Nonlinear Vibrations of Aerospace Structures

Tutorial 06
Nonlinear Simulations

- Nonlinear Modelling
- Time Integration
- Continuation

Nonlinear Identification to Design
Software

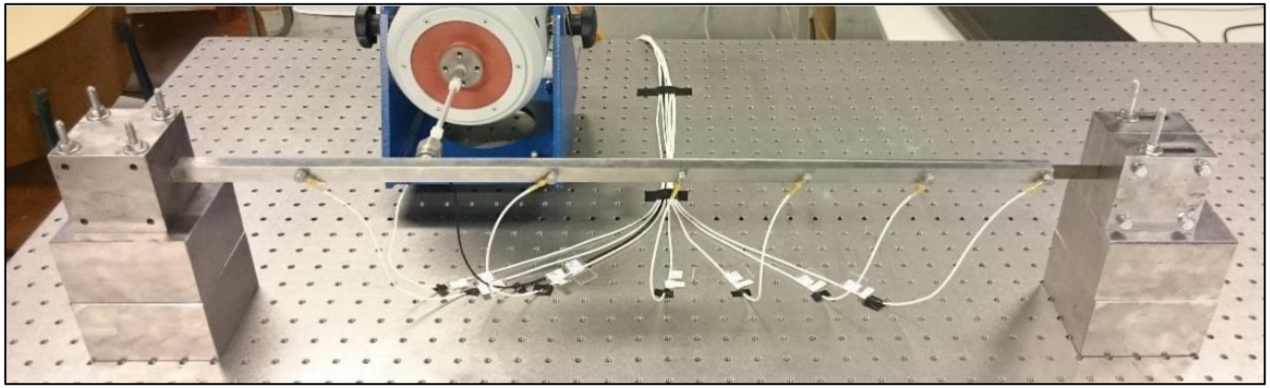
Objectives of this Tutorial

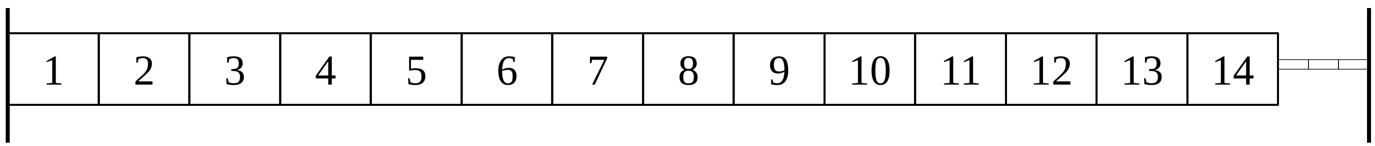
Get familiar with NI2D tools for nonlinear simulations:

- Simulate the system dynamics.
- Compute nonlinear frequency response curves (NFRCs).

Case Study: A Nonlinear Beam

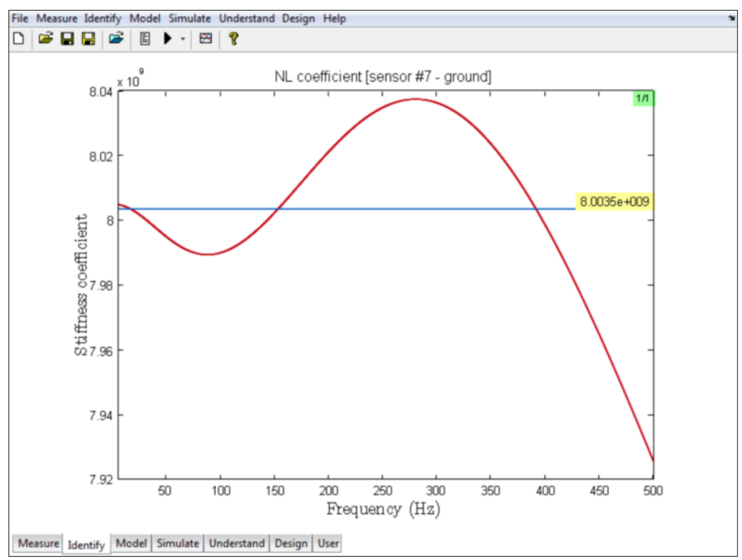
Linear
FE model





+

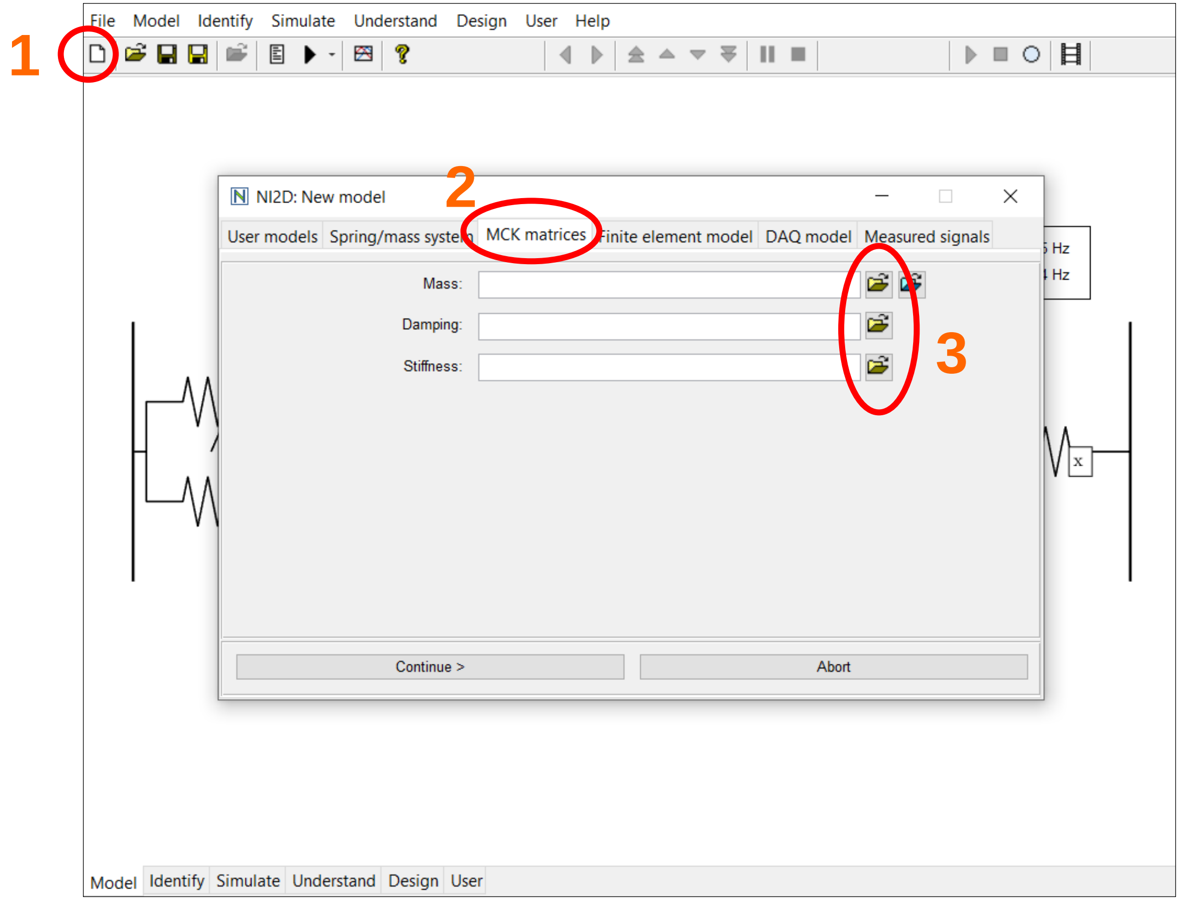
Identified
Nonlinearities



Cubic coefficient
(geometrical)

Quadratic coefficient
(clamping)

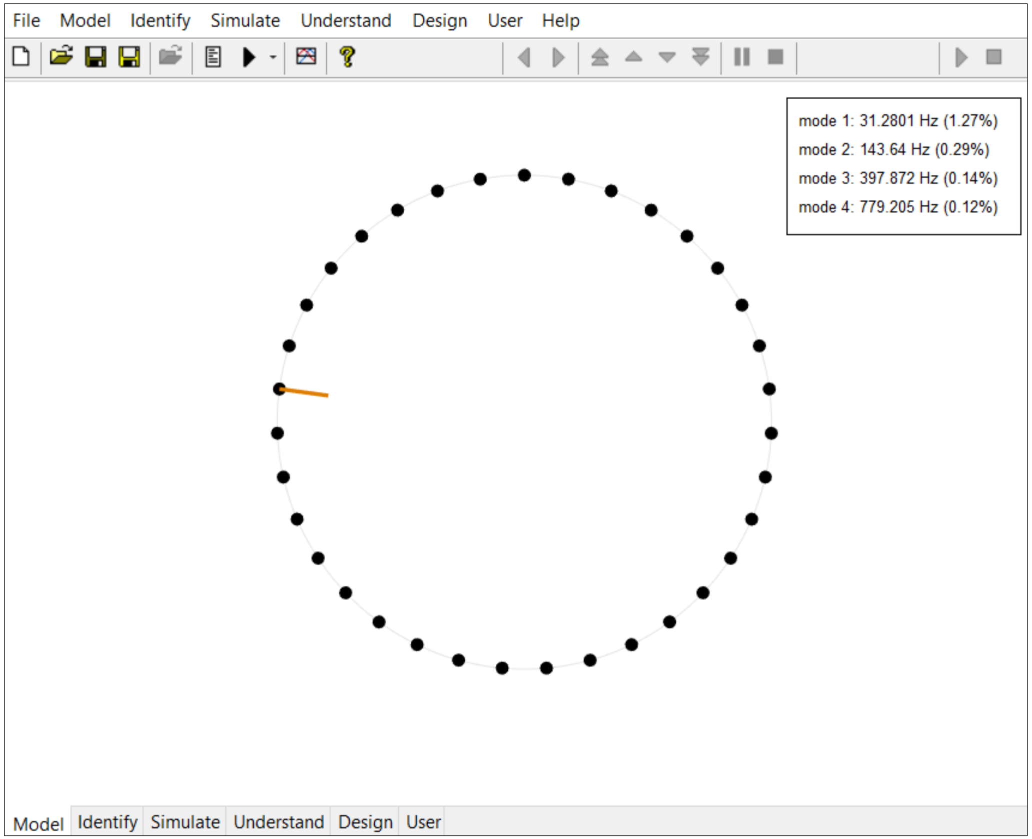
Import the Linear FE Model



- Create a new model (1) and select the matrices M,C and K (2).
- Import **M, C** and **K.**
- Name the new system ‘NLBeam’.

Create Nonlinear Connections

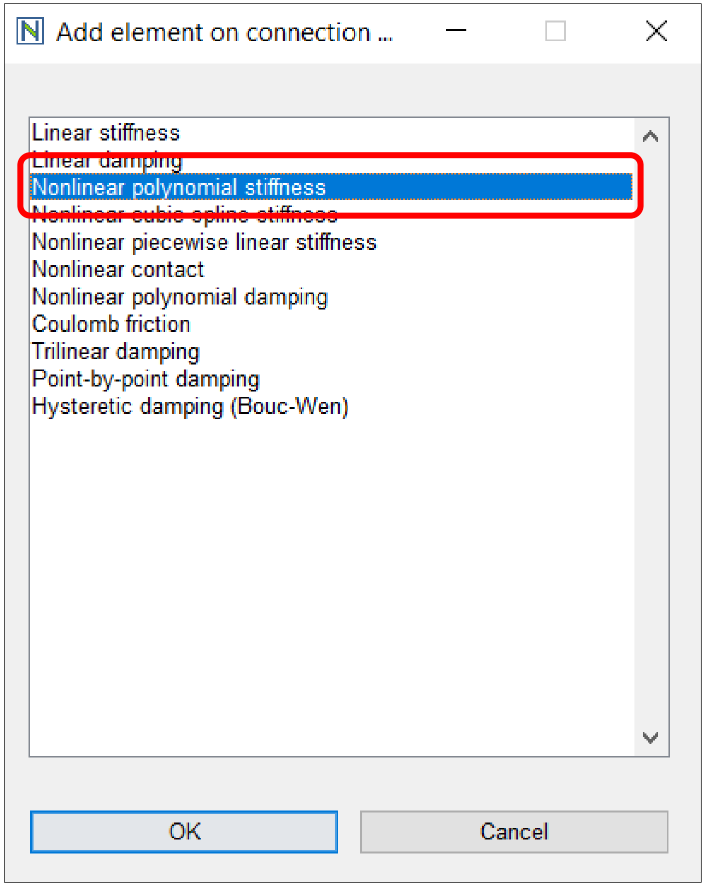
At this point, the model is linear and a nonlinear connection (cubic + quadratic) has to be created between the displacement at the tip of the main beam (DOF #28) and the ground.



