dot{y} + Ky = 0}$$

Linear elasticity	→ nonlinear materials
Small displ. and rotations	→ geometrical nonlinearity → nonlinear boundary conditions
Viscous damping	→ nonlinear damping mechanisms

Aerospace structures are nonlinear

Real-life examples

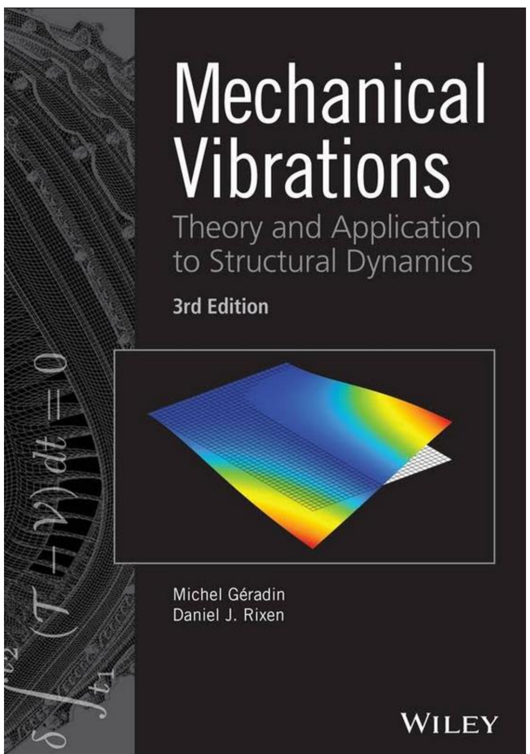


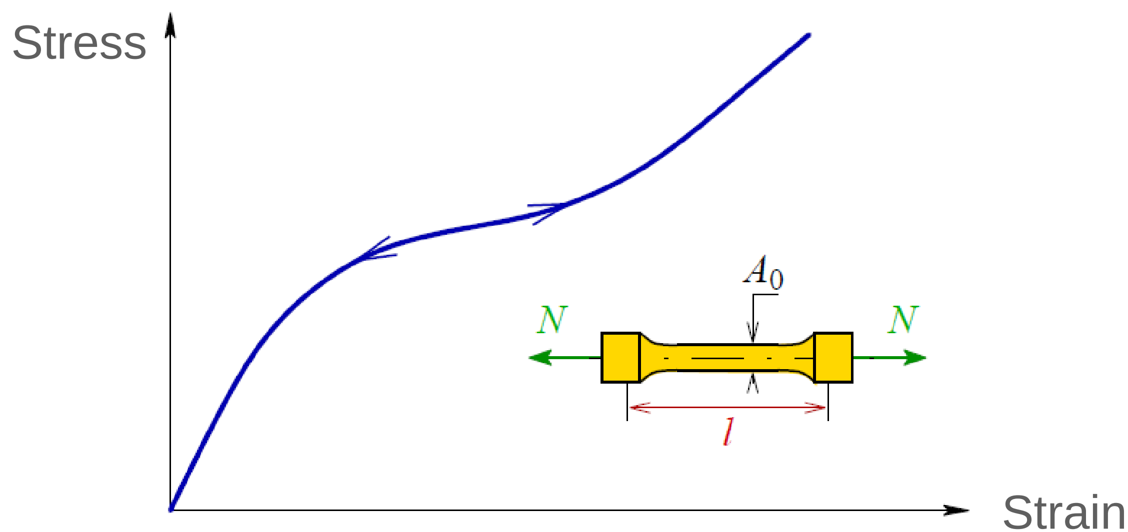
Source of nonlinearity #1: materials

Linear material

A material has linear elastic properties when the stress state remains strictly proportional to the strain state. Owing to the symmetry of the tensors σ_{ij} and ε_{kl}, such a material is characterized in the general case by 21 distinct coefficients C_{ijkl} so that:

