Nonlinear Vibrations of Aerospace Structures

University of Liège, Belgium

T03 Nonlinear Simulations

Nonlinear Modeling

Time Integration

Continuation

NI2D

Nonlinear Identification to Design
Software

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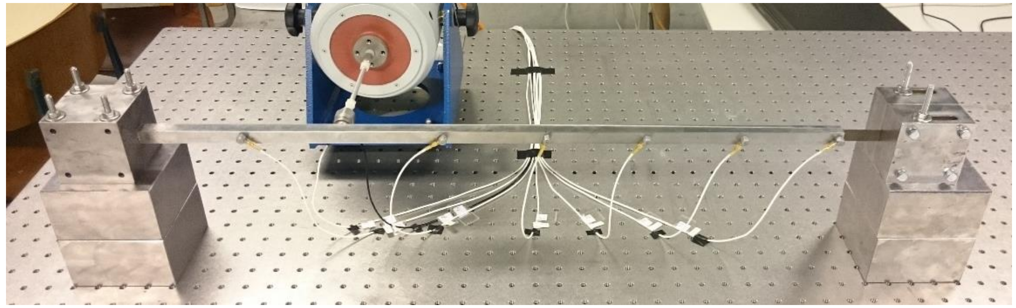
Objectives of This Tutorial

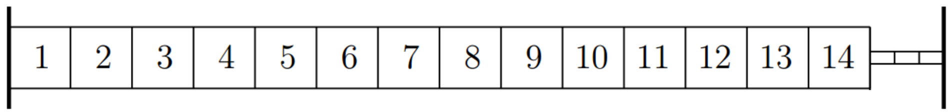
Get familiar with NI2D tools for nonlinear simulations:

- Simulate the system dynamics.
- Compute nonlinear frequency response curves (NFRCs).

Case Study: A Nonlinear Beam

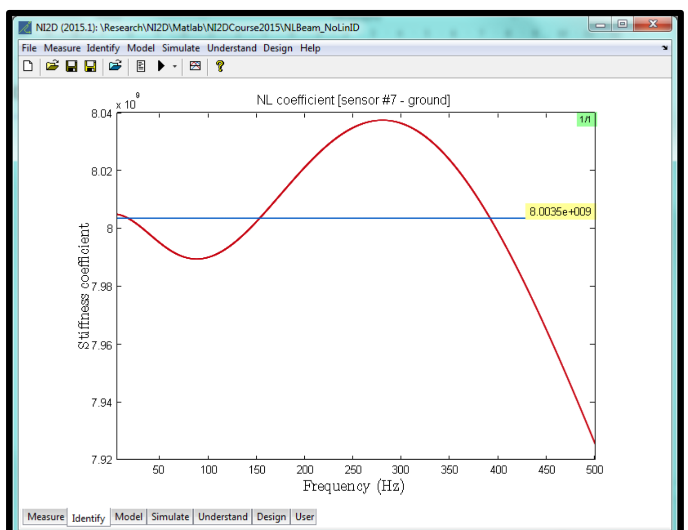
Linear
FE model





+

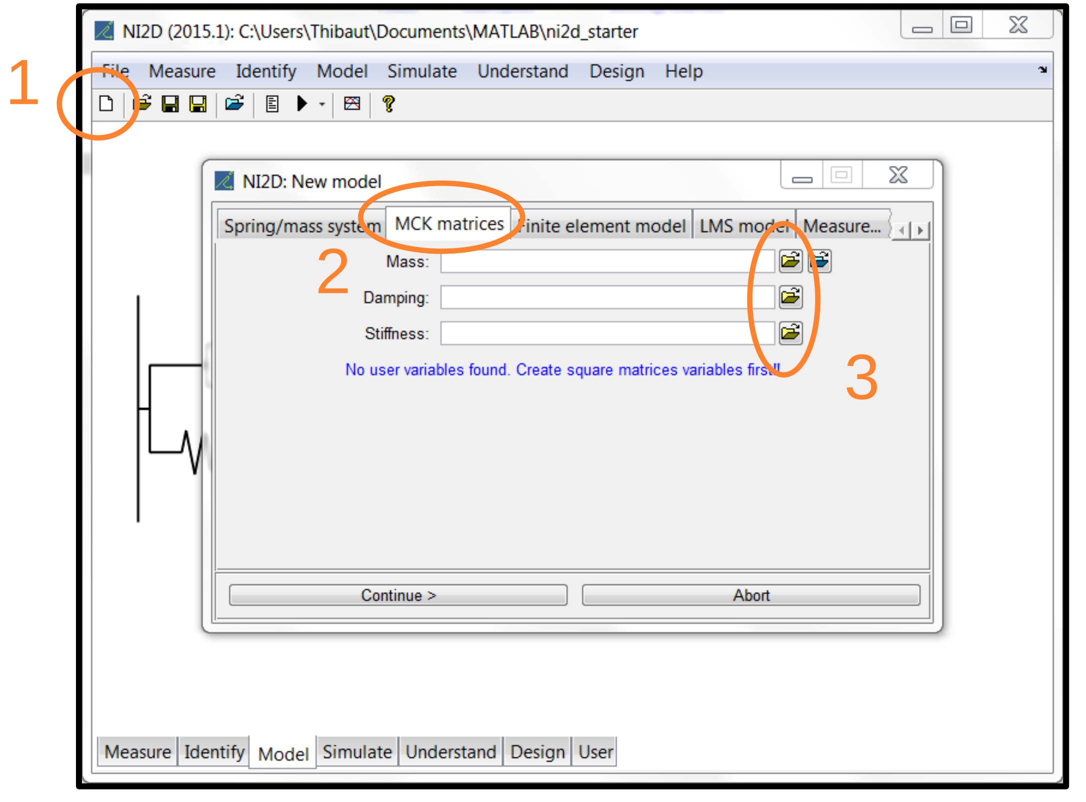
Identified
nonlinearities



Cubic coeff.
(geometrical)

Quadratic coeff.
(clamping)

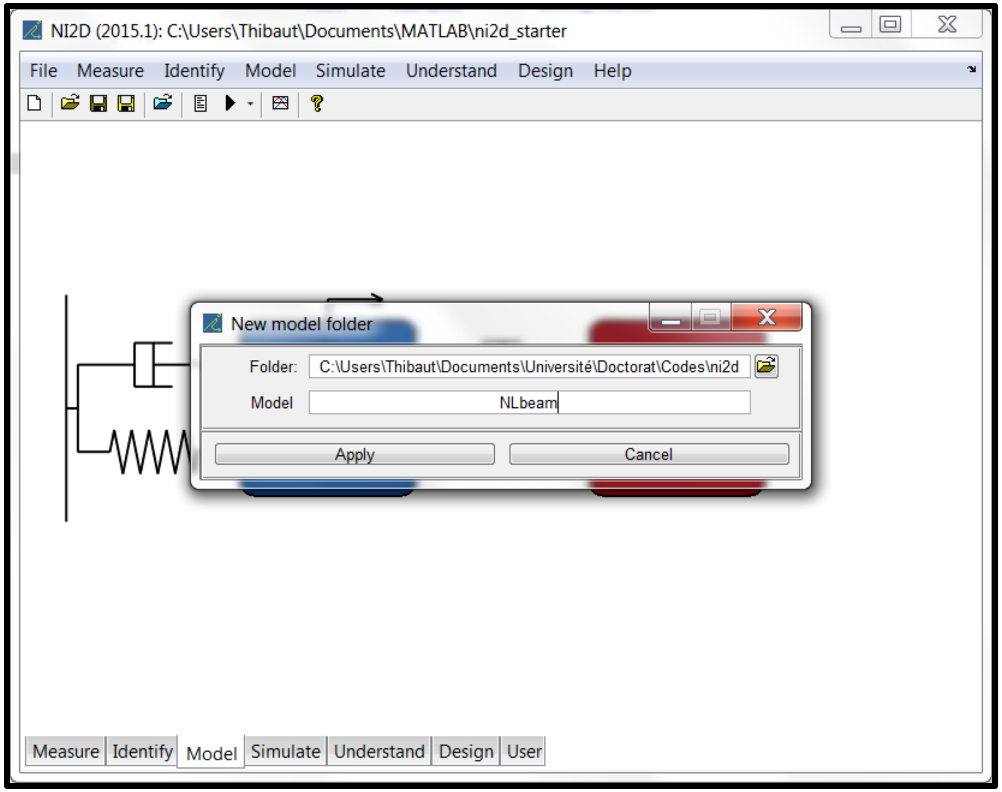
Import of the Linear FE Model



- Create a new model (1) and select “MCK matrices” (2).
- Import $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ from *NLbeam.mat* (3).

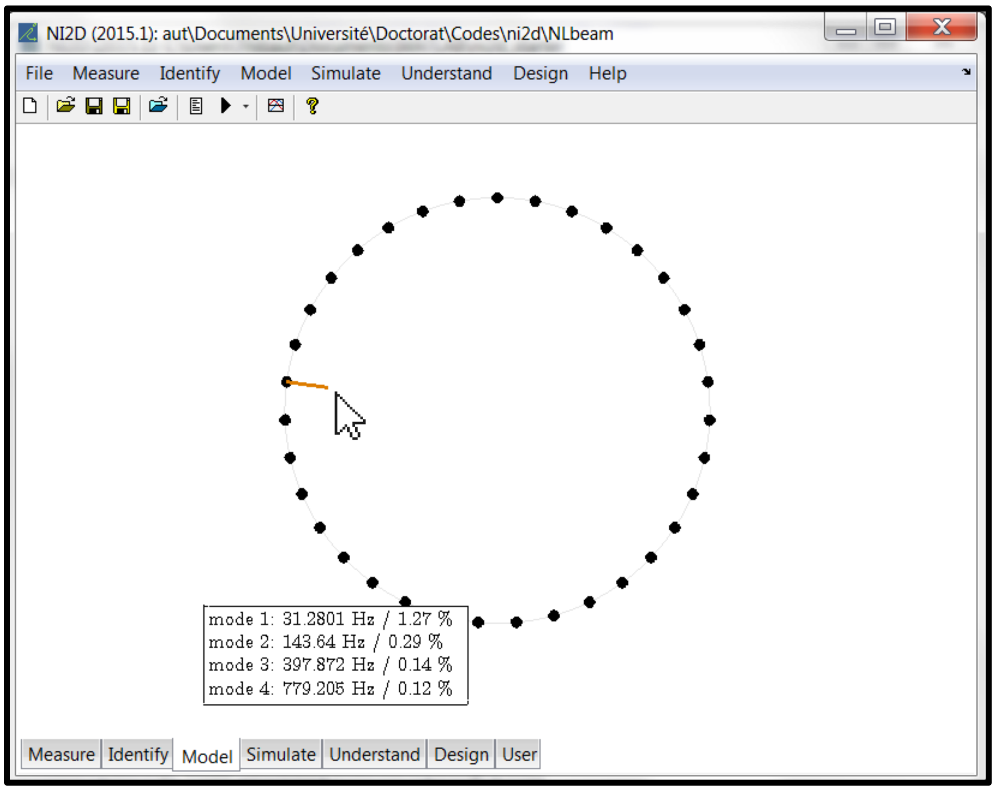
Import of the Linear FE Model

- Name this new system “NLbeam”.



Creation of Nonlinear Connections

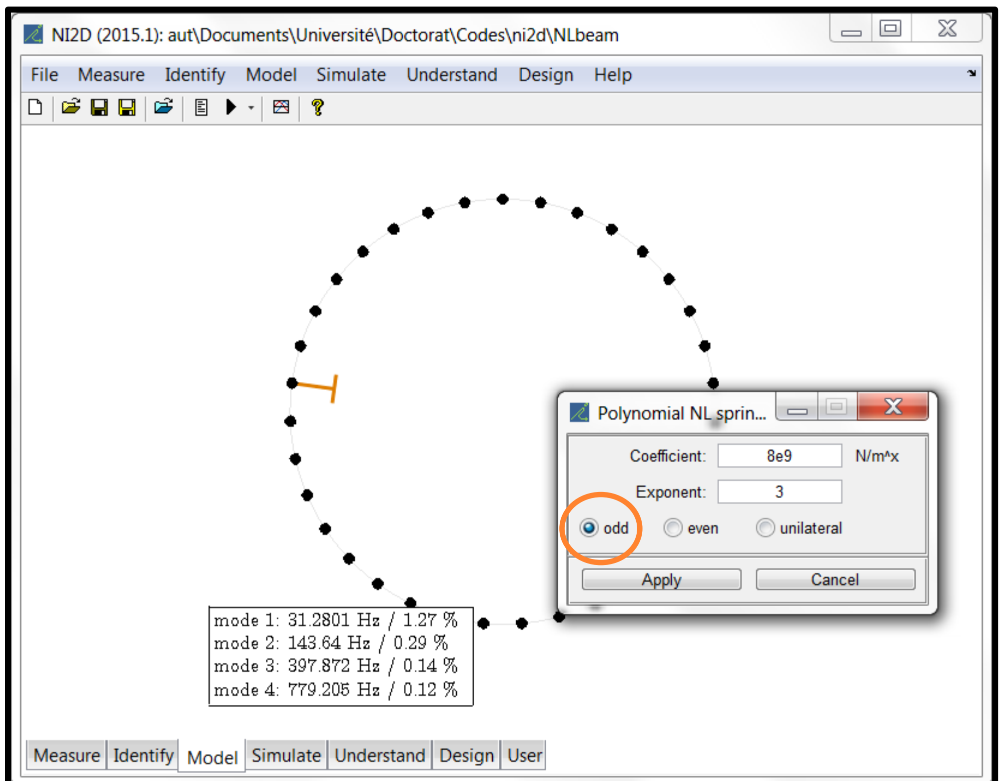
At this point, the model is linear and a nonlinear connection (cubic+quadratic) has to be created between the displacement at the tip of the main beam (dof #28) and the ground.



- Bring the cursor on dof #28 and use <CTRL+LEFT+DRAG> inside the circle to create a connection with the ground.

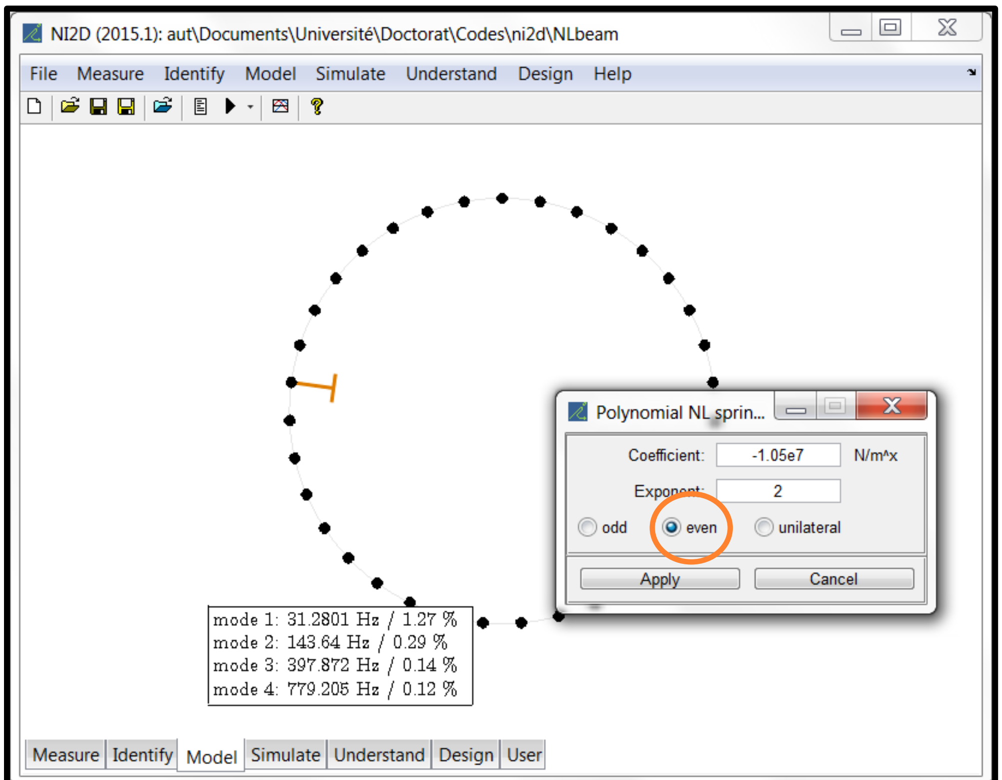
Creation of Nonlinear Connections

- Select “Nonlinear polynomial stiffness”.
- Create an odd cubic nonlinearity with stiffness set to $8 \times 10^9 \mathrm{N/m^3}$.



