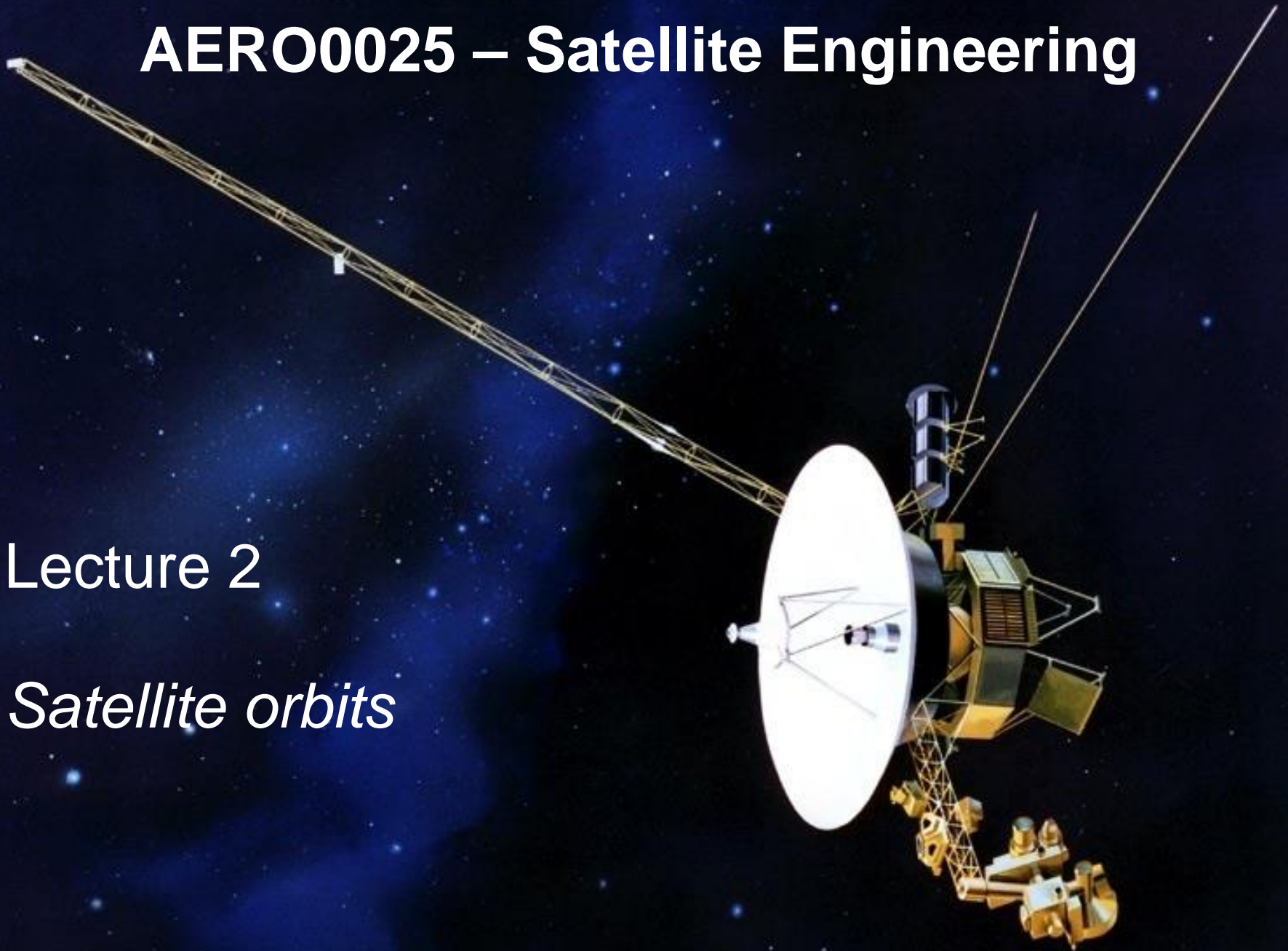


AERO0025 – Satellite Engineering

Lecture 2

Satellite orbits



Can the Orbit Affect ...

Mass of the satellite ?

Power generation ?

Amount of data that can be transferred to the ground ?

Space radiation environment ?

Revisit time of satellite to a point on Earth ?

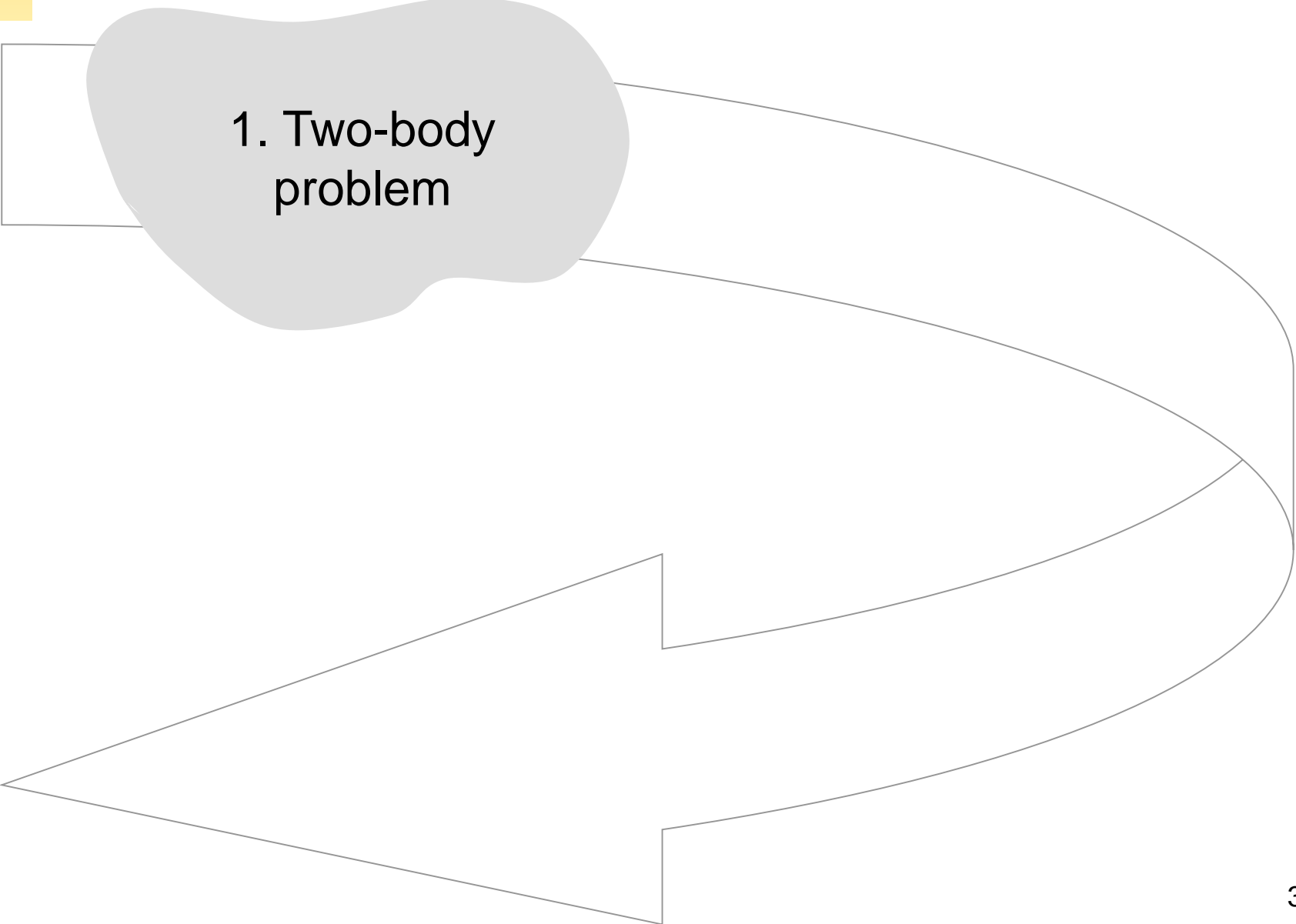
Thermal control ?

Launch costs ?

YES !

Satellite Orbits

1. Two-body
problem

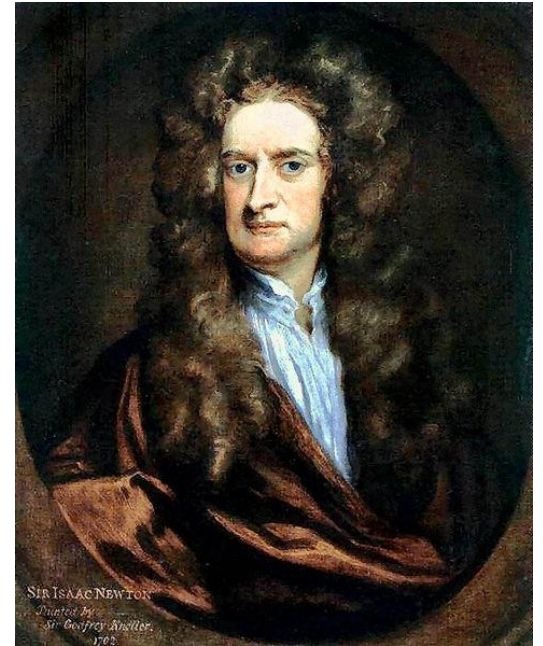


The diagram illustrates a satellite orbiting Earth. A grey, irregularly shaped blob represents Earth, with the text '1. Two-body problem' centered within it. A thin black line represents the satellite's orbit, starting from the left edge of the Earth, curving upwards and to the right, then looping back to the left, forming a large, elongated loop. The orbit is depicted as a continuous, smooth curve. The background is white, and a yellow horizontal bar is visible at the top of the slide.

1. Gravitational Force

The law of universal gravitation is an empirical law describing the gravitational attraction between bodies with mass.

It was first formulated by Newton in *Philosophiae Naturalis Principia Mathematica* (1687). He was able to relate objects falling on the Earth to the motion of the planets.



Isaac Newton (1642-1727)

1. Gravitational Force

Every point mass attracts every other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between the point masses:



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

1. Gravitational Parameter of a Body

$$\mu = GM_{\oplus}$$

The gravitational parameter of the Earth has been determined with considerable precision from the analysis of laser distance measurements of artificial satellites:

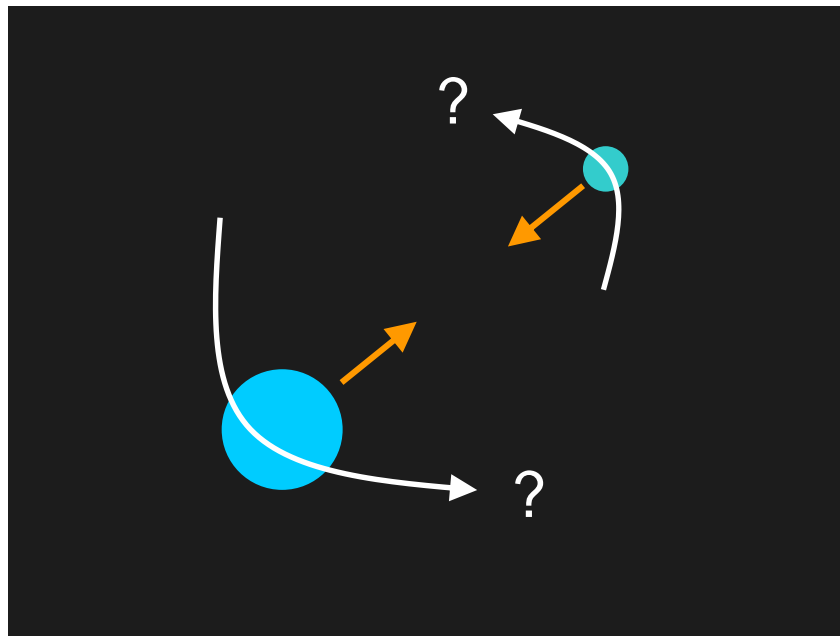
$$398600.4418 \pm 0.0008 \text{ km}^3.\text{s}^{-2}.$$

The uncertainty is 1 to 5e8, much smaller than the uncertainties in G and M separately (~ 1 to $1\text{e}4$ each).

1. Definition of the 2-Body Problem

Motion of two bodies due solely to their own mutual gravitational attraction. Also known as **Kepler problem**.

Assumption: two point masses (or equivalently spherically symmetric objects).