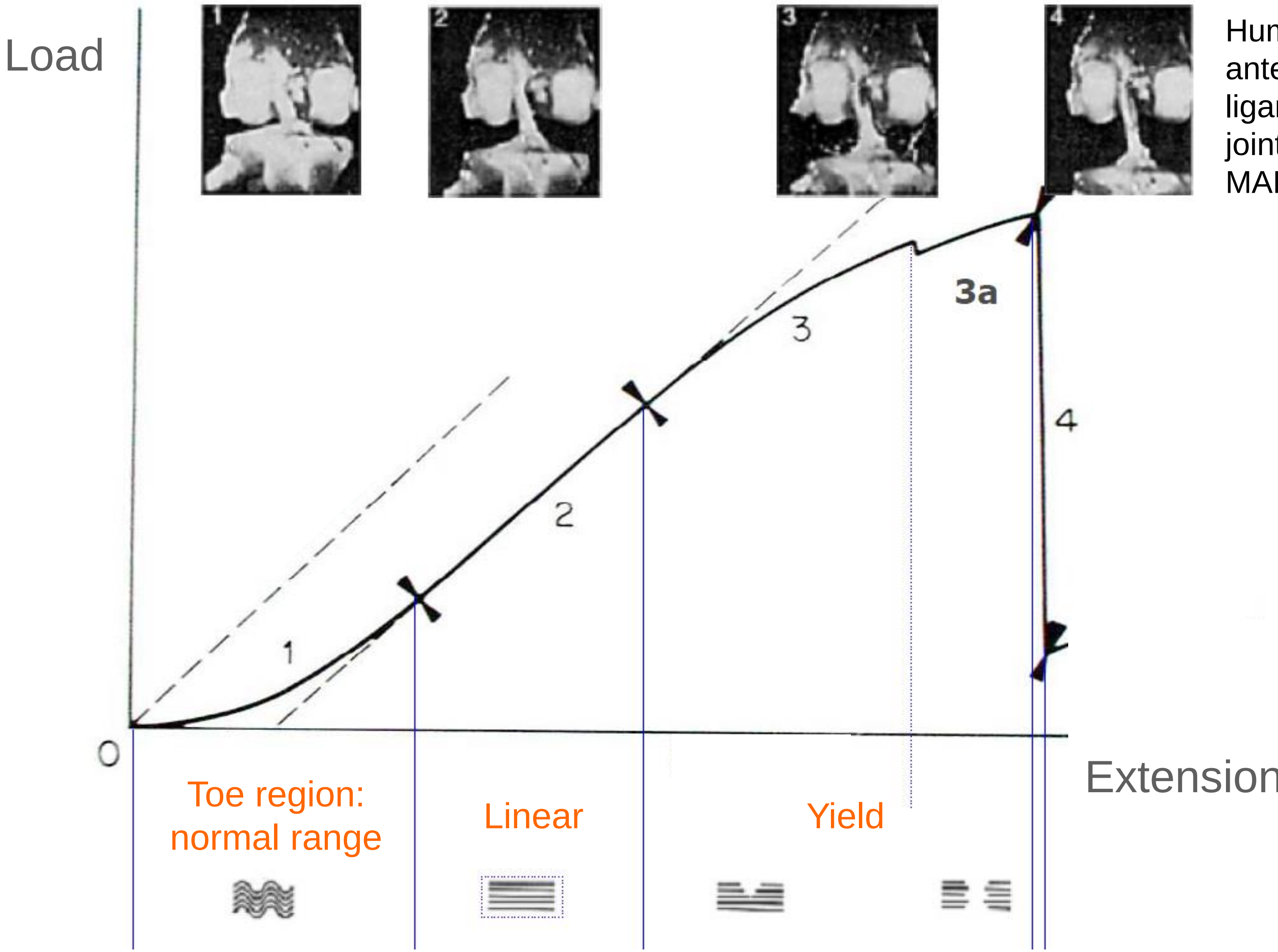
$$\sigma_{ij} = C_{ijkl}\, \varepsilon_{kl} \tag{4.14}$$





Hyperelastic material
(e.g., rubber)

Source of nonlinearity #1: ligament in your knee joint



Human cadaveric anterior cruciate ligament in knee joint (Dr. Ziv, MAE, Buffalo)

Source of nonlinearity #1: Airbus A400M

Sources of nonlinearity:

- Gaps: joints, latches, riveted structures
- Elastomeric mounts (e.g. engine mounts)
- Hydraulic actuators (e.g. control surfaces)

Implications / Complications:

- Structural excitation is no longer trivial (paths, levels are relevant)
- Basic assumptions of standard modal analysis may not hold (e.g. linearity, reciprocity)
- Nonlinearity laws may be complex
- Multiple nonlinear mechanisms may combine
- Standard curve fitting algorithms may fail

Industrial Approach:

Combination of:

a) Multi-level sine sweeps (PSM) and assumption of piecewise linearity
b) Normal Modes Tuning (PRM) for most critical modes

