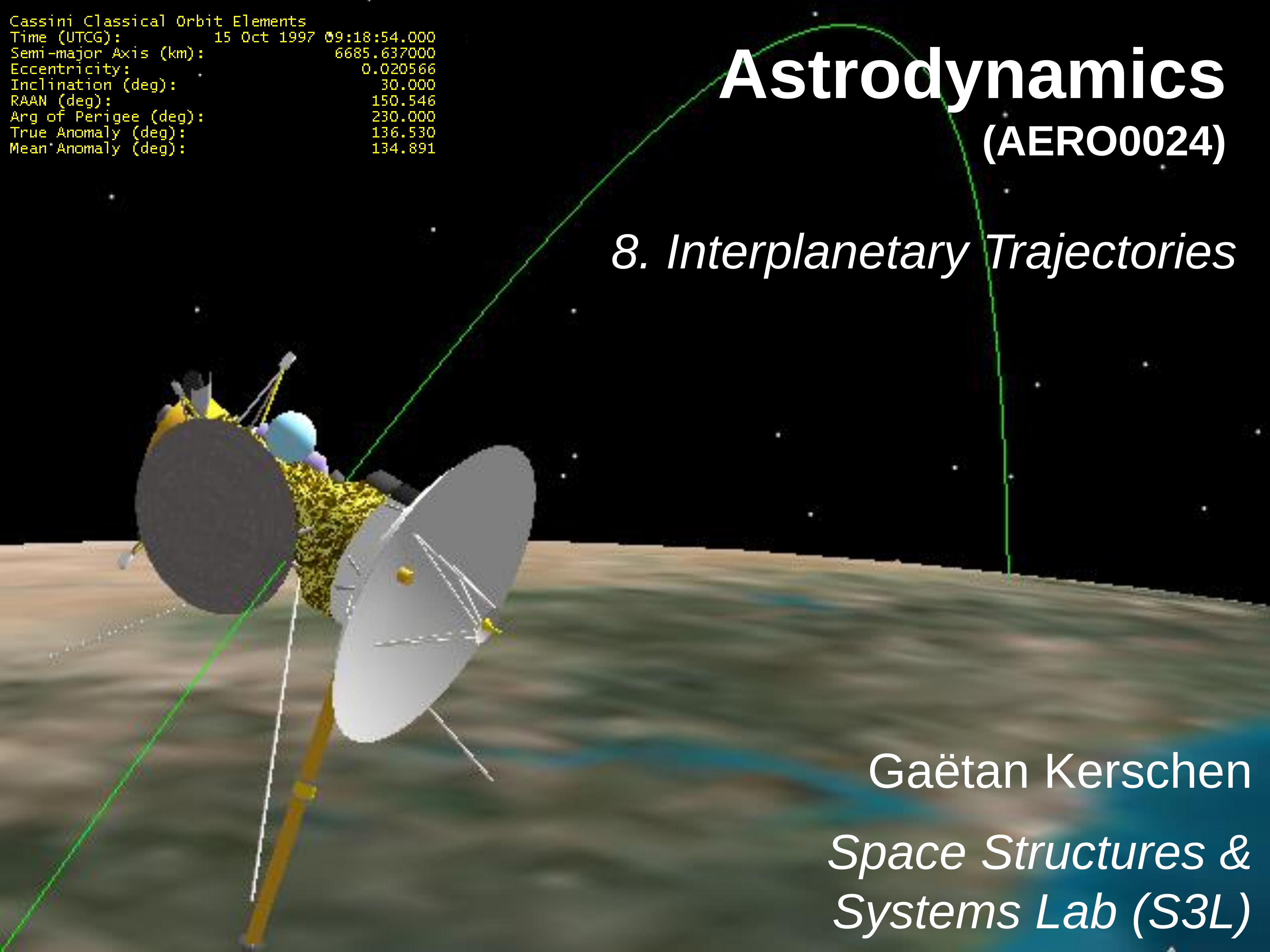

Astrodynamics

(AERO0024)