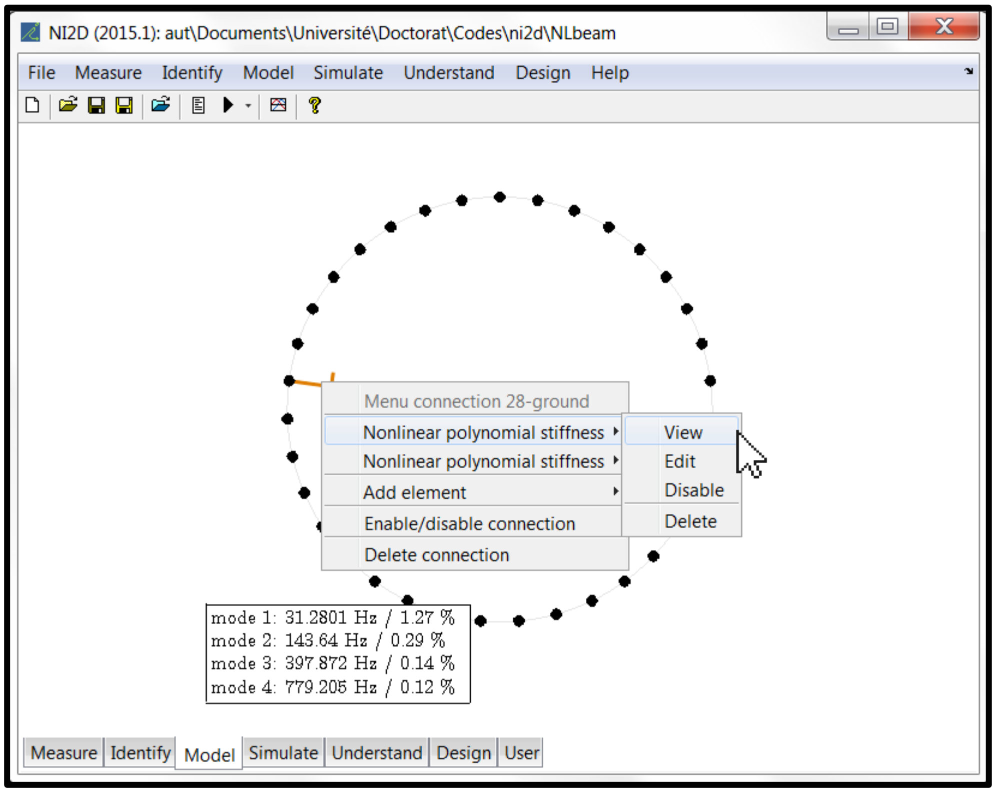
Creation of Nonlinear Connections

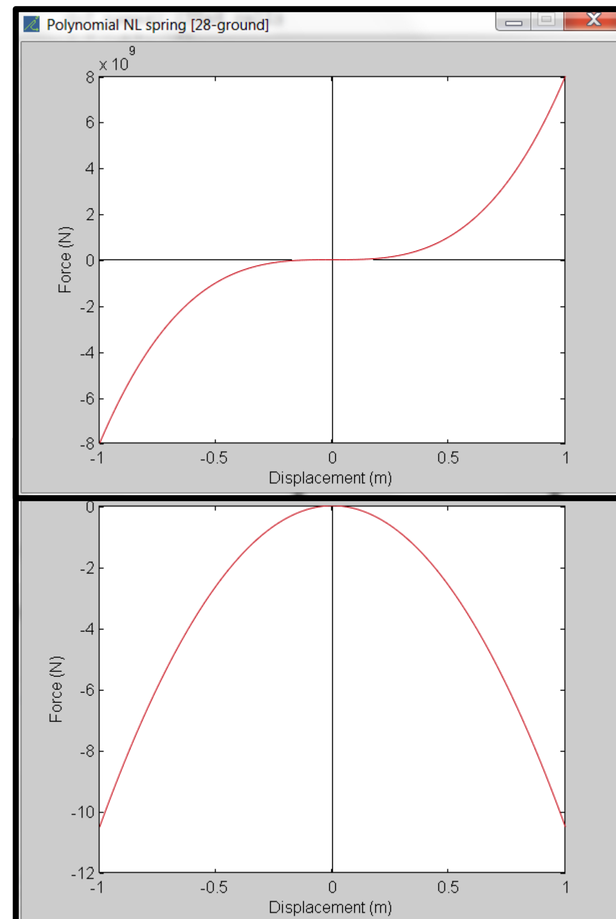
- Create a new connection between dof #28 and the ground
- Create an even quadratic nonlinearity with stiffness set to -1.05×10^7 N/m².



Visualization of Nonlinear Connections

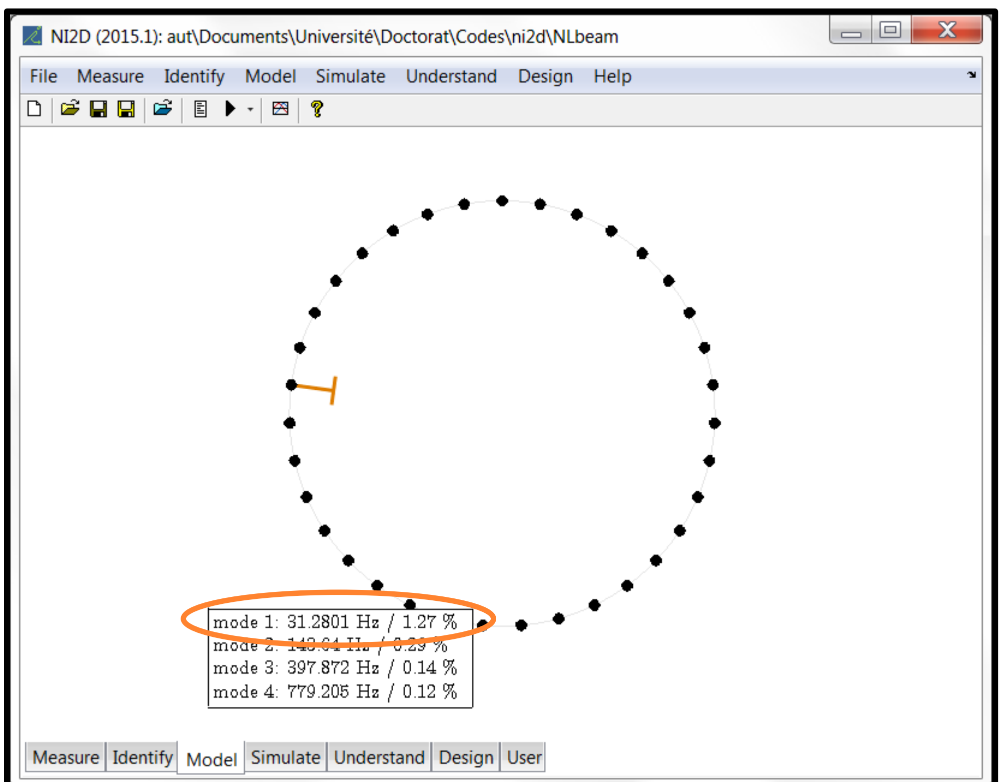
- Use <RIGHT CLICK> on the nonlinear connections and select “view” to visualize each restoring force.





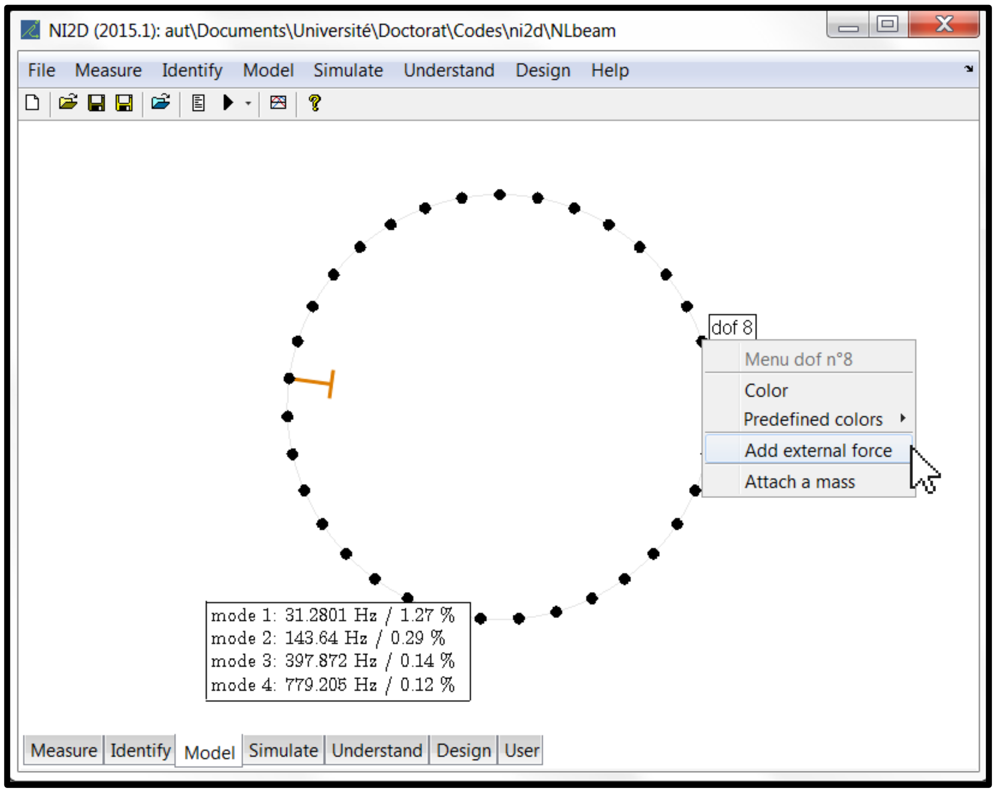
Creation of a Sine External Force

As a first exercise, we will study the system’s response to a sine excitation with a frequency close to the first resonance frequency of the beam (31.28Hz).



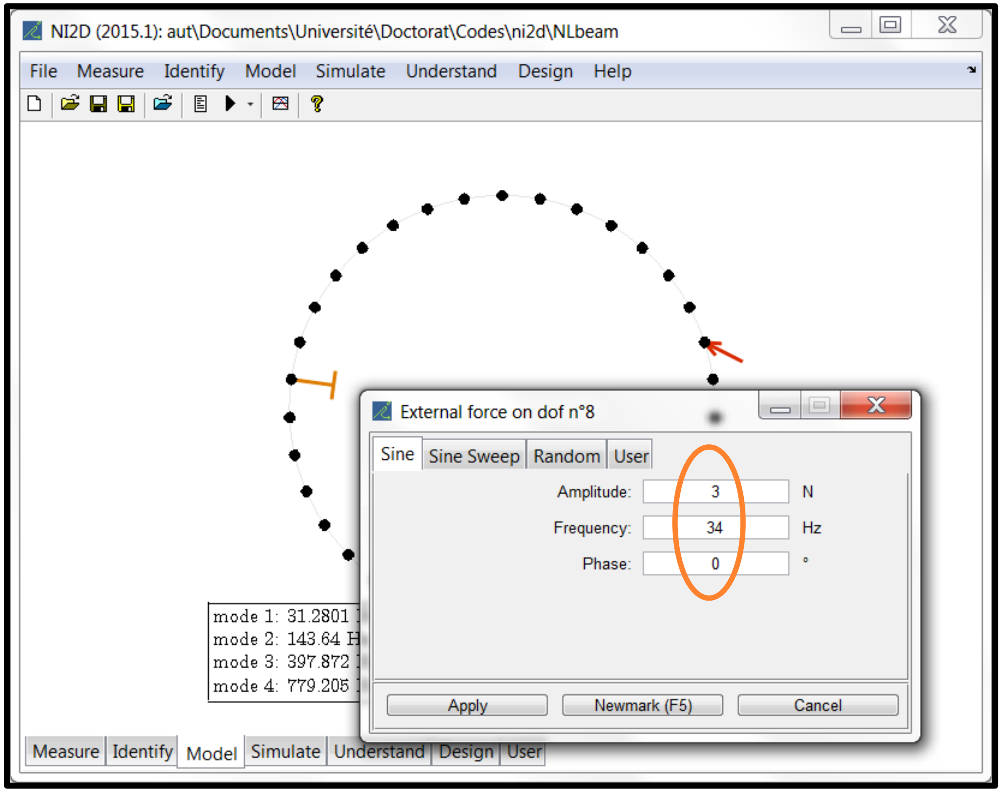
Creation of a Sine External Force

- Create a forcing by using <RIGHT CLICK> on dof #8 and selecting “Add external force”.



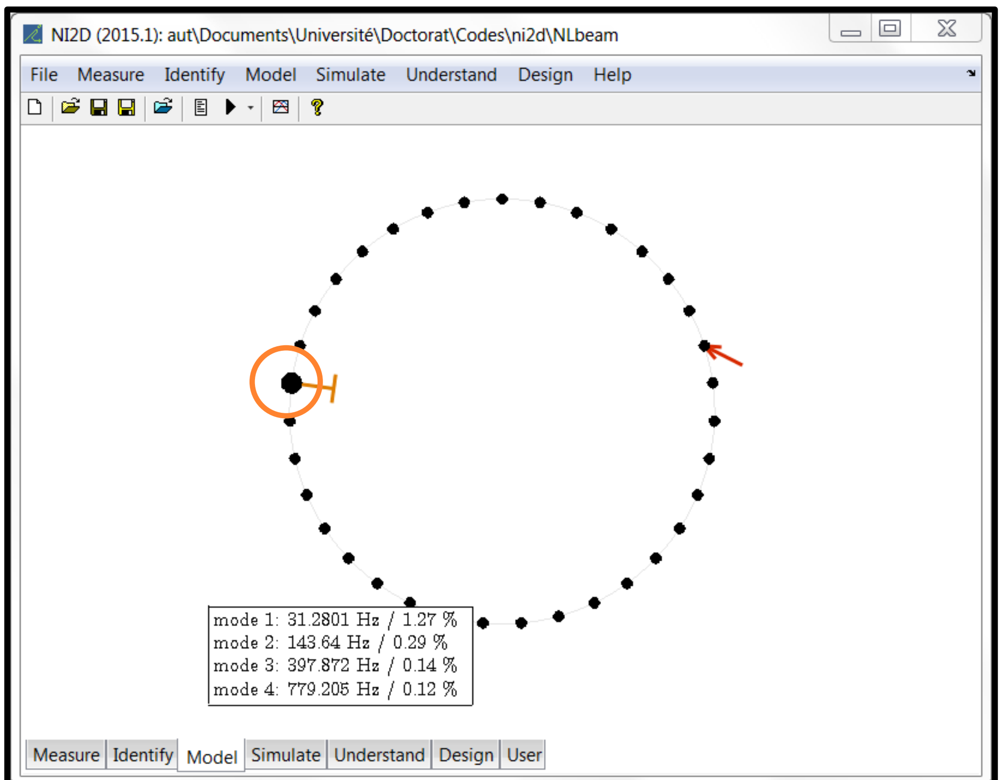
Creation of a Sine External Force

- Select “Sine” and give the following forcing parameters, then click on “Apply”.