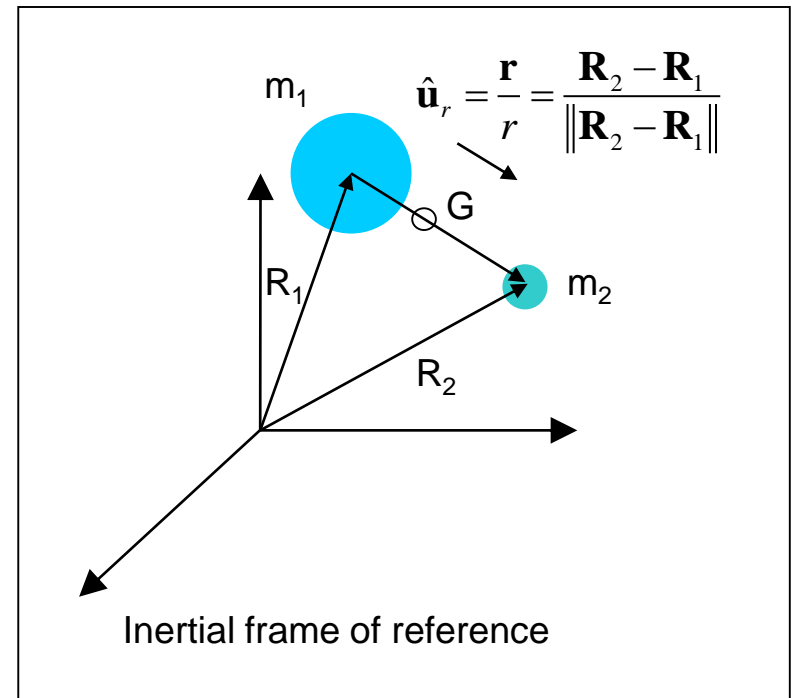


1. Motion of the Two Bodies

$$m_1 \ddot{\mathbf{R}}_1 = \frac{Gm_1m_2}{r^2} \hat{\mathbf{u}}_r$$

+

$$m_2 \ddot{\mathbf{R}}_2 = -\frac{Gm_1m_2}{r^2} \hat{\mathbf{u}}_r$$



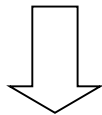
1. Equations of Relative Motion

$$-m_1 m_2 \ddot{\mathbf{R}}_1 = \frac{-G m_1 m_2^2}{r^2} \hat{\mathbf{u}}_r$$

+

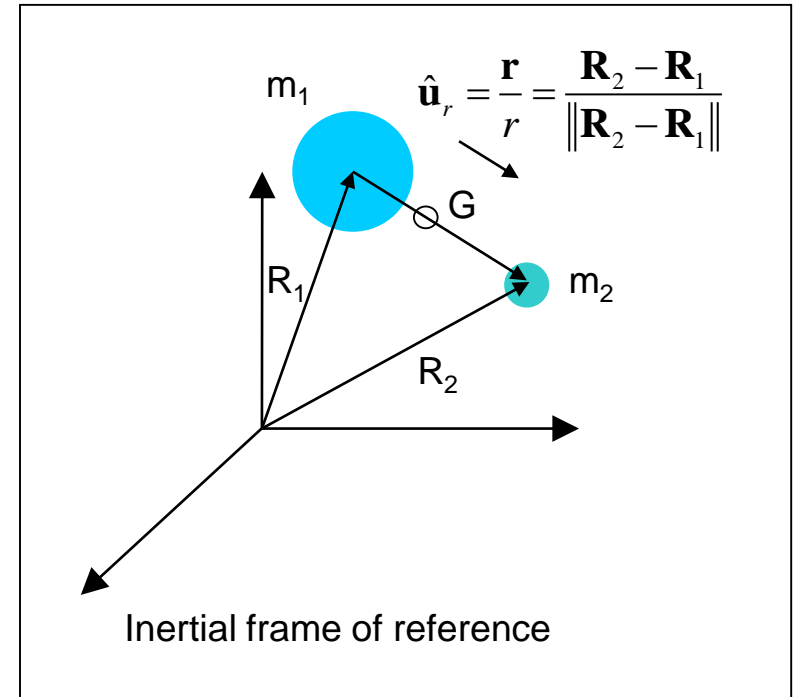
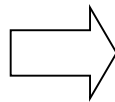
$$m_1 m_2 \ddot{\mathbf{R}}_2 = -\frac{G m_1^2 m_2}{r^2} \hat{\mathbf{u}}_r$$

$$\ddot{\mathbf{R}}_2 - \ddot{\mathbf{R}}_1 = -\frac{G(m_1 + m_2)}{r^2} \hat{\mathbf{u}}_r$$



$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r}$$

μ is the gravitational parameter

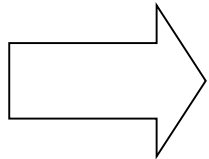


The motion of m_2 as seen from m_1 is the same as the motion of m_1 as seen from m_2 .

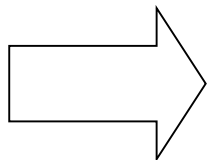
1. Equations of Relative Motion

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r}$$

This is a nonlinear dynamical system. How to solve it ?



Find constants of the motion !



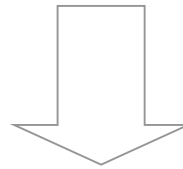
How many ?

1. Constant Angular Momentum

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r} \quad \xrightarrow{\mathbf{r} \times} \quad \mathbf{r} \times \ddot{\mathbf{r}} = \mathbf{r} \times \left(-\frac{\mu}{r^3} \mathbf{r} \right) = 0$$

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} \quad \xrightarrow{d/dt} \quad \frac{d\mathbf{h}}{dt} = \dot{\mathbf{r}} \times \dot{\mathbf{r}} + \mathbf{r} \times \ddot{\mathbf{r}} = \mathbf{r} \times \ddot{\mathbf{r}}$$

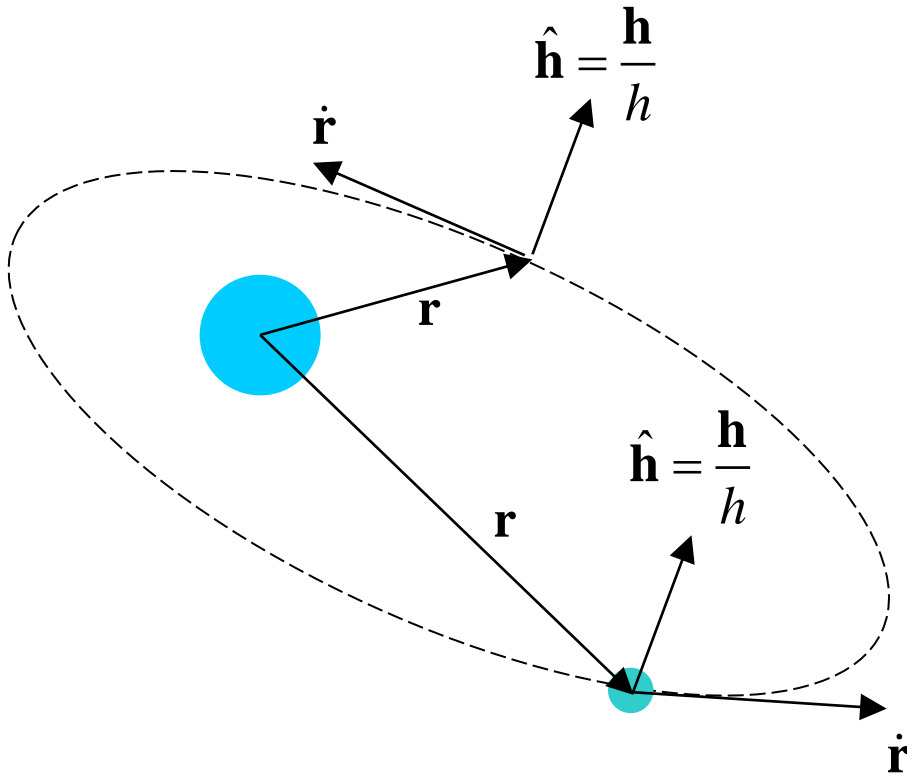
Specific angular
momentum



$$\frac{d\mathbf{h}}{dt} = 0 \rightarrow \mathbf{r} \times \dot{\mathbf{r}} = \text{constant} = \mathbf{h}$$



1. The Motion Lies in a Fixed Plane



The fixed plane is the **orbit plane** and is normal to the angular momentum vector.

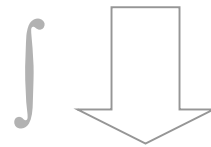
$$\mathbf{r} \times \dot{\mathbf{r}} = \text{constant} = \mathbf{h}$$

1. First Integral of Motion

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r} \quad \xrightarrow{\times \mathbf{h}} \quad \ddot{\mathbf{r}} \times \mathbf{h} = -\frac{\mu}{r^3} \mathbf{r} \times \mathbf{h} = -\frac{\mu}{r^3} \mathbf{r} \times (\mathbf{r} \times \dot{\mathbf{r}})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b}) \quad \downarrow \quad \frac{d}{dt} \left(\frac{\mathbf{r}}{r} \right) = \frac{r\dot{\mathbf{r}} - \mathbf{r}\dot{r}}{r^2} \quad \mathbf{r} \cdot \dot{\mathbf{r}} = r\dot{r}$$

$$\ddot{\mathbf{r}} \times \mathbf{h} = \frac{\mu}{r^3} [\dot{\mathbf{r}}(\mathbf{r} \cdot \mathbf{r}) - \mathbf{r}(\mathbf{r} \cdot \dot{\mathbf{r}})] = \mu \left(\frac{\dot{\mathbf{r}}}{r} - \frac{\mathbf{r}\dot{r}}{r^2} \right) = \mu \frac{d}{dt} \left(\frac{\mathbf{r}}{r} \right)$$



$$\dot{\mathbf{r}} \times \mathbf{h} - \mu \frac{\mathbf{r}}{r} = \text{constant} = \mu \mathbf{e}$$

\mathbf{e} lies in the orbit plane
 $(\mathbf{e} \cdot \mathbf{h}) = 0$: the line defined by \mathbf{e}
 is the apse line.

Its norm, e , is the eccentricity.



1. Orbit Equation

$$\frac{\dot{\mathbf{r}} \times \mathbf{h}}{\mu} = \frac{\mathbf{r}}{r} + \mathbf{e} \quad \xrightarrow{\mathbf{r} \cdot} \quad \frac{\mathbf{r} \cdot (\dot{\mathbf{r}} \times \mathbf{h})}{\mu} = \frac{\mathbf{r} \cdot \mathbf{r}}{r} + \mathbf{r} \cdot \mathbf{e}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$

$$\frac{\mathbf{r} \cdot (\dot{\mathbf{r}} \times \mathbf{h})}{\mu} = \frac{(\mathbf{r} \times \dot{\mathbf{r}}) \cdot \mathbf{h}}{\mu} = \frac{\mathbf{h} \cdot \mathbf{h}}{\mu} = \frac{h^2}{\mu} = r + \mathbf{r} \cdot \mathbf{e}$$

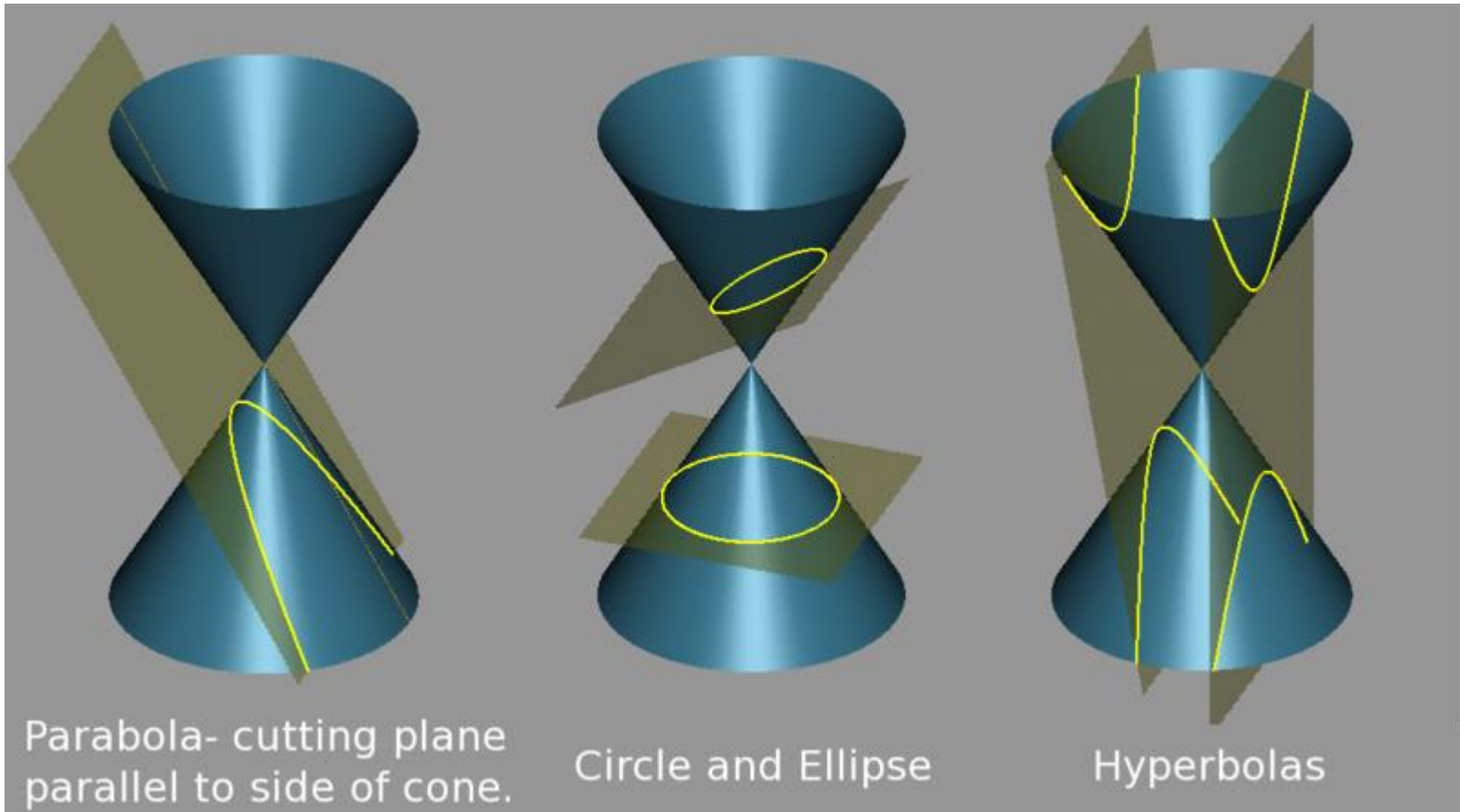


$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

Closed form of the nonlinear equations of motion (θ is the true anomaly)