8. Interplanetary Trajectories

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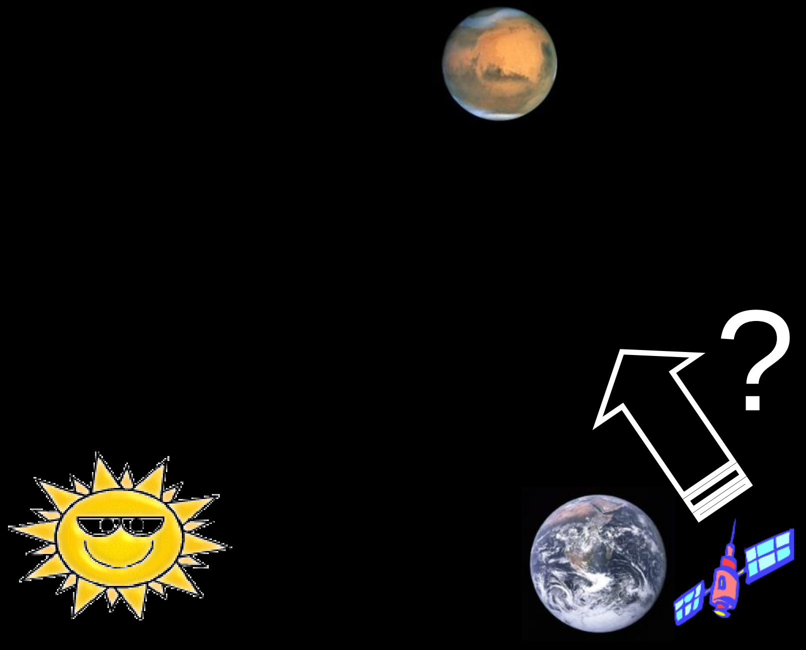
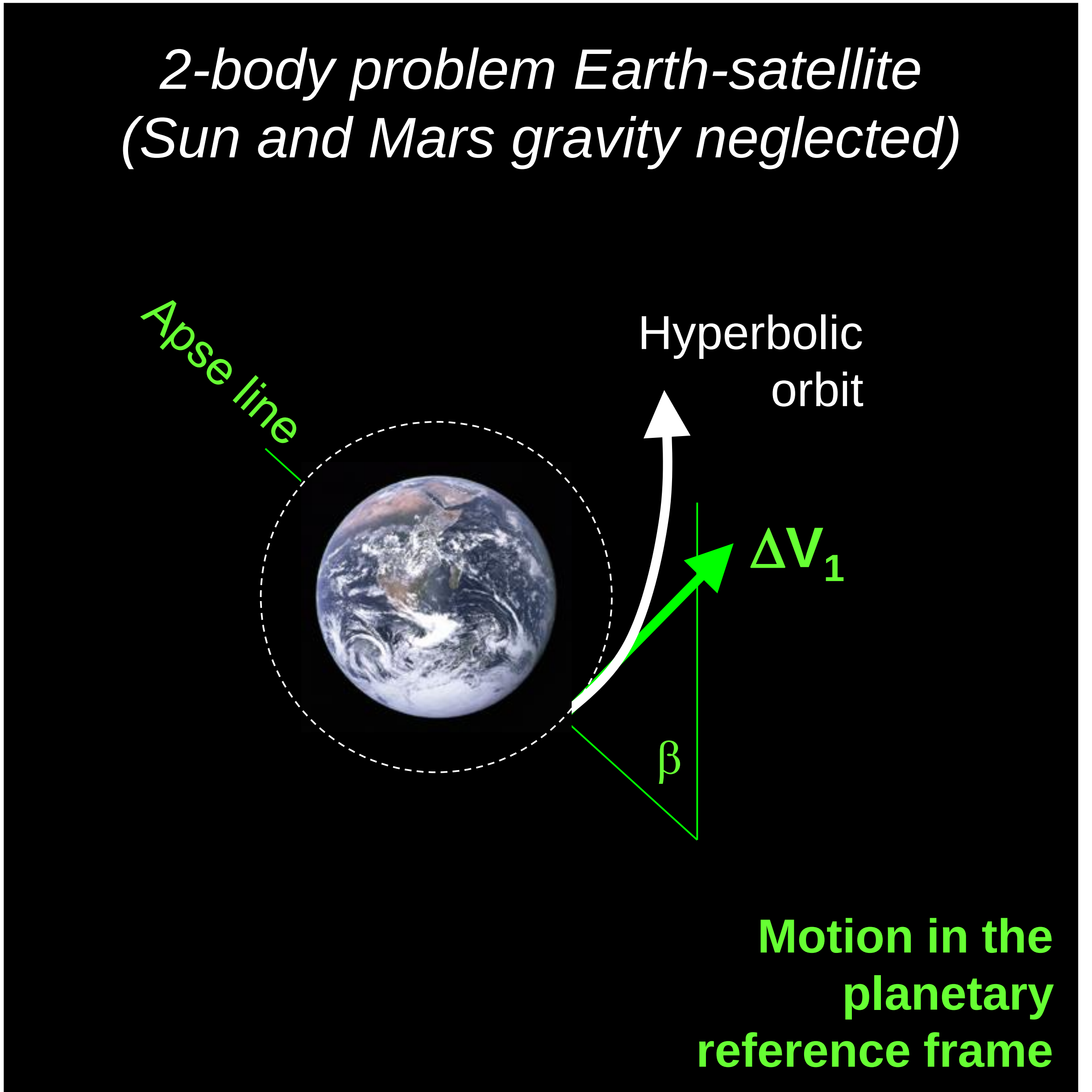
Space Structures & Systems Lab (S3L)