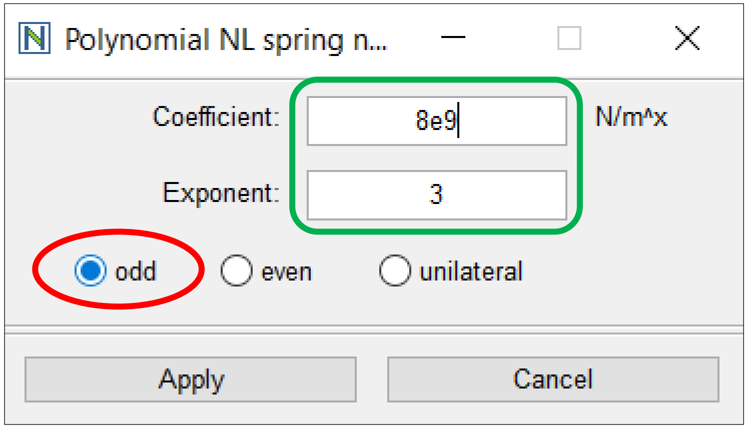
Bring the cursor on DOF #28 and use 'Ctrl + left click+ drag' inside the circle to create a connection with the ground.

Create Nonlinear Connections

1. Create 'Nonlinear polynomial stiffness'

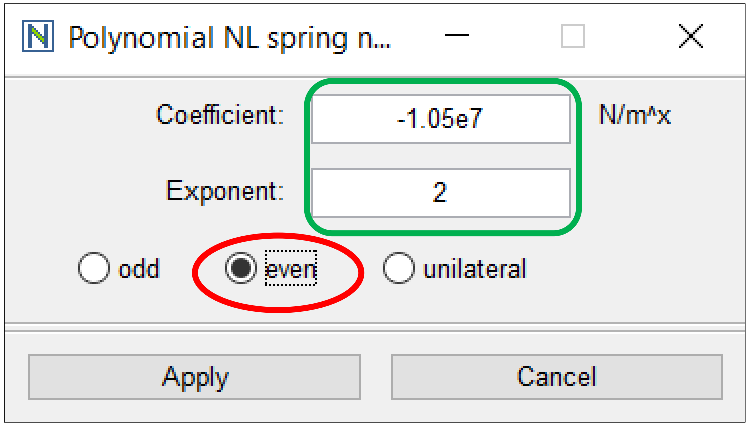


2. Create an odd cubic nonlinearity with stiffness set to $8\ x\ 10^9$ N/m^3



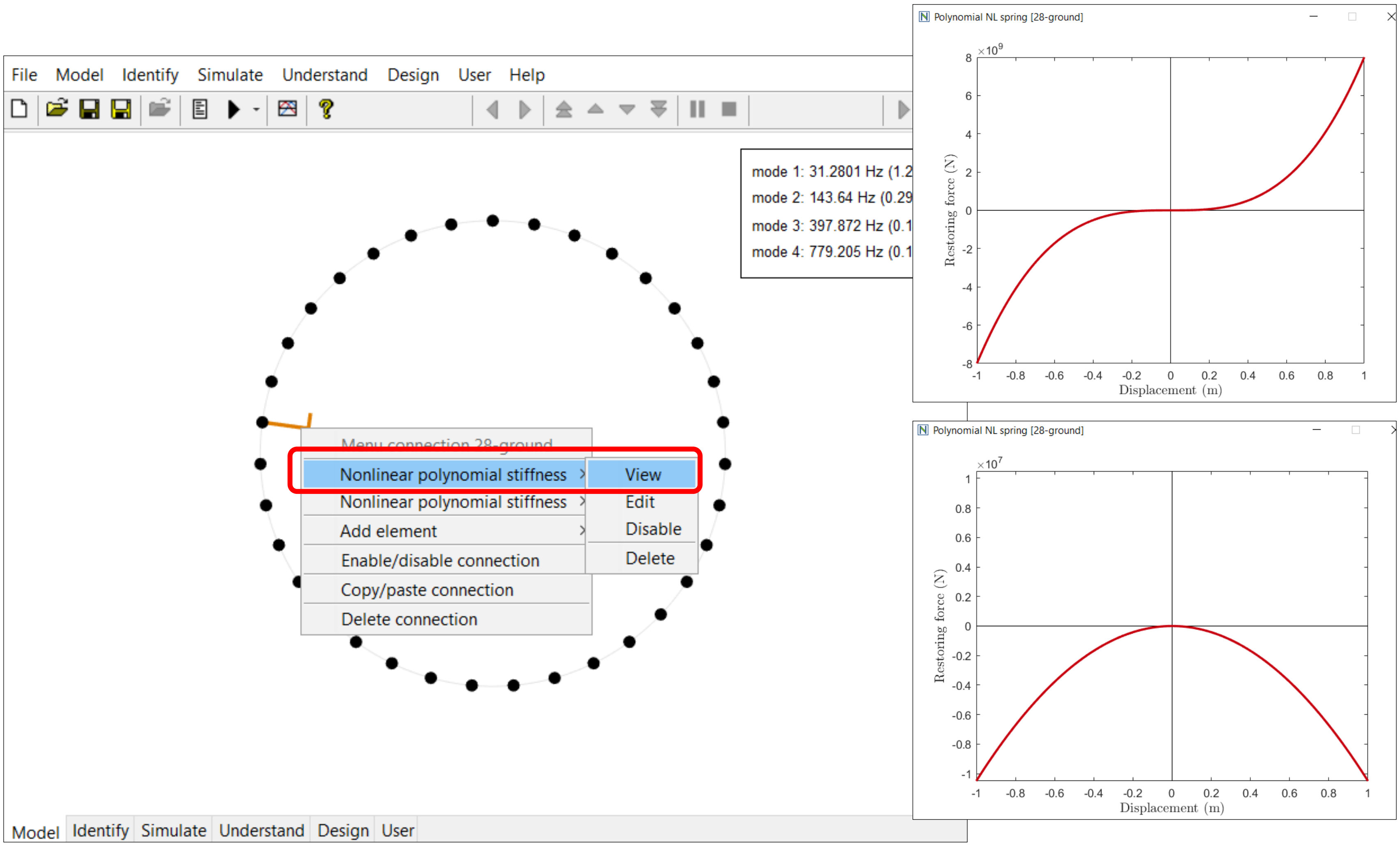
Create Nonlinear Connections

- Create a new connection between DOF #28 and the ground
- Create an even quadratic nonlinearity with stiffness set to $-1.05\ x\ 10^7$ N/m²



Visualise Nonlinear Connections

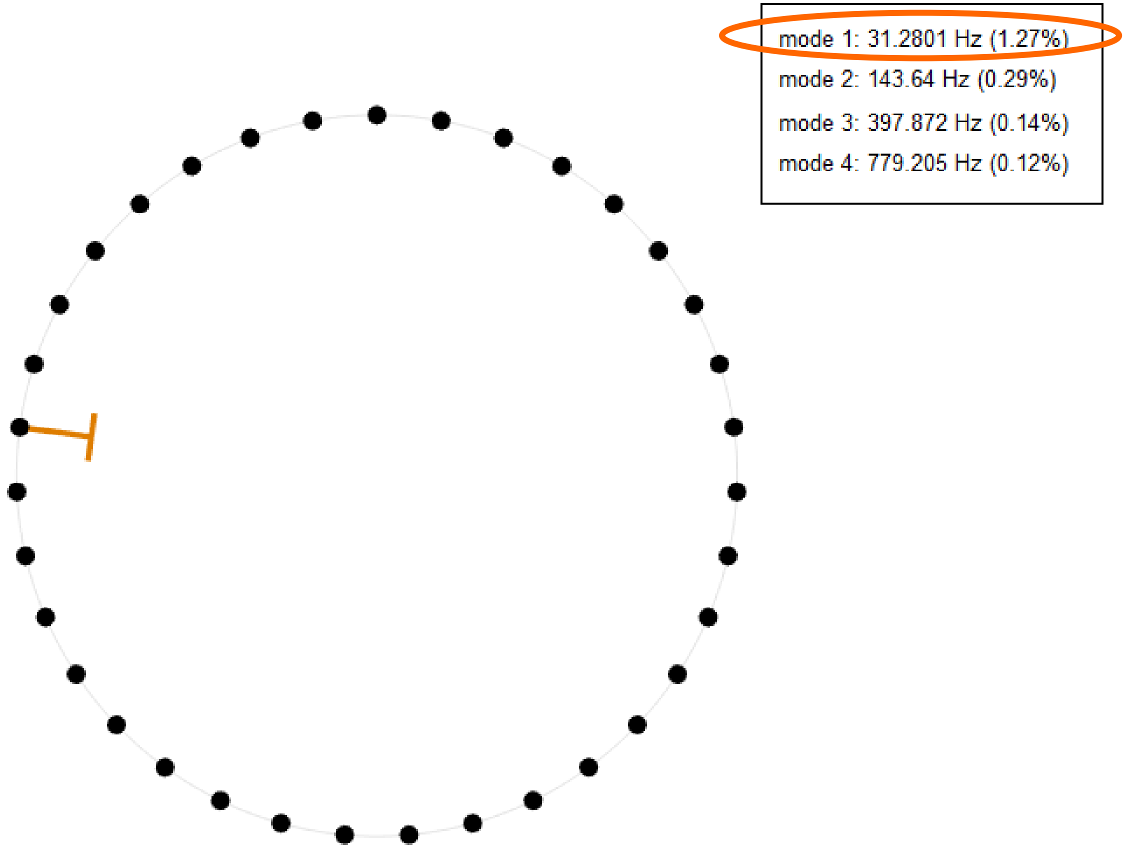
- Use ‘right click’ in the nonlinear connections and select ‘View’ to visualise each restoring force.



Exercise 1

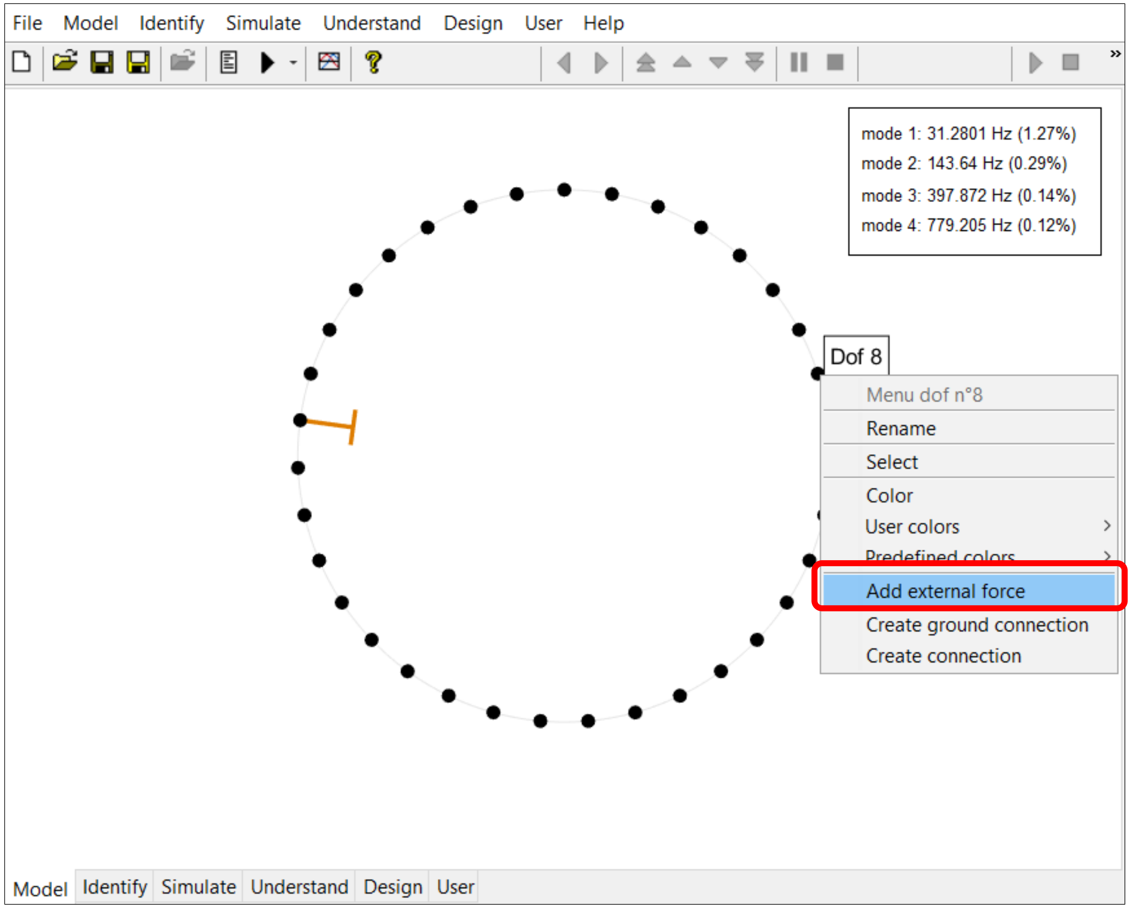
Create a Sine External Force

As a first exercise, we will study the system's response to a sine excitation with a frequency close to the first resonance frequency of the beam (31.28 Hz).