Simulation of a Response to a Sine Excitation

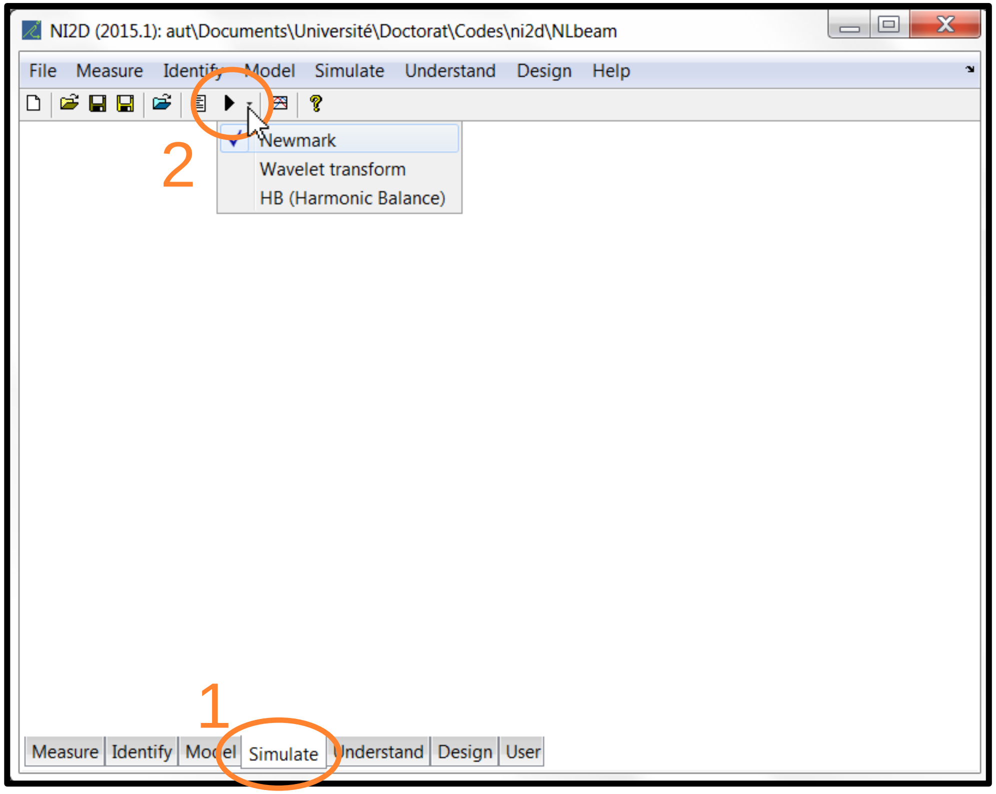
- In order to show the time series at the tip of the main beam, use <ALT+LEFT> on dof #28 to display next results on that dof.



- You can modify the color associated to dof #28 using <RIGHT CLICK> on dof #28.

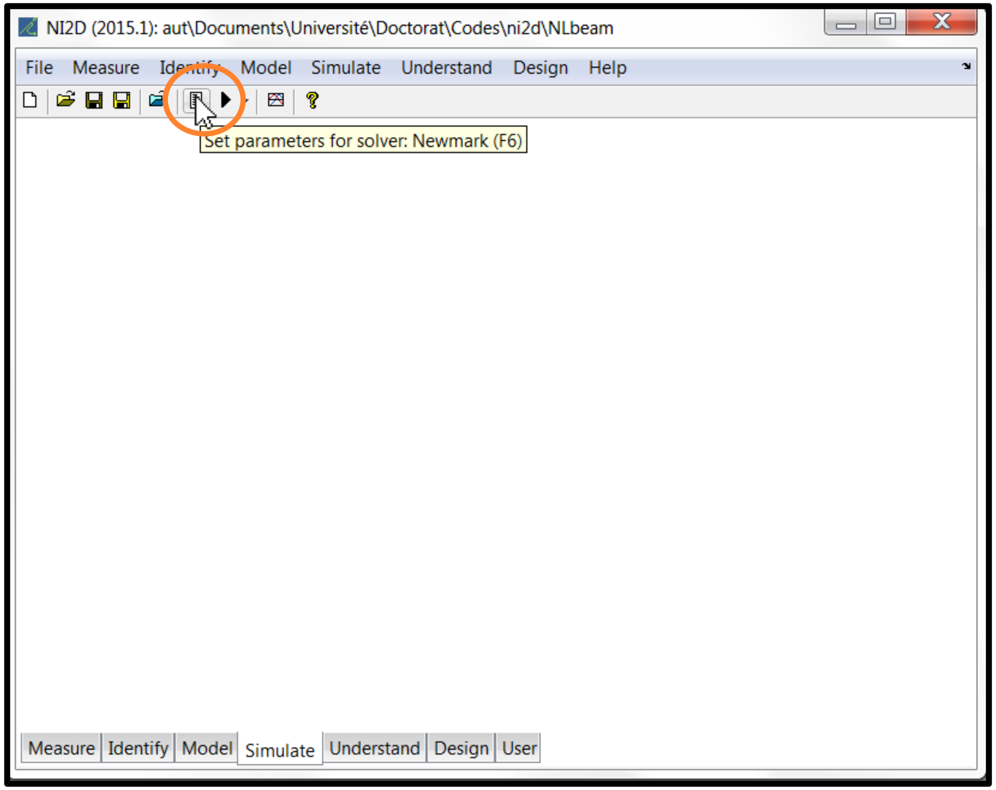
Simulation of a Response to a Sine Excitation

- Go to “Simulate” tab (1) and select “Newmark” as a solver (2).



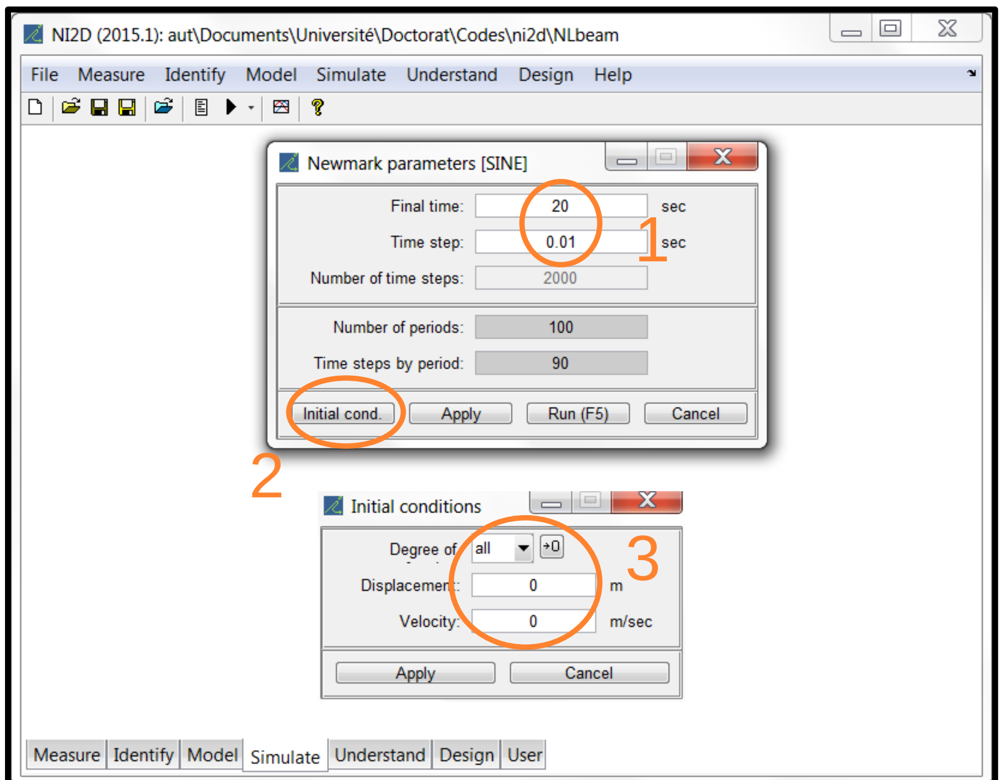
Simulation of a Response to a Sine Excitation

- Click on “Set parameters for solver”.



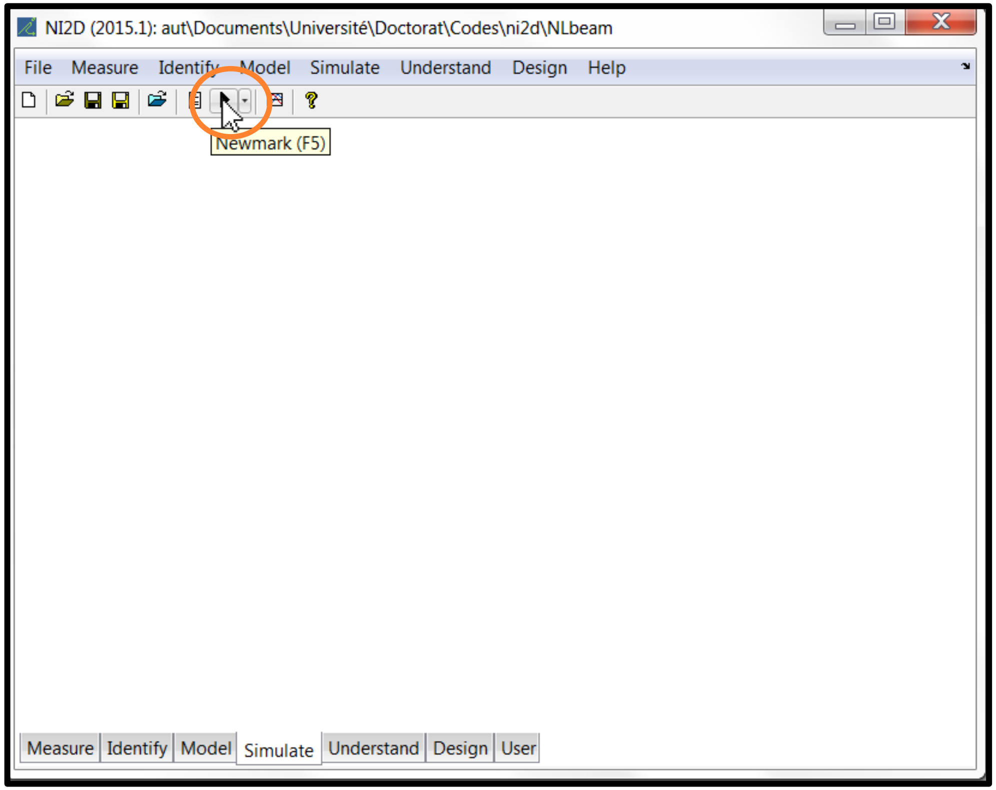
Simulation of a Response to a Sine Excitation

- Give the solver parameters as in (1) and click on “Initial conditions” (2) to set all displacements and velocities to 0 to have a system initially at rest (3). Click on “Apply” in both windows.



Simulation of a Response to a Sine Excitation

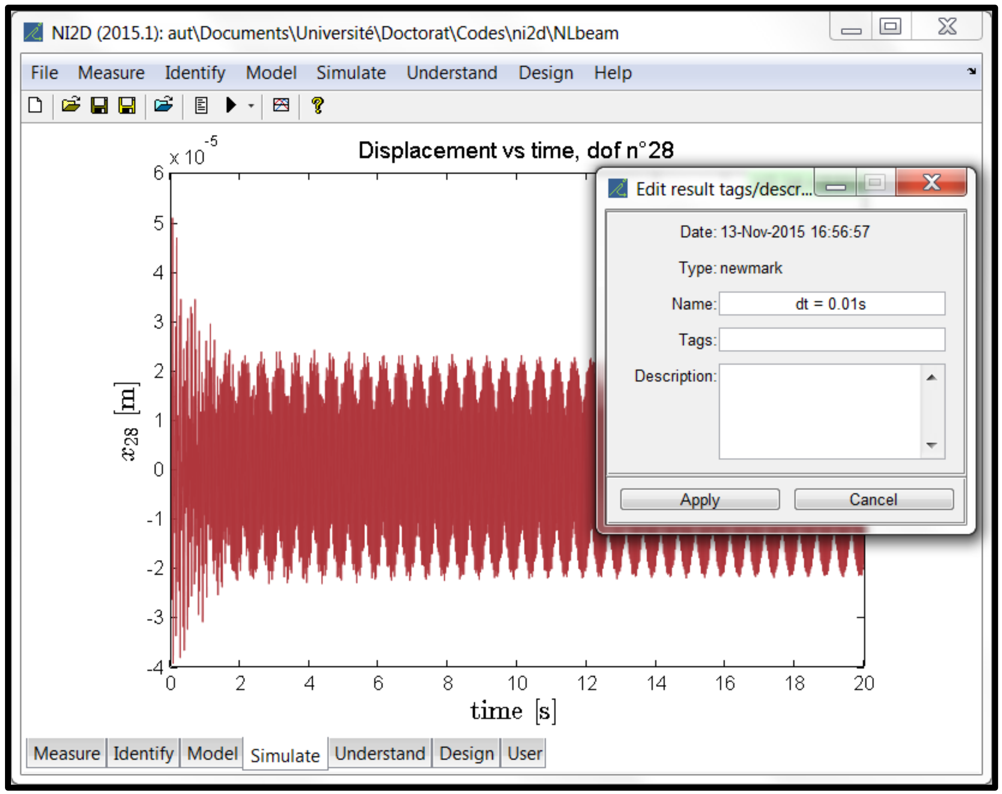
- Start the simulation.



WARNING: in this tutorial, you could have several heavy time series to store; make sure to have at least 2Gb available in your disk, or delete previous time series before computing new ones.

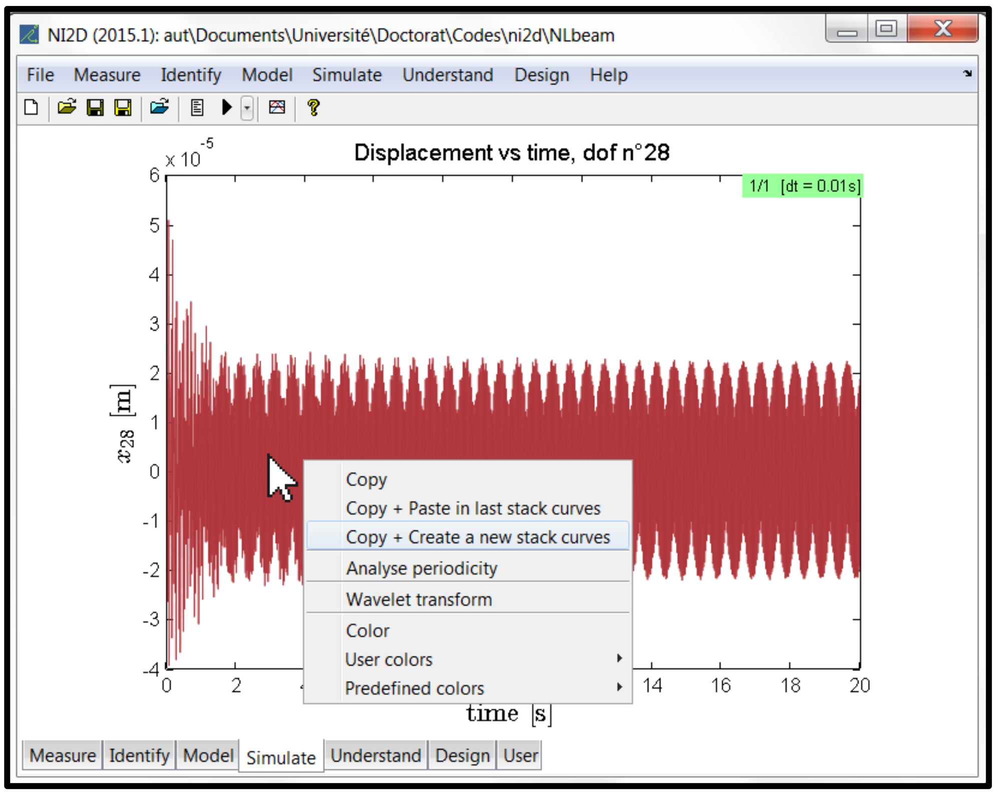
Simulation of a Response to a Sine Excitation

- Tag your result “*dt = 0.01s*” using <F11>.



Simulation of a Response to a Sine Excitation

- Create a new stack curve for this curve (which will be available in the “User” tab) using <RIGHT CLICK> on the curve.



