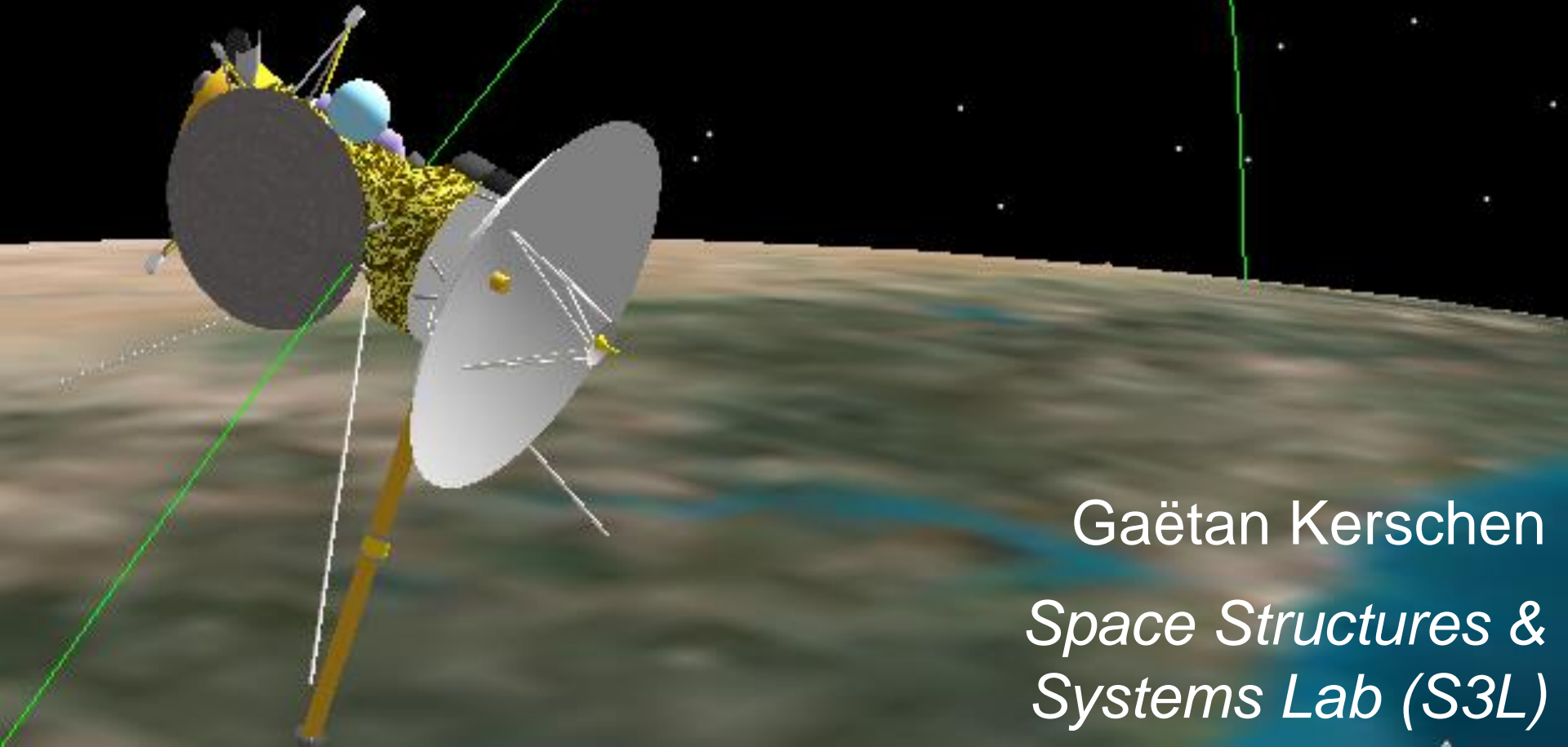


Cassini Classical Orbit Elements
Time (UTCG): 15 Oct 1997 09:18:54.000
Semi-major Axis (km): 6685.637000
Eccentricity: 0.020566
Inclination (deg): 30.000
RAAN (deg): 150.546
Arg of Perigee (deg): 230.000
True Anomaly (deg): 136.530
Mean Anomaly (deg): 134.891