1. Conic Section

$$r = \frac{p}{1 + e \cos \theta}$$



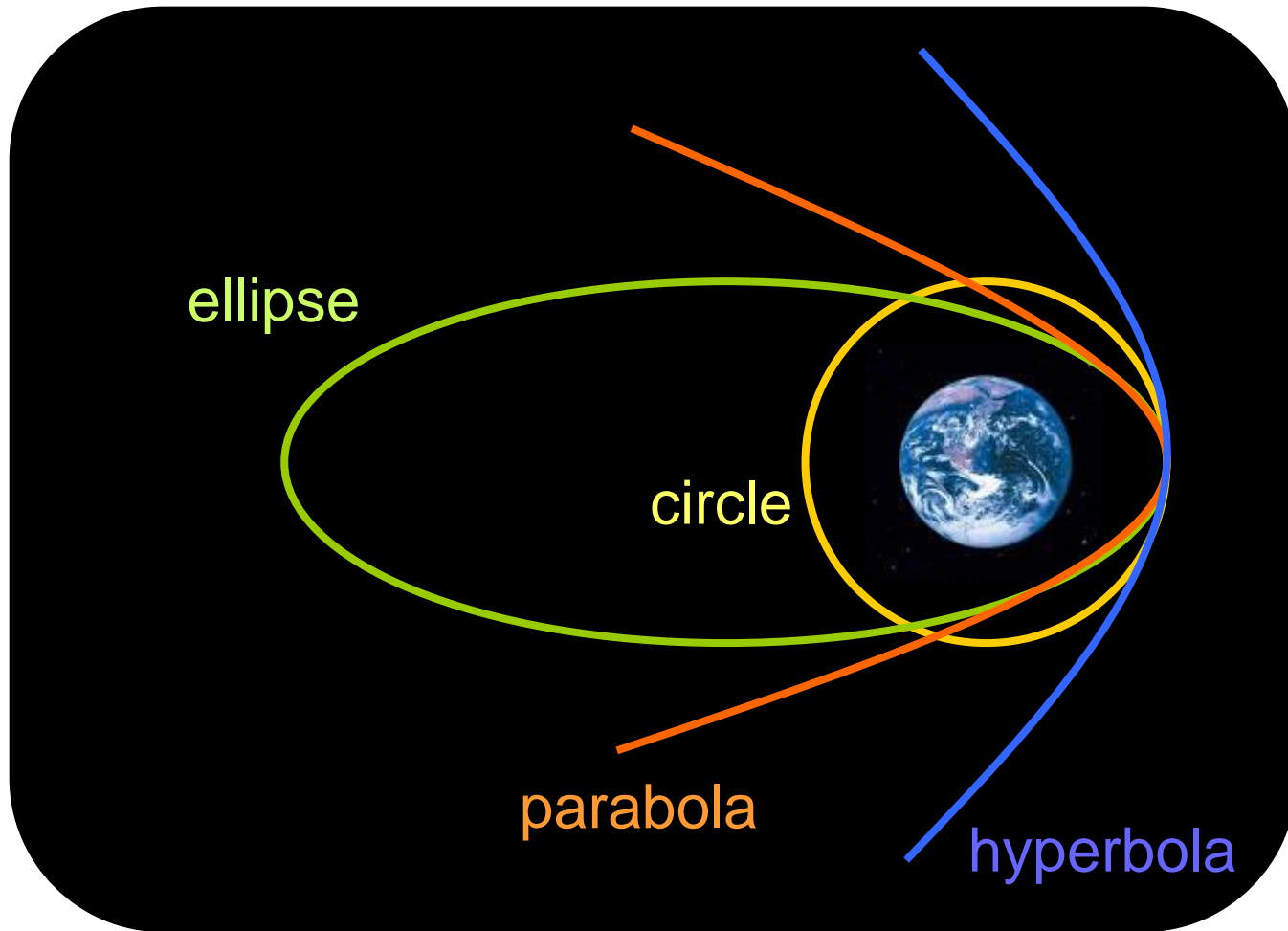
$e=1$

$e=0$

$0 < e < 1$

$e > 1$

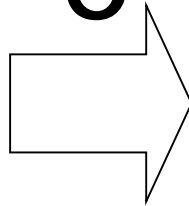
1. Possible Motions in the 2-Body System









1. How Many Variables to Define An Orbit ?

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r}$$

6



3 ODEs of second-order

X:	526.375 km	
Y:	-5773.15 km	
Z:	3409.81 km	
X Velocity:	5.76514 km/sec	
Y Velocity:	-2.21072 km/sec	
Z Velocity:	-4.60703 km/sec	

*Useful parametrization
of the orbit ?*

ISS cartesian parameters on March 4,
2009, 12:30:00 UTC (Source: Celestrak)

1. Cartesian Coordinates ?

\mathbf{r} and $\dot{\mathbf{r}}$ do not directly yield much information about the orbit.

We cannot even infer from them what type of conic the orbit represents or what is the orbit altitude !

Another set of six variables, which is much more descriptive of the orbit, is needed.

1. Six Orbital (Keplerian) Elements

1. e : shape of the orbit

definition of the ellipse

2. a : size of the orbit

3. i : orients the orbital plane with respect to the ecliptic plane

definition of the orbital plane

4. Ω : longitude of the intersection of the orbital and ecliptic planes

5. ω : orients the semi-major axis with respect to the ascending node

orientation of the ellipse within the orbital plane

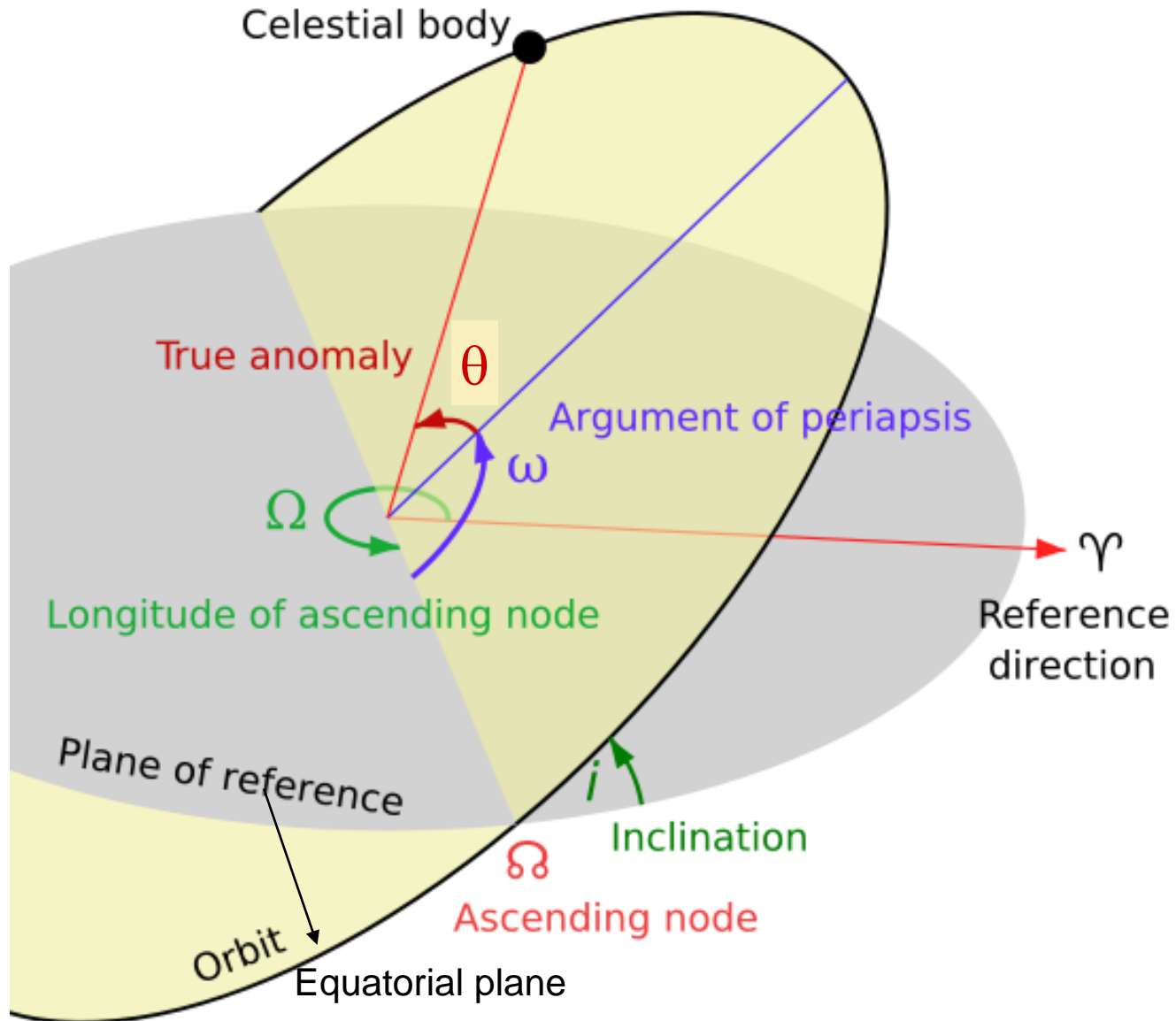
6. θ : orients the celestial body in space

position of the satellite on the ellipse

● Orbital plane

● orientation of the ellipse

● position of the satellite



1. In Summary

+

We can calculate r for all values of the true anomaly.

+

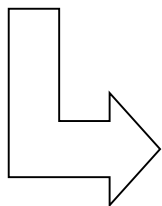
The orbit equation is a mathematical statement of Kepler's first law.

-

We only know the relative motion (however, e.g., the motion of our sun relative to other parts of our galaxy is of little importance for missions within our solar system).

-

The solution of the “simple” problem of two bodies cannot be expressed in a closed form, explicit function of time.



Do we have 6 independent constants ?

The two vector constants \mathbf{h} and \mathbf{e} provide only 5 independent constants: $\mathbf{h} \cdot \mathbf{e} = 0$

Satellite Orbits

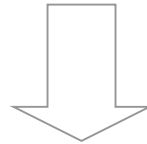
1. Two-body
problem

2. Orbit types

2.1 Circular Orbits (e=0)

$$r = \frac{h^2}{\mu} = \text{Constant}$$

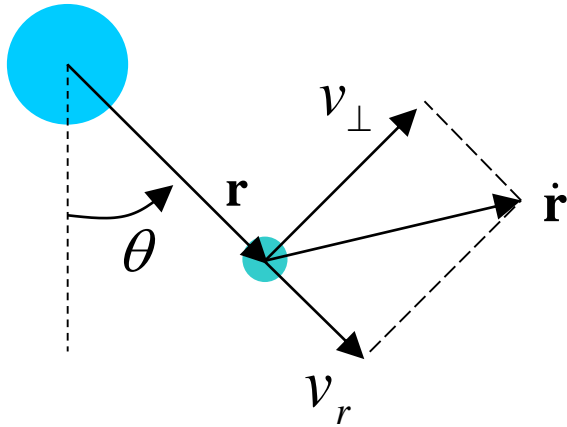
$$h = rv_{\perp} = rv_{\text{circular}}$$



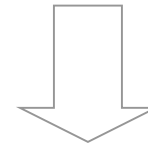
$$v_{\text{circ}} = \sqrt{\frac{\mu}{r}}$$

$$T_{\text{circ}} = 2\pi r / \sqrt{\frac{\mu}{r}} = \frac{2\pi}{\sqrt{\mu}} r^{3/2}$$

2.1 Digression: Angular Momentum



$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} = r \hat{\mathbf{u}}_r \times (v_r \hat{\mathbf{u}}_r + v_\perp \hat{\mathbf{u}}_\perp) = r v_\perp \hat{\mathbf{h}}$$