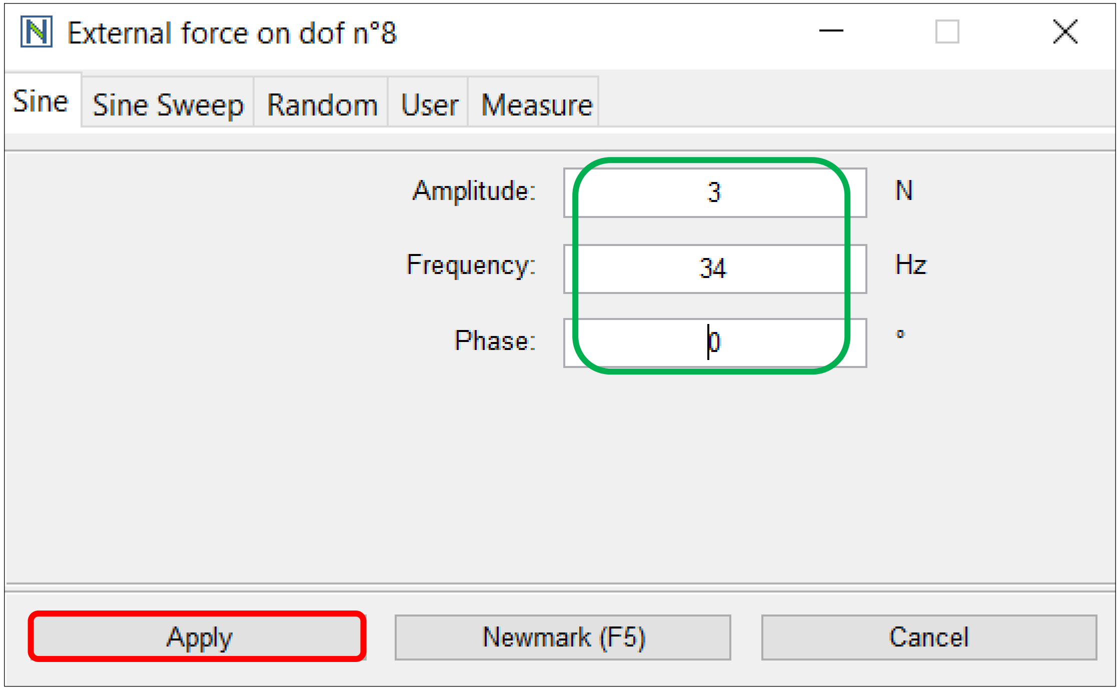
Create a Sine External Force

- Create a forcing by using ‘right click‘ DOF #8 and selecting ‘Add external force‘.



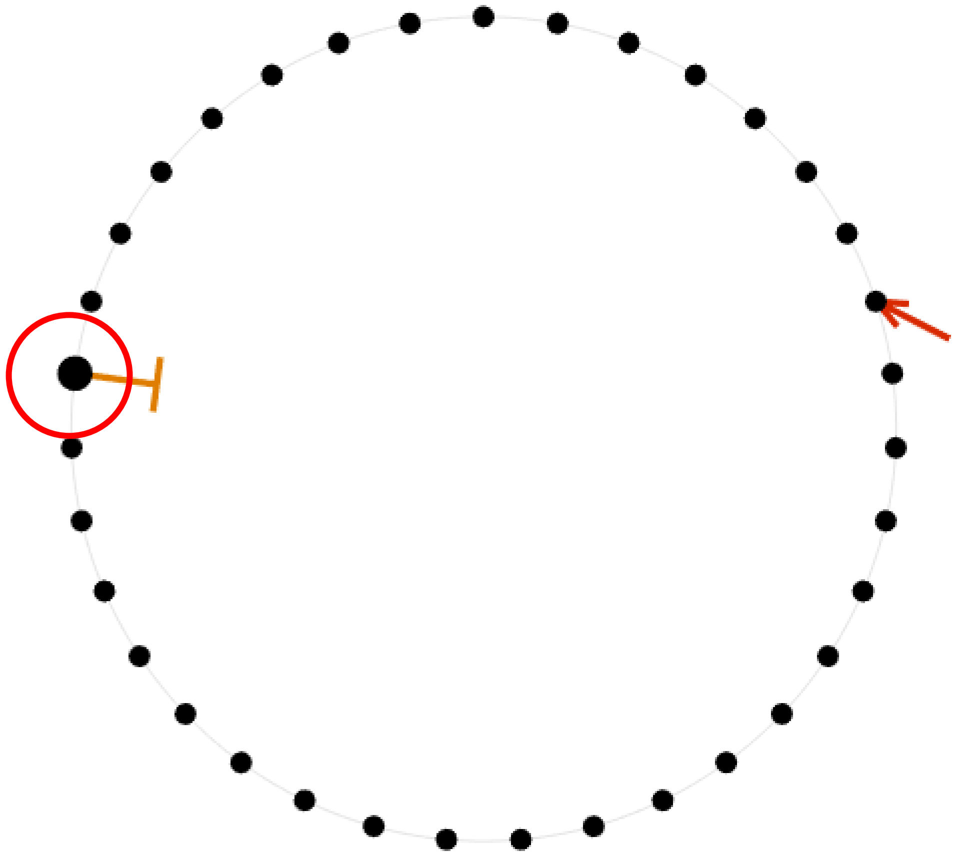
Create a Sine External Force

- Select ‘Sine‘ and give the following forcing parameters, then ‘Apply‘.



Simulation of a Response to a Sine Excitation

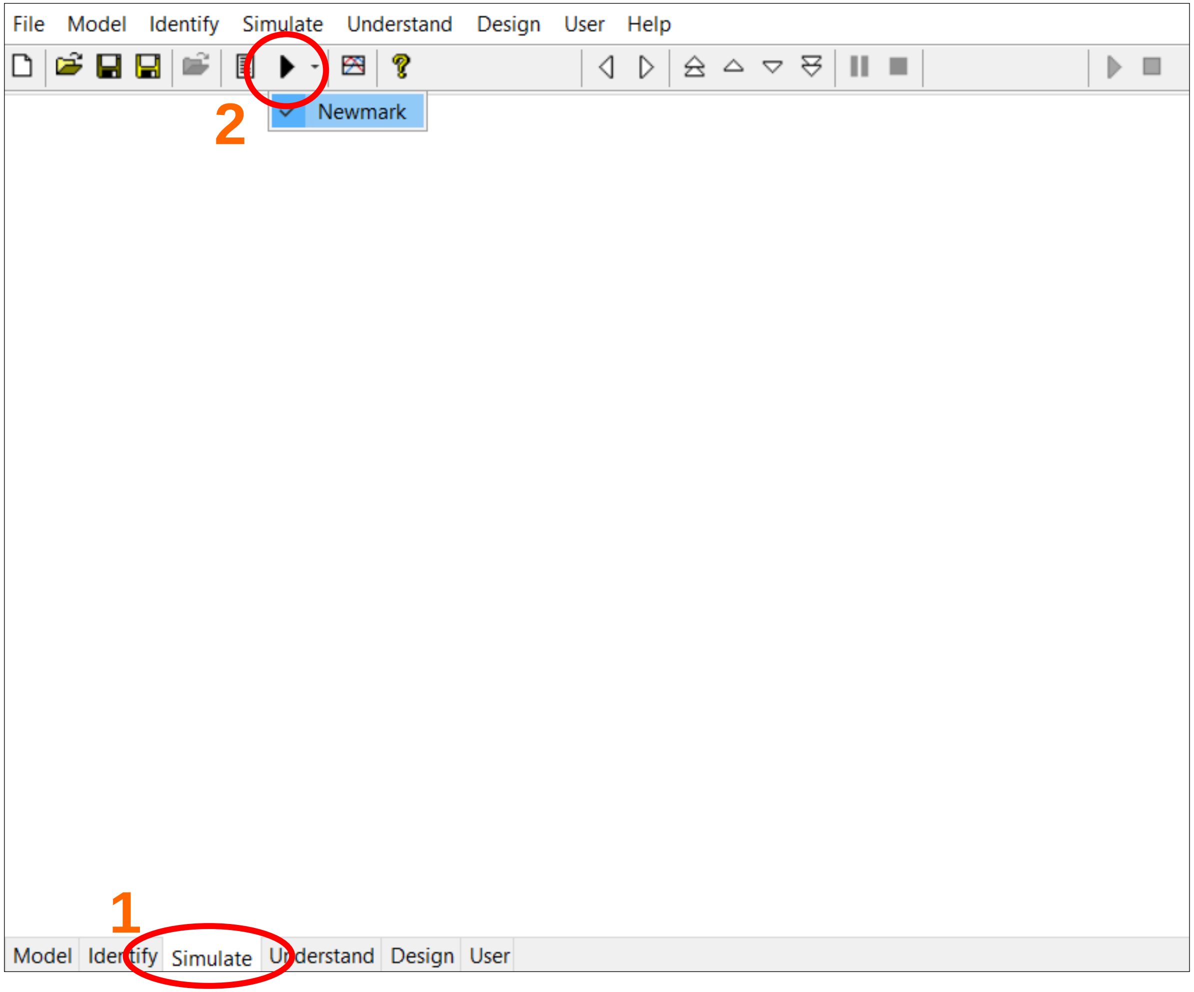
- In order to show the time series at the tip of the main beam, use ‘Alt + double left click‘ on DOF #28 to display next results on that DOF.



You can modify the colour associated to DOF #28 using ‘right click’ on that DOF.

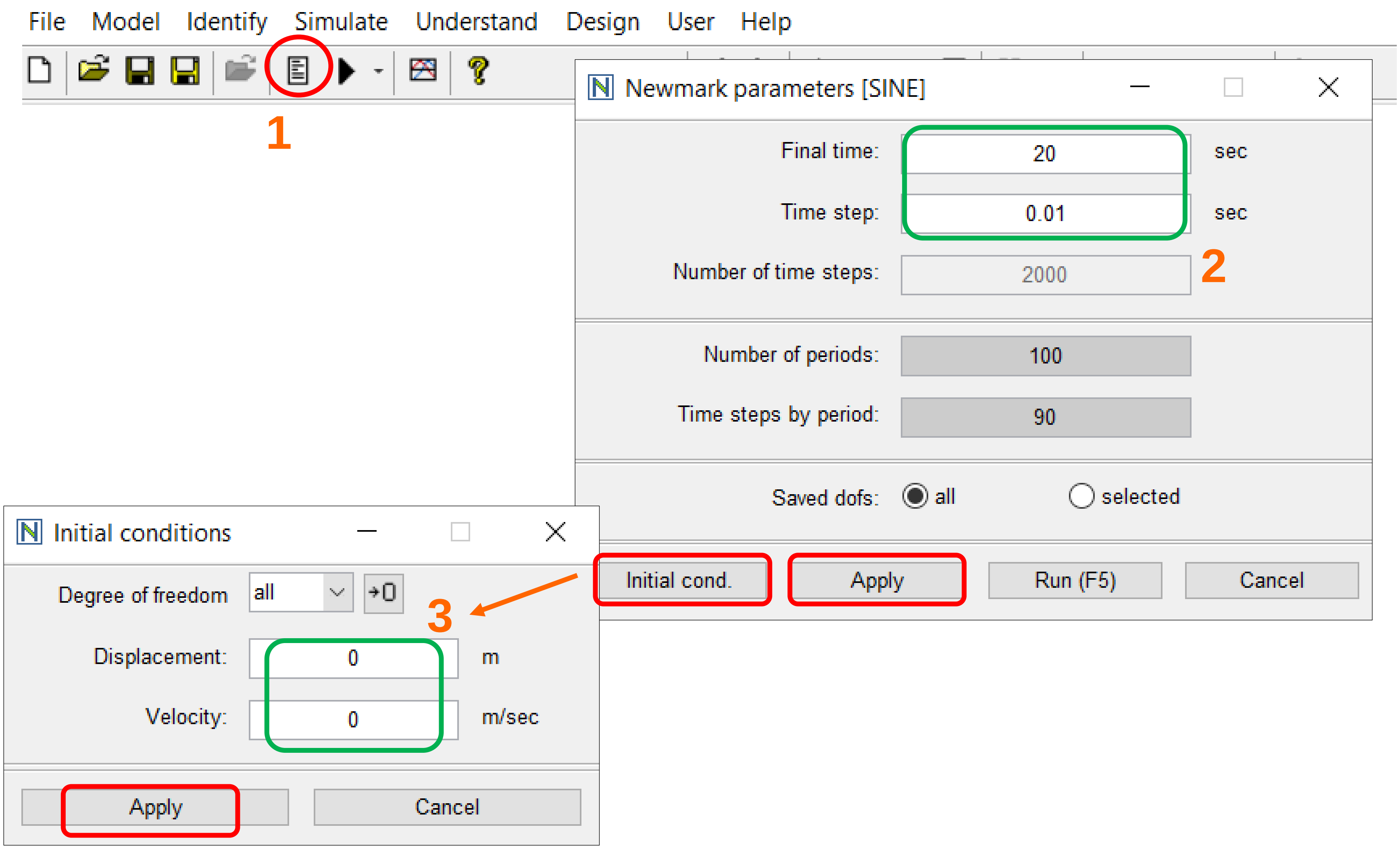
Simulation of a Response to a Sine Excitation

- Go to the 'Simulate' tab (**1**) and select 'Newmark' as a solver (**2**).