Step 1: escape the Earth



SOI: Correct Definition due to Laplace

It is the surface along which:

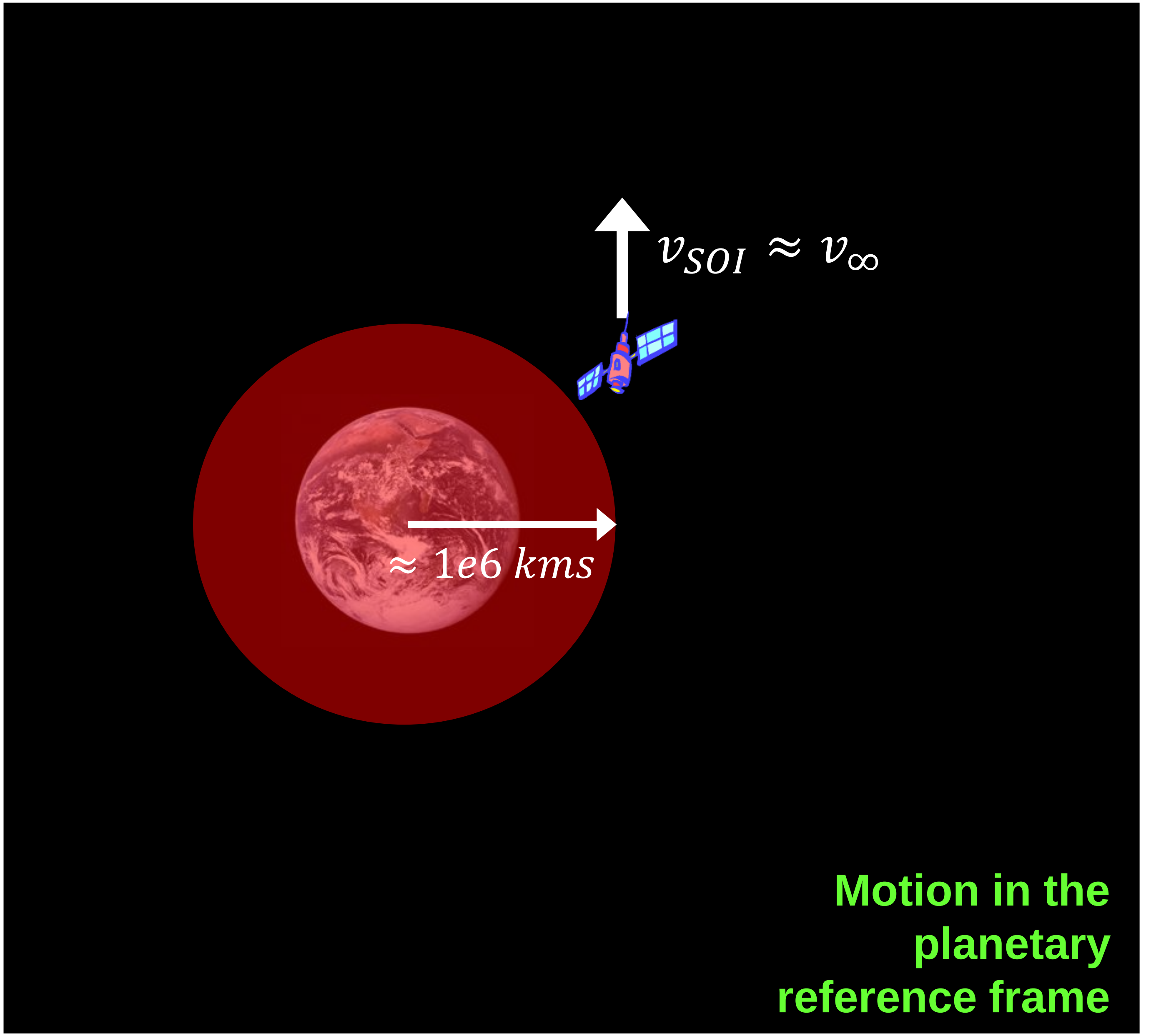
$$\frac{P_p}{A_s} = \frac{P_s}{A_p}$$

Measure of the planet's influence on the orbit of the vehicle relative to the sun ($\frac{P_p}{A_s}$)