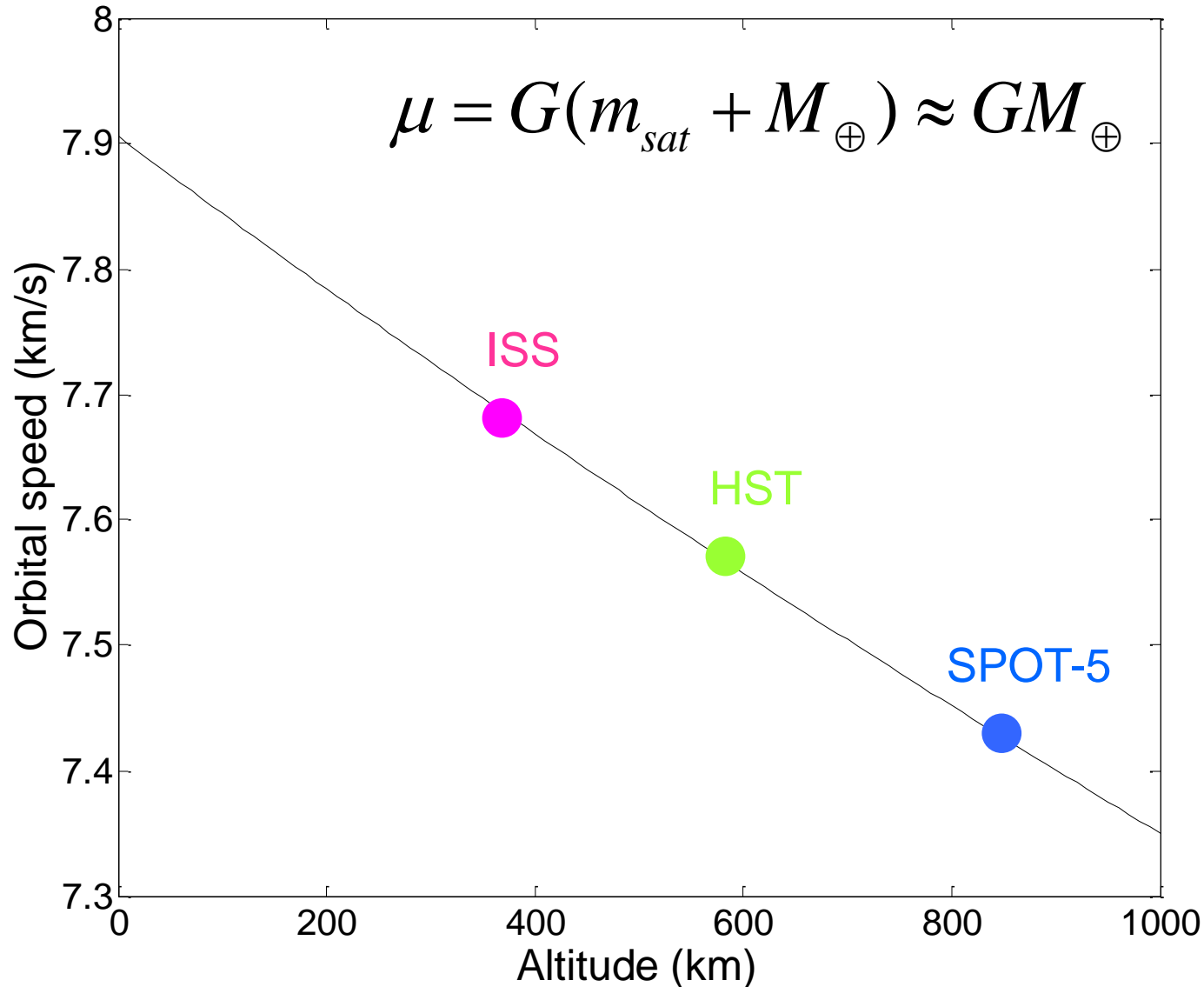


$$h = r v_\perp = r^2 \dot{\theta}$$

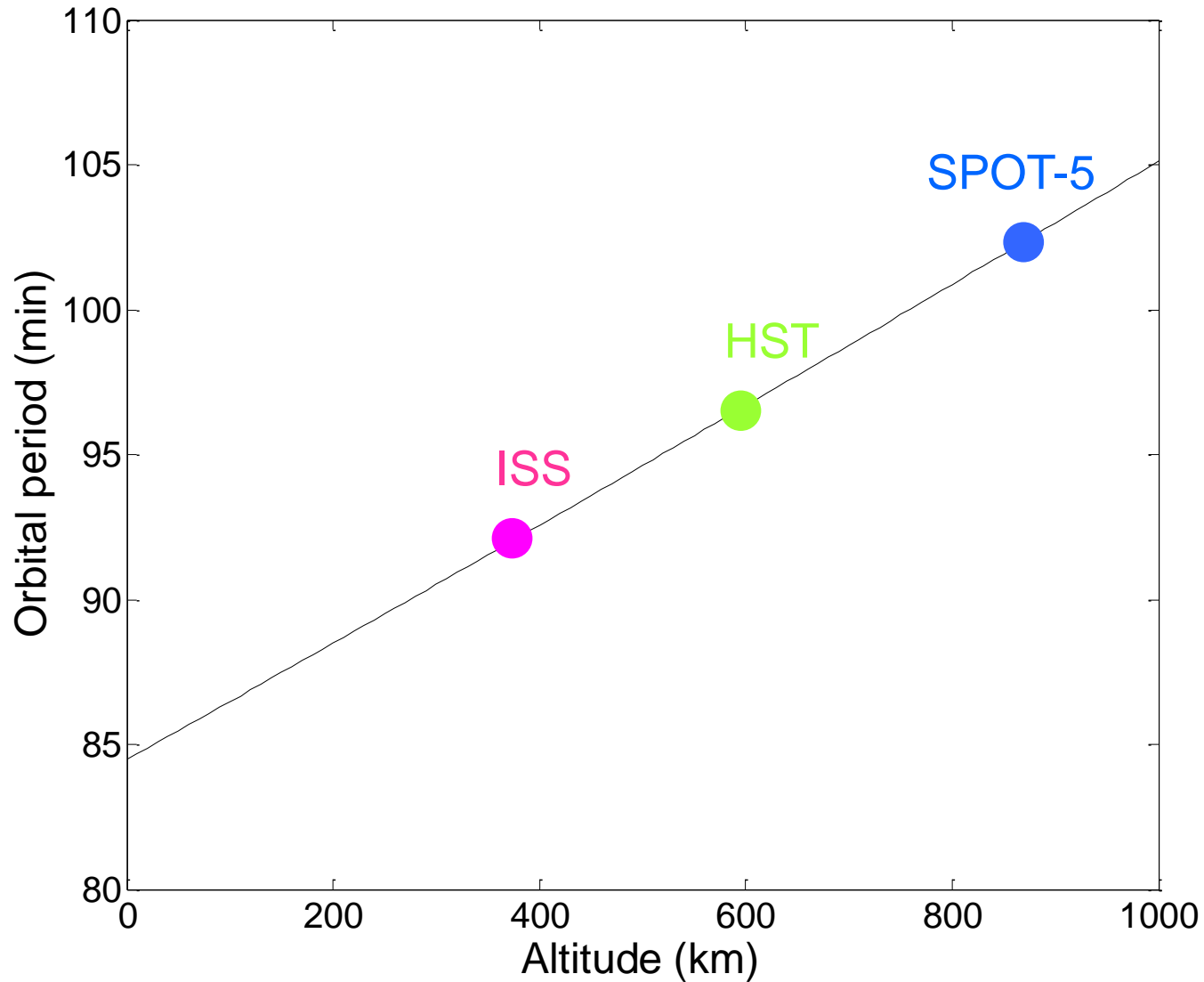
The angular momentum depends only on the azimuth component of the relative velocity

[End of digression]²⁴

2.1 Orbital Speed Decreases with Altitude



2.1 Orbital Period Increases With Altitude



2.1 Two Important Cases

1. 7.9 km/s is the **first cosmic velocity**; i.e., the minimum velocity (theoretical velocity, $r=6378$ km) to orbit the Earth.
2. 35786 km is the altitude of the **geostationary orbit**. It is the orbit at which the satellite angular velocity is equal to that of the Earth, $\omega=\omega_E=7.292 \cdot 10^{-5}$ rad/s, in inertial space (*).

$$r_{GEO} = \left(\frac{T_{circ} \sqrt{\mu}}{2\pi} \right)^{2/3}$$

* A sidereal day, 23h56m4s, is the time it takes the Earth to complete one rotation relative to inertial space. A synodic day, 24h, is the time it takes the sun to apparently rotate once around the Earth. They would be identical if the earth stood still in space.

2.2 Elliptic Orbits ($0 < e < 1$)

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

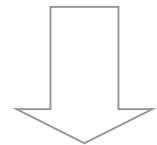
The relative position vector remains bounded.

$\theta=0$, minimum separation, **periapse**

$$r_p = \frac{h^2}{\mu(1+e)}$$

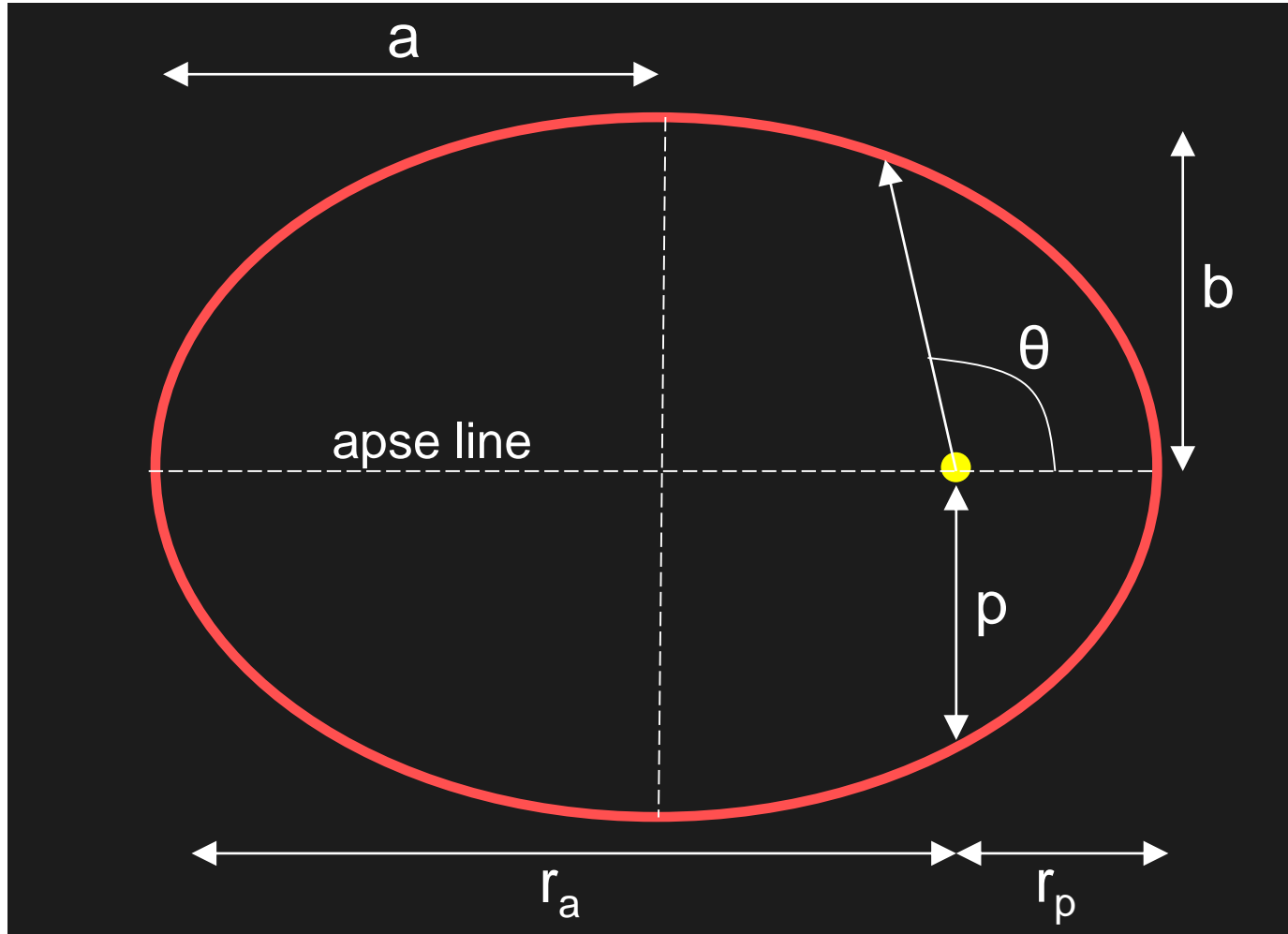
$\theta=\pi$, greatest separation, **apoapse**

$$r_a = \frac{h^2}{\mu(1-e)}$$



$$e = \frac{r_a - r_p}{r_a + r_p}$$

2.2 Geometry of the Elliptic Orbit



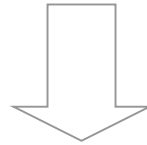
2.2 Digression: Angular Momentum

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$

Orbit equation

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$

Polar equation of an ellipse
(a , semimajor axis)

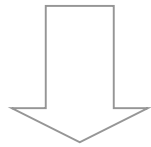


$$h = \sqrt{\mu a(1 - e^2)}$$

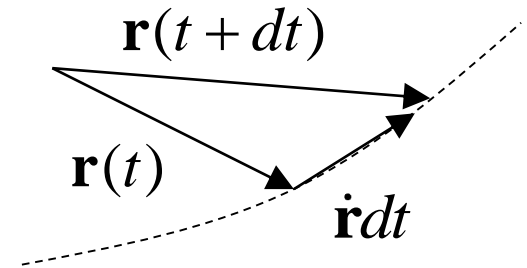
[End of digression] ³⁰

2.2 Kepler's Second Law

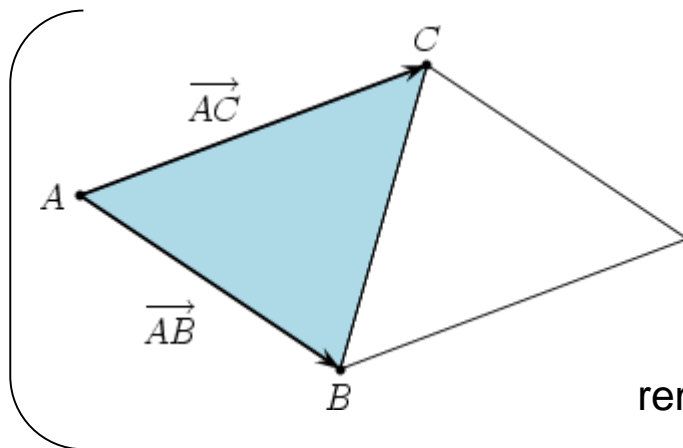
$$dA = \frac{1}{2} |\mathbf{r} \times \dot{\mathbf{r}} dt| = \frac{1}{2} |\mathbf{h}| dt = \frac{1}{2} h dt$$



$$\frac{dA}{dt} = \frac{h}{2} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{constant}$$



The line from the sun to a planet sweeps out equal areas inside the ellipse in equal lengths of time.



reminder

$$Area = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$$

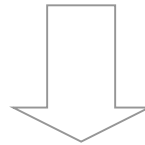
2.2 Kepler's Third Law

$$T = \frac{\text{enclosed area}}{dA / dt} = \frac{2\pi ab}{h}$$

$$h = \sqrt{\mu a(1-e^2)} \quad \Downarrow \quad b = a\sqrt{1-e^2}$$

$$T_{\text{ellip}} = 2\pi \sqrt{\frac{a^3}{\mu}}$$

The elliptic orbit period depends only on the semimajor axis and is independent of the eccentricity.



$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

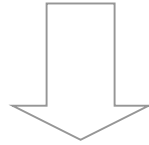
The squares of the orbital periods of the planets are proportional to the cubes of their mean distances from the sun.

2.2 Vis-Viva Equation

$$v_{ellip} = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

2.2 Example (1447km x 354km)

$$r_p = 354 + 6378 = 6732 \text{ km} \quad r_a = 1447 + 6378 = 7825 \text{ km}$$



$$\left\{ \begin{array}{l} e = \frac{r_a - r_p}{r_a + r_p} = 0.075, \quad a = \frac{r_a + r_p}{2} = 7278.5 \text{ km} \end{array} \right.$$

$$\left\{ \begin{array}{l} T = 2\pi \sqrt{\frac{a^3}{\mu}} = 6179.79 \text{ s} = 103 \text{ min} \end{array} \right.$$

$$\left\{ \begin{array}{l} v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)} \end{array} \right. \begin{array}{l} \nearrow v_p = 7.98 \text{ km/s} \\ \searrow v_a = 6.86 \text{ km/s} \end{array}$$

2.3 Parabolic Orbits (e=1)

$$r = \frac{h^2}{\mu} \frac{1}{1 + \cos \theta} \quad \theta \rightarrow \pi, r \rightarrow \infty$$

$$v_{parab} = \sqrt{\frac{2\mu}{r}}$$

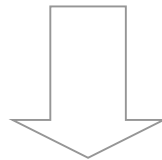
The satellite will coast to infinity, arriving there with zero velocity relative to the central body.