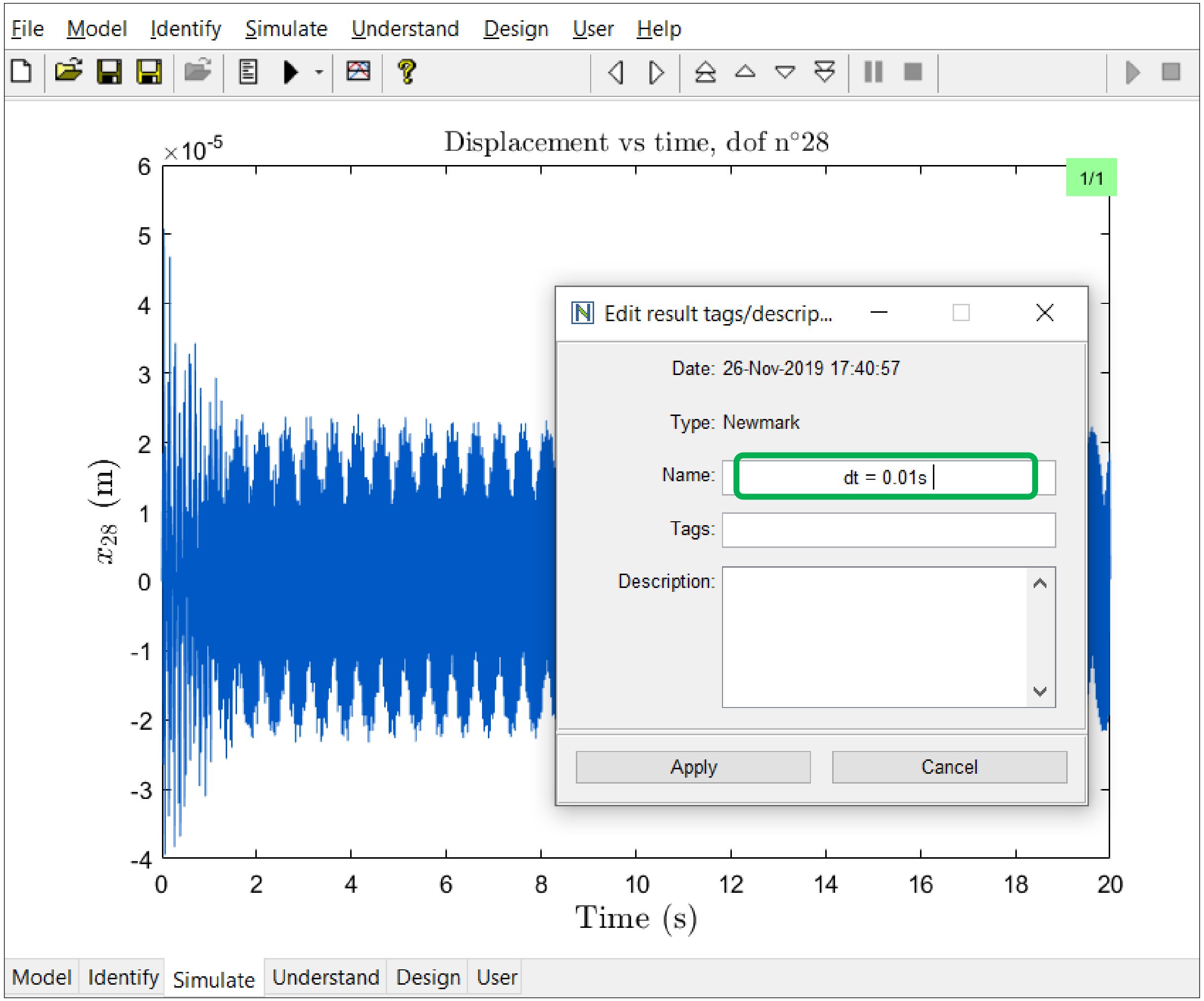
Simulation of a Response to a Sine Excitation

- Set the parameters for the solver and click on ‘Apply‘ for both windows.



Simulation of a Response to a Sine Excitation

- Tag your result 'dt = 0.01s' using 'F11' and save it in a new curve stack.



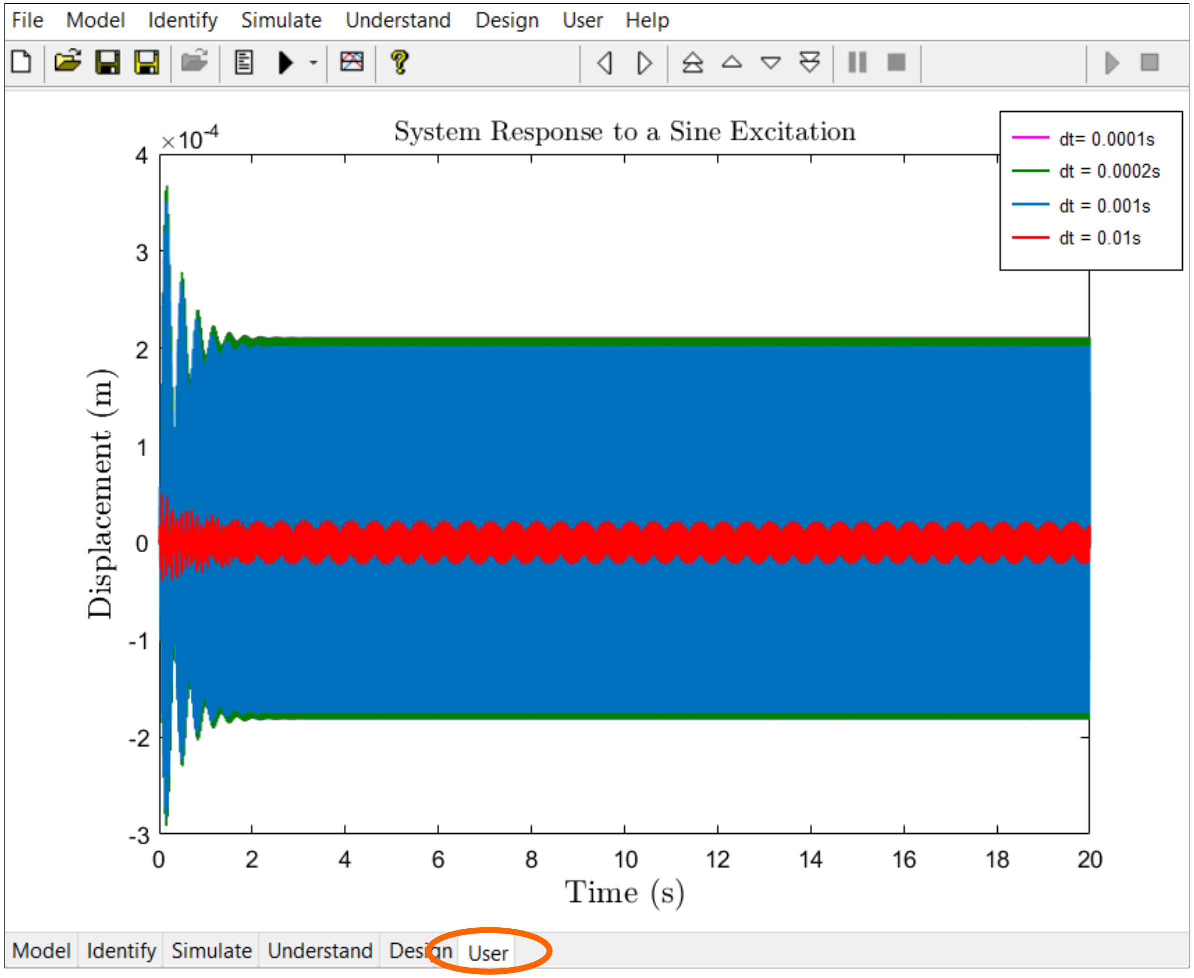
Influence of the Time Step

- Perform another simulation for a smaller time step of $0.001s$, tag it, and select a different colour using ‘right click’ on the curve. Compare the curves by saving them in the same curve stack.

- Repeat the same operations for smaller time steps of $0.0002s$ and $0.0001s$.

Influence of the Time Step

- Go to the ‘User’ tab, and use ‘right click’ and ‘Stack up/down’ or ‘Auto stack’ to observe the different curves in the figure.



Question:
Can you explain why the time series are different?

Influence of the Time Step

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Can you explain why the time series are different?

Answer:

Selecting a small time step is crucial for obtaining accurate time series. For example, linear Newmark's scheme has a periodicity error of

$$\Delta\omega = \frac{4\,\pi^2 f^2}{12\, f_s^2}\,.$$

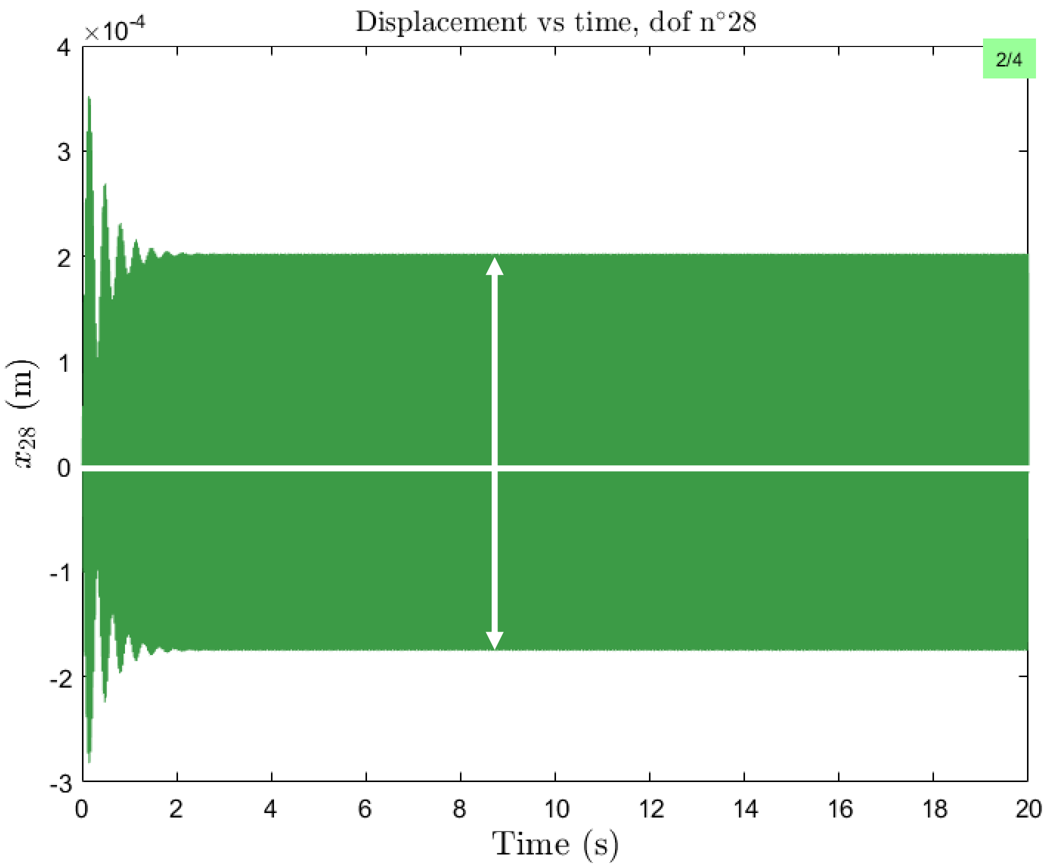
Frequency of interest in the signal (f)

Sampling frequency (f_s)

For an error of less than 1%, and for $f = 34$ Hz, one should have larger f_s than 620 Hz.

Influence of the Time Step

Considering that third harmonics are present in the response, f_s should be even larger. Here we can choose 0.0002s as an optimal time step.



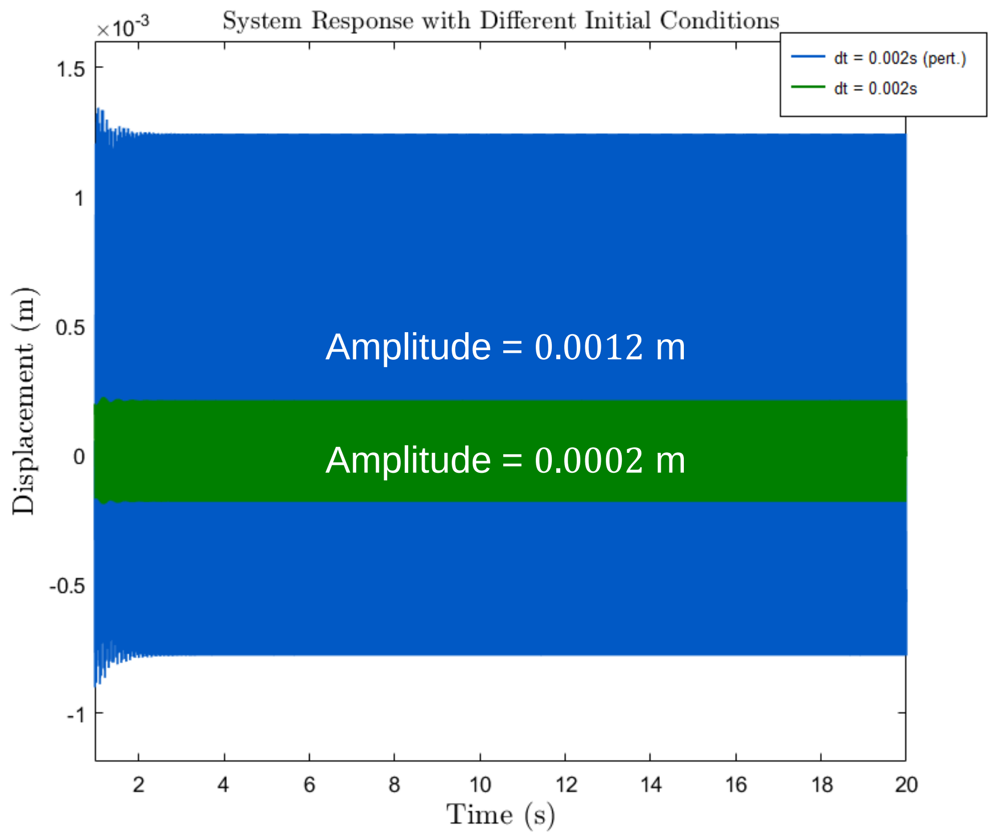
You can notice that the response is slightly asymmetric due to the presence of the quadratic nonlinearity.