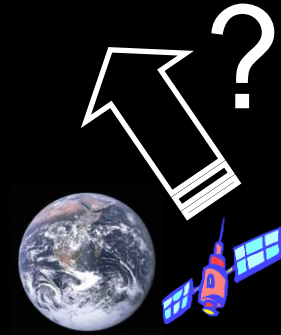
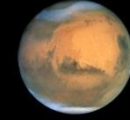
Aerodynamics

(AERO0024)

8. Interplanetary Trajectories

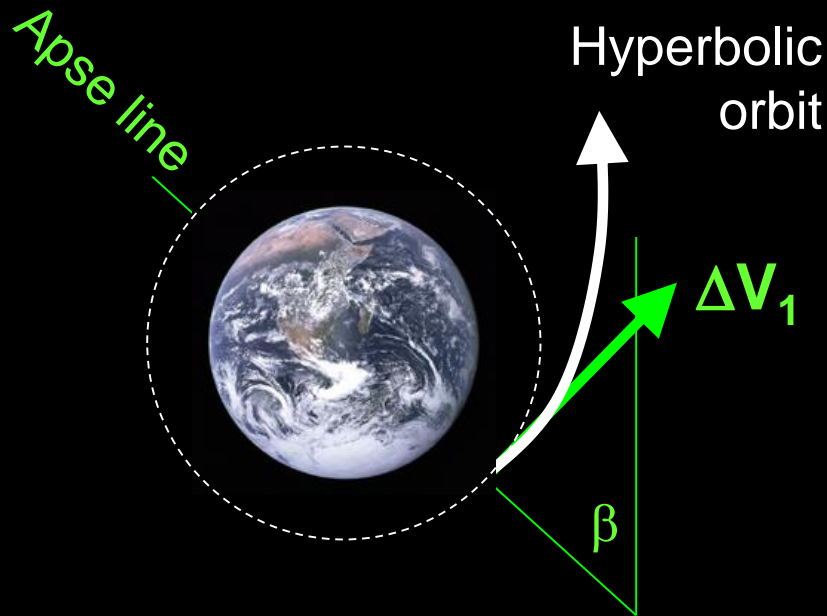


Gaëtan Kerschen
*Space Structures &
Systems Lab (S3L)*



Step 1: escape the Earth

*2-body problem Earth-satellite
(Sun and Mars gravity neglected)*



**Motion in the
planetary
reference frame**

SOI: Correct Definition due to Laplace

It is the surface along which:

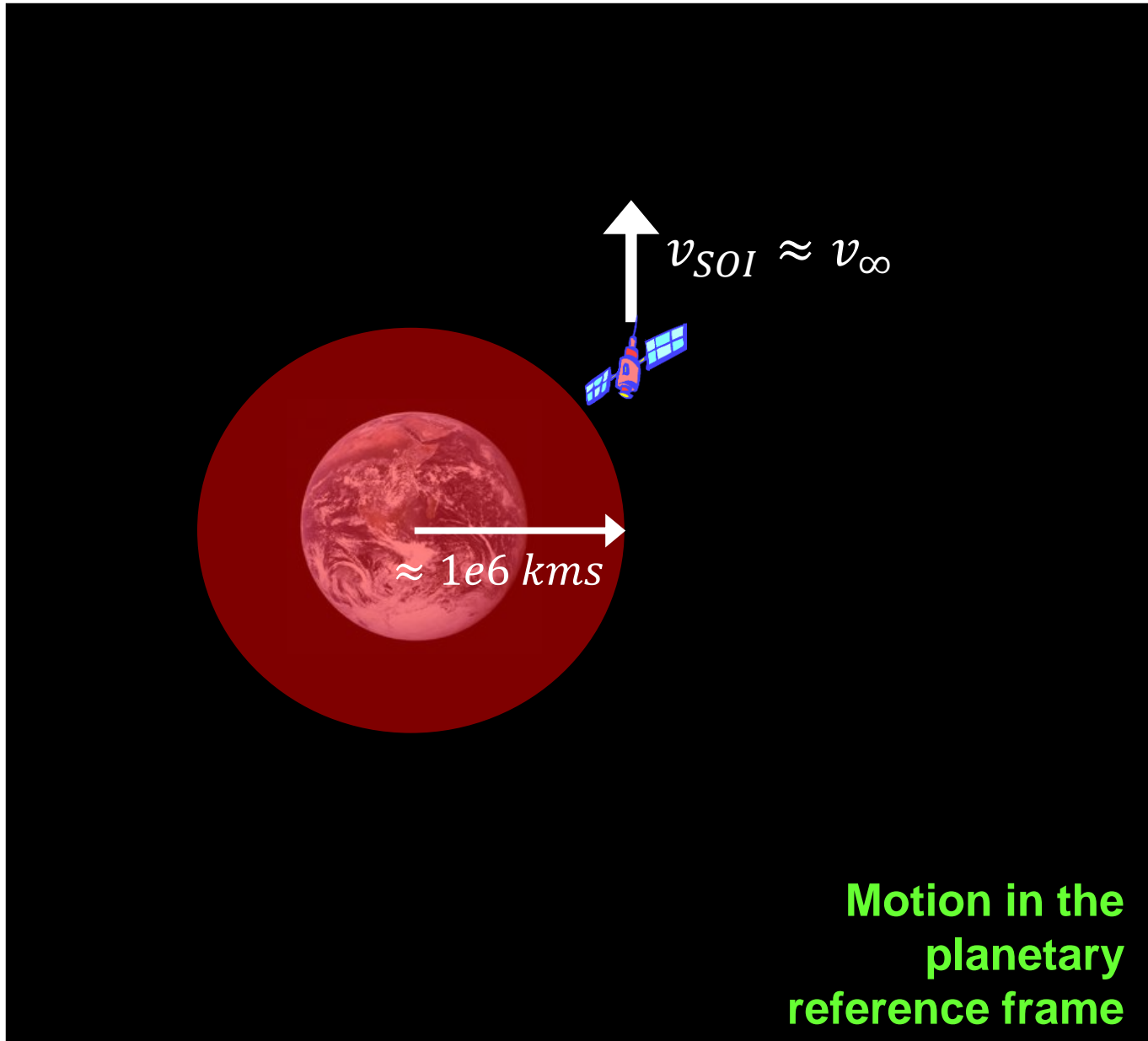
$$\frac{P_p}{A_s} = \frac{P_s}{A_p}$$

Measure of the planet's influence on the orbit of the vehicle relative to the sun

Measure of the deviation of the vehicle's