2.3 Escape Velocity, V_{esc}

11.2 km/s is the **second cosmic velocity**; i.e., the minimum velocity (theoretical velocity, $r=6378\text{km}$) to escape the gravitational attraction of the Earth.

$$v_{\text{circ}} = \sqrt{\frac{\mu}{r}}$$

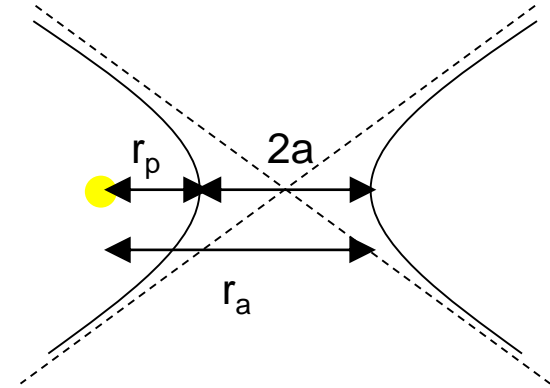
$$v_{\text{parab}} = \sqrt{\frac{2\mu}{r}}$$



$$11.2\text{km/s} = \sqrt{2} \times 7.9\text{km/s}$$

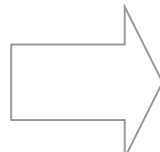
2.4 Hyperbolic Orbits ($e > 1$)

$$r = \frac{h^2}{\mu} \frac{1}{1 + e \cos \theta}$$



$$v_{\infty} = \sqrt{\frac{\mu}{a}}$$

Hyperbolic
excess speed



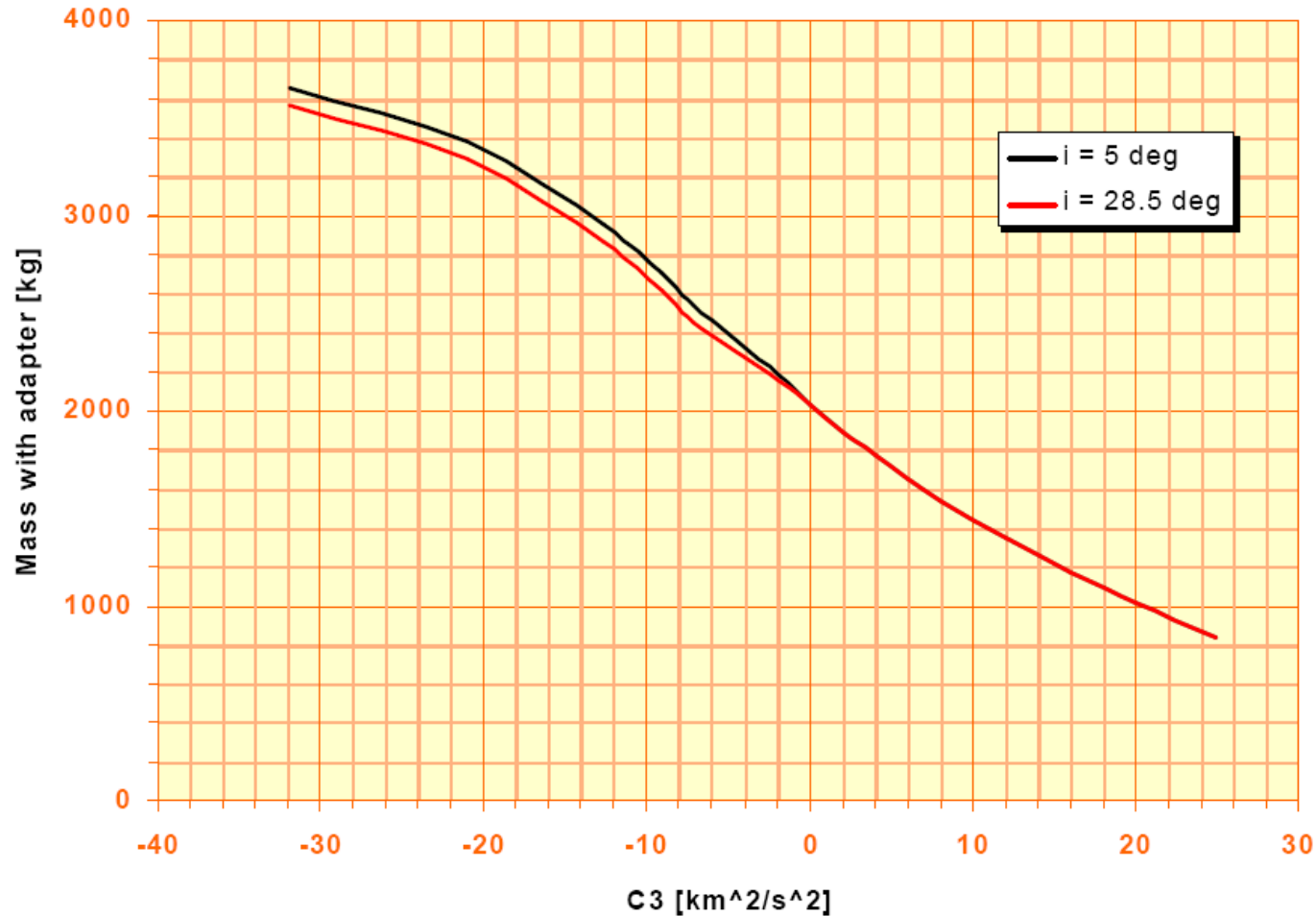
$$v^2 = v_{\infty}^2 + v_{esc}^2 = C_3 + v_{esc}^2$$

C_3 is a measure of the energy for an
interplanetary mission:

16.6 km²/s² (Cassini-Huygens)

8.9 km²/s² (Solar Orbiter, phase A)

2.4 Soyuz ST v2-1b (Kourou Launch)



2.4 Proton

Table 2.9.1-1: Earth Escape Proton M Breeze M Missions

C3 Parameter (km ² /s ²)	Payload Systems Mass (kg)
-5	6270
-2	5890
0	5650
5	5090
10	4580
15	4110
20	3685
25	3295
30	2920
35	2575
40	2260
45	1990
50	1750
55	1525
60	1305
65	1120
<p>C3 Parameter = $V^2 - 2\mu/R$. Performance based on the use of 15255 mm PLF (standard). At fairing jettison, FMHF shall be no more than 1135 W/m². PSM includes LV adapter system mass. PSM is calculated assuming a 2.33-sigma LV propellant margin.</p>	

What Do you Think ?

Assume we have a circular or elliptic orbit for our satellite.

Will it stay there ???

Satellite Orbits

1. Two-body
problem

2. Orbit types

3. Orbit
perturbations

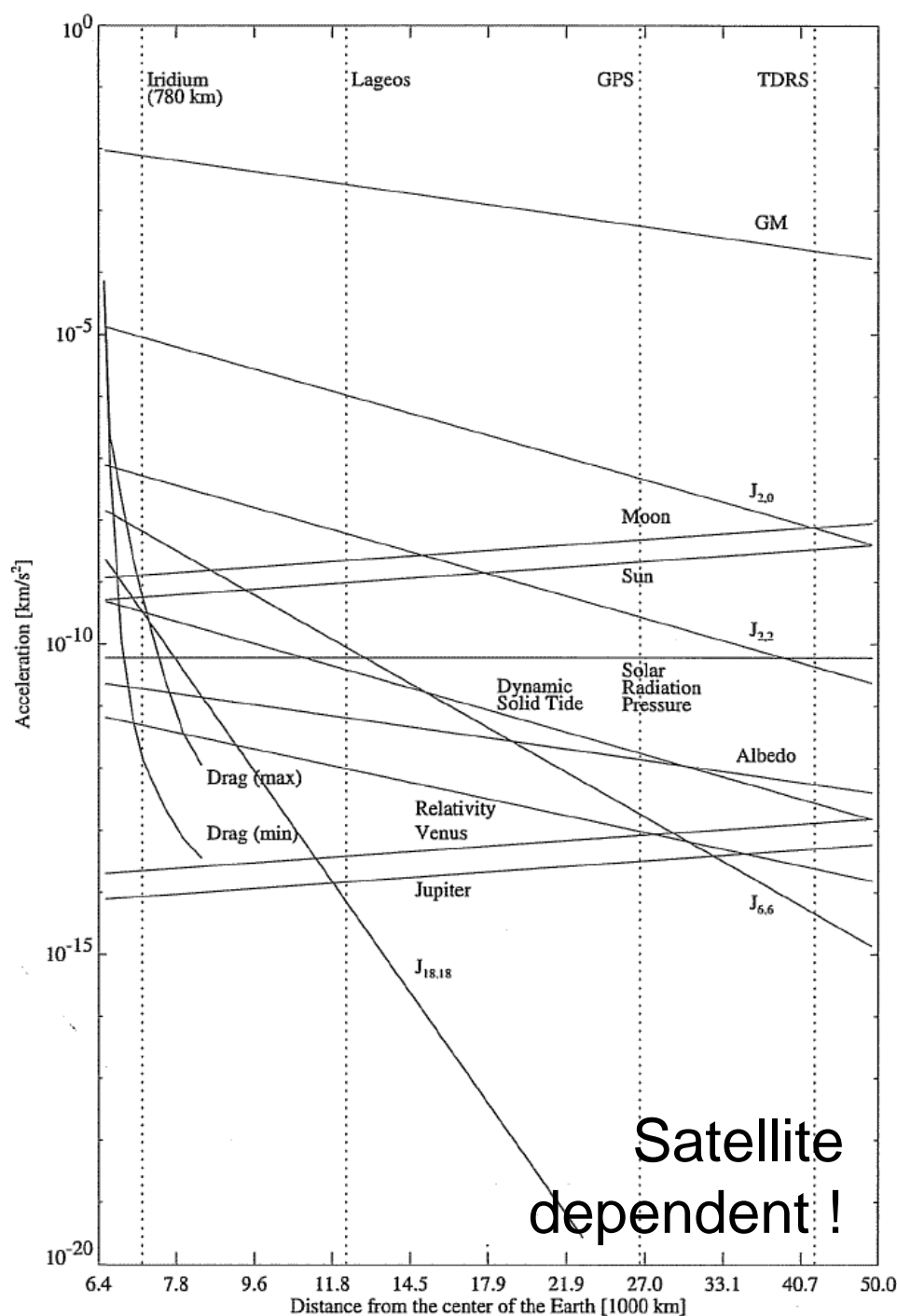
3.1 Non-Keplerian Motion

In many practical situations, a satellite experiences significant perturbations (accelerations).

These perturbations are sufficient to cause predictions of the position of the satellite based on a Keplerian approach to be in significant error in a brief time.

Different Perturbations ?

LEO ? GEO ?



Montenbruck and Gill, *Satellite orbits*, Springer, 2000

3.1 Respective Importance

400 kms

1000 kms

36000 kms

Oblateness

Drag

Oblateness

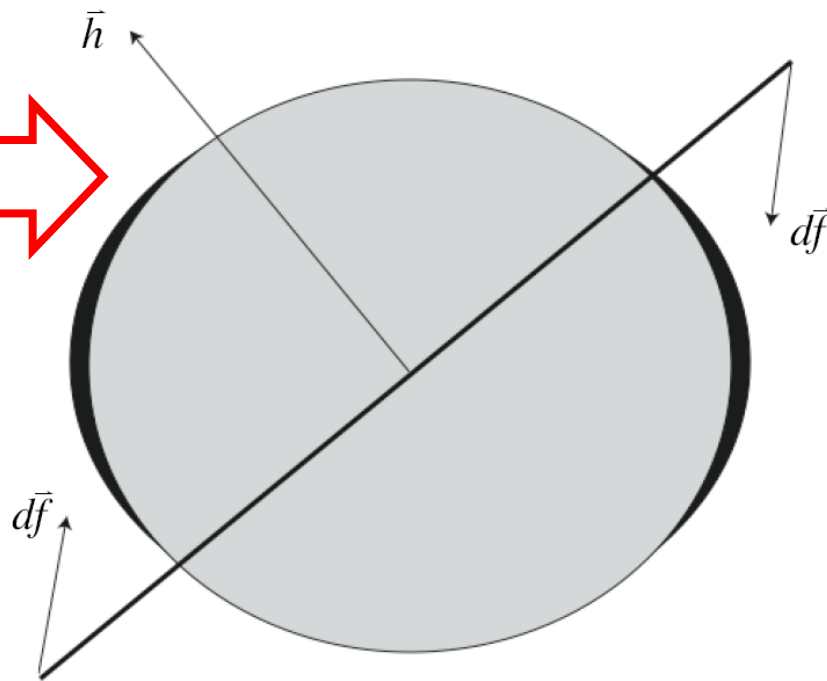
Sun and moon

Oblateness

Sun and moon

SRP

3.2 The Earth is not a Sphere...



$$\ddot{\vec{r}} = -\frac{\mu\vec{r}}{r^3} + d\vec{f}$$

$$\vec{r} \times \ddot{\vec{r}} = -\vec{r} \times \frac{\mu\vec{r}}{r^3} + \vec{r} \times d\vec{f}$$

$$\frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = \dot{\vec{h}} = \vec{r} \times d\vec{f}$$

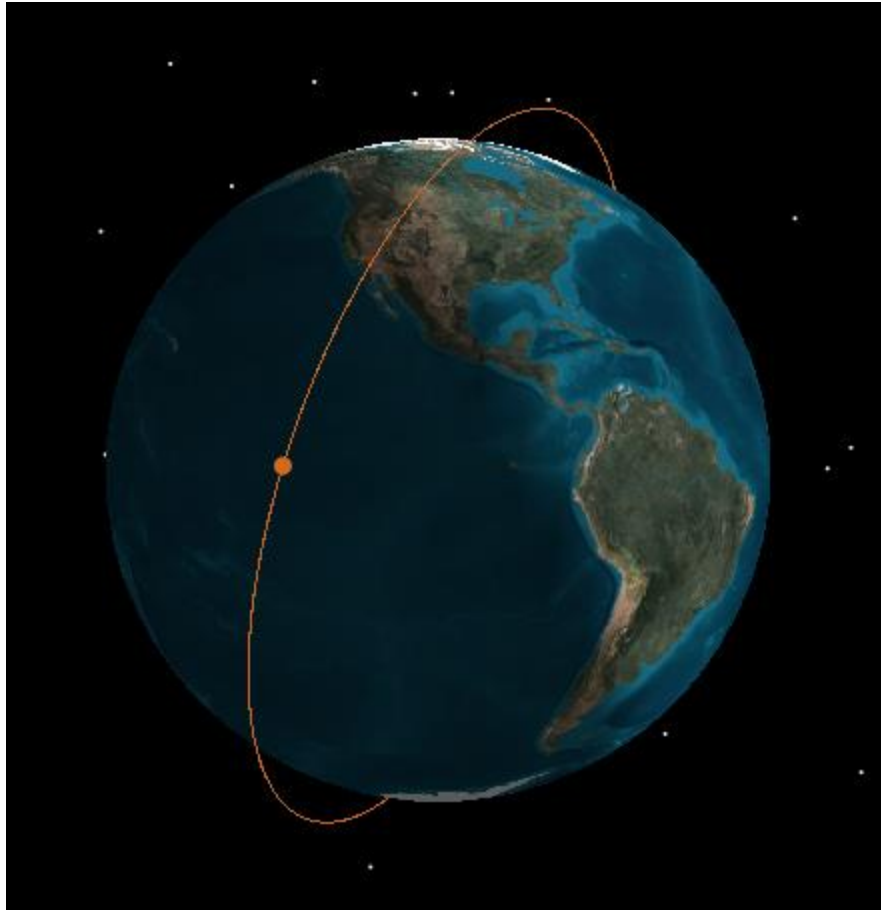
3.2 Physical Interpretation

The force of gravity is no longer within the orbital plane:
non-planar motion will result.