Influence of the Initial Conditions

- Select a time step of 0.002s, and consider an initially perturbed system using an initial velocity of 1 m/s for DOF #28.
- By using tags and colours, compare the initially perturbed and unperturbed responses.

Depending on the initial conditions, the system can have small or large amplitude oscillations.

▶ Bistable behaviour



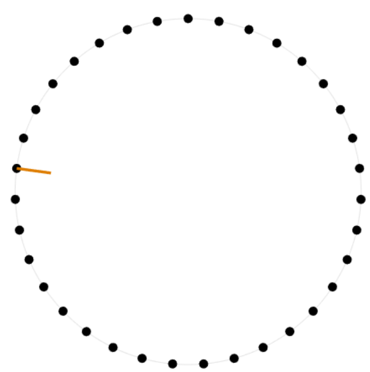
Exercise 2

Create a Swept-sine External Force

We consider the system from exercise 1 again:

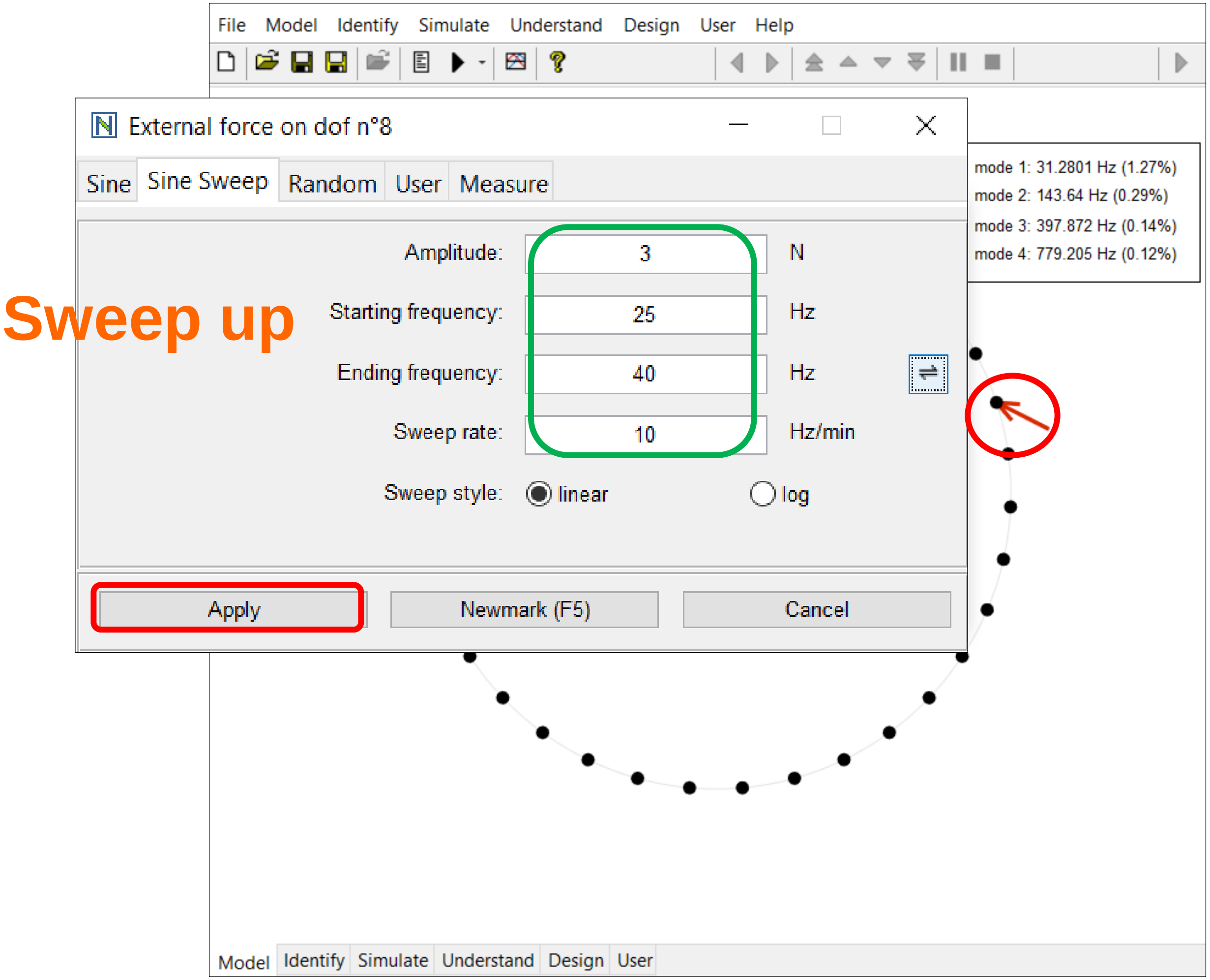
As a second exercise, we will study the system's response to a **swept-sine excitation** with a frequency range between 25 Hz and 40 Hz, which encompasses the first resonance frequency of the beam (31.28 Hz).

We will study the effect of the sweep rate, and compare responses to sweep up and down in order to highlight the bistable region in the frequency response.



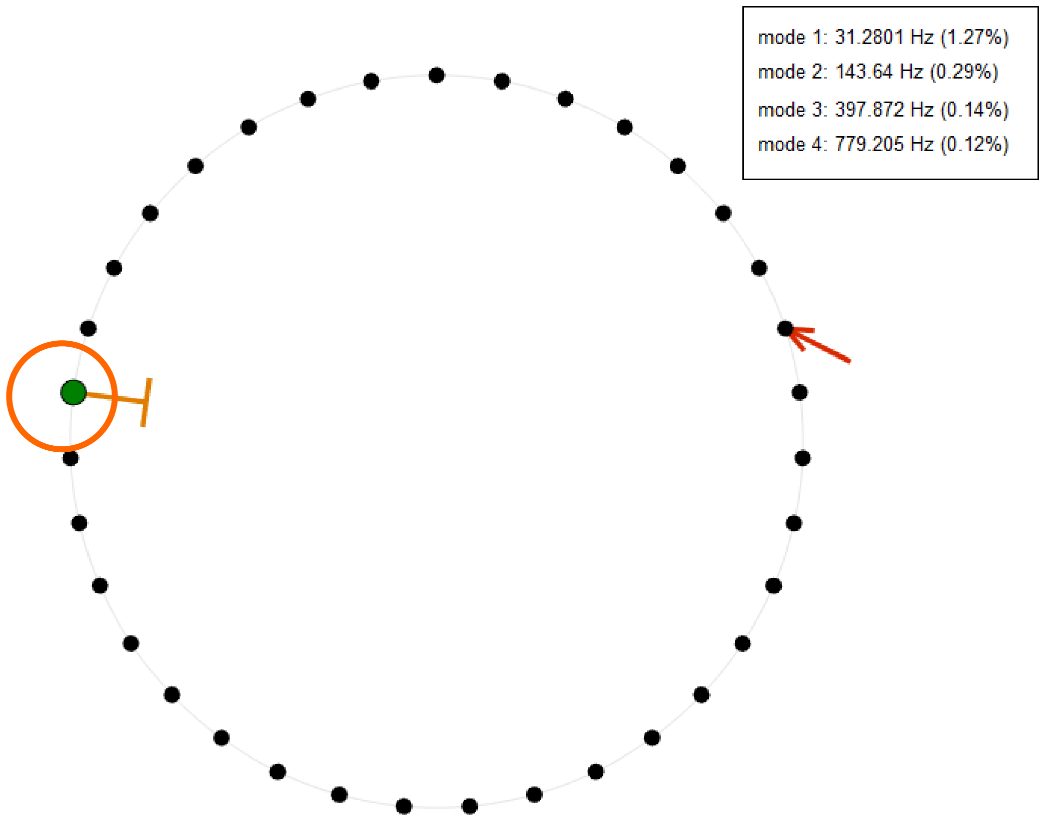
Create a Swept-sine External Force

- Select ‘Sine Sweep’ on DOF #8 and give the following forcing parameters. Apply.



Simulation of a Response to a Swept-sine Excitation

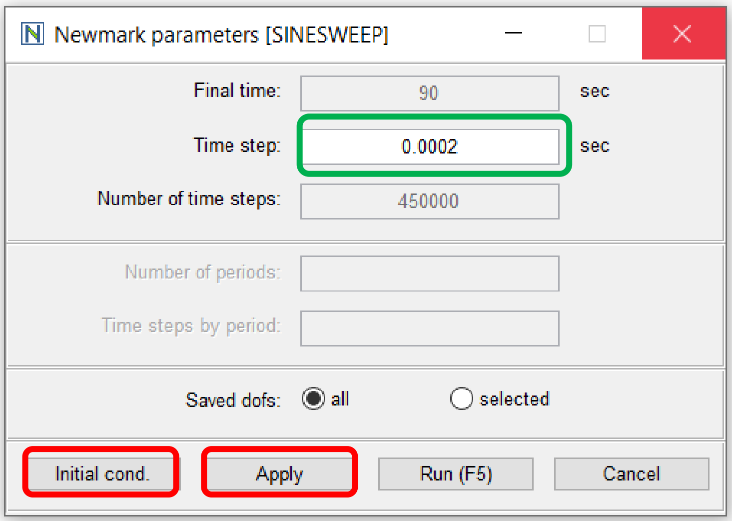
- In order to show the time series at the tip of the main beam, use 'Alt + left click' in DOF #28 to display next results on that DOF.



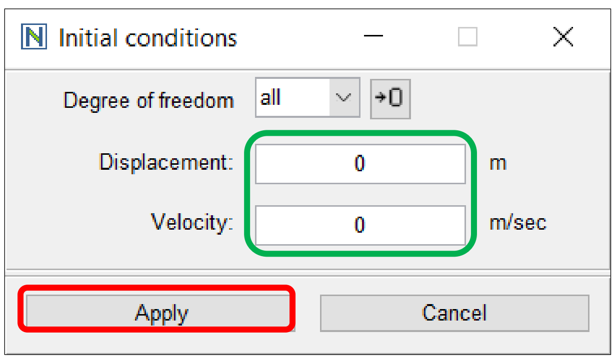
Simulation of a Response to a Swept-sine Excitation

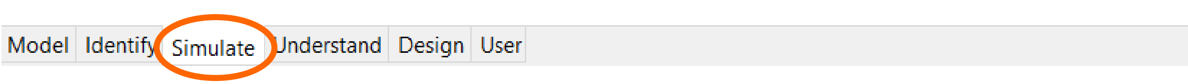
- In the ‘Simulate’ tab, modify the solver parameters to set all displacements and velocities to zero, and the time step to 0.0002s. Apply.