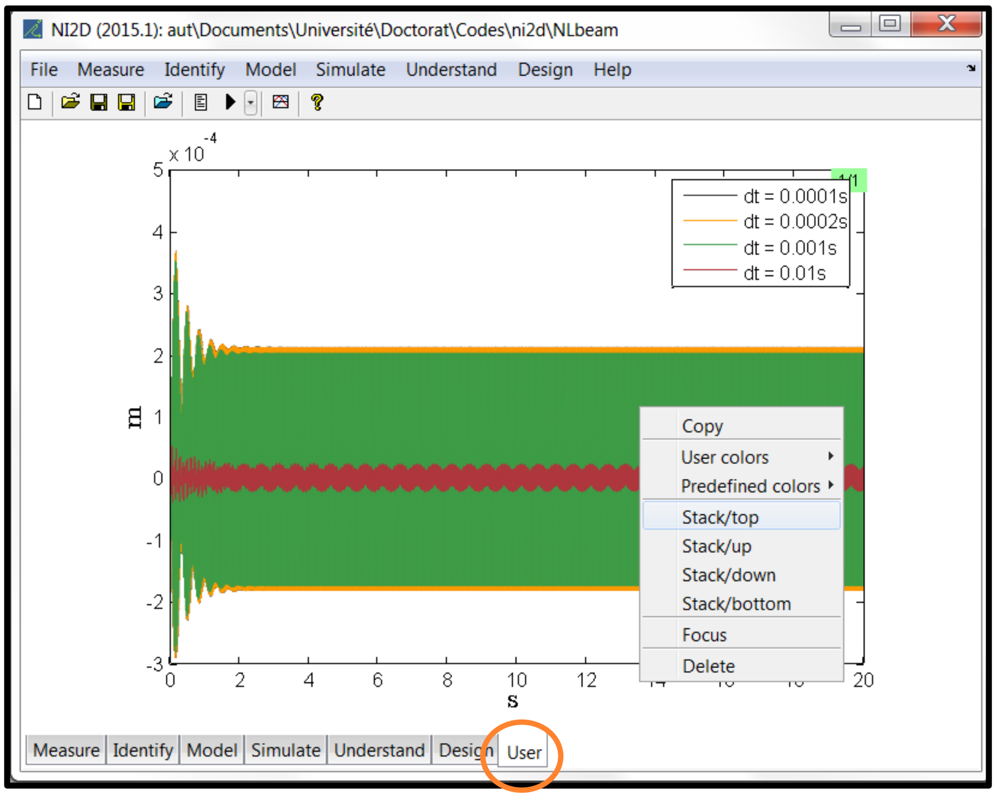
Influence of the Time Step

- Perform another simulation for a smaller time step of 0.001s, tag it, select a different color using <RIGHT CLICK> on the curve and “Color”, and compare in the stack curves using <RIGHT CLICK> on the curve and “Paste in last stack curves”.

- Repeat the same operations for smaller time steps of 0.0002s and 0.0001s.

Influence of the Time Step

- Go to “User” tab, and use <RIGHT CLICK> and “Stack up/down” to observe the different curves in the figure.



Q: Can you explain why the time series are different?

Influence of the Time Step

Q: Can you explain why the time series are different?

A: Selecting a small time step is crucial for obtaining accurate time series. For example, linear Newmark's scheme has a periodicity error of

Frequency of interest in the signal

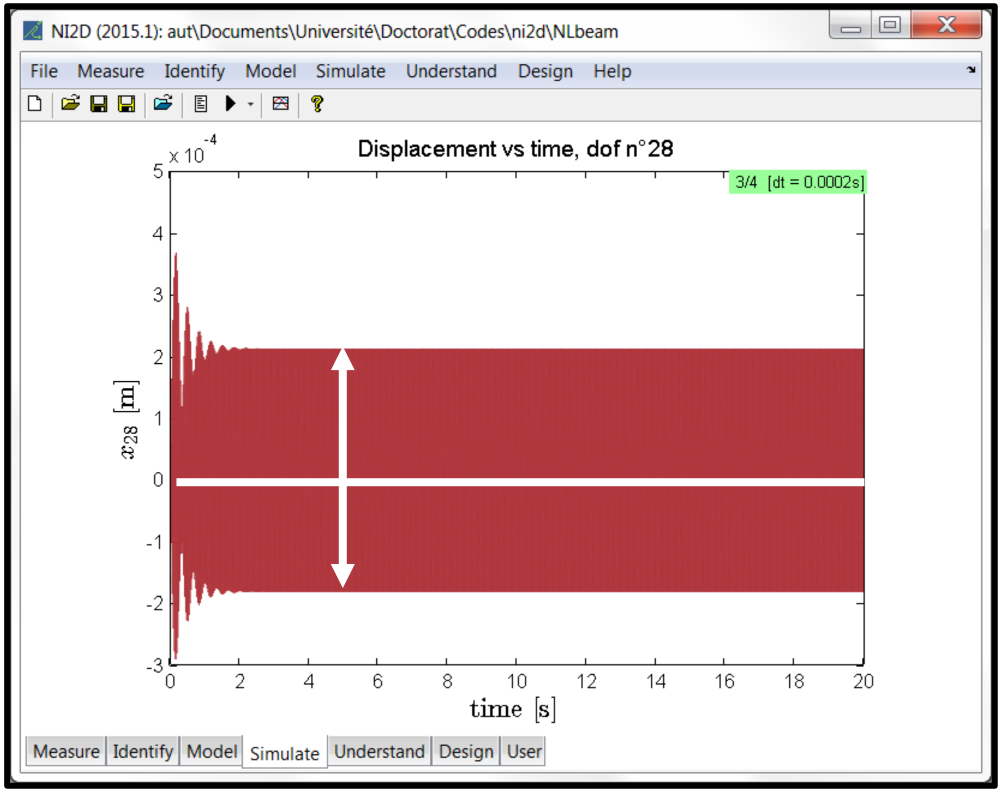
$$\Delta\omega = \frac{4\pi^2 f^2}{12 f_s^2}$$

Sampling frequency

For an error of less than 1%, and for $f = 34$Hz, one should have f_s larger than 620Hz.

Influence of the Time Step

Considering than third harmonics are present in the response, f_s should be even larger. Here we can choose 0.0002s as an optimal time step.



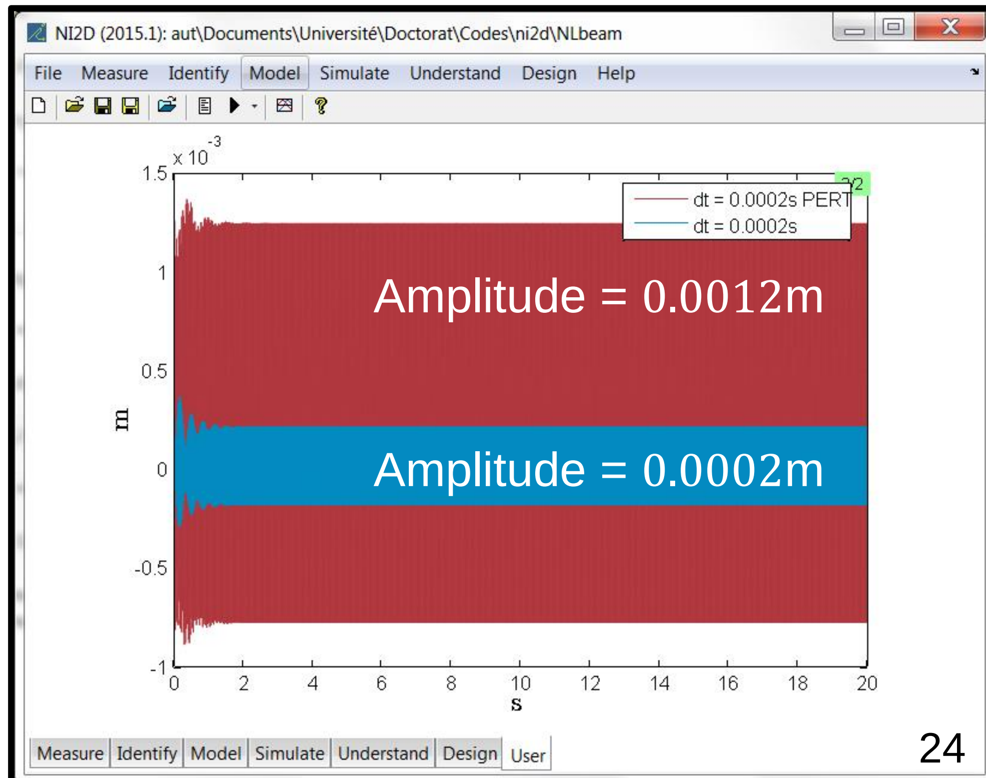
You can notice that the response is slightly asymmetric due to the presence of the quadratic nonlinearity.

Influence of Initial Conditions

- Select a time step of 0.0002s, and consider an initially perturbed system using an initial velocity of 1m/s for dof #28.
- Using tags and colors, compare the initially perturbed and unperturbed responses.

Depending on the initial conditions, the system can have small or large amplitude oscillations.

▶ Bistable behavior



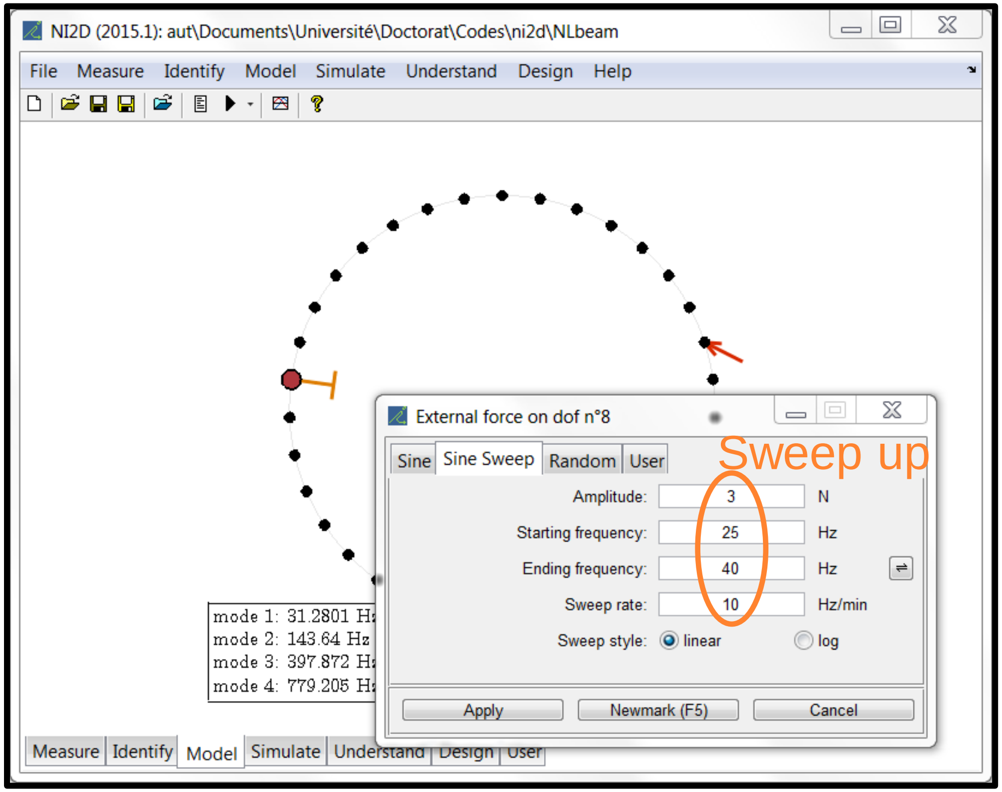
Creation of a Swept-Sine External Force

As a second exercise, we will study the system's response to a swept-sine excitation with a frequency range between 25Hz and 40Hz, which encompasses the first resonance frequency of the beam (31.28Hz).

We will study the effect of the sweep rate, and compare responses to sweep up and down in order to highlight the bistable region in the frequency response.

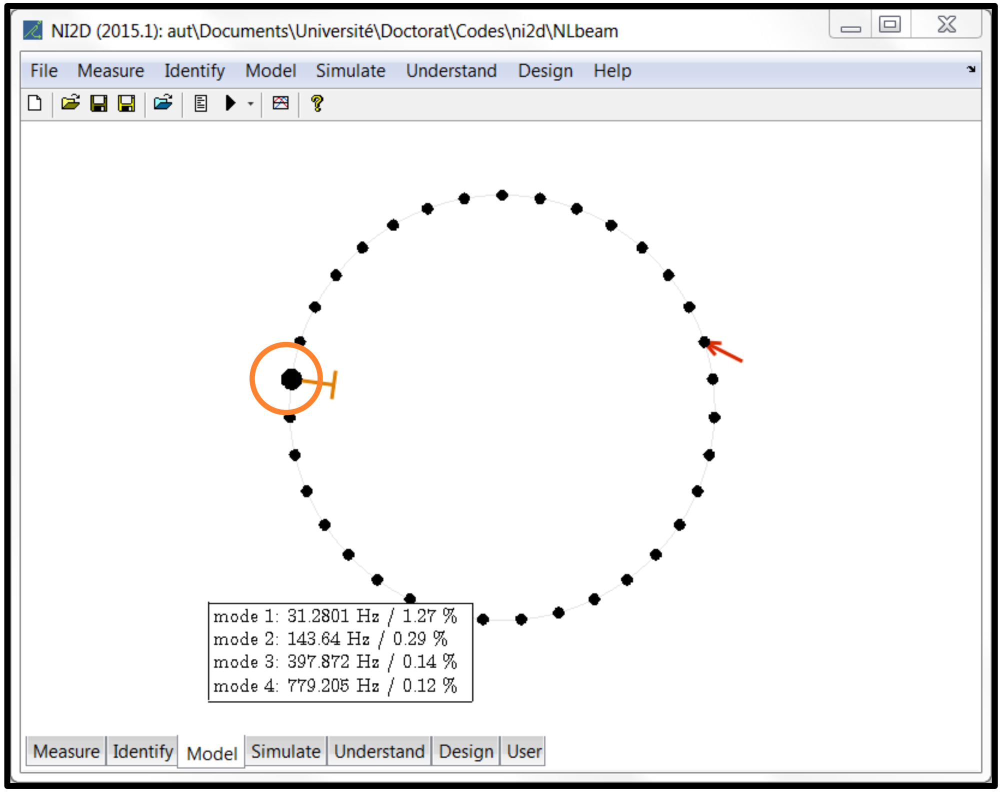
Creation of a Swept-Sine External Force

- Select “Sine Sweep” and give the following forcing parameters, then click on “Apply”.



Simulation of a Response to a Swept-Sine Excitation

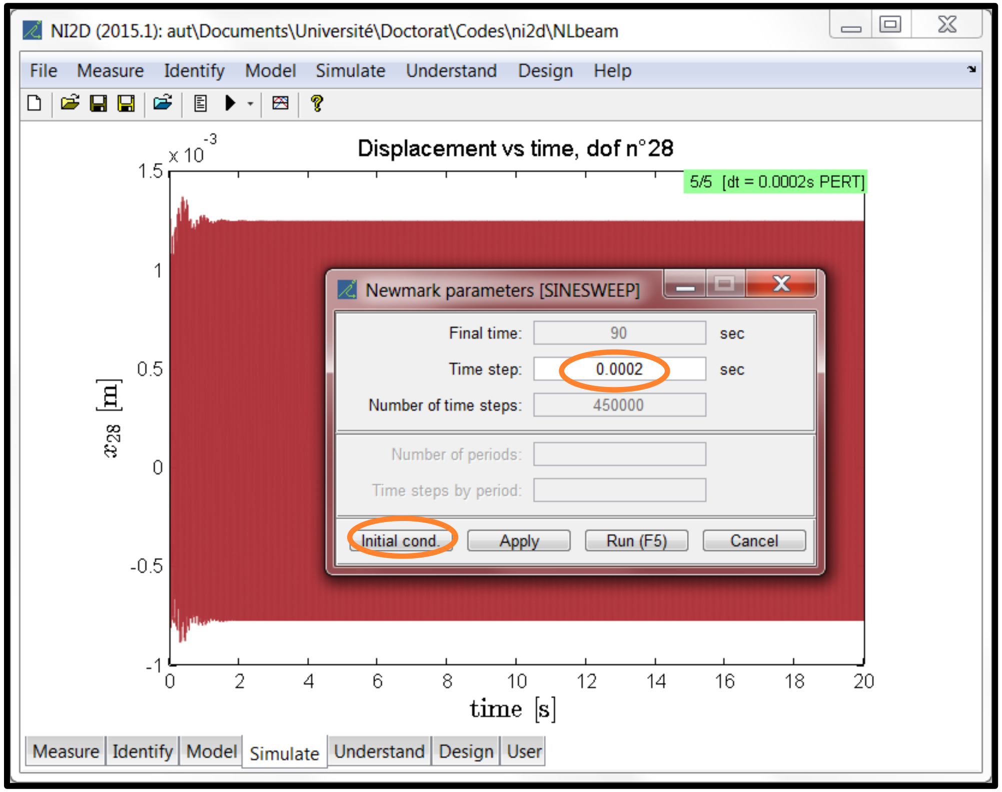
- In order to show the time series at the tip of the main beam, use <ALT+LEFT> on dof #28 to display next results on that dof.