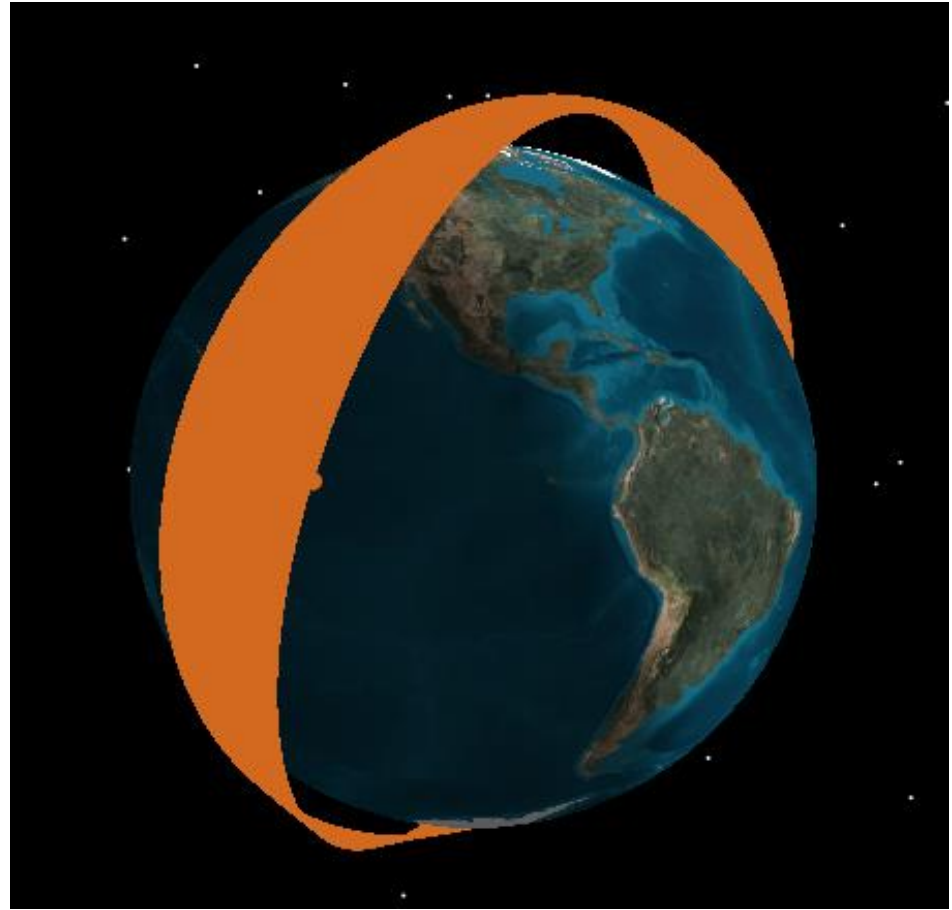
The equatorial bulge exerts a force that pulls the satellite back to the equatorial plane and thus tries to align the orbital plane with the equator.

Due to its angular momentum, the orbit behaves like a spinning top and reacts with a precessional motion of the orbital plane (the orbital plane of the satellite rotates in inertial space).

3.2 What Do You See ?



Two-body propagator



J2 propagator

3.3 The Earth Has an Atmosphere


Atmospheric forces represent the largest nonconservative perturbations acting on low-altitude satellites.


The drag is directly opposite to the velocity of the satellite.

The lift force can be neglected in most cases.

3.3 Effects of Atmospheric Drag




Start Time: 21 Sep 2009 10:00:00.000 UTC 

Stop Time: 1 Mar 2010 14:45:53.000 UTC 

Apogee Altitude  2000 km 

Perigee Altitude  250 km 

Inclination 71 deg 

Argument of Perigee 0 deg 

RAAN  180 deg 

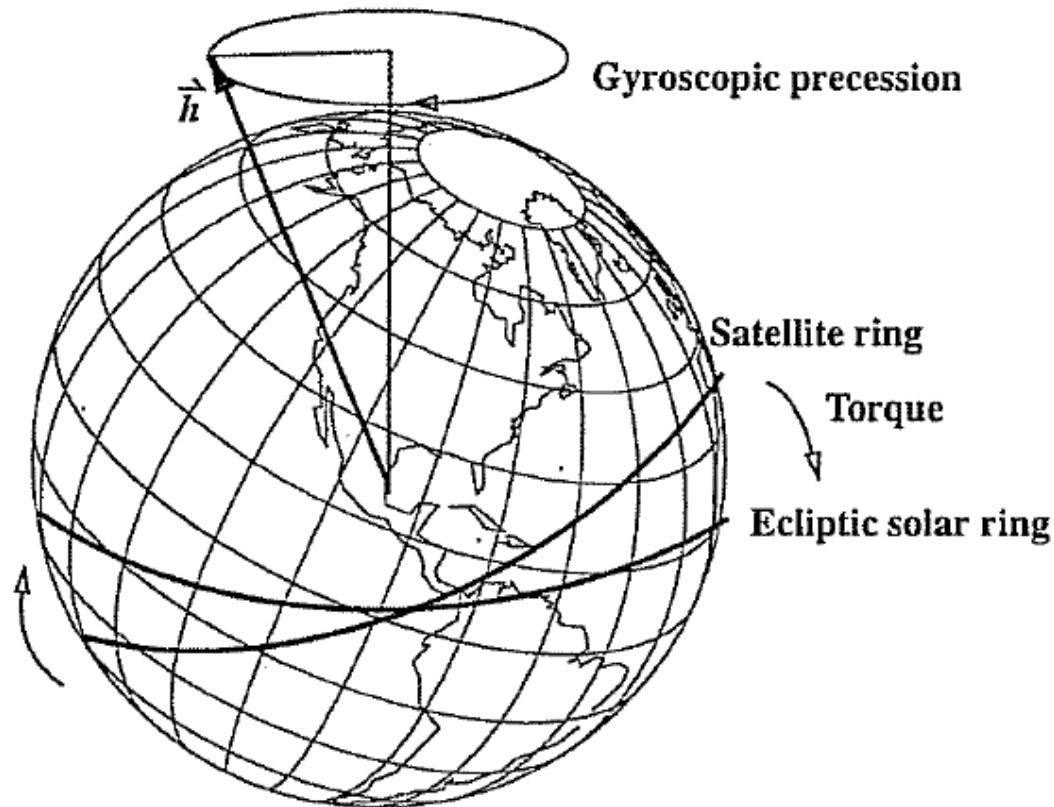
True Anomaly  0 deg 

3.4 Third-Body Perturbations

For an Earth-orbiting satellite, the Sun and the Moon should be modeled for accurate predictions.

Their effects become noticeable when the effects of drag begin to diminish.

3.4 Effects of Third-Body Perturbations



The Sun's attraction tends to turn the satellite ring into the ecliptic. The orbit precesses about the pole of the ecliptic.

Third-Body Interactions. Imagine that the entire mass of a third body (the Sun, for instance) occupies a band about the planet. The resulting torque causes the satellite's orbit to precess like a gyroscope.

3.5 Solar Radiation Pressure

Solar radiation
(photons)

\neq

Solar wind
(particles)

It produces a nonconservative perturbation on the spacecraft, which depends upon the distance from the sun.

It is usually very difficult to determine precisely, but the effects are usually small for most satellites.

800km is regarded as a transition altitude between drag and SRP.

What Do you Think ?

We have an orbit.

Do we have a launch vehicle ???

Satellite Orbits

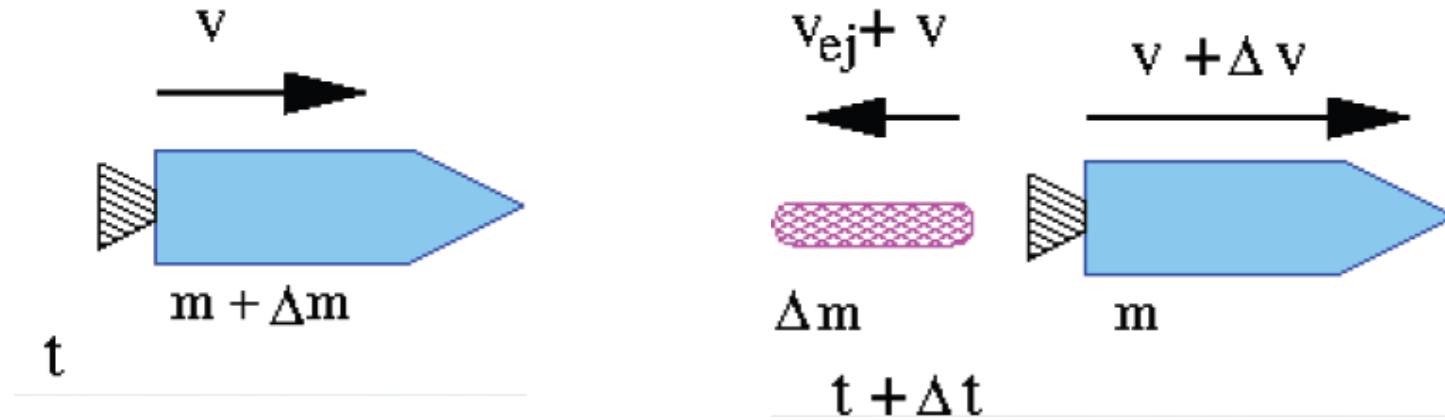
1. Two-body
problem

2. Orbit types

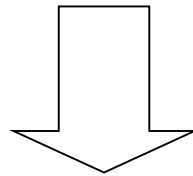
3. Orbit
perturbations

4. Orbit transfer

4. Tsiolkovsky's Rocket Equation (1903)



$$\Delta(m\vec{v}) = 0$$



$$\Delta v = v_{ej} \ln \left(\frac{m_i}{m_f} \right) = I_{sp} g_0 \ln \left(\frac{m_i}{m_f} \right)$$

4. Single-Stage Rocket

$$\Delta v = v_{ej} \ln \left(\frac{100}{100 - 80} \right) = v_{ej} \ln 5 = 1.61 v_{ej}$$
$$\in [5 - 7] \text{ km/s}$$

80%: fuel

10%: dry mass

10%: payload

$V_{ej} = [3-4] \text{ km/s}$

What to conclude ?

4. Specific Impulse

TABLE E1.2.2
THEORETICAL ROCKET PROPELLANT PERFORMANCE

OXIDIZER					FUEL					SEA LEVEL ⁽⁴⁾			VACUUM ⁽⁵⁾		
Name	Sym- bol	Boiling Point °F	Freez. Point °F	Density lbs/ft ³	Name	Sym- bol	Boiling Point °F	Freez. Point °F	Density lbs/ft ³	Specific Impulse Sec	Mixture Ratio O/F	Bulk Density lbs/ft ³	Specific Impulse Sec	Mixture Ratio O/F	Bulk Density lbs/ft ³
Oxygen	LO ₂	-297	-361	71.4	Hydrogen	LH ₂	-423	-434	4.4	388	4.00	17.7	454	4.50	19.0
					RP-1	RP-1	422	- 76	49.7	299	2.55	63.5	351	2.60	63.7
					Hydrazine	N ₂ H ₄	236	35	62.7	313	0.90	66.5	367	0.95	66.6
					UDMH ⁽²⁾	UDMH	146	- 71	49.1	310	1.67	60.9	364	1.67	60.9
					50-50 ⁽³⁾		158	19	56.1	312	1.29	63.8	366	1.32	63.9
Fluorine	LF ₂	-306	-363	94.3	Hydrogen	LH ₂	-423	-434	4.4	410	8.00	28.9	473	10.00	33.1
					RP-1	RP-1	422	- 76	49.7	318	2.56	75.3	377	2.56	75.3
					Hydrazine	N ₂ H ₄	236	35	62.7	364	2.25	81.6	422	2.30	81.8
					UDMH	UDMH	146	- 71	49.1	339	2.45	74.4	400	2.45	74.4
					50-50		158	19	56.1	351	2.40	78.5	412	2.40	78.5
Nitrogen Tetroxide	N ₂ O ₄	70	12	89.5	Hydrazine	N ₂ H ₄	236	35	62.7	292	1.33	75.6	342	1.36	75.8
					UDMH	UDMH	146	- 71	49.1	287	2.60	72.8	336	2.70	73.2
					50-50		158	19	56.1	289	2.00	74.6	339	2.05	74.9
Inhibited Red Fuming Nitric Acid	IRFNA (1)	150	- 63	97.1	RP-1	RP-1	422	- 76	49.7	263	5.00	83.8	309	5.10	84.0
					UDMH	UDMH	146	- 71	49.1	272	3.10	70.9	320	3.20	78.8
Hydrogen Peroxide	H ₂ O ₂	302	31	90.0	Hydrazine	N ₂ H ₄	236	35	62.7	288	2.27	79.4	338	2.32	79.6
Ozone	LO ₃	-169	-315	91.1	Hydrogen	LH ₂	-423	-434	4.4	423	3.80	17.9	493	4.10	18.8

(1) 83% HNO₃
2% H₂O
15% N₂O₄

(2) Unsymmetrical
Dimethylhydrazine

(3) 50% Hydrazine
50% UDMH

(4) P_c = 1000 psia
Opt. ε

(5) P_c = 100 psia
ε = 40

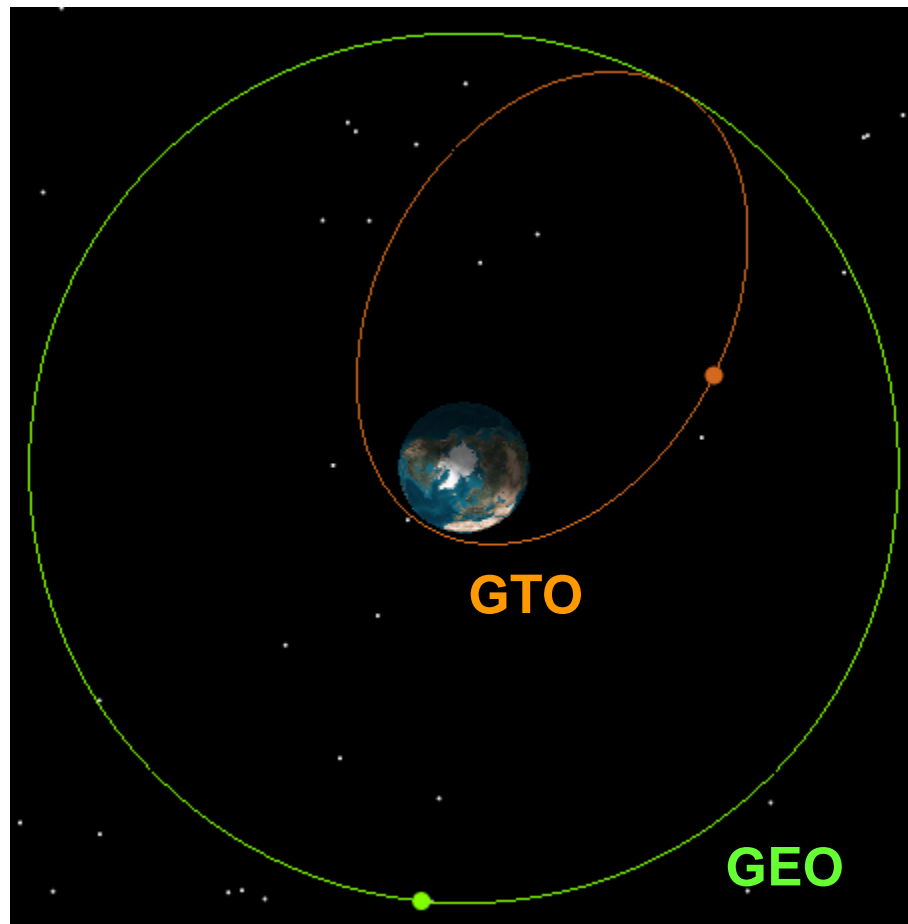
4. Motivation

Without maneuvers, satellites could not go beyond the close vicinity of Earth.