1



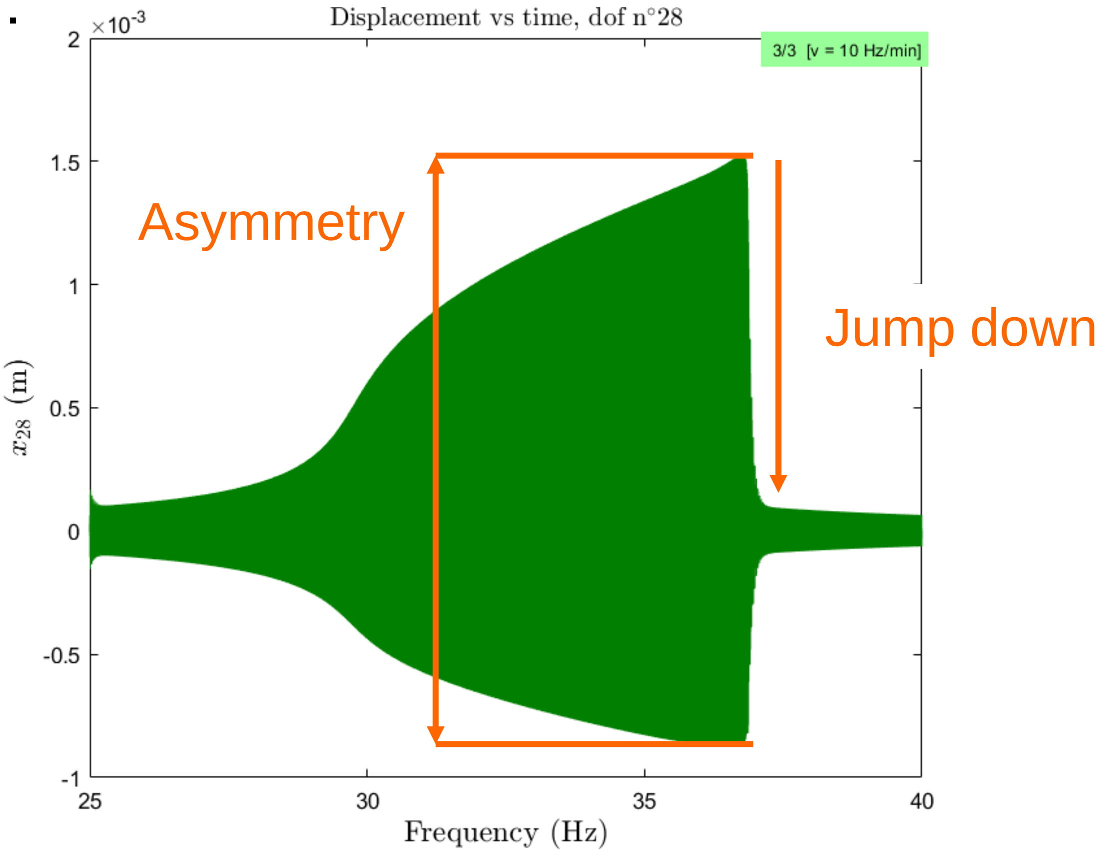
2





Simulation of a Response to a Swept-sine Excitation

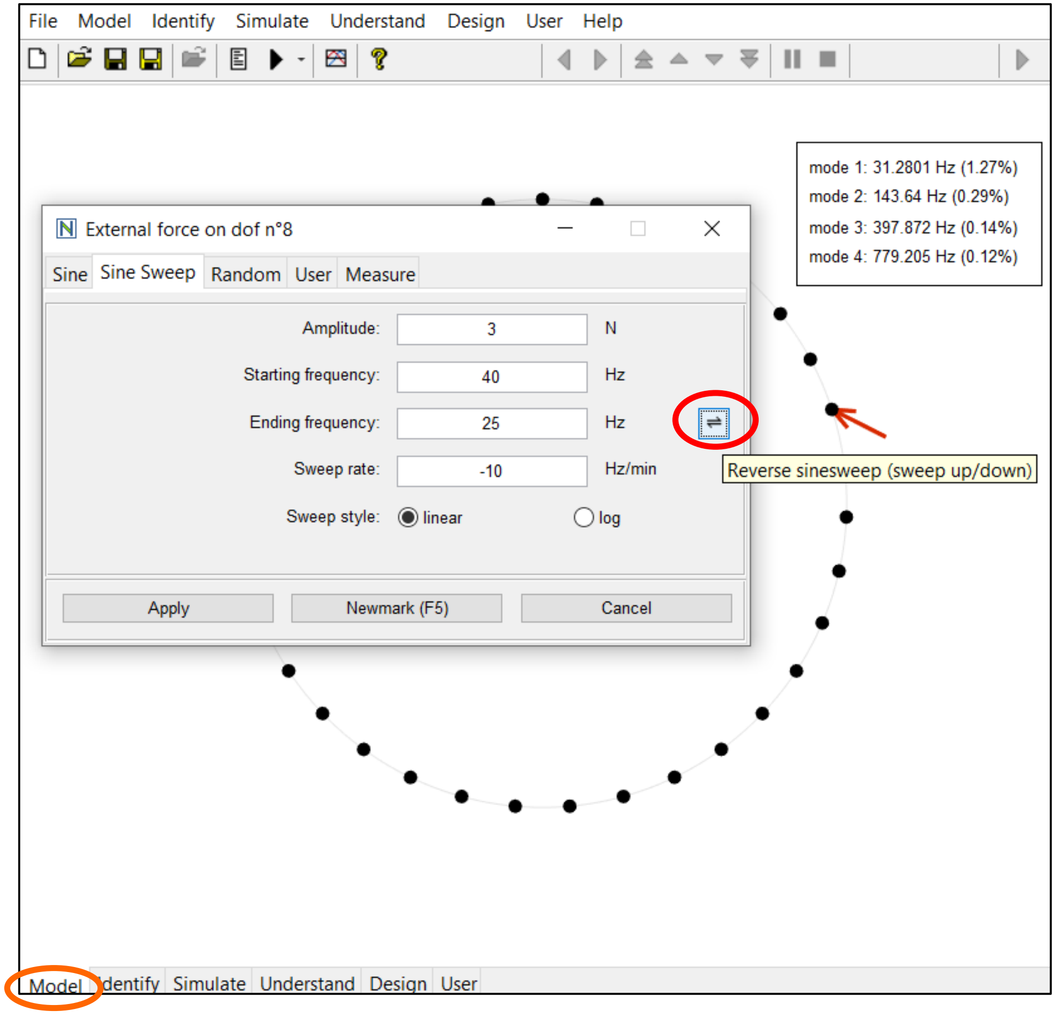
- Start the simulation and tag your result as ‘v = 10 Hz/min’ by using ‘F11’.



The response is asymmetric due to the presence of the quadratic nonlinearity. The jump down occurs due to hardening behaviour.

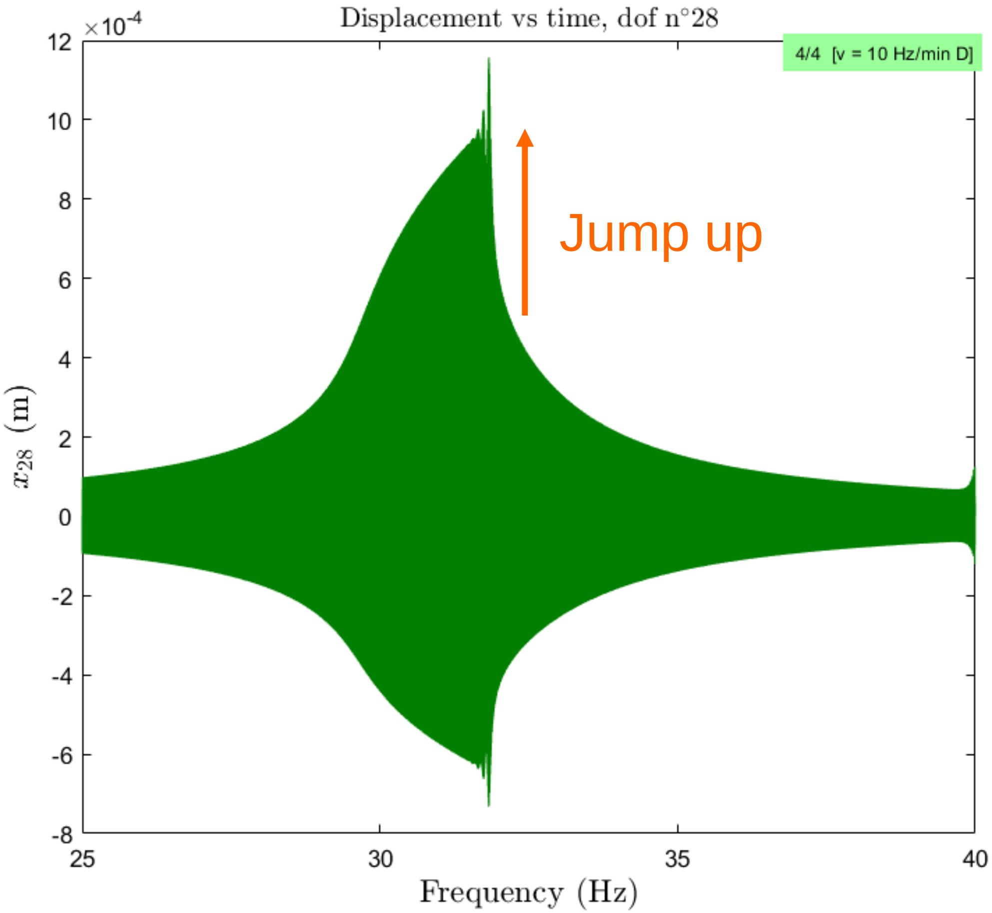
Simulation of a Response to a Swept-sine Excitation

- Reverse the sweep direction.



Comparison between Sweep Up and Sweep Down

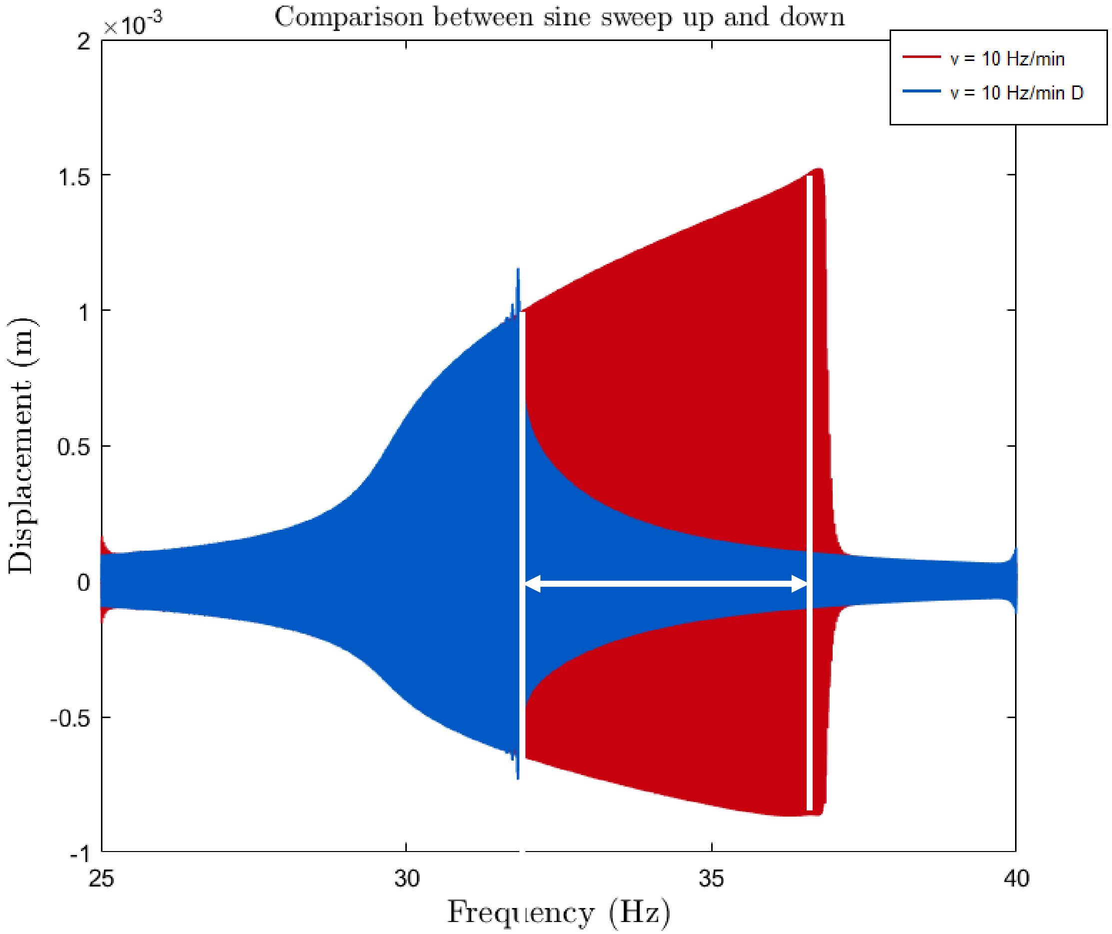
- Start the simulations and tag the result as ‘v = 10 Hz/min D’.



The jump up occurs due to hardening behaviour.

Comparison between Sweep Up and Sweep Down

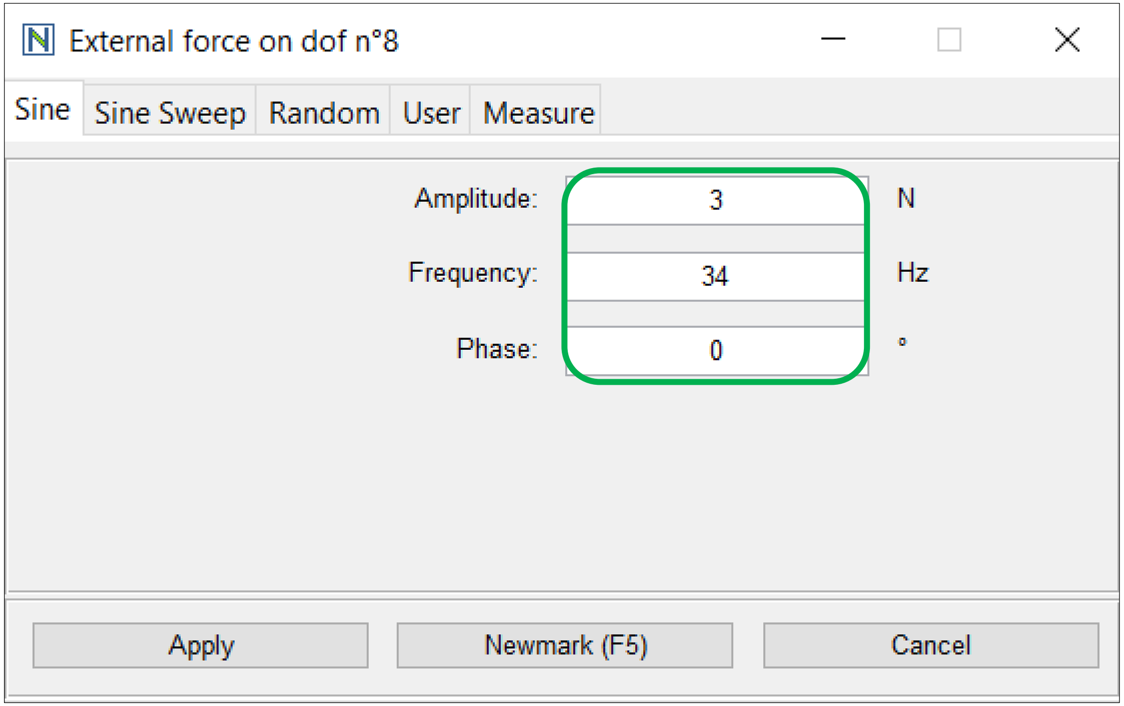
- Compare the responses for a sweep up and sweep down.