- You can modify the color associated to dof #28 using <RIGHT CLICK> on dof #28.

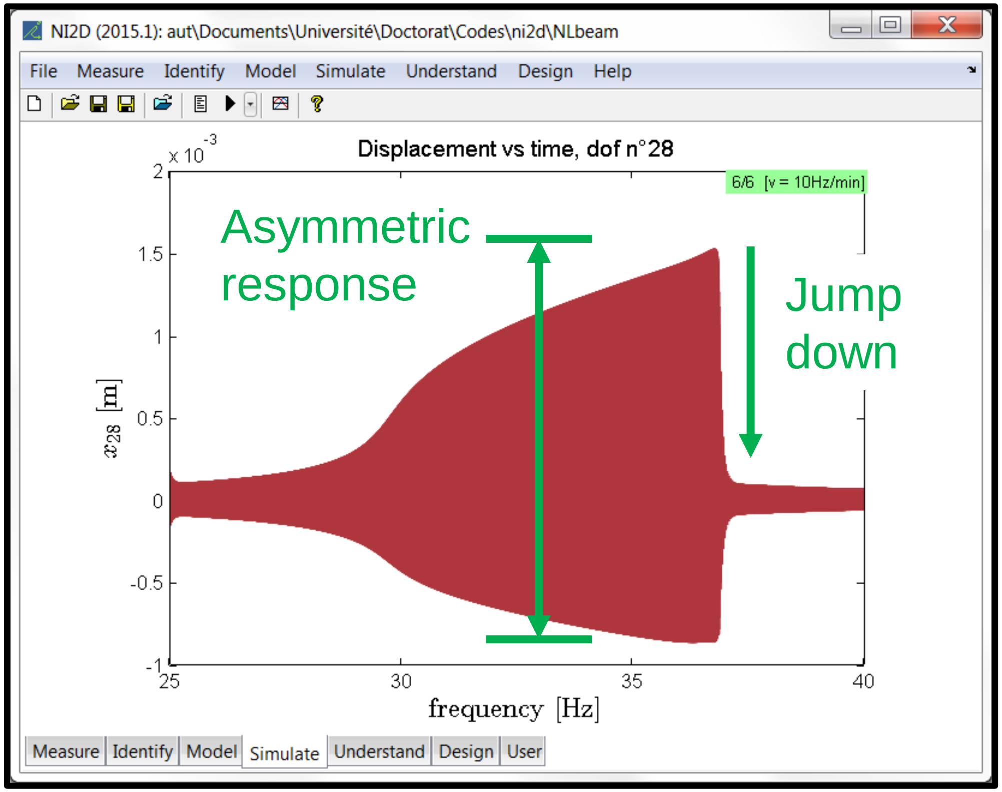
Simulation of a Response to a Swept-Sine Excitation

- In “Simulate” tab, modify the solver parameters to set all displacements and velocities to 0, and the time step to $0.0002s$.



Simulation of a Response to a Swept-Sine Excitation

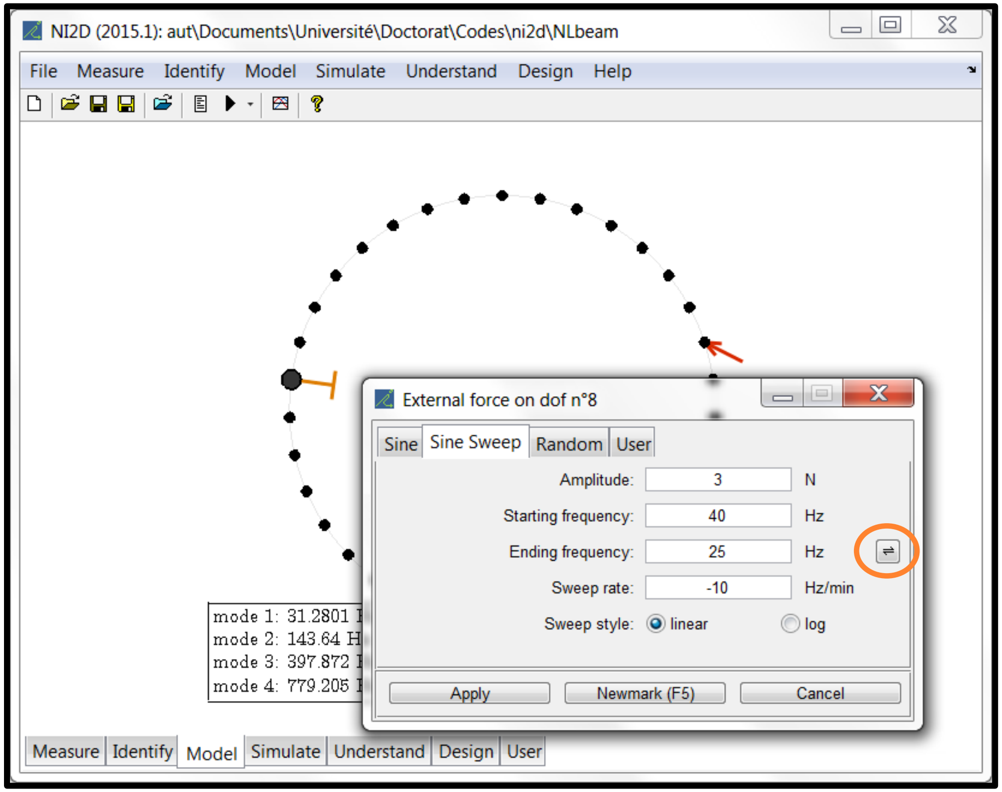
- Start the simulation and tag your result as “*v = 10Hz/min*” using F11.



The response is asymmetric due to the presence of the quadratic nonlinearity. The jump down occurs because of the hardening behavior.

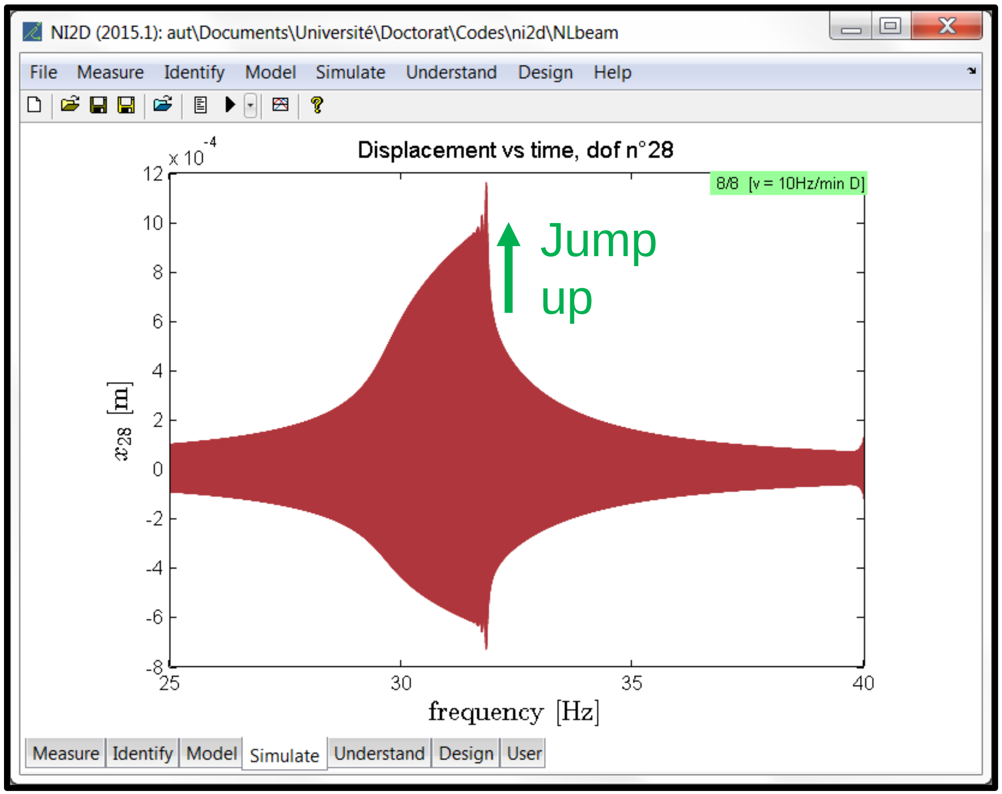
Comparison Between Sweep Up and Sweep Down

- Reverse the sweep direction.



Comparison Between Sweep Up and Sweep Down

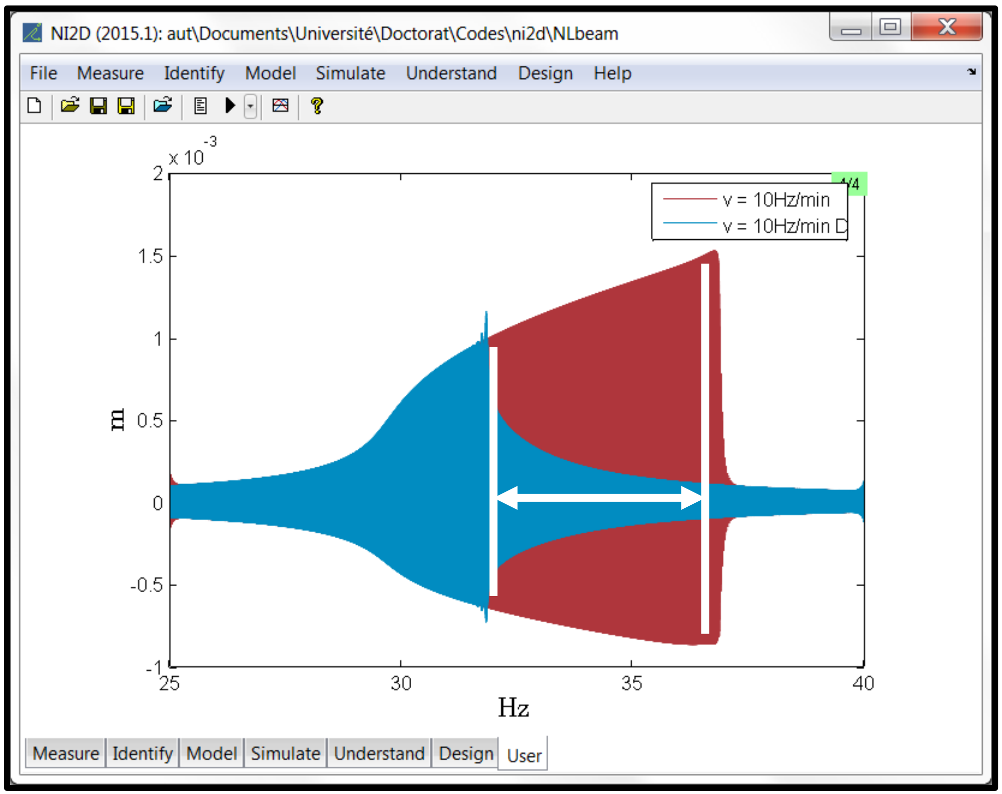
- Start the simulation and tag your result as "*v = 10Hz/min D*".



The jump up occurs because of the hardening behavior.

Comparison Between Sweep Up and Sweep Down

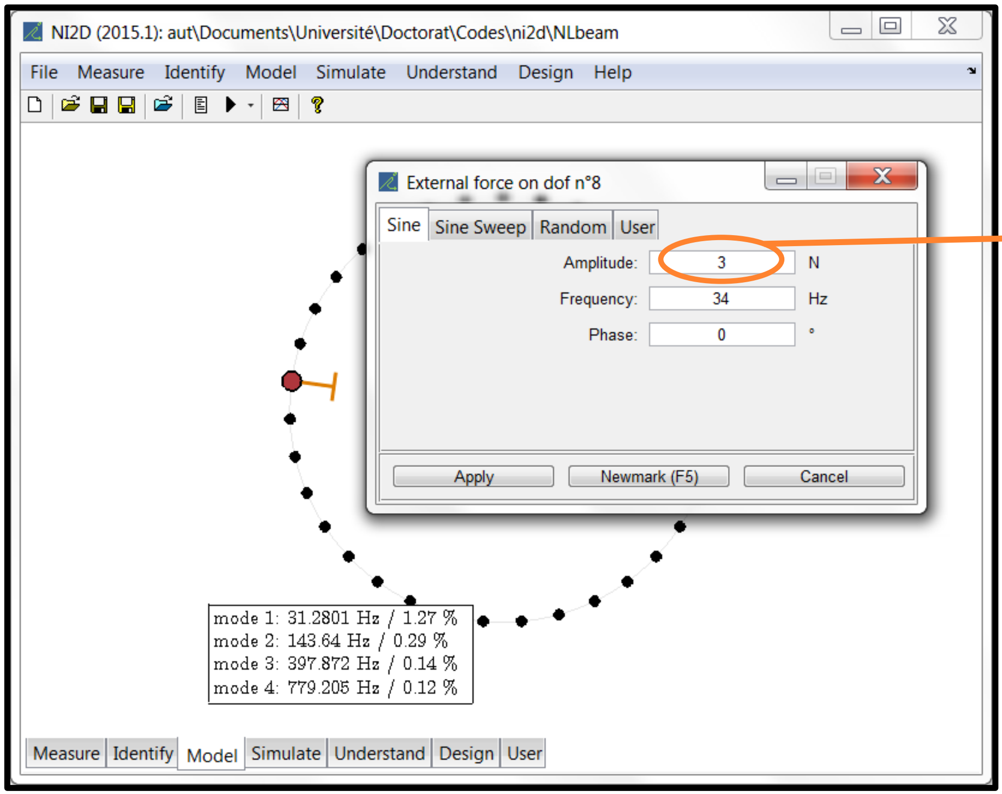
- Compare the response for sweep up and sweep down.



The bistable region spans between 32Hz and 37Hz.

Computation of the NFRC

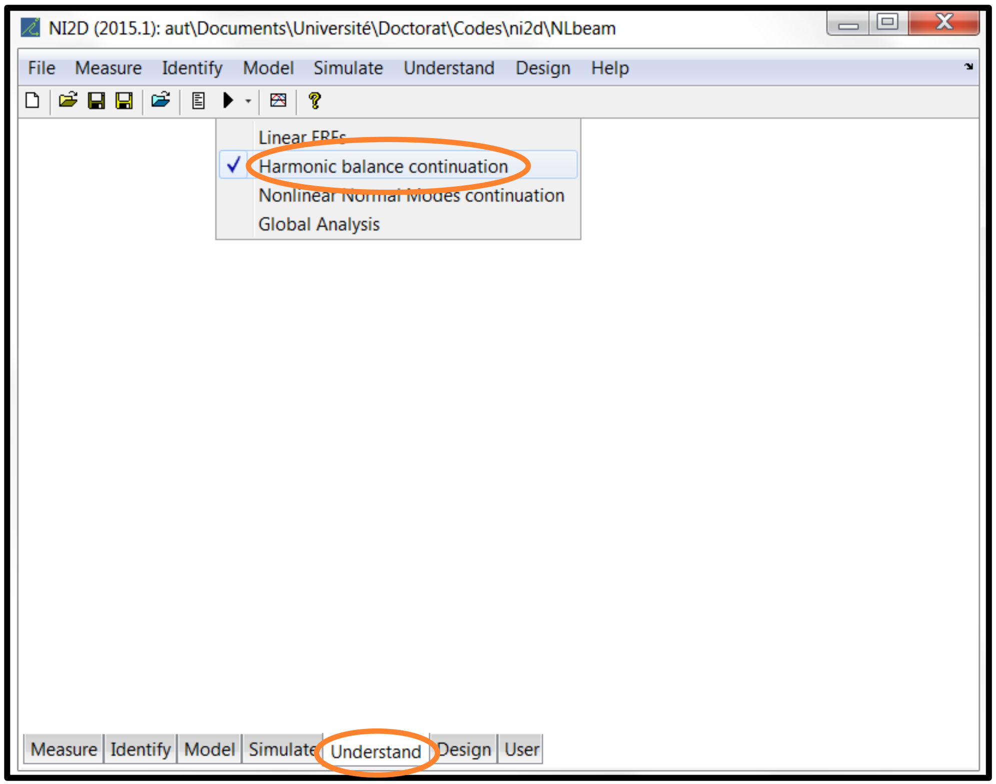
- Select a sine excitation with the following parameters.



This forcing amplitude will be considered for all the periodic solutions along the branch.