AIRBUS MILITARY

Source of nonlinearity #2: large displacements

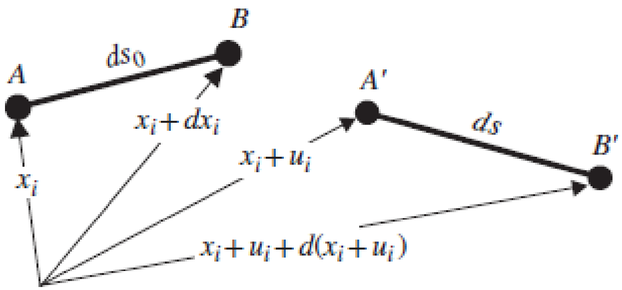
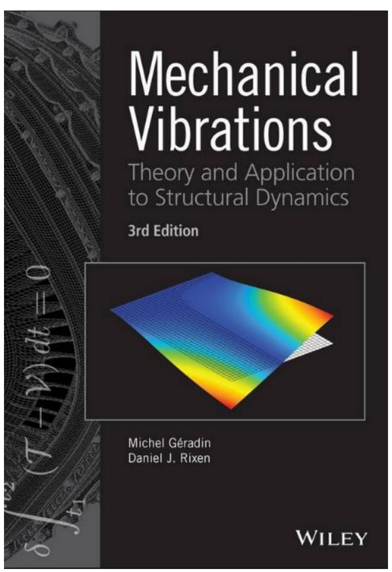


Figure 4.2 Strain measure of a segment.



Let us denote ds_0 and ds as the lengths of segment $\overline{AB}$ before and after deformation:[1]

$$ds_0^2 = dx_i\, dx_i$$
$$ds^2 = d(x_i + u_i)\, d(x_i + u_i)$$

By taking account of:

$$du_i = \frac{\partial u_i}{\partial x_j} dx_j \qquad i = 1, 2, 3 \tag{4.1}$$

one may write:

$$\begin{aligned} ds^2 &= \left(dx_i + \frac{\partial u_i}{\partial x_j} dx_j \right) \left(dx_i + \frac{\partial u_i}{\partial x_m} dx_m \right) \\ &= dx_i\, dx_i + \frac{\partial u_i}{\partial x_j} dx_j\, dx_i + \frac{\partial u_i}{\partial x_m} dx_m\, dx_i + \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_m} dx_j\, dx_m \\ &= dx_i\, dx_i + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_m}{\partial x_j} \frac{\partial u_m}{\partial x_i} \right) dx_i\, dx_j \end{aligned}$$

Let us next define the components of *Green's symmetric strain tensor* ε_{ij} in such a way that the length increment of segment $\overline{AB}$ is expressed as:

$$ds^2 - ds_0^2 = 2\varepsilon_{ij}\, dx_i\, dx_j \tag{4.2}$$

Hence, according to the previous expressions, the strain components may be written:

Green's strain tensor

$$\boxed{\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_m}{\partial x_i} \frac{\partial u_m}{\partial x_j} \right)} \tag{4.3}$$

It is a symmetric second-order tensor, quadratic in the displacements u_i. Relations (4.2) and (4.3) directly imply the following:

Source of nonlinearity #2: large displacements

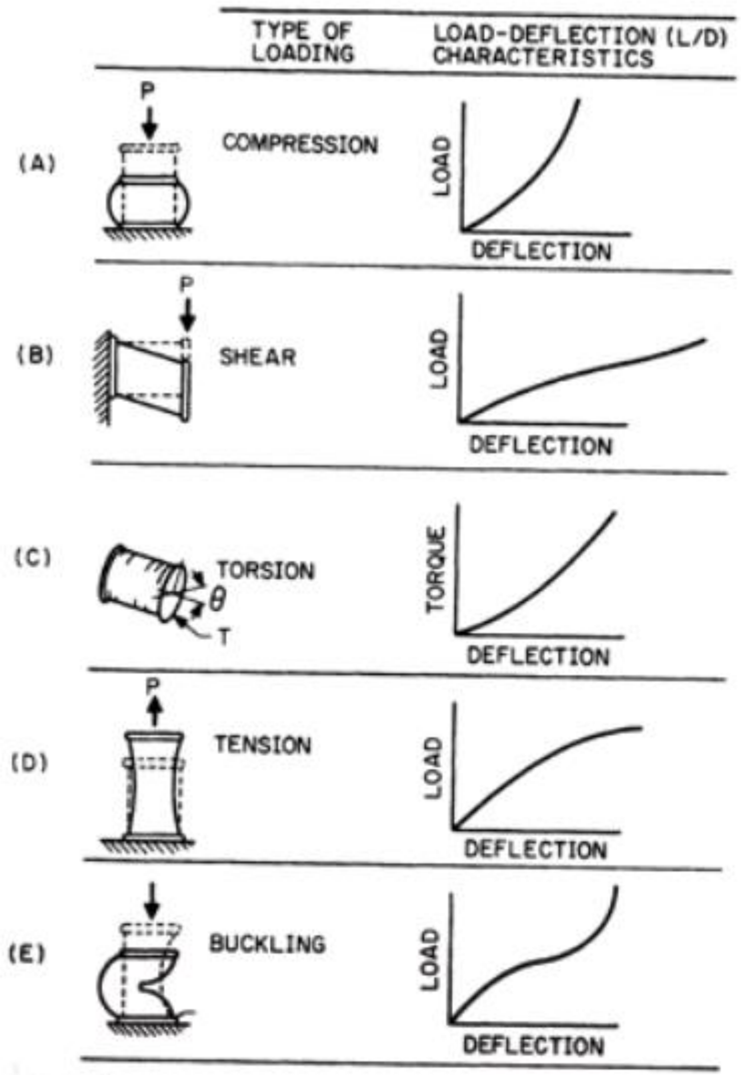
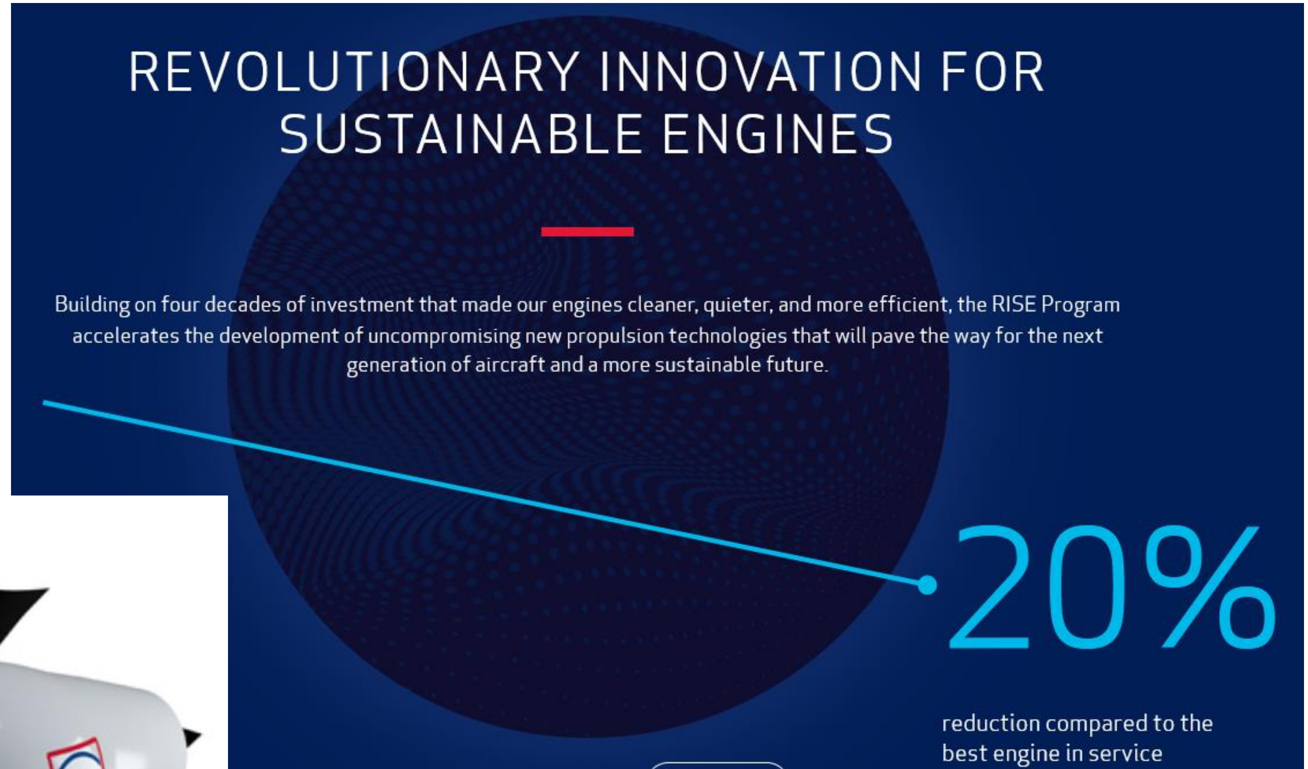