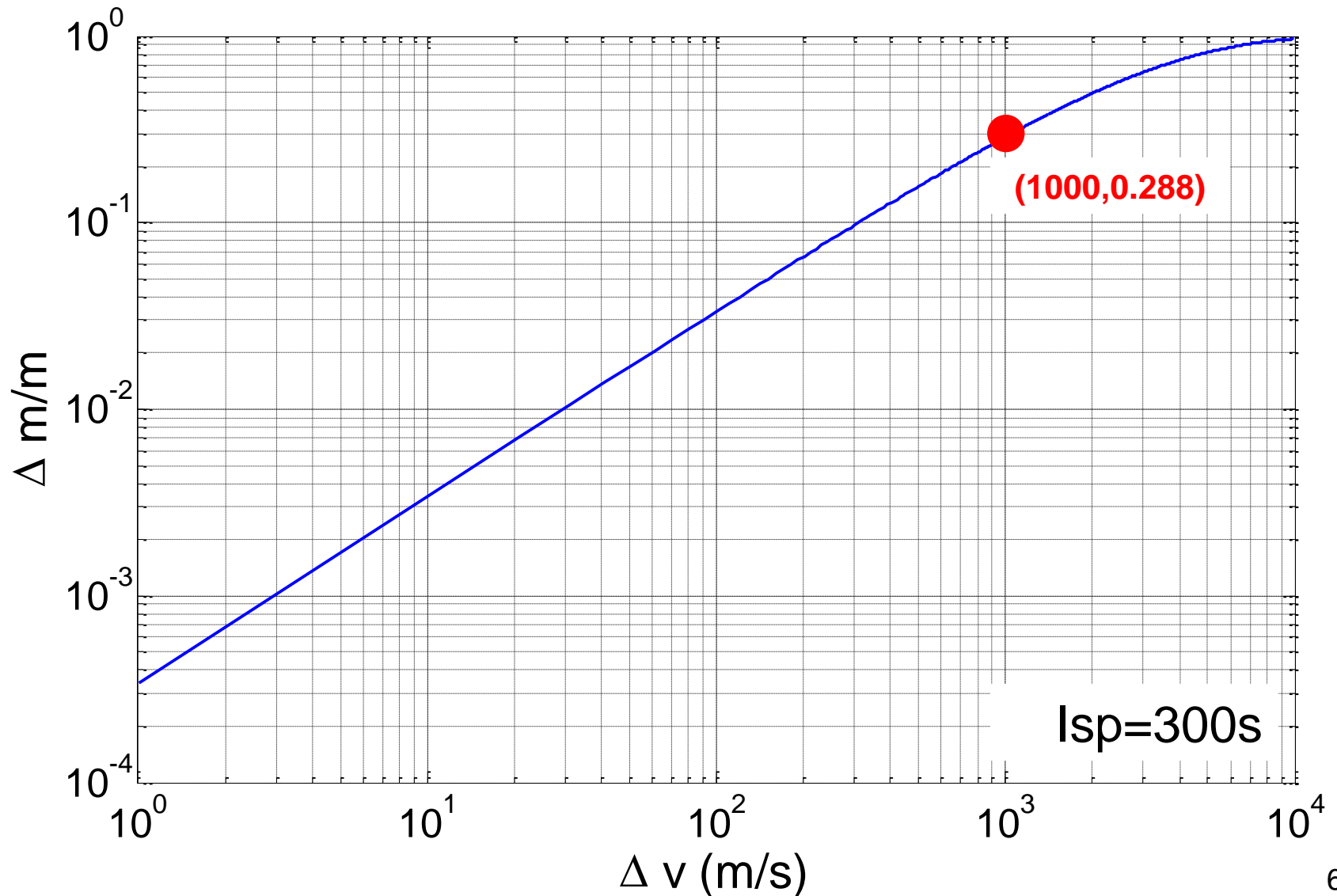
For instance, a GEO spacecraft is usually placed on a transfer orbit (LEO or GTO).

4. From GTO to GEO: Ariane V

Ariane V is able to place heavy GEO satellites in GTO:
perigee: 200-650 km and apogee: ~35786 km.



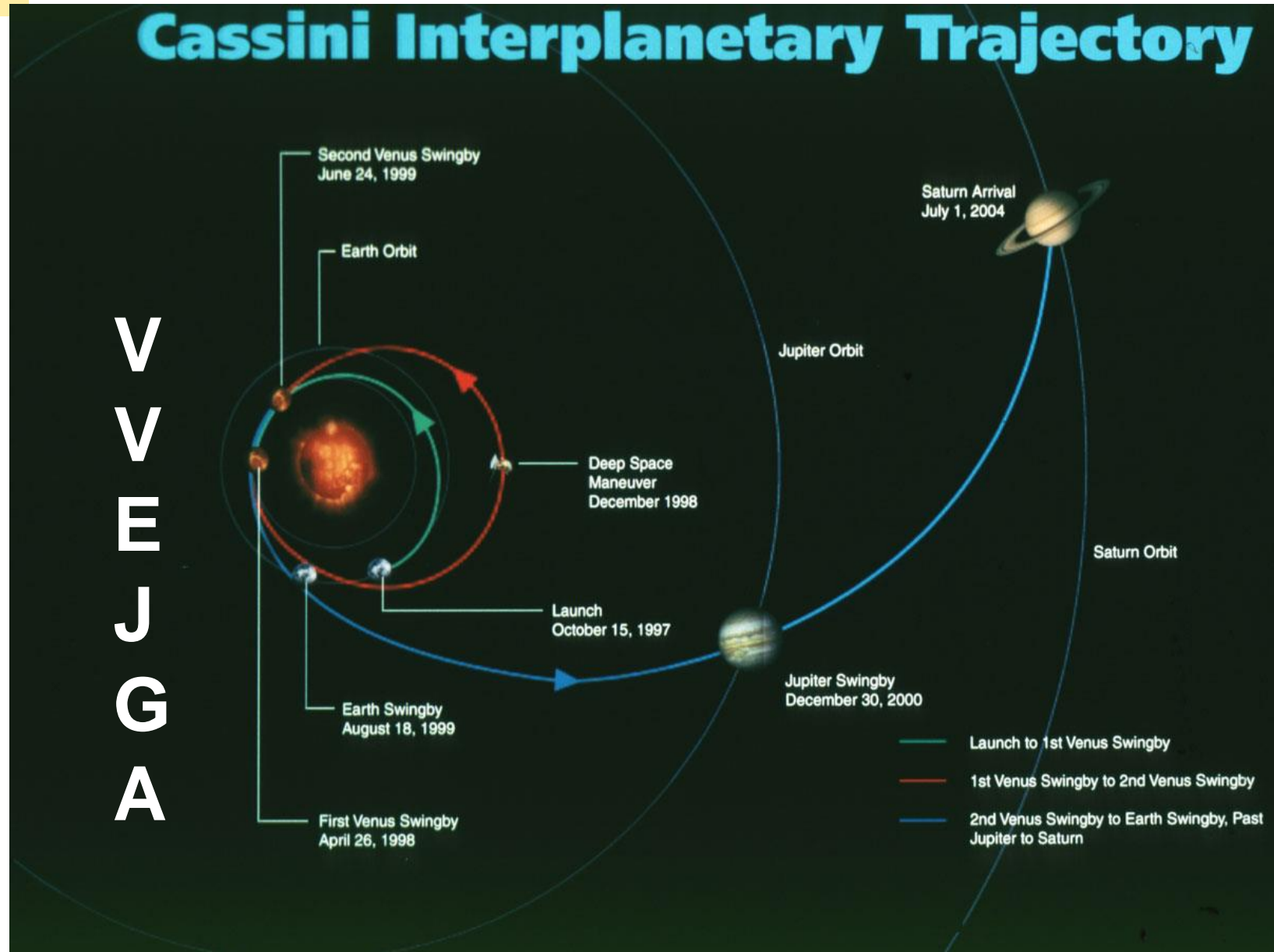
4. Delta-V: Order of Magnitudes



4. Delta-V Budget: GEO

Mission orbit	Geostationary	(Allowable deviation from nominal position 0,1 deg)			
Launcher	Proton				
Launch in GTO					
Mission duration (yrs)	15				
Manoeuvre	delta v/manoeuvre (m/s)	cycle time (days)	no. of maneuvers (-)	delta v/yr (m/s)	total delta V (m/s)
Apogee kick	1836,49	*	1,0	*	1836,5
10 yr average NSSK	10,73	86,1	63,6	45,5	682,0
Worst Case NSSK	10,90	77,4	70,7	51,4	770,7
EWSK	0,13	35,3	155,3	1,33	19,9
Worst Case EWSK	NA	NA	NA	1,74	26,1
Orbit Maneuvres	0,00	*	0,0	*	0,0
Disposal	10,88	*	1,0	*	10,9
Total Delta V (most favourable)					2549,3
Total Delta V (worst case EWSK)					2555,5
Total Delta V (worst case NSSK & EWSK)					2644,2

4. How to Go to Saturn ?



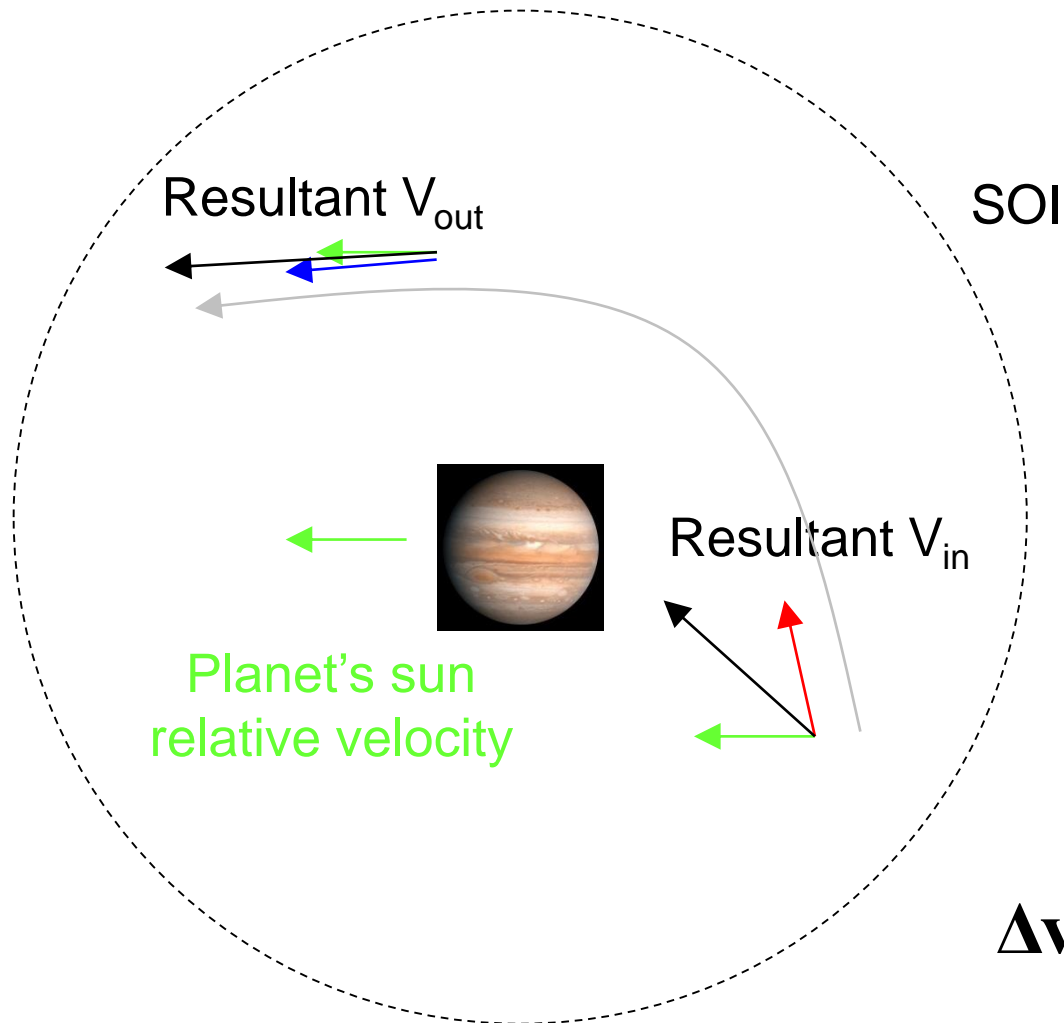
4. Gravity Assist

Also known as planetary flyby trajectory, slingshot maneuver and swingby trajectory.

Useful in interplanetary missions to obtain a velocity change without expending propellant.