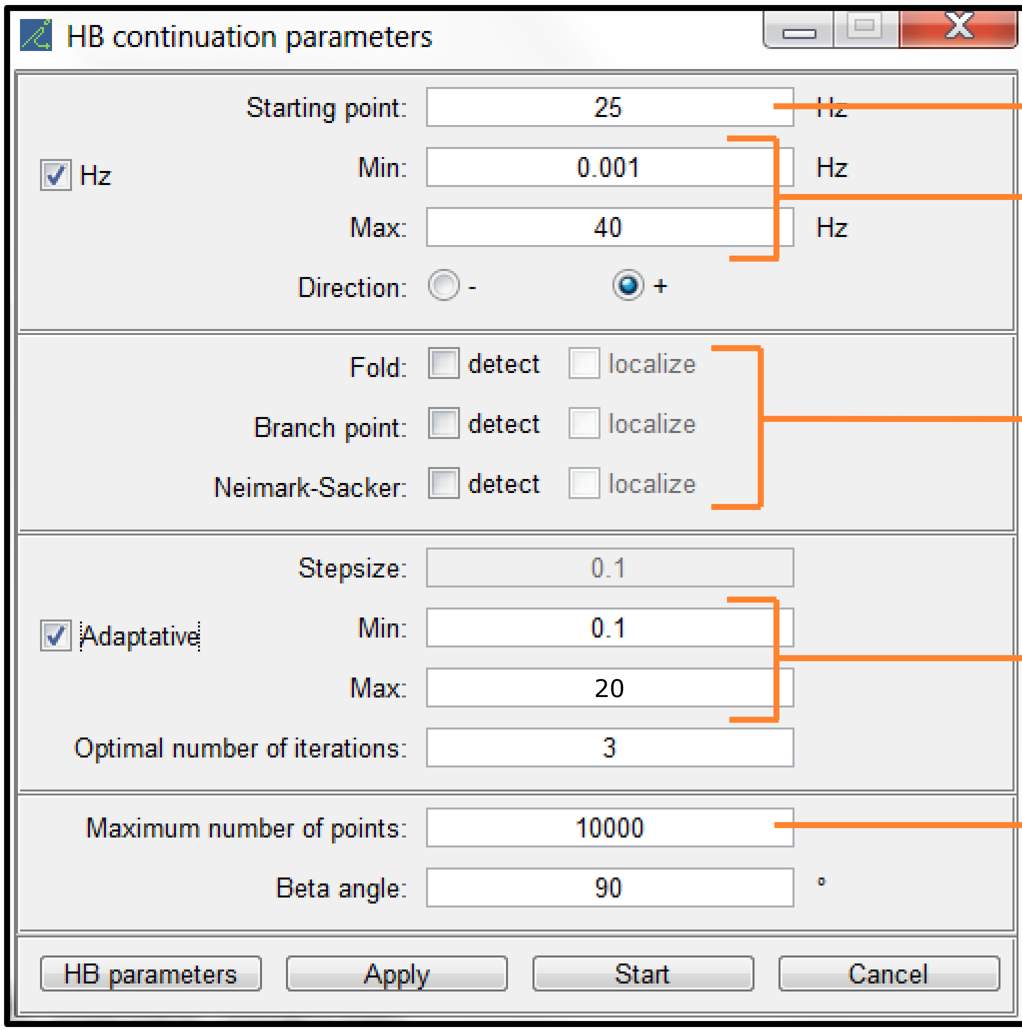
Computation of the NFRC

- Go to “Understand” tab and select “Harmonic balance continuation”.



Computation of the NFRC

- Modify the continuation parameters as follows.



Frequency of the initial point.

Continuation stops when “Min” or “Max” frequencies are reached.

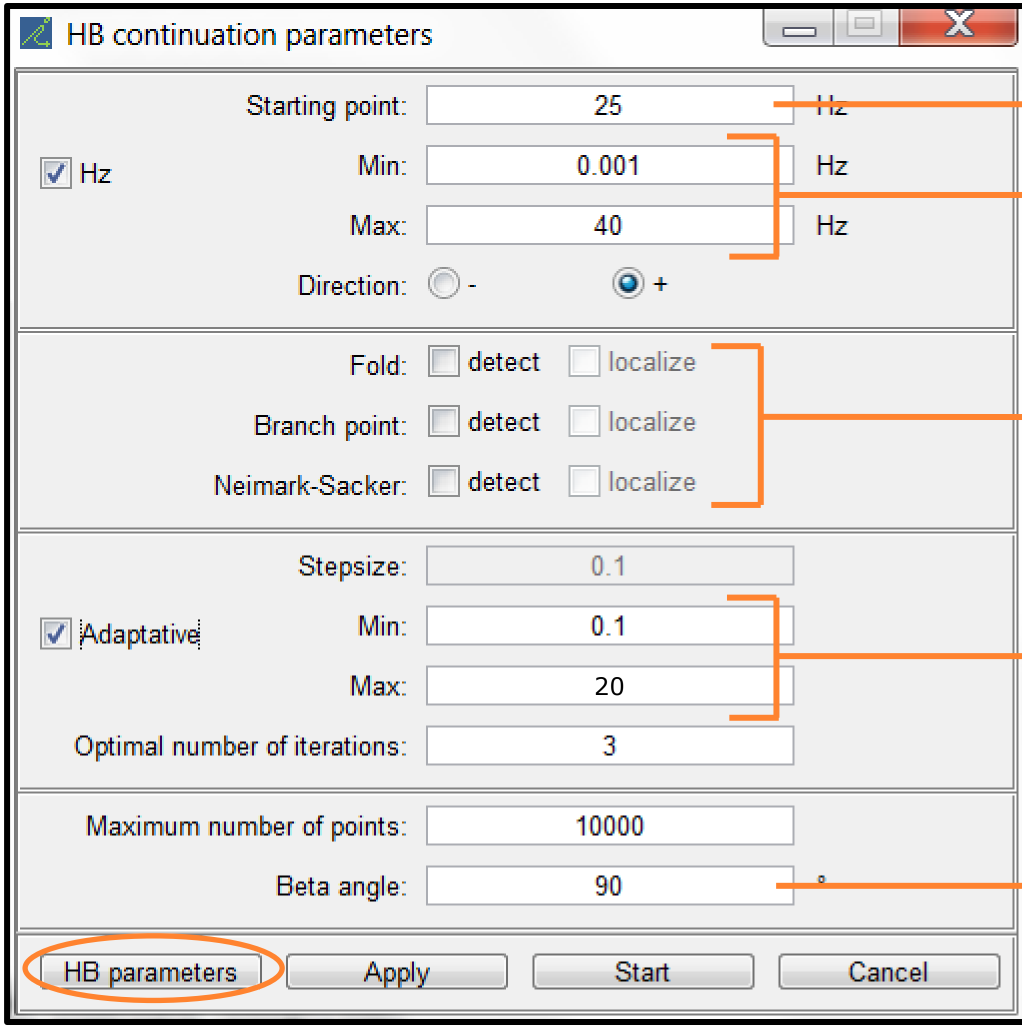
Bifurcations monitoring.

Min. and max. stepsize for the adaptative strategy.

Continuation stops when this number of points is reached.

Computation of the NFRC

- Modify the continuation parameters as follows.



Frequency of the initial point.

Continuation stops when “Min” or “Max” frequencies are reached.

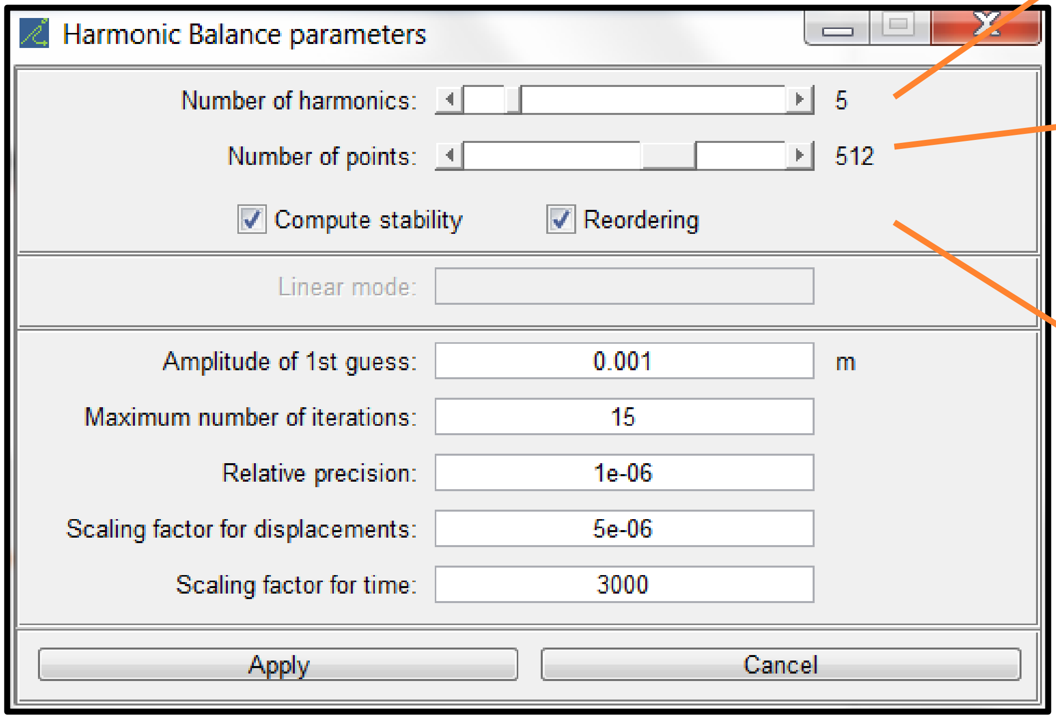
Bifurcations monitoring.

Min. and max. stepsize for the adaptative strategy.

If the angle between two consecutive tangents is larger than this angle, the prediction vector is reversed.

Computation of the NFRC

- Modify the HB parameters as follows.



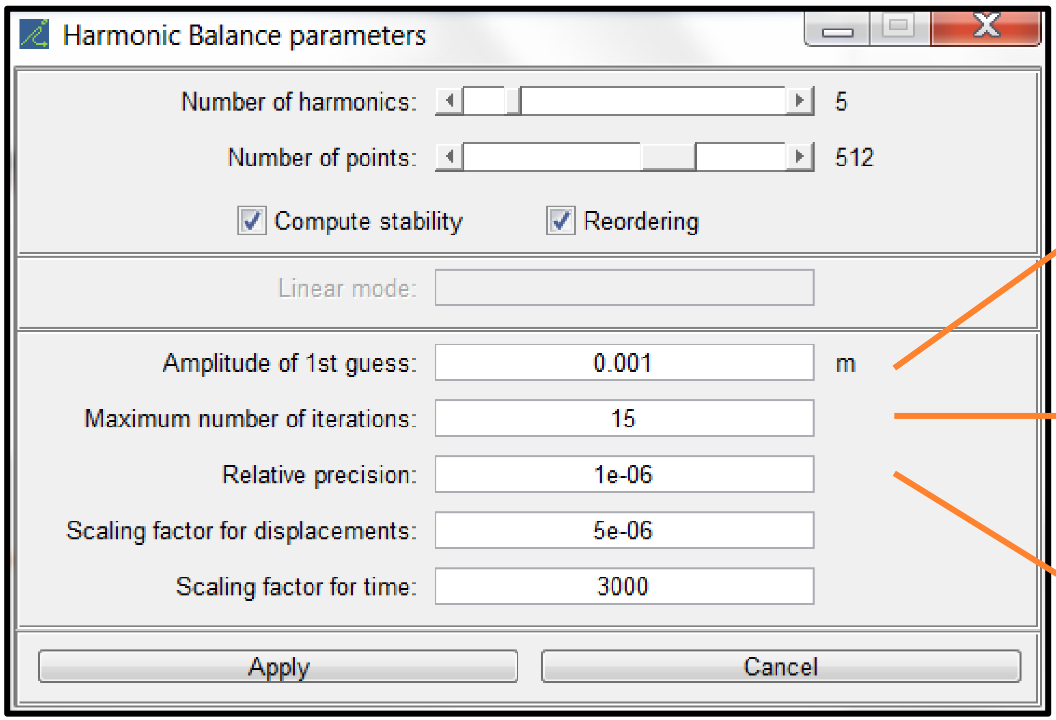
Number of harmonics N_H retained in the Fourier series.

Number of time samples N in the Fourier transform.

Use of symmetric reverse Cuthill-McKee permutation to accelerate the eigenvalue problem resolution.

Computation of the NFRC

- Modify the HB parameters as follows.



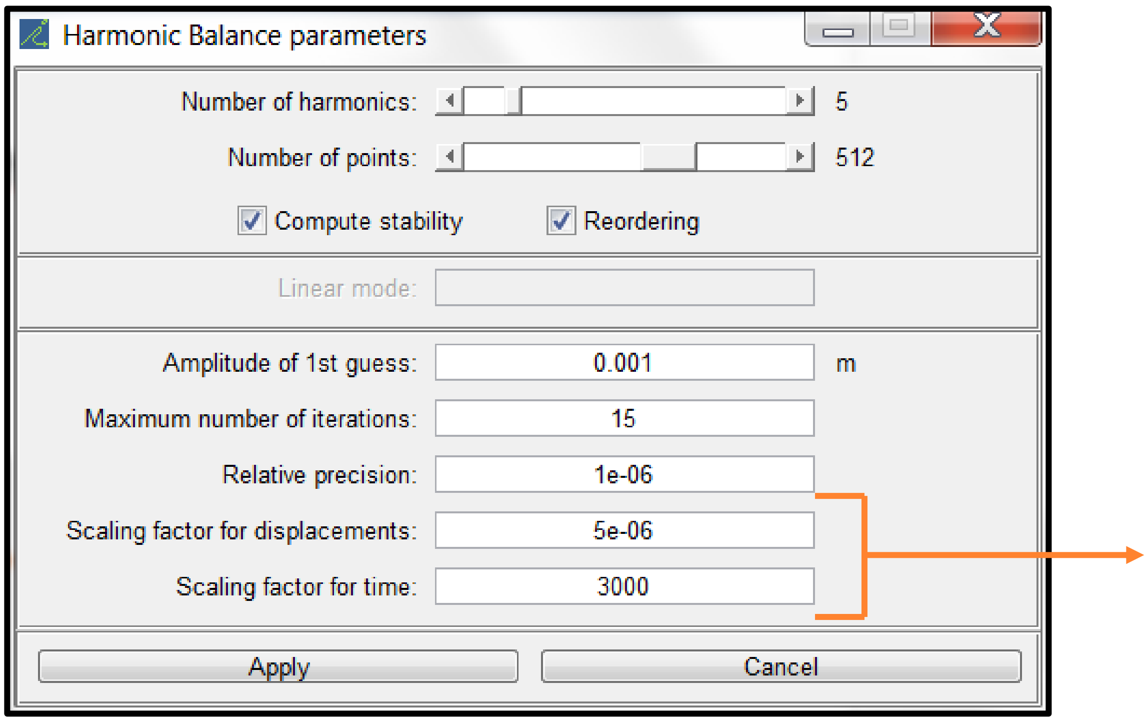
Amplitude of the sine series used as initial guess for all dofs.

The Newton-Raphson procedure fails if this number of iterations is exceeded.

The Newton-Raphson procedure stops if the relative error is smaller than this precision.