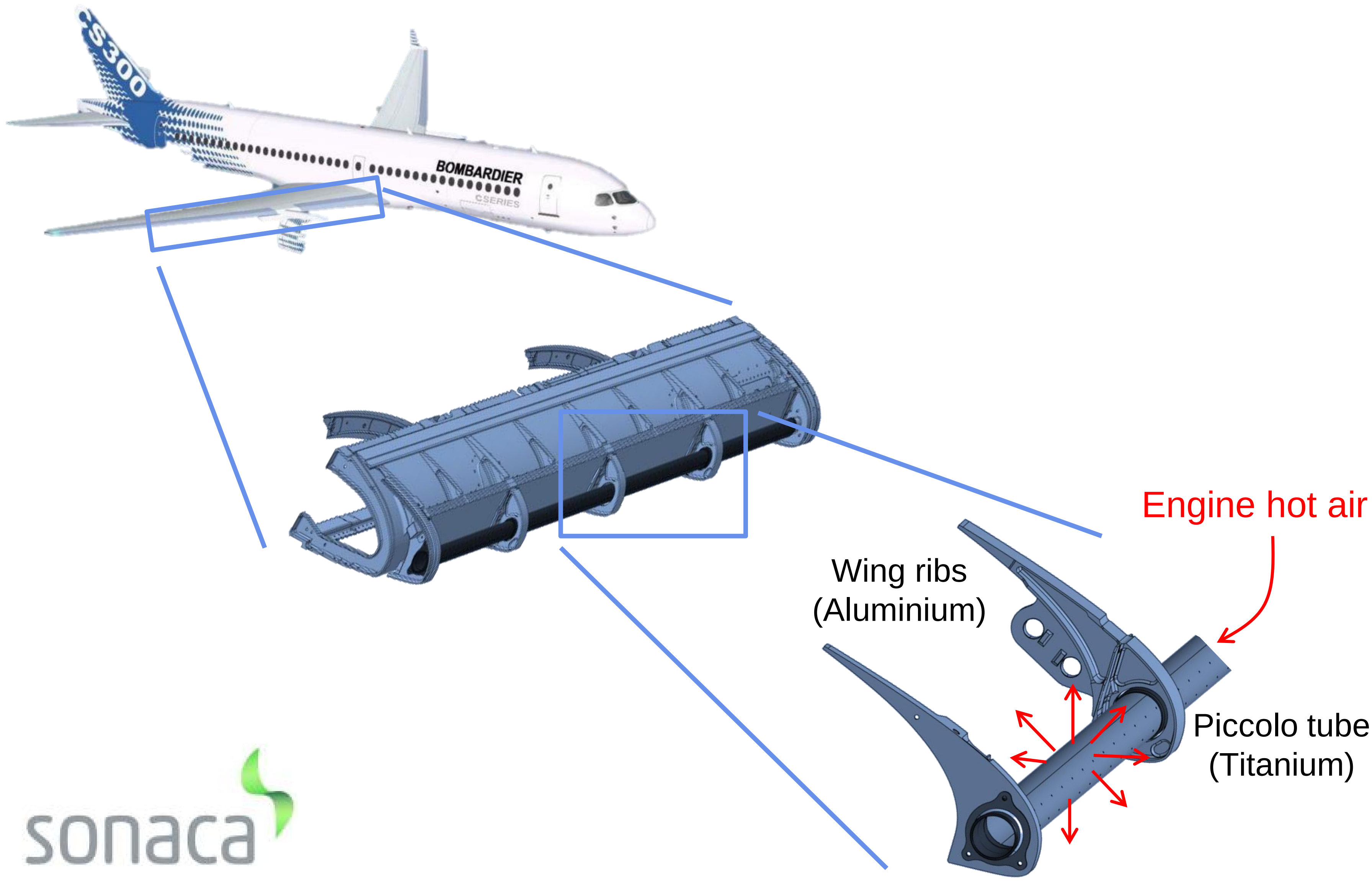
FIGURE 32.2 Load-deflection characteristics of typical elastomeric isolators. (*After R. Racca.*[1])