orbit from the Keplerian orbit arising from the planet acting by itself

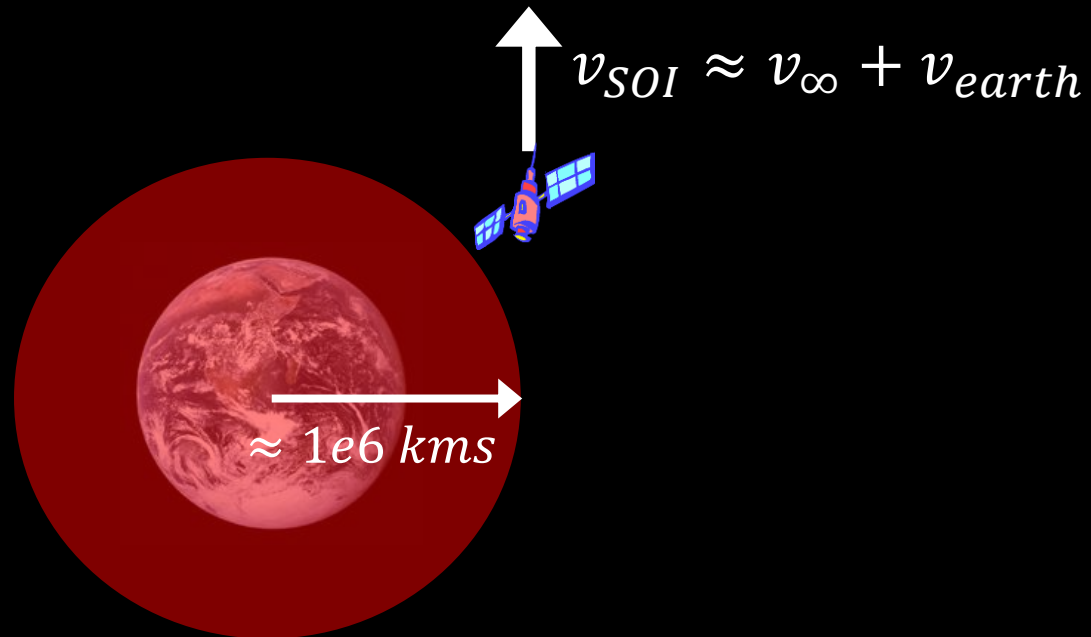
If $\frac{P_p}{A_s} > \frac{P_s}{A_p}$ the spacecraft is inside the SOI of the planet.