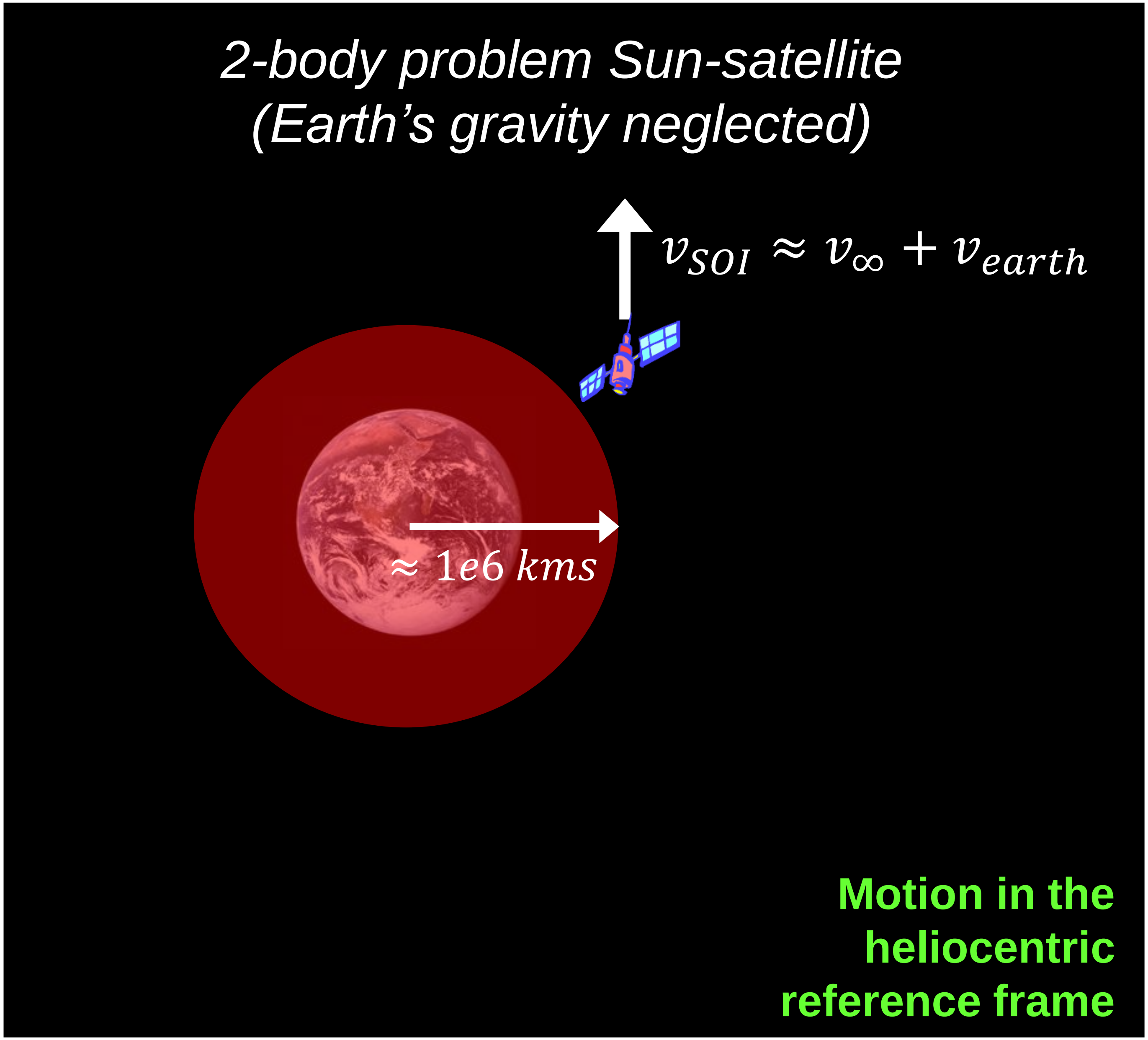
Measure of the deviation of the vehicle's orbit from the Keplerian orbit arising from the planet acting by itself ($\frac{P_s}{A_p}$)

If $\frac{P_p}{A_s} > \frac{P_s}{A_p}$ the spacecraft is inside the SOI of the planet.