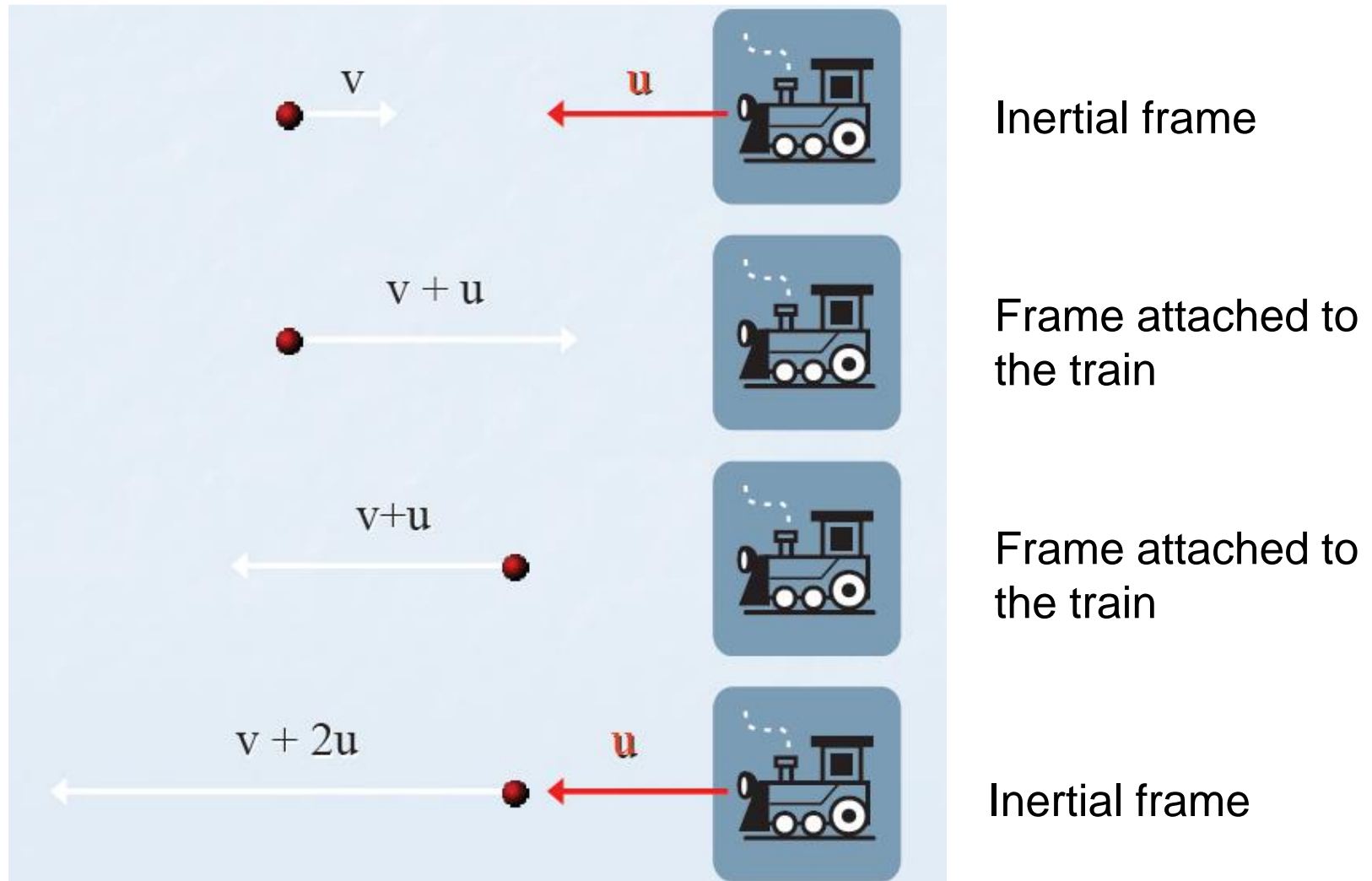
This free velocity change is provided by the gravitational field of the flyby planet and can be used to lower the delta-v cost of a mission.

4. Basic Principle



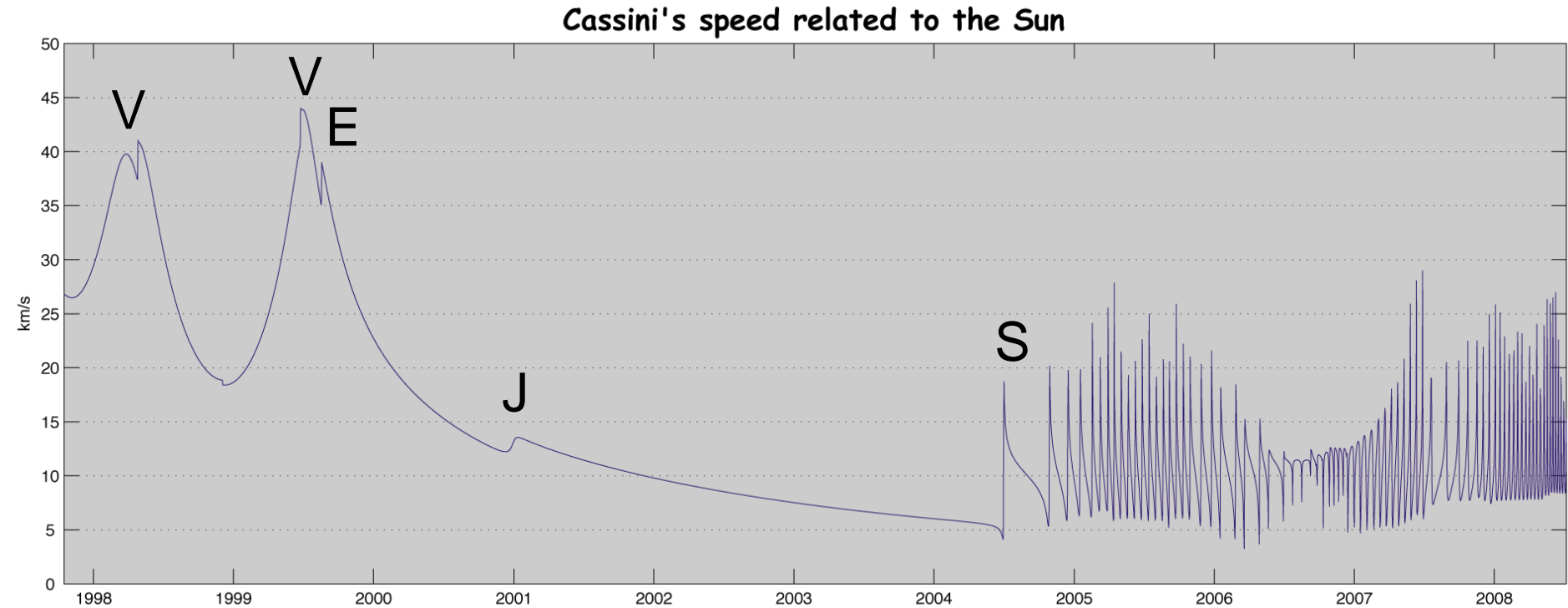
$$\Delta \mathbf{v} = \mathbf{v}_{\infty, out} - \mathbf{v}_{\infty, in}$$

4. Basic Principle

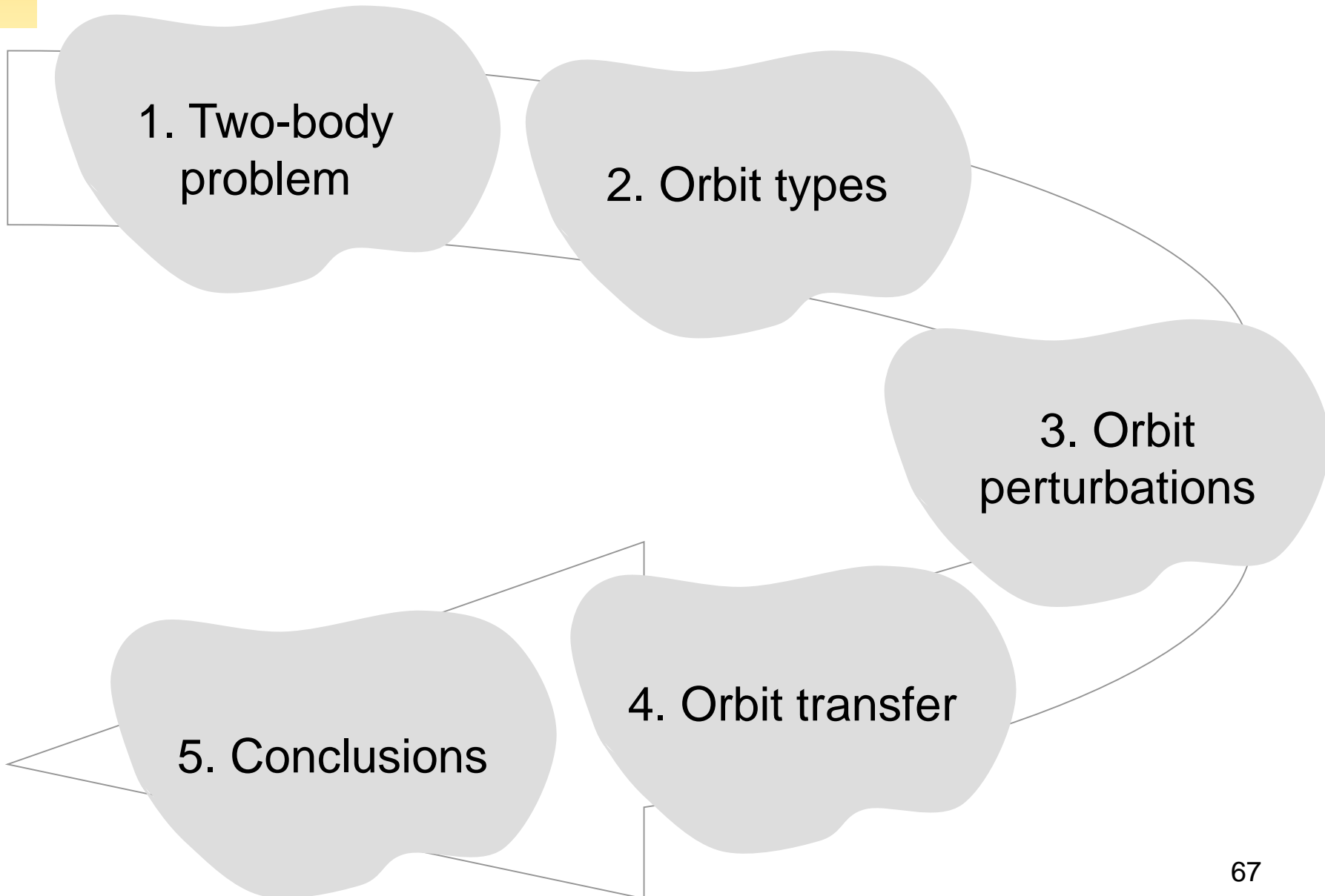


A gravity assist looks like an elastic collision, although there is no physical contact with the planet.

4. Cassini: Swingby Effects



Satellite Orbits



Satellite Orbits

Gentle introduction to satellite orbits; more details in the astrodynamics course.

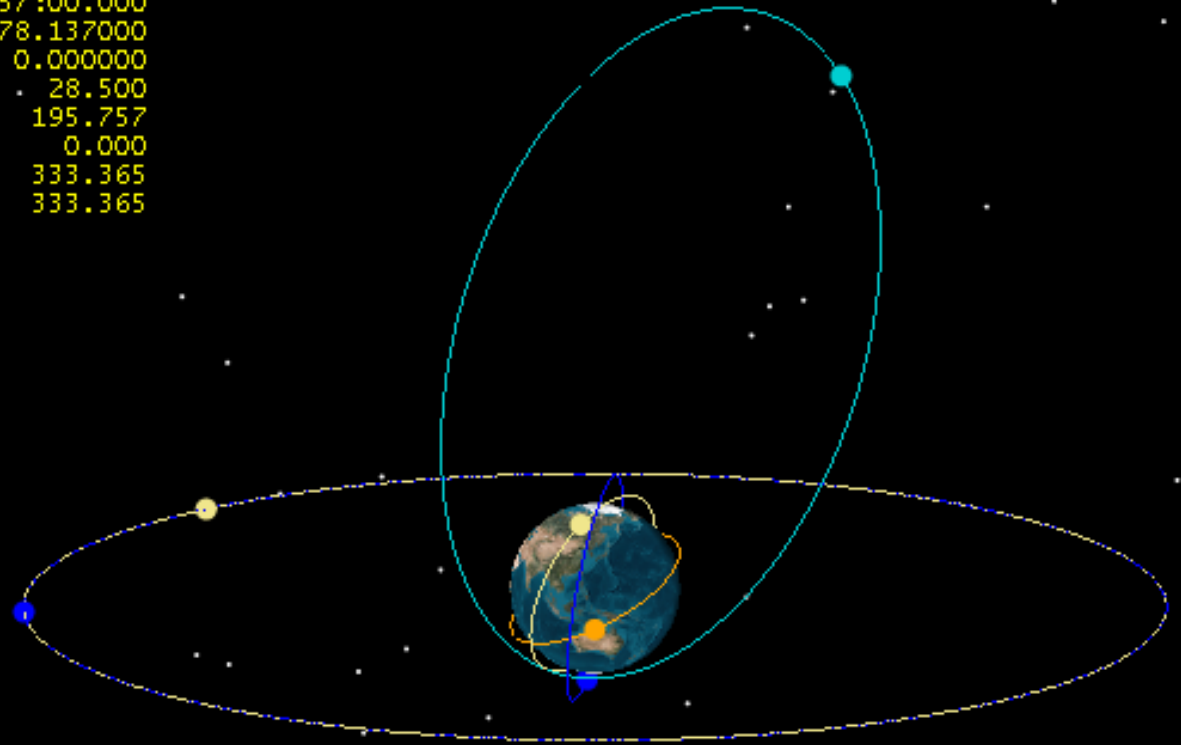
Closed-form solution of the 2-body problem from which we deduced Kepler's laws.

Orbit perturbations cannot be ignored for accurate orbit propagation and for mission design.

Orbit transfers are commonly encountered. Satellite must often have their own propulsion.

HST Classical Orbit Elements
 Time (UTCG): 2 Sep 2008 03:57:00.000
 Semi-major Axis (km): 6978.137000
 Eccentricity: 0.000000
 Inclination (deg): 28.500
 RAAN (deg): 195.757
 Arg of Perigee (deg): 0.000
 True Anomaly (deg): 333.365
 Mean Anomaly (deg): 333.365

Educational Use Only



METEOSAT 6-7, HST, OUFIT-1, SPOT-5, MOLNIYA



GEO

$i=0$



LEO

$i=28.5$



LEO

$i=71$



(LEO) SSO

$i=98.7$



Molniya

$i=63.4$

AERO0025 – Satellite Engineering

Lecture 2

Satellite orbits