Source of nonlinearity #2: large displacements

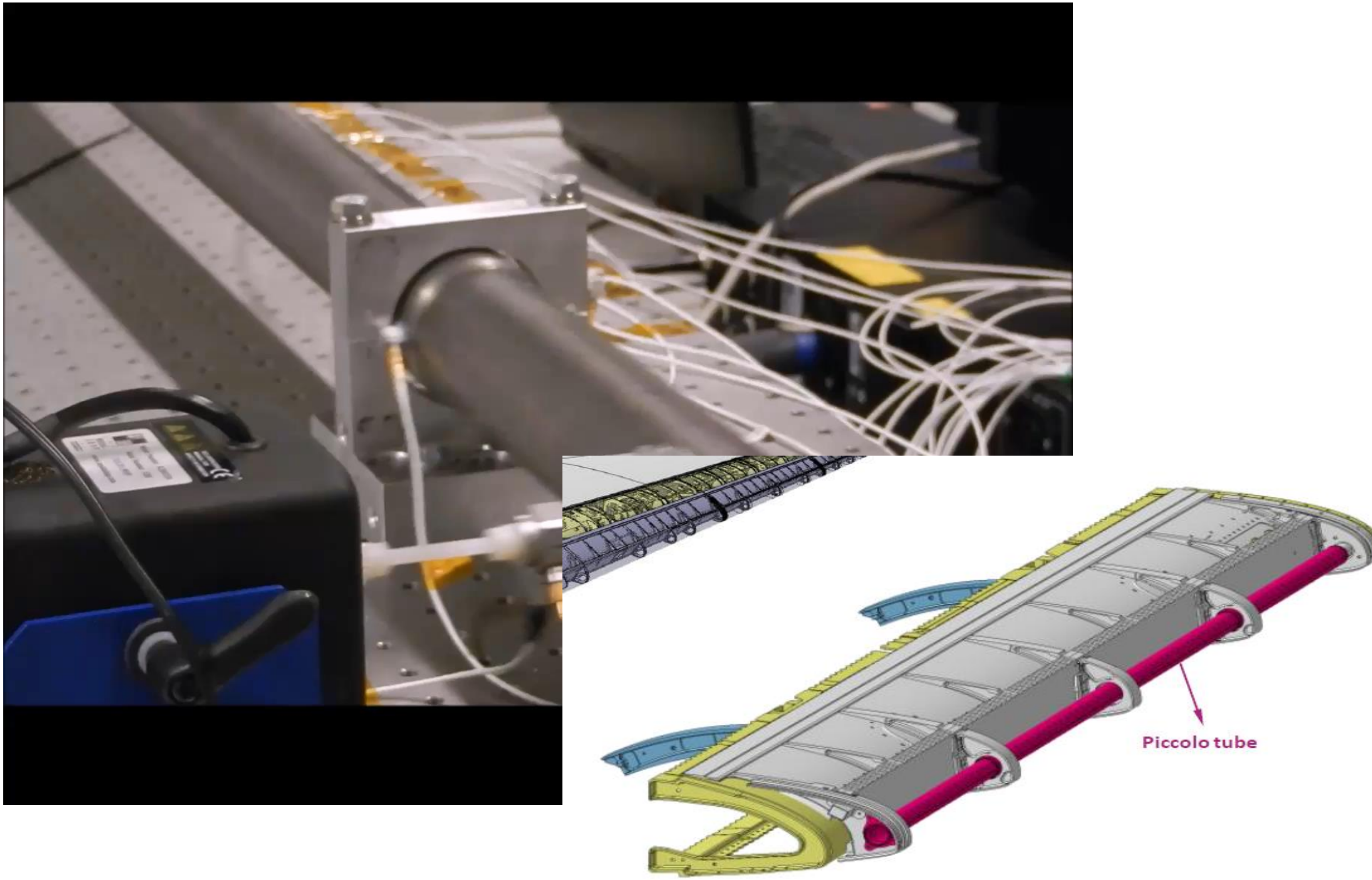


To guarantee excellent structural properties with limited weight, lighter materials are used. Since the structures are light and slender, they are more flexible. As a result, they are prone to significant structural displacements outside the linear regime, known as geometric non-linearities.

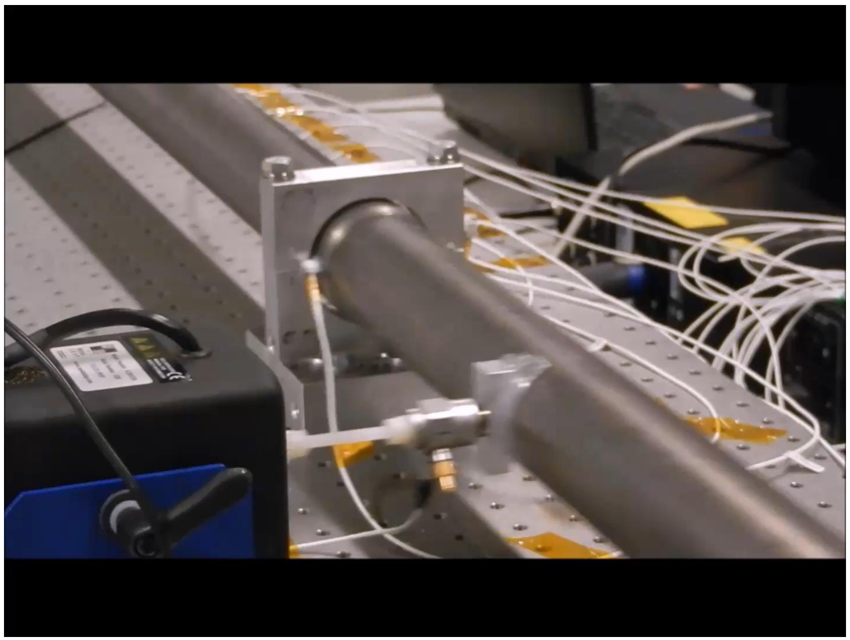
Source of nonlinearity #3: boundary conditions