At the sphere of influence, far from the Earth



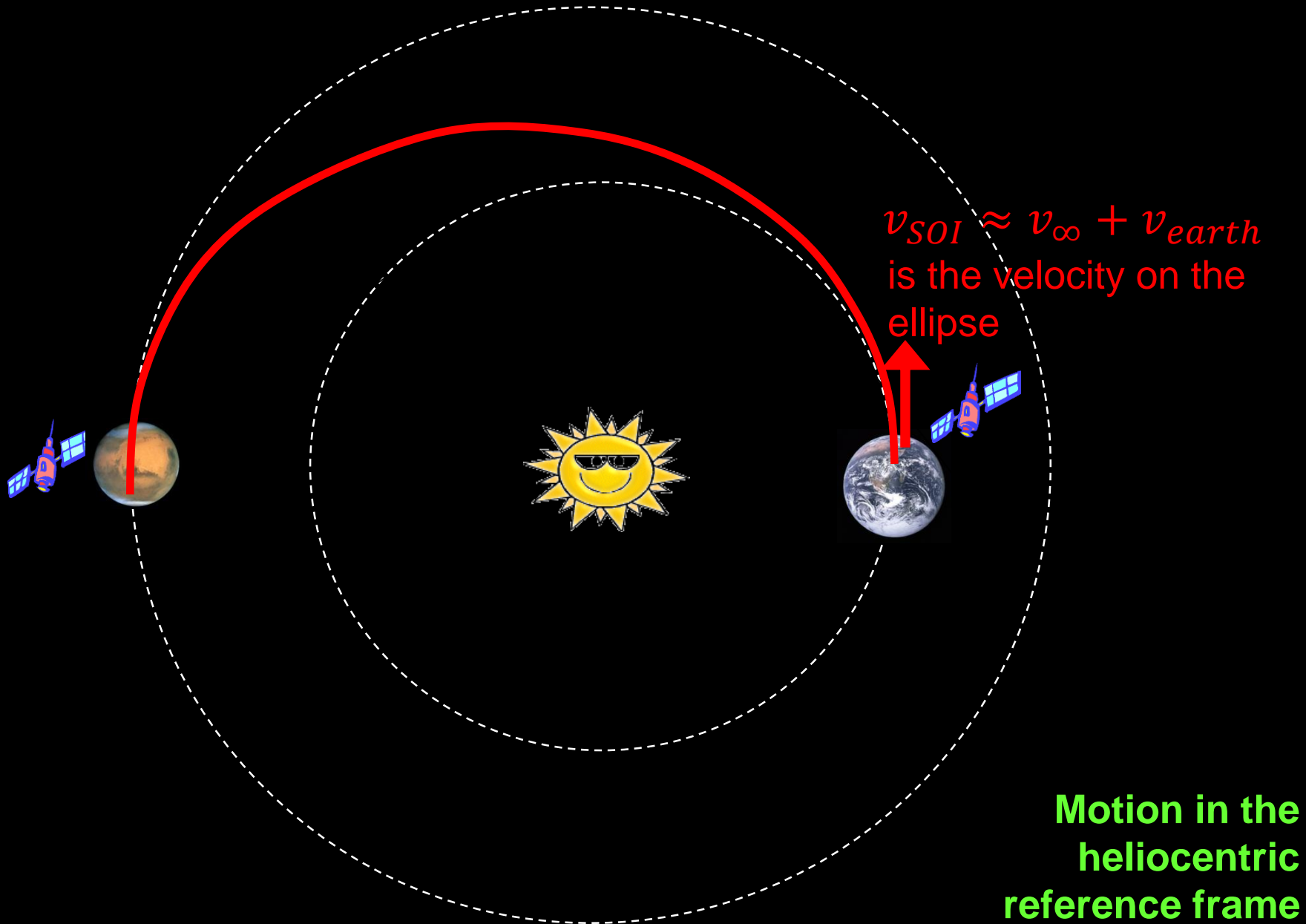
Planetary to heliocentric frame

*2-body problem Sun-satellite
(Earth's gravity neglected)*



**Motion in the
heliocentric
reference frame**

Step 2: Hohmann transfer



Hohmann transfer design

$$\Delta V = \sqrt{\frac{2\mu_{sun}R_{mars}}{R_{earth}(R_{earth} + R_{mars})}} - \sqrt{\frac{\mu_{sun}}{R_{earth}}}$$

Velocity of the satellite
on the elliptical orbit
around the Sun

$$v_{\infty} + v_{earth}$$

Velocity of the Earth
around the Sun

$$v_{earth}$$

$$\sqrt{\frac{2\mu_{sun}R_{mars}}{R_{earth}(R_{earth} + R_{mars})}} - \sqrt{\frac{\mu_{sun}}{R_{earth}}} \approx v_{\infty} + v_{earth} - v_{earth} \approx v_{\infty}$$