The bistable region spans between 32 Hz and 37 Hz.

Computation of the NFRC

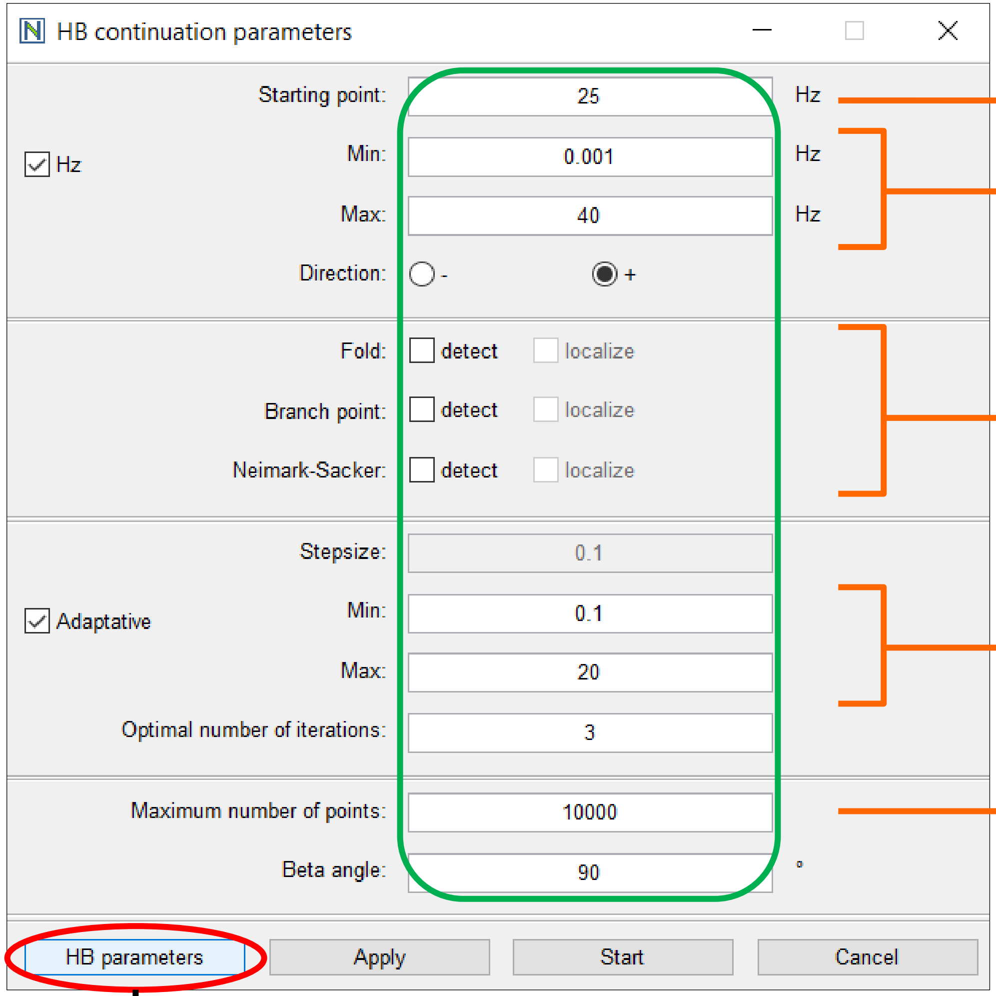
- Go back to your model. Select a sine excitation with the following parameters.



The chosen forcing amplitude of 3 N will be considered for all the periodic solutions along the branch.

Computation of the NFRC

- Modify the continuation parameters:



Frequency of the initial point.

Continuation stops when ‘Min’ or ‘Max’ frequencies are reached.

Bifurcations monitoring.

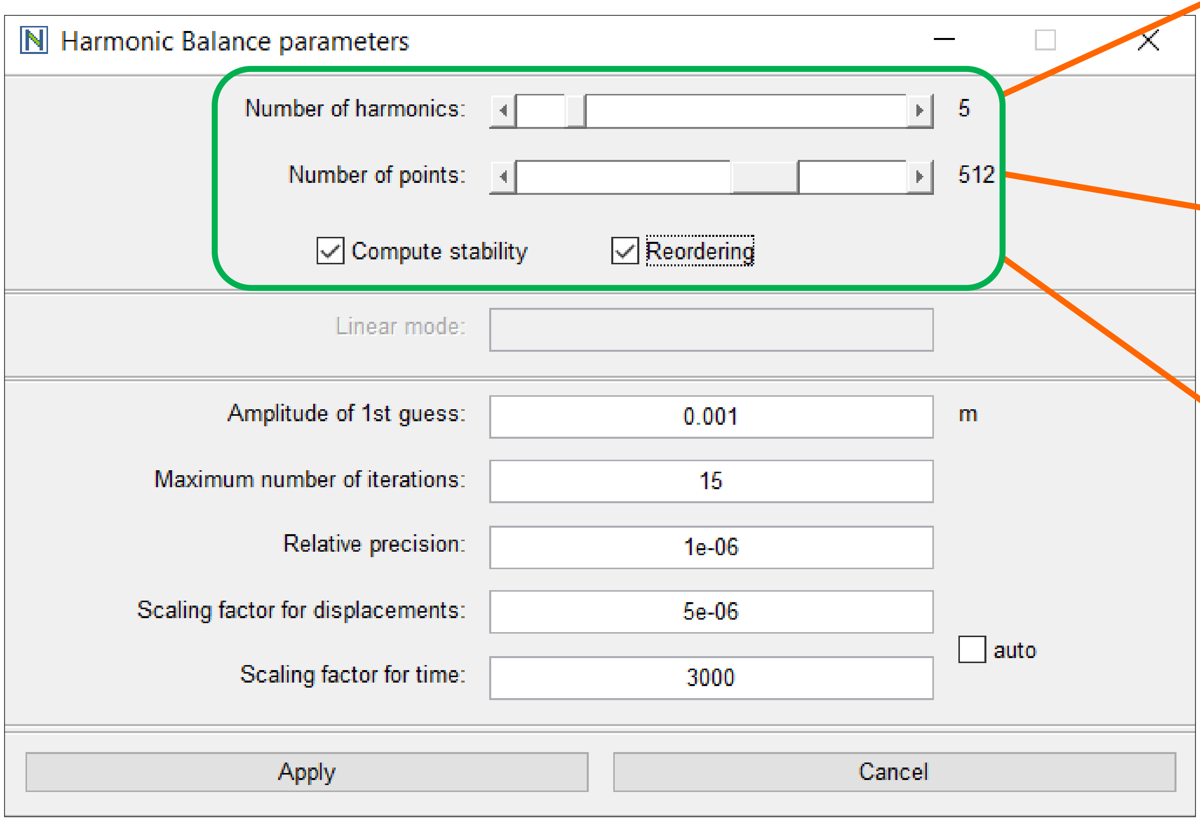
Min. and max. step size for the adaptive strategy.

Continuation stops when this number of points is reached.

Go to HB parameters afterwards.

Computation of the NFRC

- Modify the HB parameters:



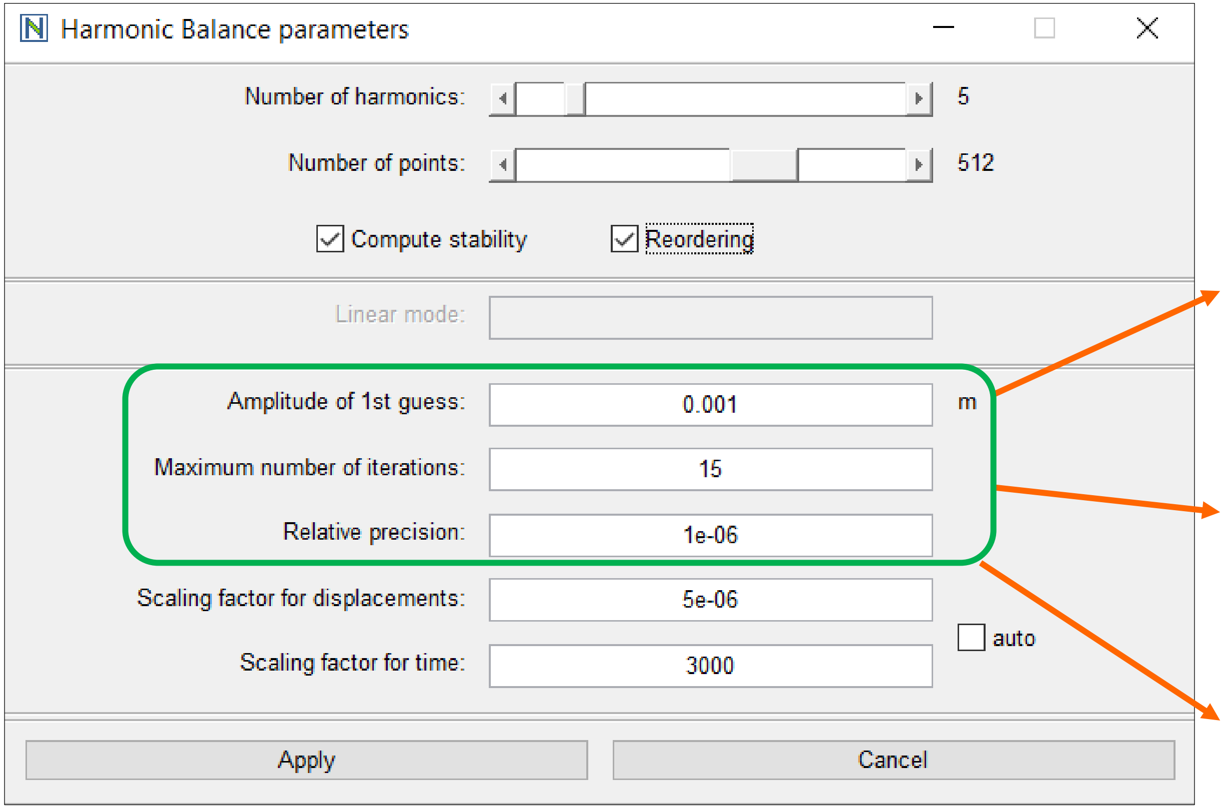
Number of harmonics N_H retained in the Fourier series

Number of the samples N in the Fourier transform.

Use of symmetric reverse Cuthill-McKee permutation to accelerate the eigenvalue problem resolution.

Computation of the NFRC

- Modify the HB parameters:



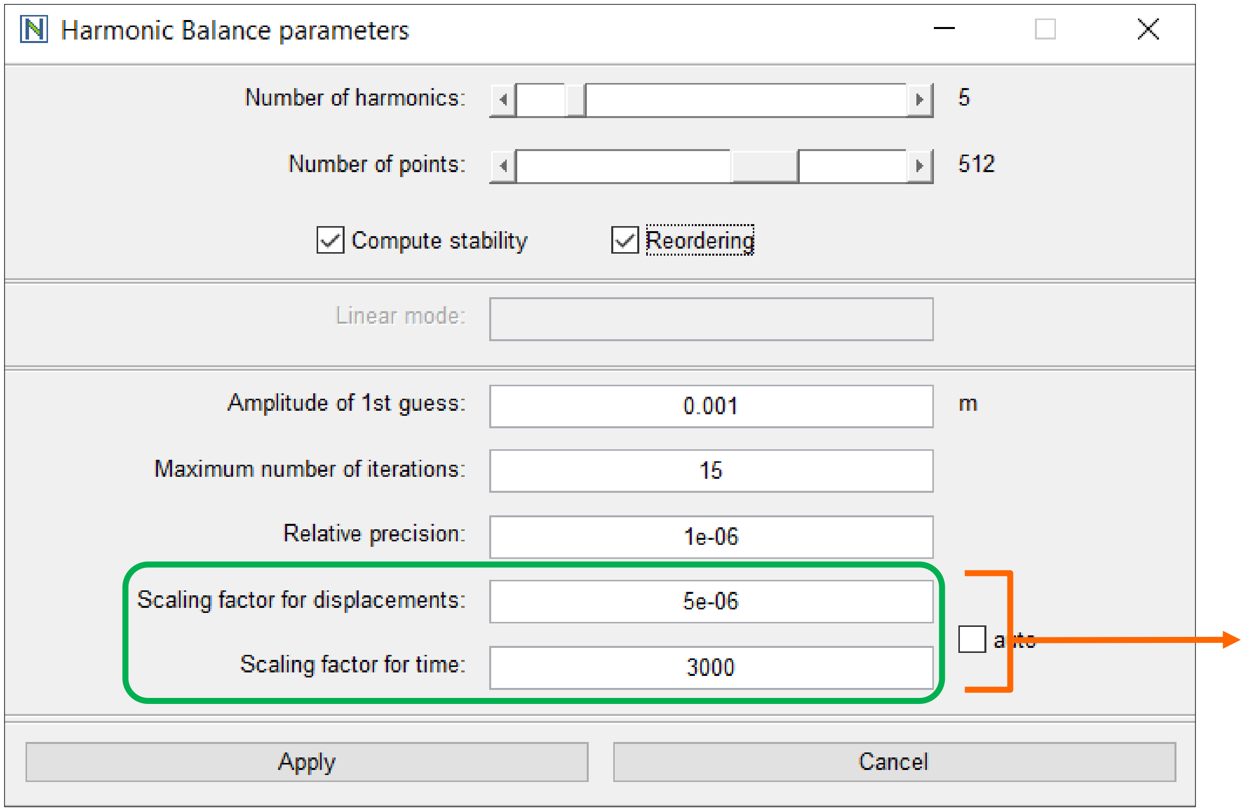
Amplitude of the sine series used as an initial guess for all DOFs.