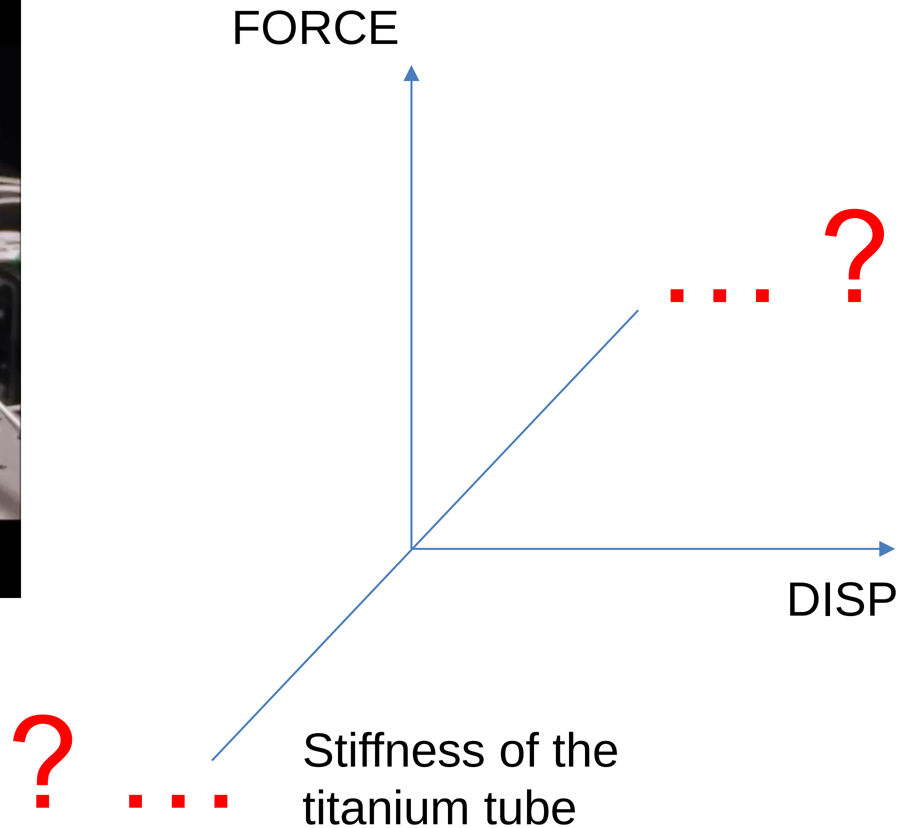


Source of nonlinearity #3: boundary conditions



Source of nonlinearity #3: guess the NL function ?





Source of nonlinearity #4: damping

Linear viscours damping friction

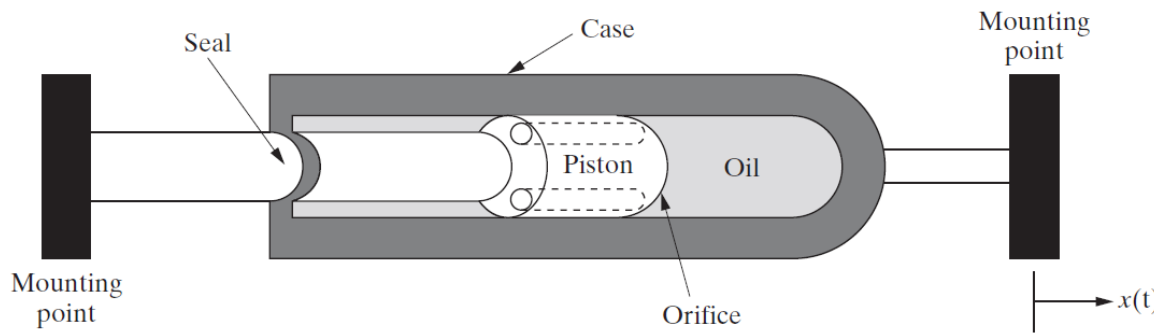
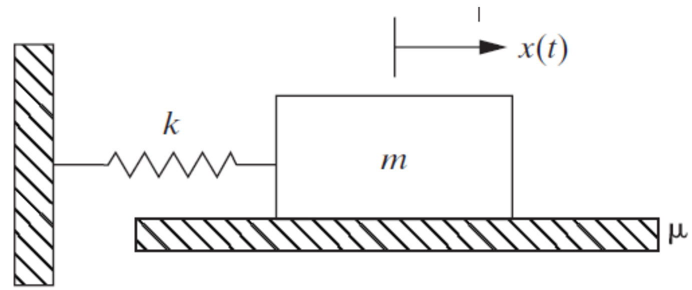


Figure 1.8 A schematic of a dashpot that produces a damping force $f_c(t) = c\dot{x}(t)$, where $x(t)$ is the motion of the case relative to the piston.