Computation of the NFRC

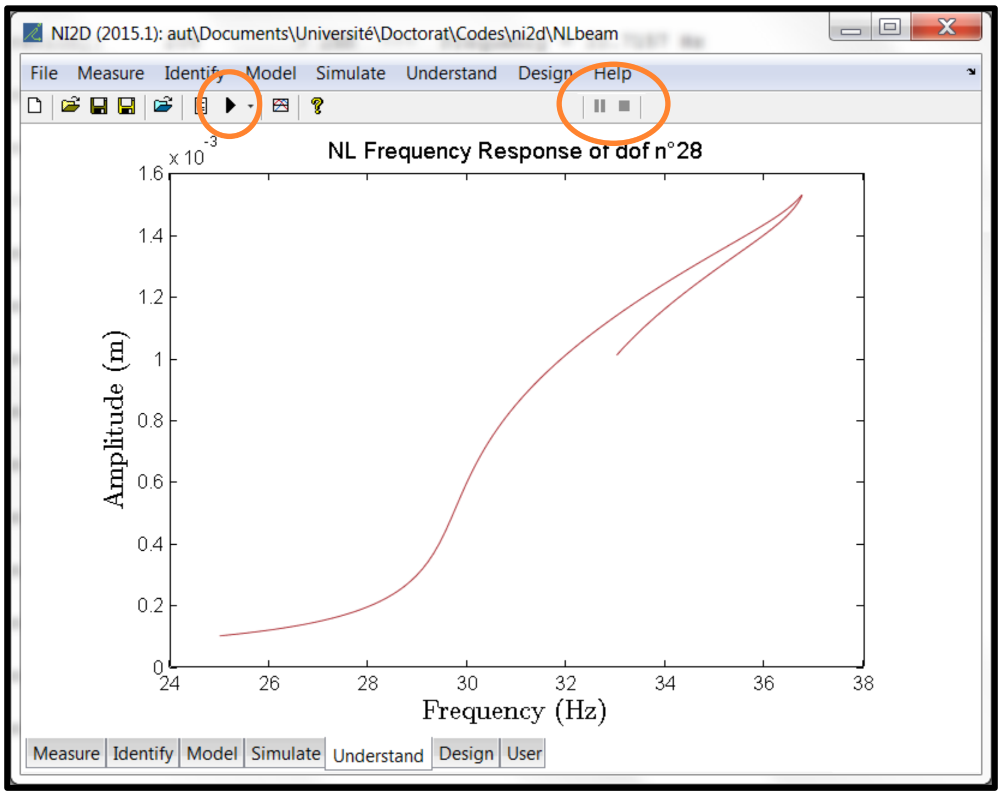
- Modify the HB parameters as follows.



Because the frequency (here, around 30Hz = 188rad/s) and the amplitude (here, around 0.001m) have different orders of magnitude, time and displacements have to be rescaled to avoid ill conditioning.

Computation of the NFRC

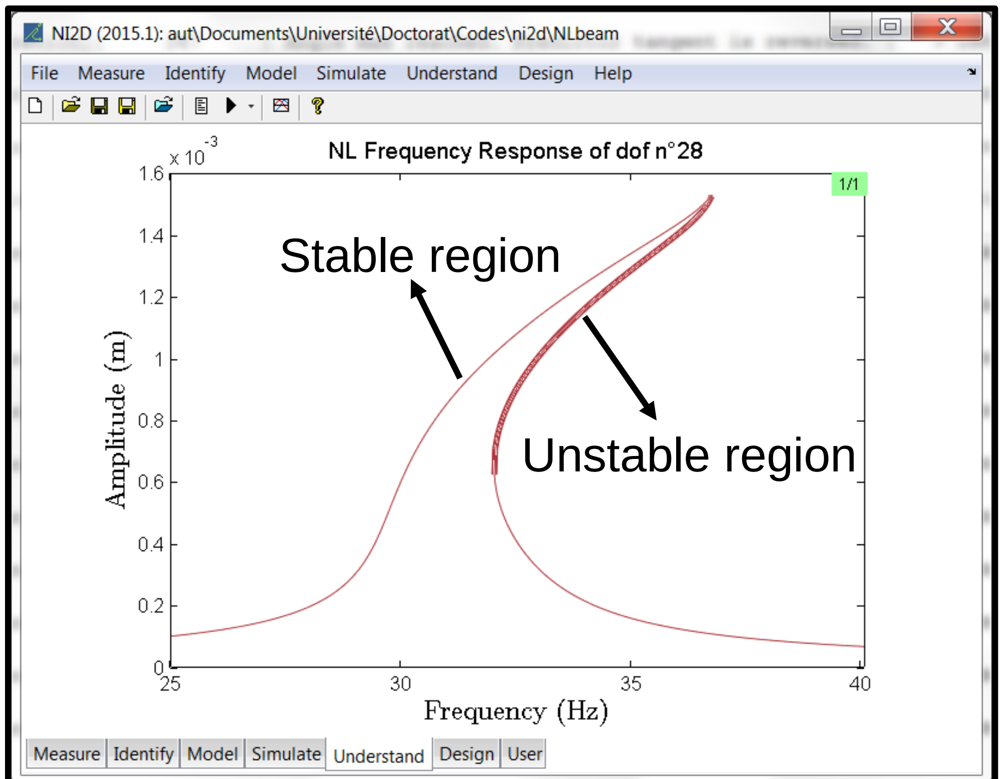
- Start the continuation procedure and wait until the maximum frequency (40Hz) is reached.



You can also pause/unpause the procedure, or stop and record it at its current stage.

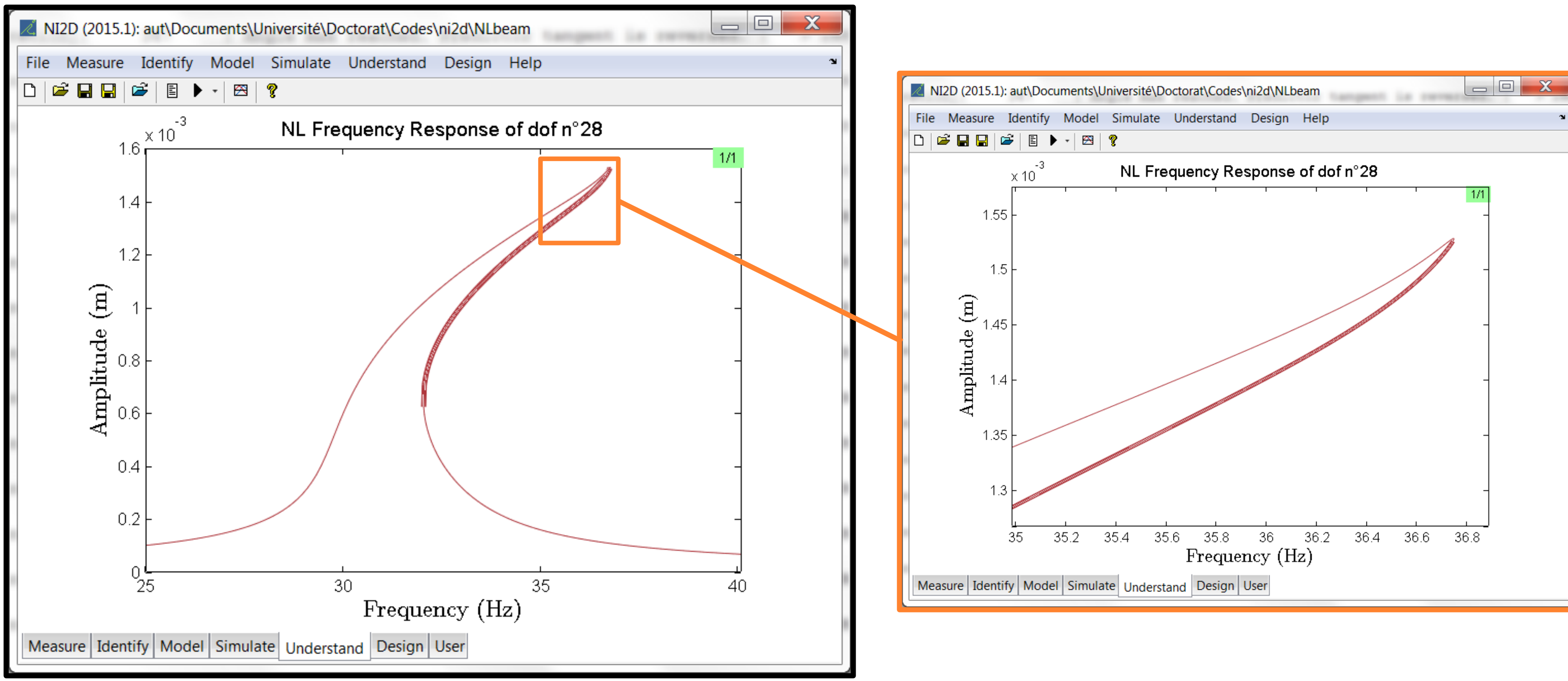
Computation of the NFRC

- Start the continuation procedure and wait until the maximum frequency (40Hz) is reached.



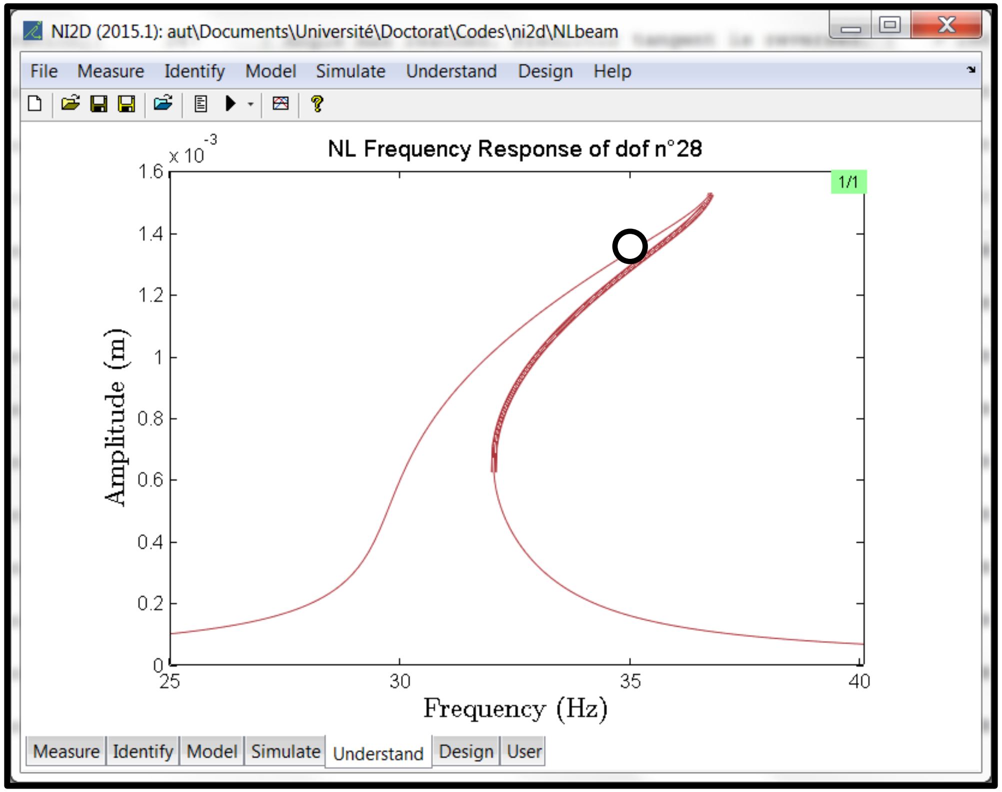
Analysis of the NFRC

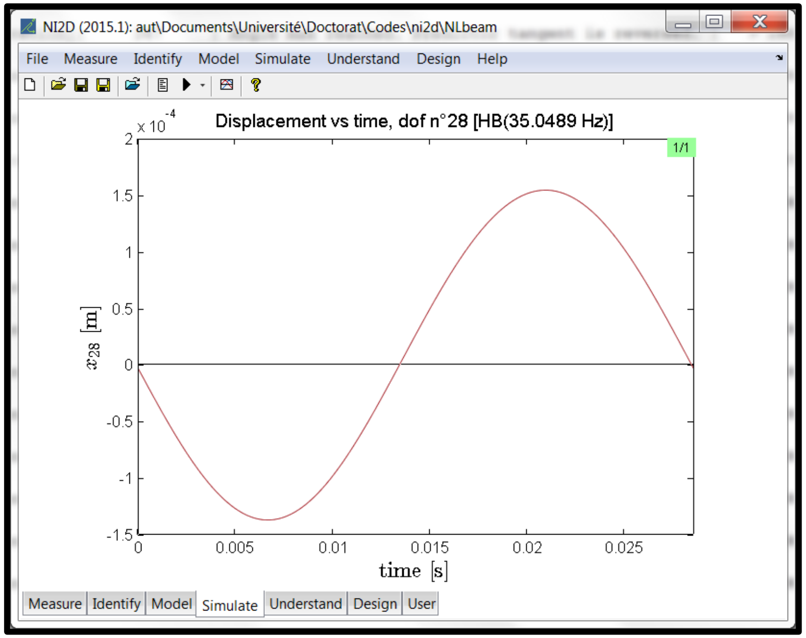
- Use <LEFT CLICK+DRAG> to zoom on a region of interest, and <R> to reset the view.



Analysis of the NFRC

- Use <DOUBLE LEFT CLICK> on a point to represent its time series reconstructed from the Fourier coefficients, and its Floquet exponents/multipliers.





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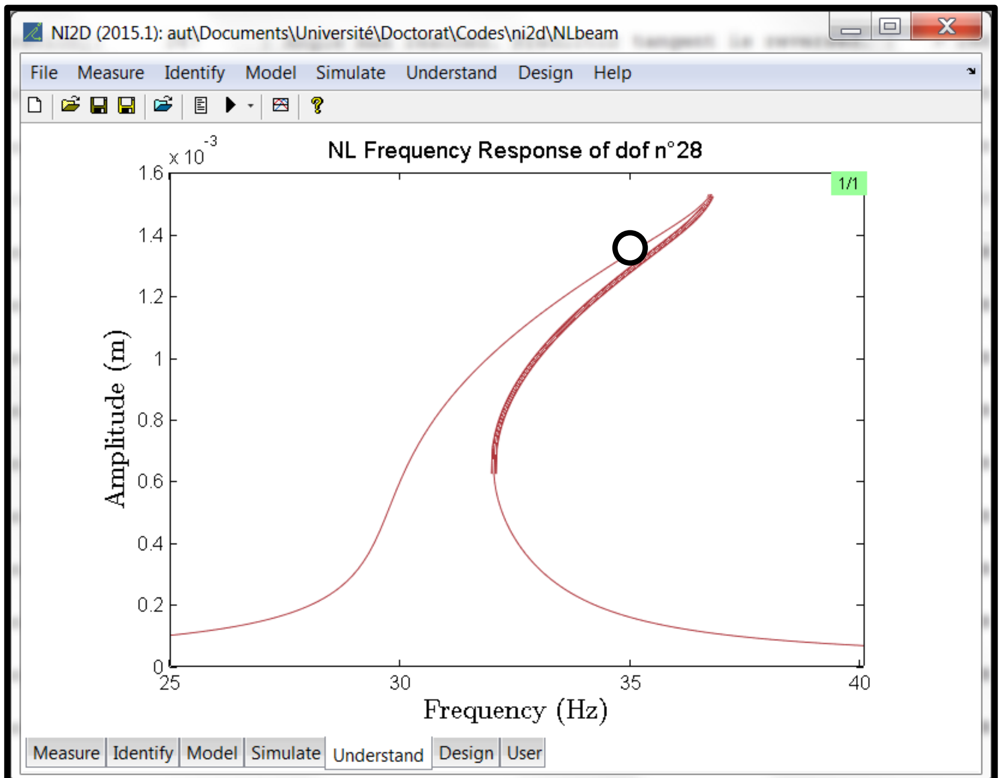

