Nonlinear Vibrations of Aerospace Structures

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T02 Nonlinear FRFs

Nonlinear symptoms
Nonlinear Model of the 1\textsuperscript{st} Beam Mode

Linear model identified at low level (31 Hz, 0.12\%):

\[ 0.289\ddot{x} + 0.1357\dot{x} + 11009x = F \sin \omega t \]

Nonlinearity identified at high level: 2.37 \times 10^9 x^3
We Built the Model in NI2D
2.1: let’s revisit what we observed last week
Nonlinearity Introduces a Fundamental Change
Let’s Reverse the Sweep.
We Can Jump Up Too!
Calculate the Nonlinear FRF
Set Appropriate Numerical Parameters
Set Appropriate Numerical Parameters
The Nonlinear FRFs
The Jump Phenomenon Can Be Understood
The Jump Phenomenon Can Be Understood
The Jump Phenomenon in Practice
2.2: sine with a fixed frequency in the multi-valued region
Consider an Excitation with a Fixed Frequency
Run and Save
Change Initial Conditions

![Image of a diagram showing a model with initial conditions and parameters. The dialog box is open, showing Newmark parameters, including final time, time step, number of time steps, number of periods, and time steps per period. The diagram also shows mathematical expressions for initial conditions such as $0.06 \sin (2\pi \cdot 34.4t)$ and $2.3 \times 10^9 x^3$. A model is represented with modes and initial conditions, labeled as mode 1: 31.0631 Hz / 0.12%.]
Apply Initial Impulse and Change Colors
Run and Compare
What Do You Think?
Usefulness of Nonlinear FRFs
2.3: a new puzzling result
Create a New System in NI2D

Equations:

\[ x'' + 1.9x' + 0.1x = 0.048 \sin(2\pi \cdot 0.0716197 \cdot t) \]

Mode 1: 0.159155 Hz / 5.00%
Parameters for Nonlinear FRF Calculation

- **Starting point:** 0.3 rad/s
- **Min:** 0.2 rad/s
- **Max:** 1.2 rad/s
- **Direction:**
  - **Fold:** detect
  - **Branch point:** detect
  - **Neimark-Sacker:** detect
- **Stepsize:** 2
- **Adaptive**
  - **Min:** 0.1
  - **Max:** 20000
  - **Optimal number of iterations:** 3
- **Maximum number of points:** 10000
- **Beta angle:** 90°

**Harmonic Balance parameters**

- **Number of harmonics:** 15
- **Number of points:** 256
- **Compute stability**
- **Reordering**
- **Linear mode:**

- **Amplitude of 1st guess:** 0.001 m
- **Maximum number of iterations:** 15
- **Relative precision:** 1e-06
- **Scaling factor for displacements:** 1e-06
- **Scaling factor for time:** 1

**Buttons:**
- HB parameters
- Apply
- Start
- Cancel
The Nonlinear FRF

NL Frequency Response of dof n°1

Amplitude (m)

Pulsation (rad/s)

0.45 rad/s
First Nonlinear Time Integration

Displacement vs time, dof n°1

\[ x(0) = 0.1 \]

OK !
Second Nonlinear Time Integration

Displacement vs time, dof n°1

$x(0) = 2$

NL Frequency Response of dof n°1

???

Amplitude (m)

Pulsation (rad/s)