Coulomb friction (interfacial damping)

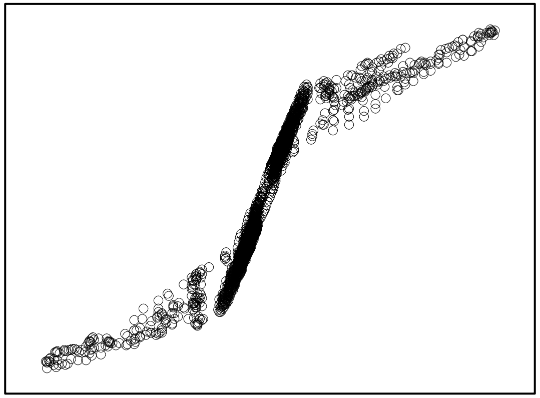


$$m\ddot{x} + \mu mg\,\mathrm{sgn}(\dot{x}) + kx = 0$$

Aerodynamic damping and material damping

Source of nonlinearity #4: F-16 aircraft

Force

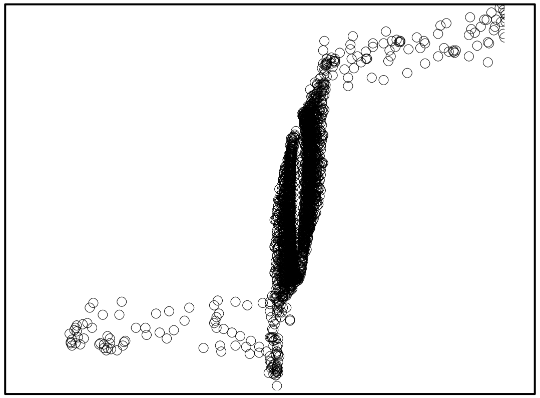


Displacement

Right missile

Connection with the wing

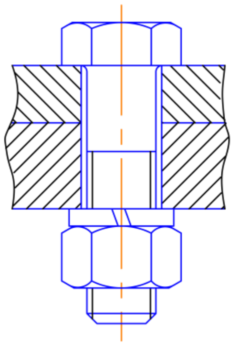
Force



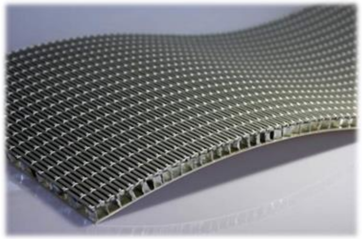
Velocity

Common sources of nonlinearity

Bolts, joints and gaps



Elastomers and composites



Friction and contact

Large amplitudes

Can I be safe with a linearized model ?

What did Richard Feynman mean about the Second Law of Motion? Where was the error?

JANUARY 17, 2021 / FRANCES48 / 0 COMMENTS

Richard Feynman writes about Newton's Second Law of Motion in his work "Lectures on Physics" (Chapter 15):

„For over 200 years the equations of motion enunciated by Newton were believed to describe nature correctly, and the first time that an error in these laws was discovered, the way to correct it was also discovered. Both the error and its correction were discovered by Einstein in 1905.

Newton's Second Law, which we have expressed by the equation

$$F = d(mv)/dt$$

was stated with the tacit assumption that m is a constant, but we now know that this is not true, and that the mass of a body increases with velocity. In Einstein's corrected formula m has the value

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$

where the rest mass represents the mass of a body that is not moving and c is the speed of light […].

Nonlinear: how ?

Course objectives

At the end of this course, you will

- Understand the impact of nonlinearity on system dynamics.
- Master the concepts of mode shape, resonance frequency and frequency response of nonlinear systems.
- Be familiar with new nonlinear concepts including stability and bifurcations.
- Recognize nonlinearity in real-world (aerospace) structures.
- Know how to use the NI2D software.

3 main parts in the NLVIB course

1. THEORY
- 1DOF systems (free and forced response)
- 2DOF systems (free and forced response)

2. NUMERICAL COMPUTATION
- How to calculate the response of a nonlinear system numerically ?
- How to calculate bifurcation diagrams ?

3. EXPERIMENTAL DATA ANALYSIS
- How to recognize the presence of nonlinearity in experimental data ?
- How to know more about the nonlinearity ?

COURSE PROJECT
- Experimental data analysis
- Develop your own numerical solver

Course evaluation

January: course project and its oral defense (100%)

August: oral exam based on the course slides

Course philosophy