$$\mu_{sun} = 1.33e20$$

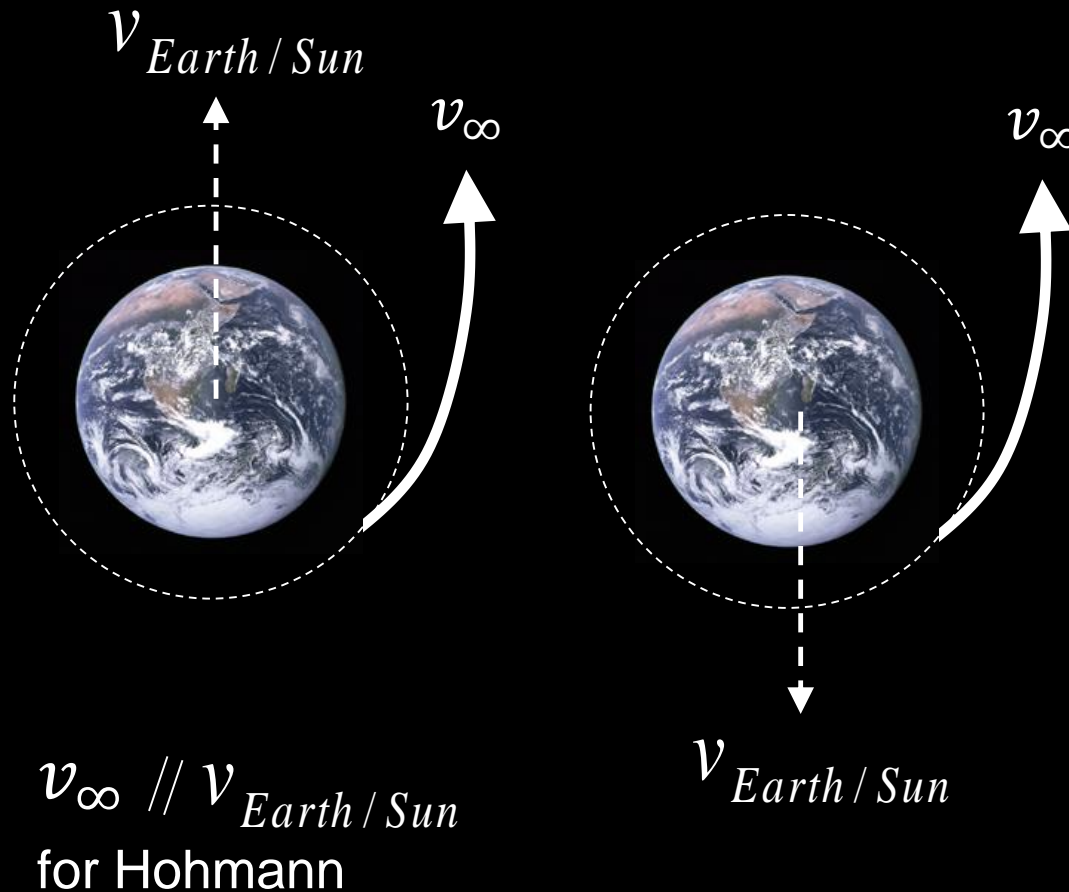
$$R_{earth} = 149.6e9$$

$$R_{mars} = 227e9$$

We can calculate $v_{\infty} \approx 2.9 \text{ km/s}$!

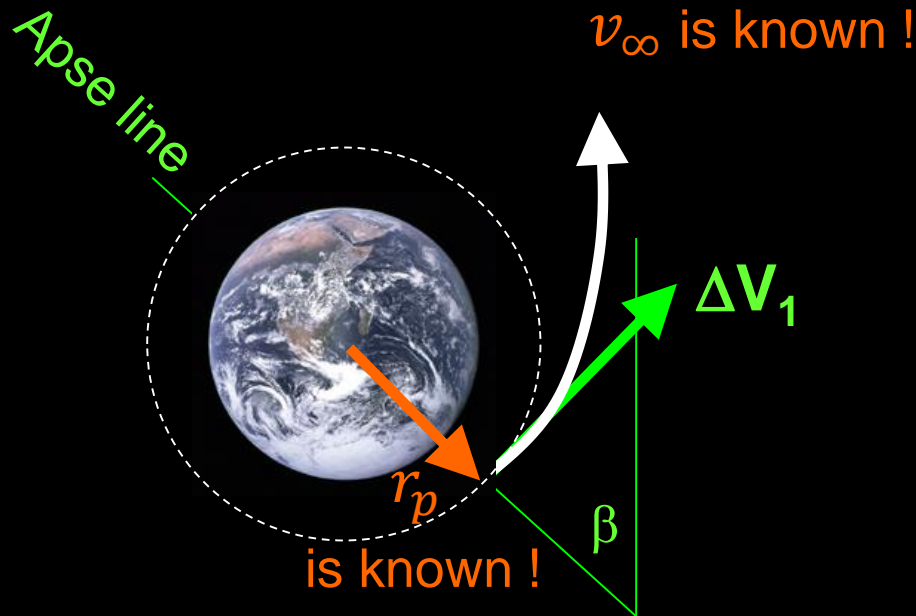
Let's go back to our initial hyperbola

Which one is a transfer to an outer (inner) planet ?



We can now design the initial hyperbola

*2-body problem Earth-satellite
(Sun's gravity neglected)*



**Motion in the
planetary
reference frame**

ΔV magnitude and location

Lecture 02:

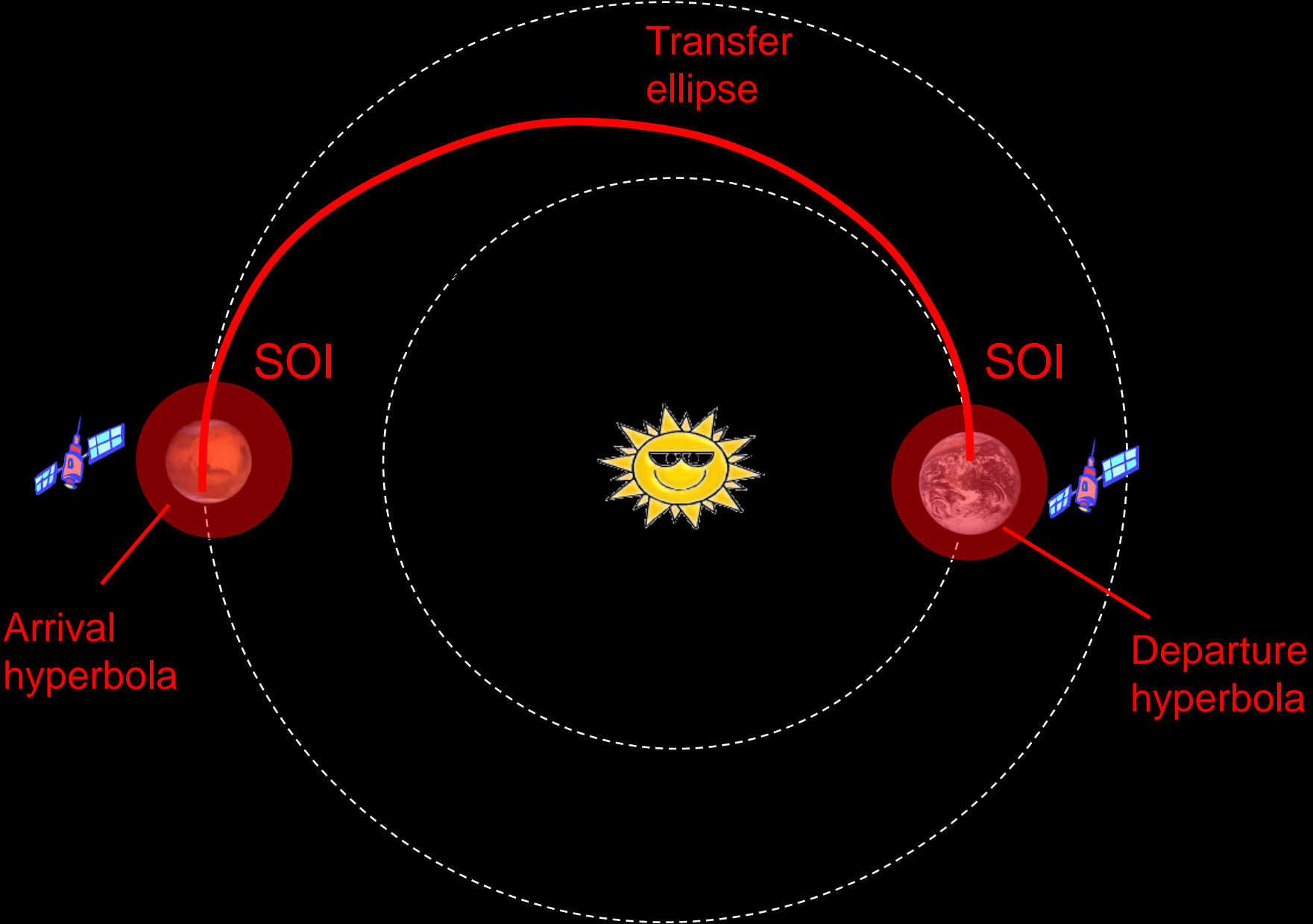
$$\left. \begin{array}{l}
 \text{known } r_p = \frac{h^2}{\mu(1+e)} \\
 \text{known } v_\infty = \sqrt{\frac{\mu}{a}} \\
 a = \frac{h^2}{\mu} \frac{1}{e^2 - 1}
 \end{array} \right\}
 \left. \begin{array}{l}
 h = \frac{\mu \sqrt{e^2 - 1}}{v_\infty} \\
 h = r_p \sqrt{v_\infty^2 + \frac{2\mu}{r_p}}
 \end{array} \right\}
 e = 1 + \frac{r_p v_\infty^2}{\mu}$$

$$\Rightarrow \Delta v = v_p - v_c = \frac{h}{r_p} - \sqrt{\frac{\mu}{r_p}} \quad \beta = \cos^{-1} \frac{1}{e}$$

↓
↓
 Hyper. Circular

If we apply this Delta-V at the angle beta, we will arrive directly at Mars (but with the wrong velocity)

Step 3: Planetary arrival → similar reasoning



Patched conic method: in summary

Sequence of 2-body problems: outbound hyperbola (departure), Hohmann transfer ellipse (interplanetary travel) and inbound hyperbola (arrival) with one body always being the spacecraft.

Approximate method: if the spacecraft is close enough to one celestial body, the gravitational forces due to other planets are neglected.

Very useful for preliminary mission design (delta-v requirements and flight times). But actual mission design employs the accurate numerical integration techniques.