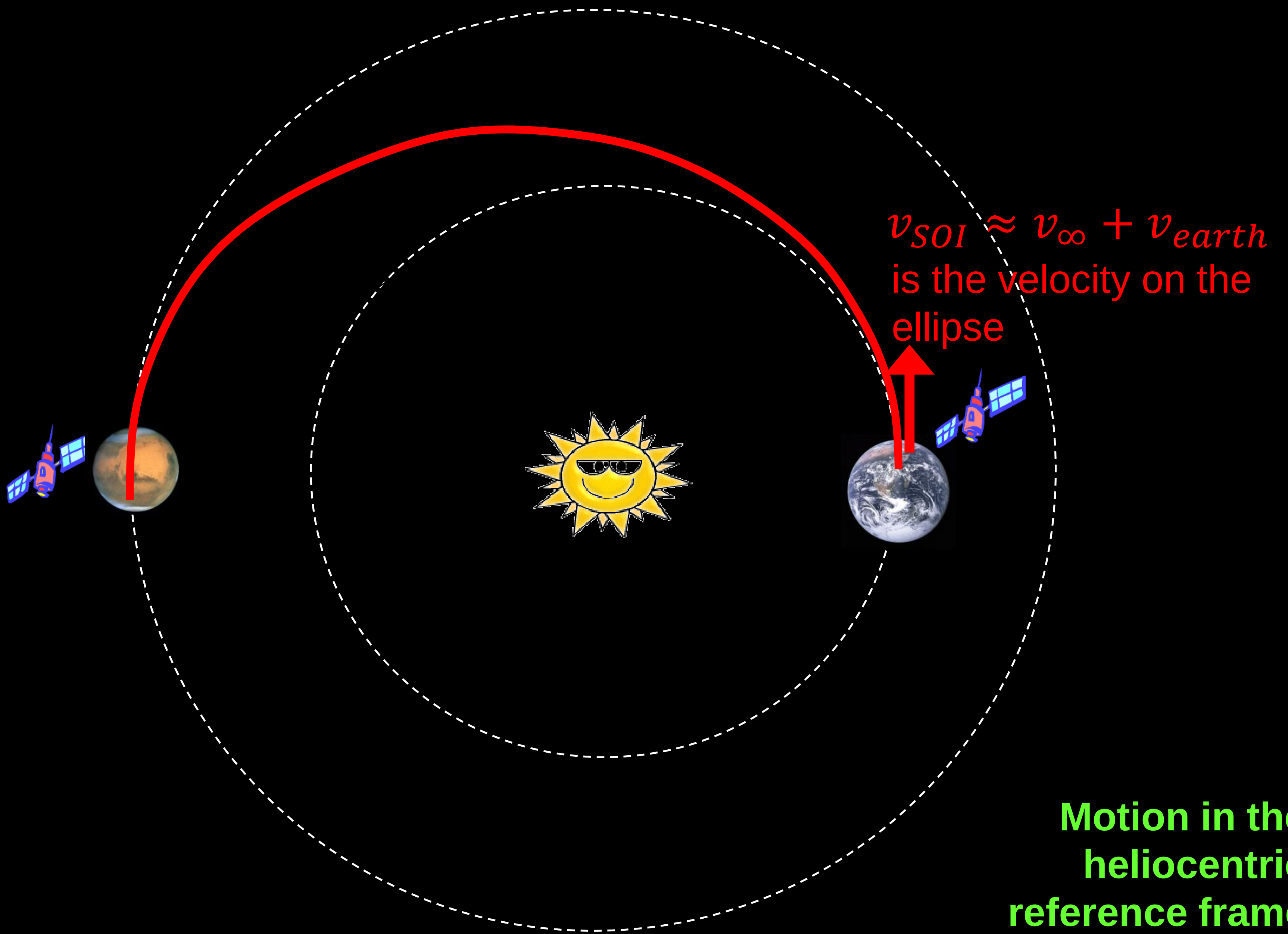
At the sphere of influence, far from the Earth



Planetary to heliocentric frame



Step 2: Hohmann transfer



Motion in the heliocentric reference frame

Hohmann transfer design

$$\Delta V = \sqrt{\frac{2\mu_{sun} R_{mars}}{R_{earth}(R_{earth} + R_{mars})}} - \sqrt{\frac{\mu_{sun}}{R_{earth}}}$$

Velocity of the satellite on the elliptical orbit around the Sun

$v_\infty + v_{earth}$

Velocity of the Earth around the Sun

v_{earth}

$$\sqrt{\frac{2\mu_{sun} R_{mars}}{R_{earth}(R_{earth} + R_{mars})}} - \sqrt{\frac{\mu_{sun}}{R_{earth}}} \approx v_\infty + v_{earth} - v_{earth} \approx v_\infty$$