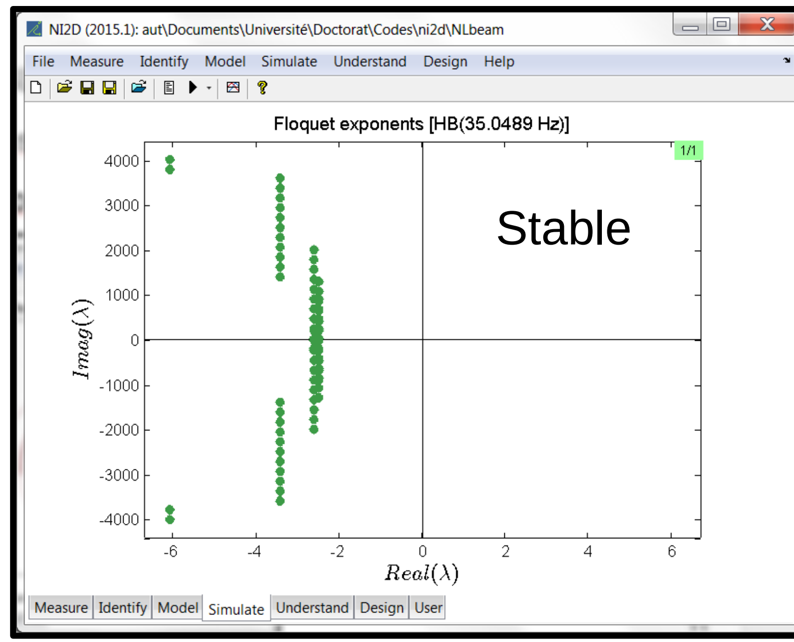

Analysis of the NFRC

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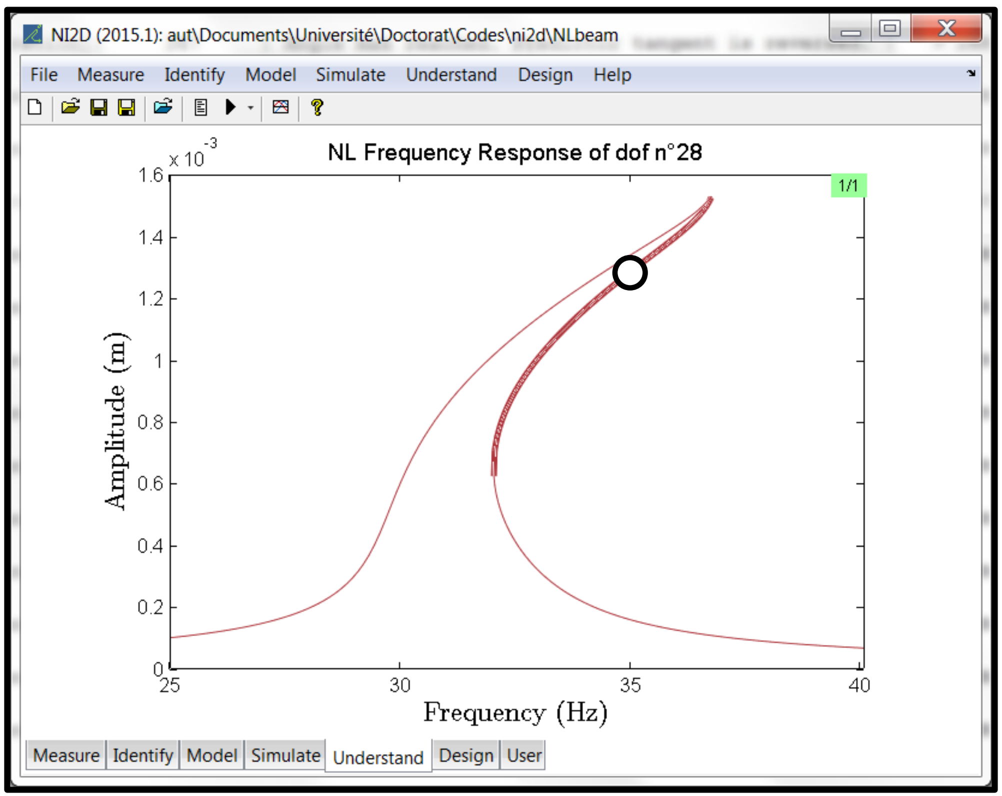
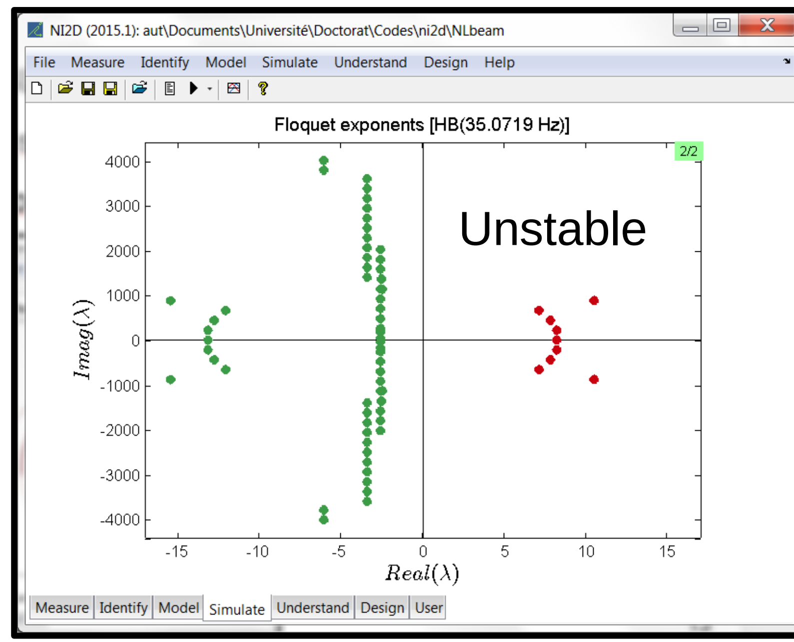
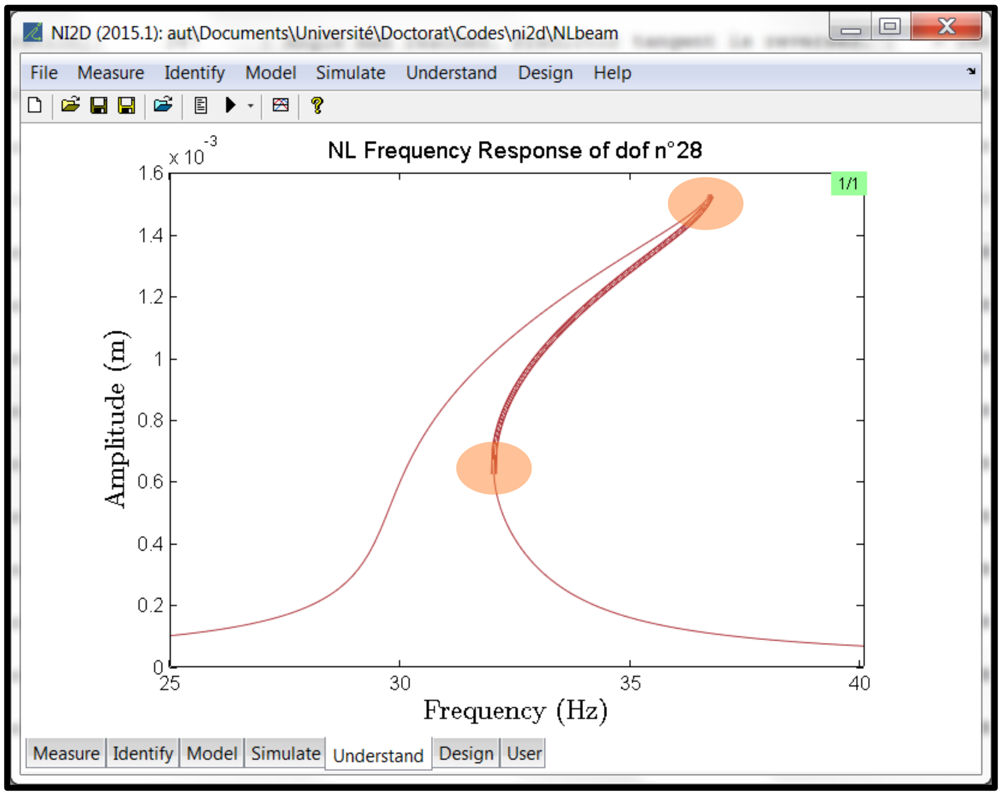




Analysis of the NFRC

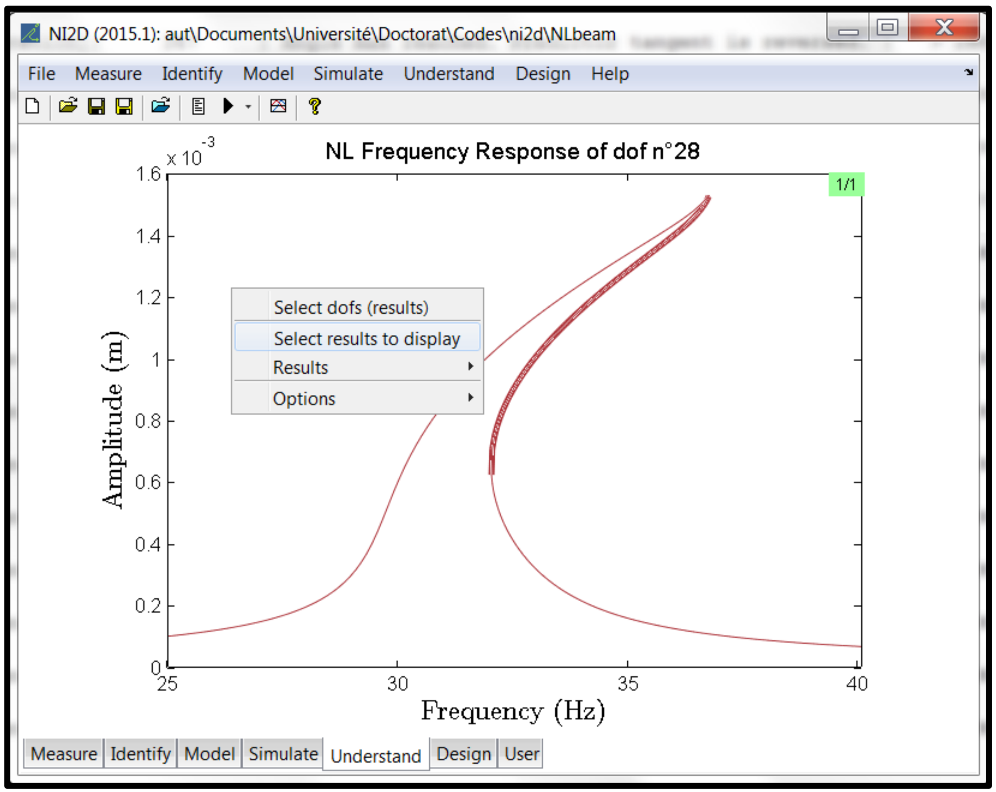


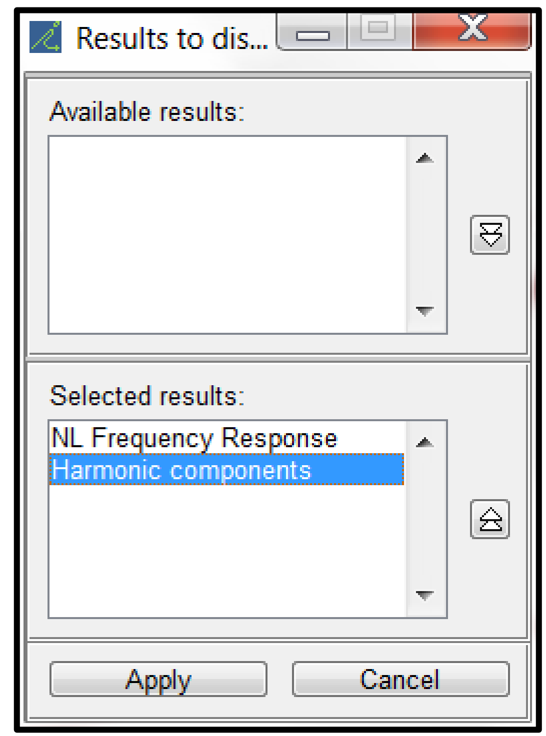
What happens at the transitions between stable/unstable regions?

See next lectures…

Analysis of the NFRC

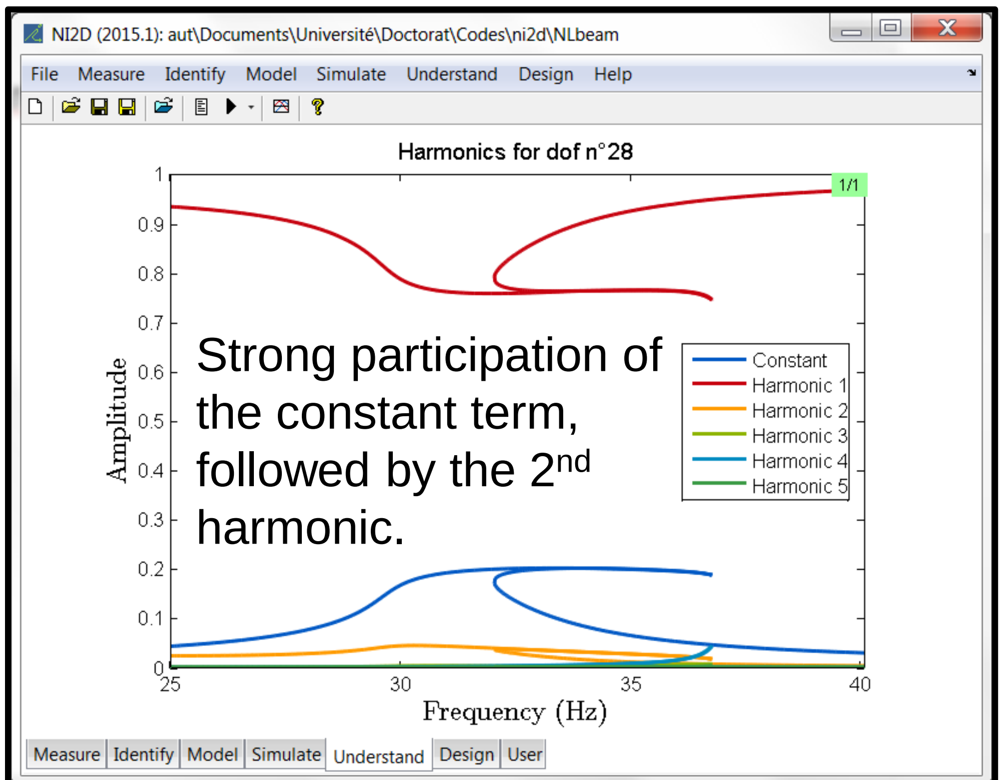
- Among the results to display, select the evolution of the harmonic components along the curve.





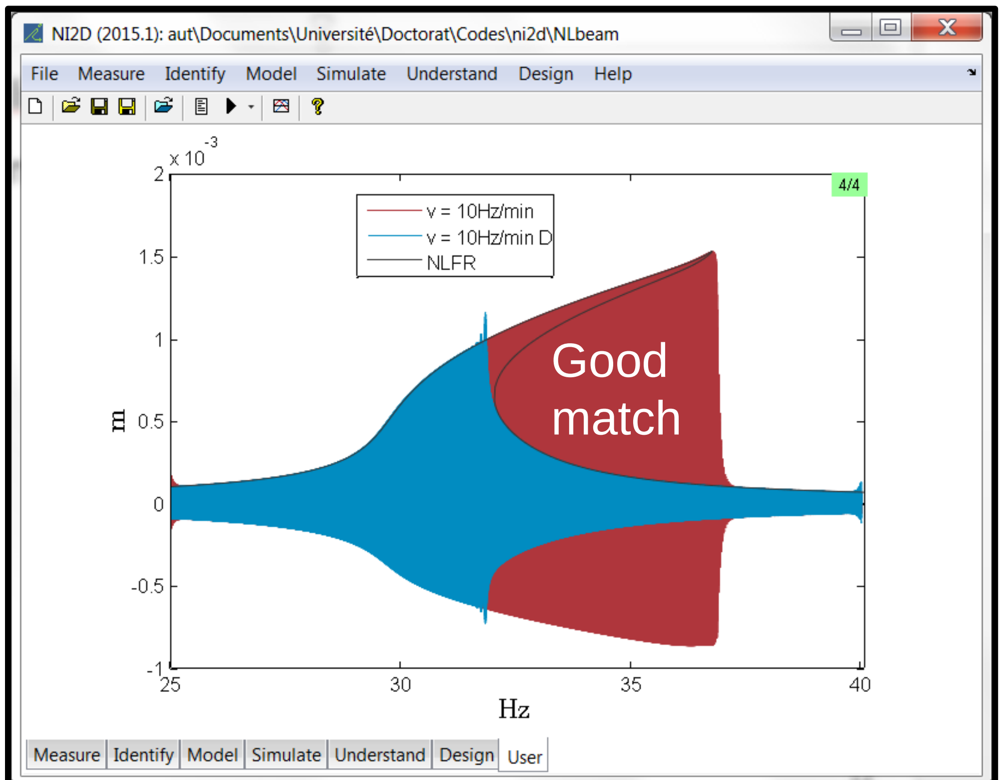
Analysis of the NFRC

- Among the results to display, select the evolution of the harmonic components along the curve.



Analysis of the NFRC

- Add the nonlinear frequency response in the stack curves of the swept-sine responses.



Conclusions

- A small step size is necessary for accurate time integrations (sampling frequency should be approx. 200 times higher than frequency of interest).
- A small sweep rate is necessary to accurately represent amplitude jumps up and down.
- Sine and swept-sine excitations can reveal coexisting solutions.