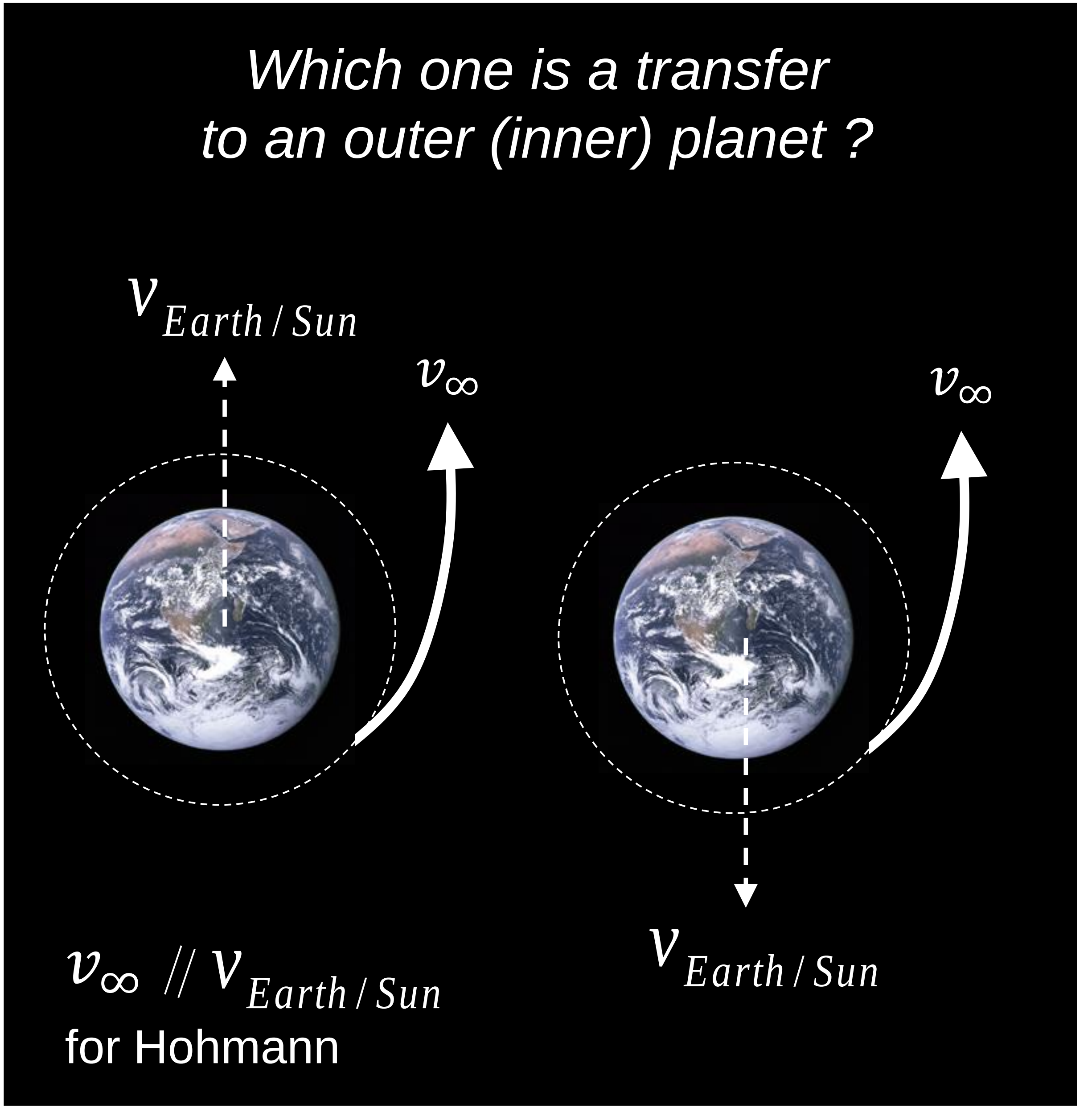
μ_{sun}=1.33e20

R_{earth}=149.6e9

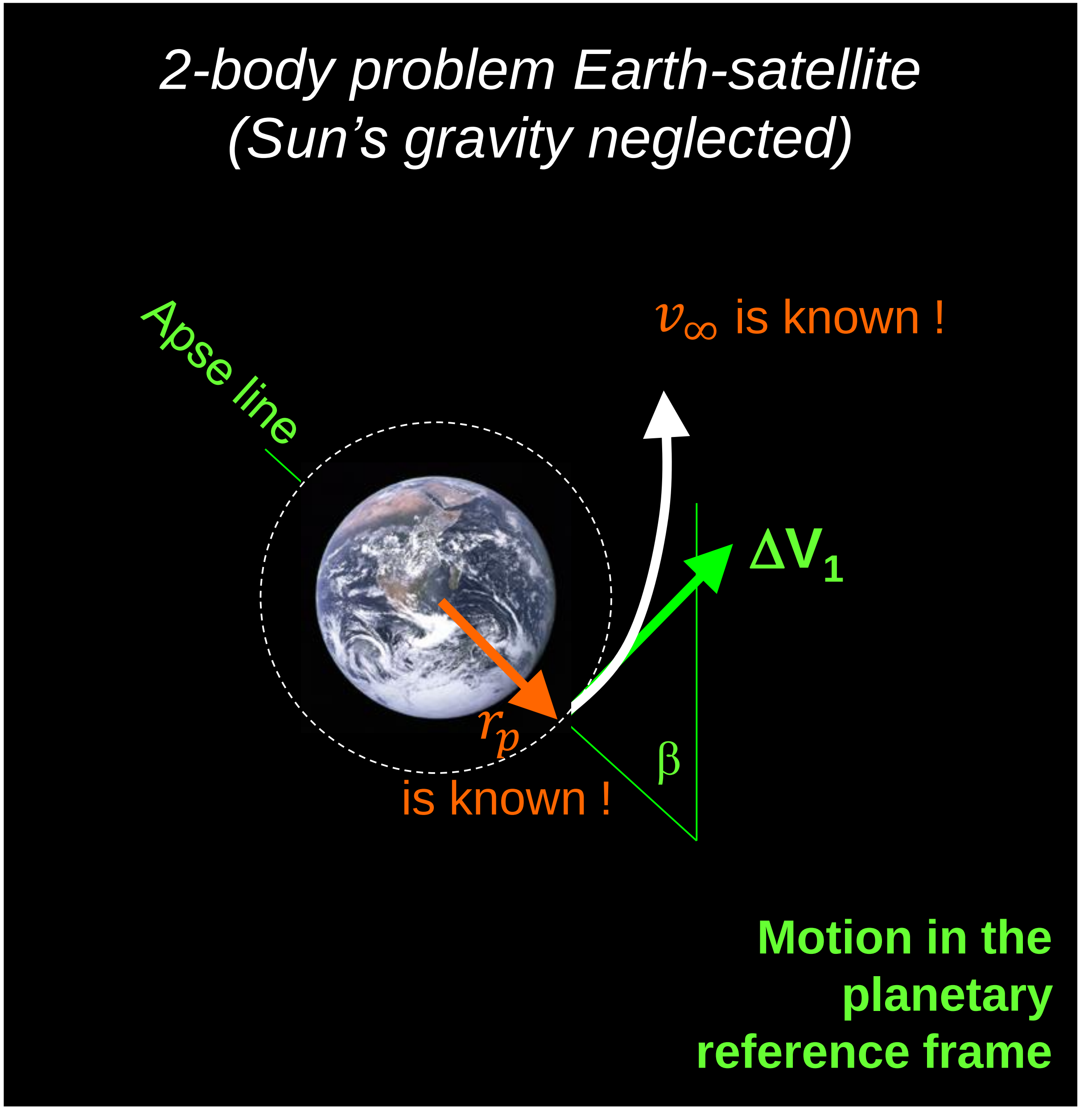
R_{mars}=227e9

We can calculate $v_\infty \approx 2.9 km/s$!

Let's go back to our initial hyperbola



We can now design the initial hyperbola



ΔV magnitude and location

Lecture 02:

known

$$v_\infty = \sqrt{\frac{\mu}{a}}$$

$$a = \frac{h^2}{\mu}\frac{1}{e^2 - 1}$$

known

$$r_p = \frac{h^2}{\mu(1+e)}$$

$$h = \frac{\mu\sqrt{e^2 - 1}}{v_\infty}$$

$$e = 1 + \frac{r_p v_\infty^2}{\mu}$$

$$h = r_p\sqrt{v_\infty^2 + \frac{2\mu}{r_p}}$$

$$\Delta v = v_p - v_c = \frac{h}{r_p} - \sqrt{\frac{\mu}{r_p}}$$

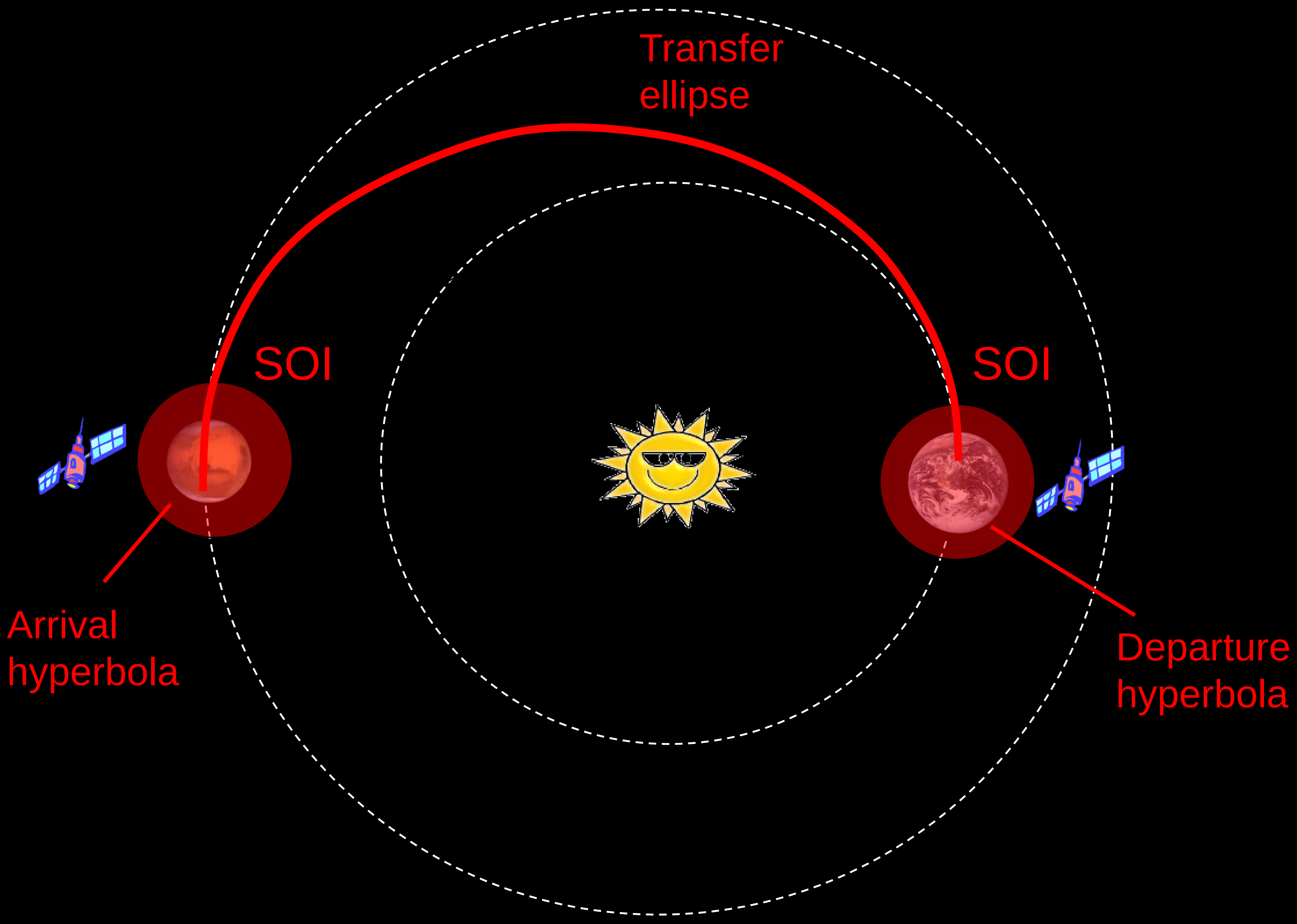
$$\beta = \cos^{-1}\frac{1}{e}$$

v_p: Hyper.

v_c: Circular

If we apply this Delta-V at the angle beta, we will arrive directly at Mars (but with the wrong velocity)

Step 3: Planetary arrival → similar reasoning



Patched conic method: in summary

Sequence of 2-body problems: outbound hyperbola (departure), Hohmann transfer ellipse (interplanetary travel) and inbound hyperbola (arrival) with one body always being the spacecraft.

Approximate method: if the spacecraft is close enough to one celestial body, the gravitational forces due to other planets are neglected.

Very useful for preliminary mission design (delta-v requirements and flight times). But actual mission design employs the accurate numerical integration techniques.