The Newton-Raphson procedure fails if this number of iteration is exceeded.

The Newton-Raphson procedure stops if the relative error is smaller than this precision.

Computation of the NFRC

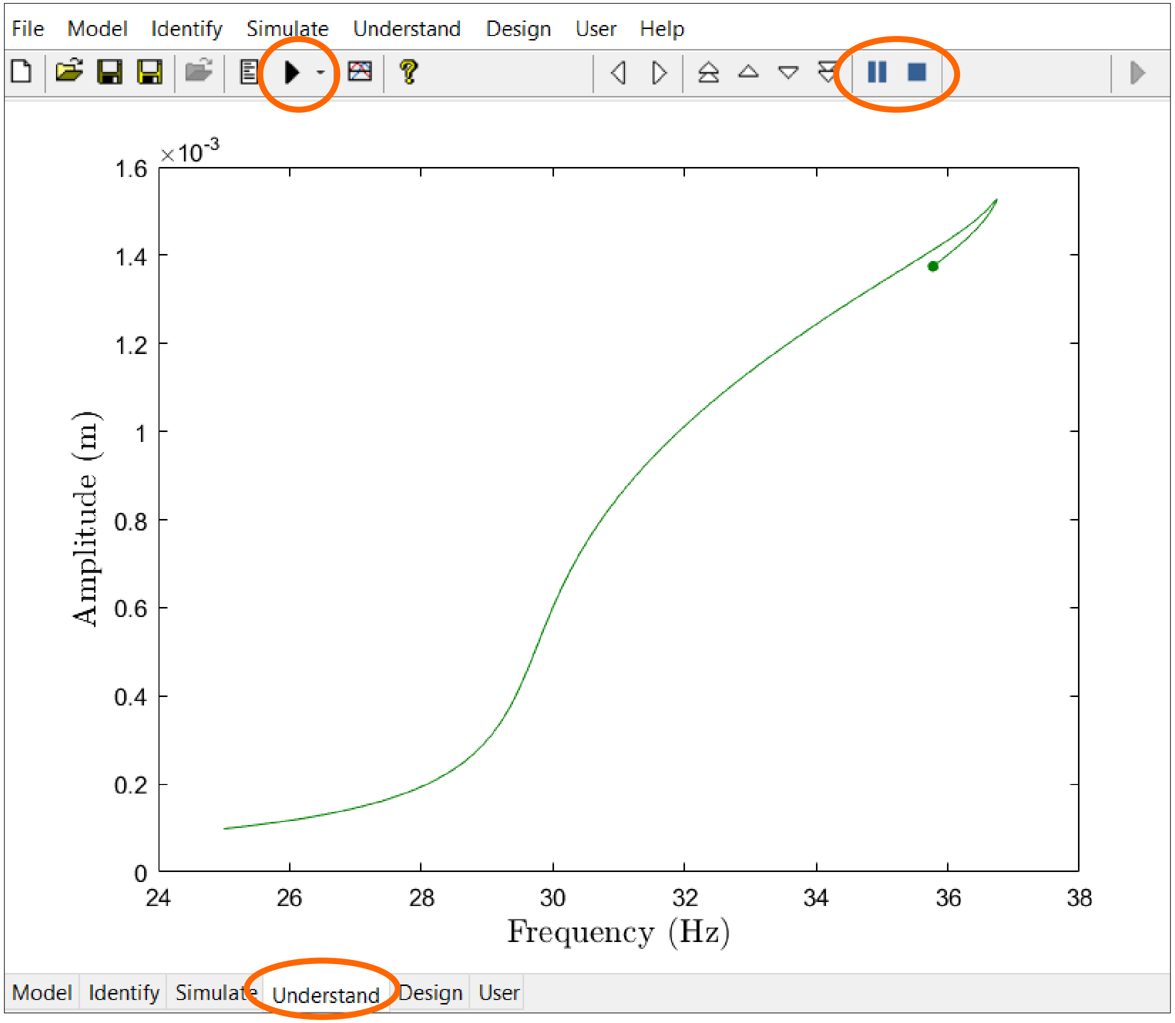
- Modify the HB parameters:



Because the frequency (here, around 30 Hz = 188 rad/s) and the amplitude (here, around 0.001 m) have different order of magnitude, time and displacements have to be rescaled to avoid ill conditioning.

Computation of the NFRC

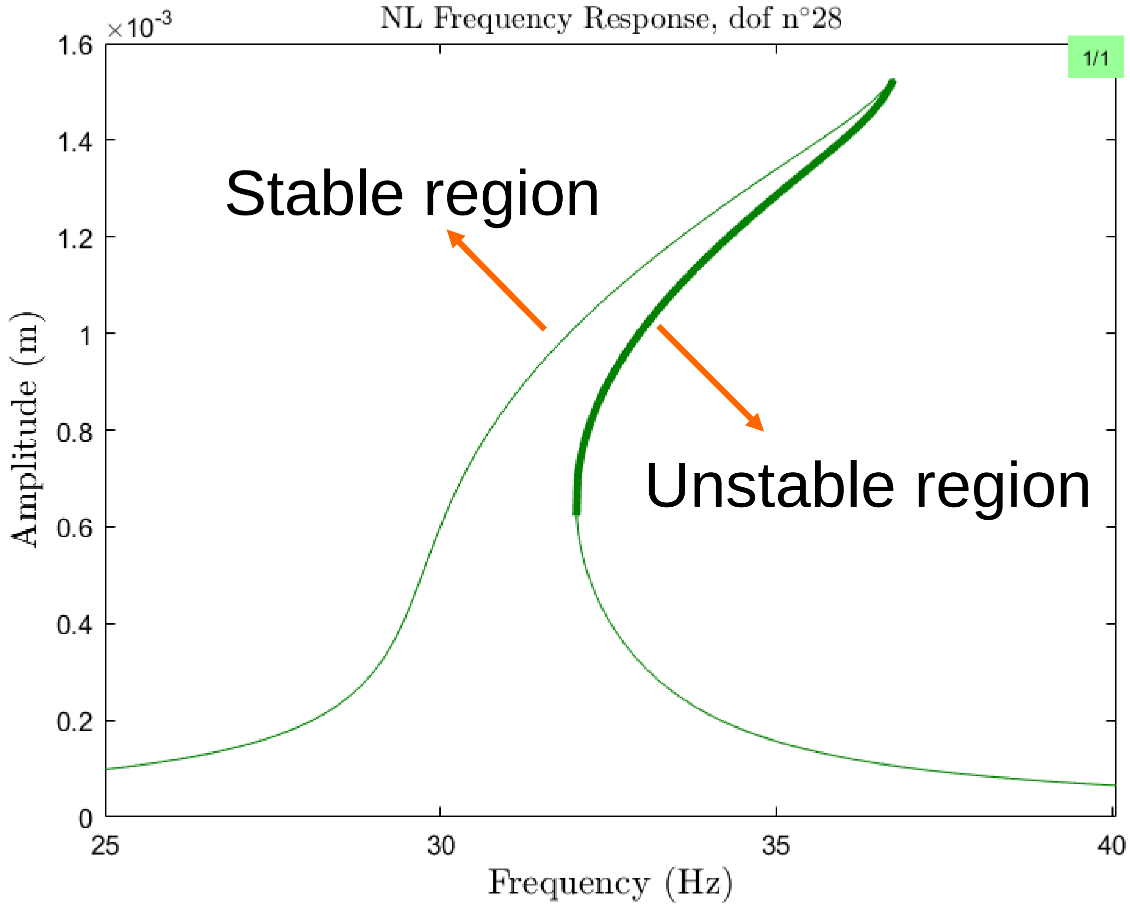
- Start the continuation procedure and wait until the maximum frequency (40 Hz) is reached.



You can also pause/unpause the procedure, or stop and record it at its current stage.

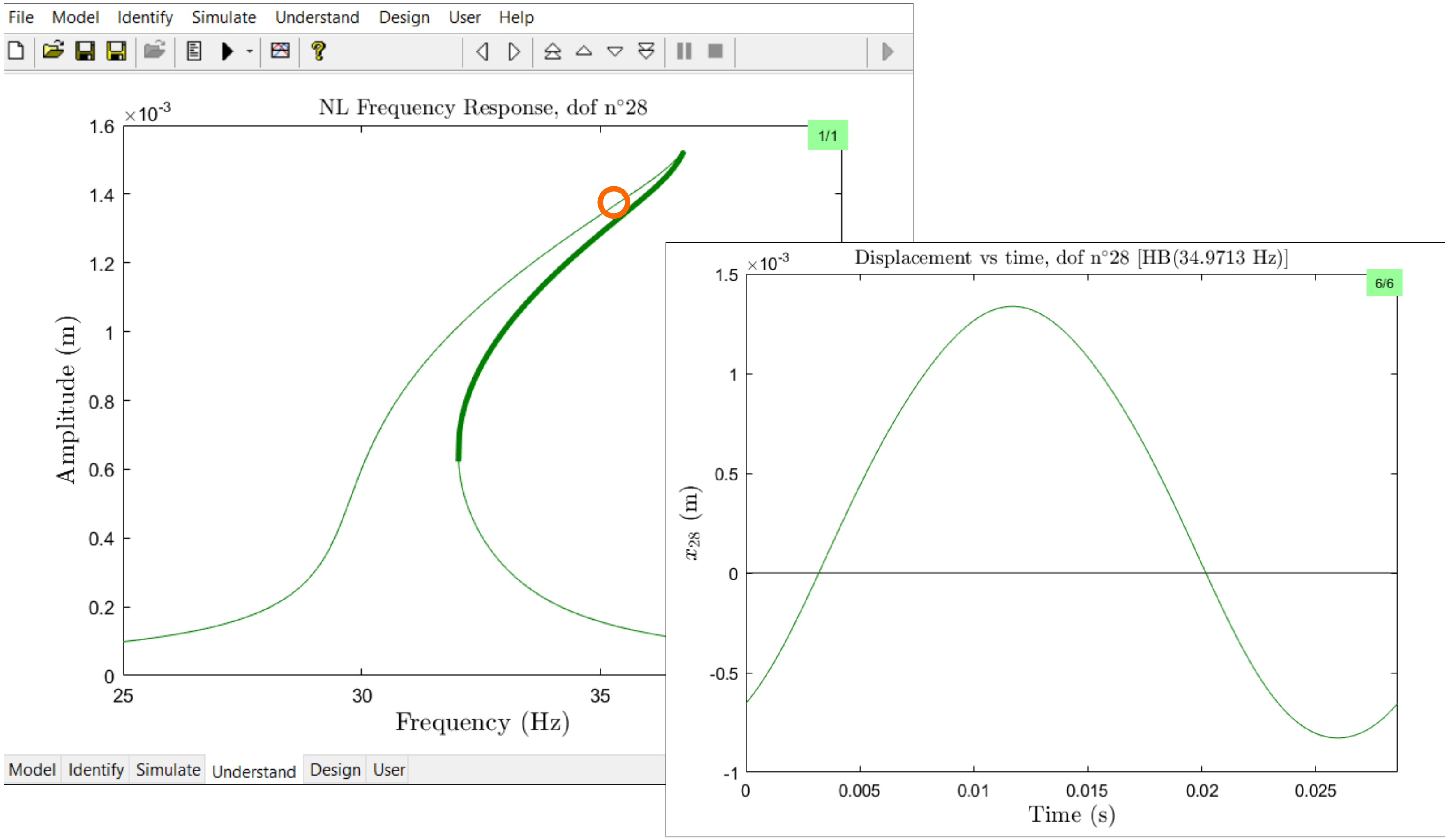
Computation of the NFRC

- Start the continuation procedure and wait until the maximum frequency (40 Hz) is reached.



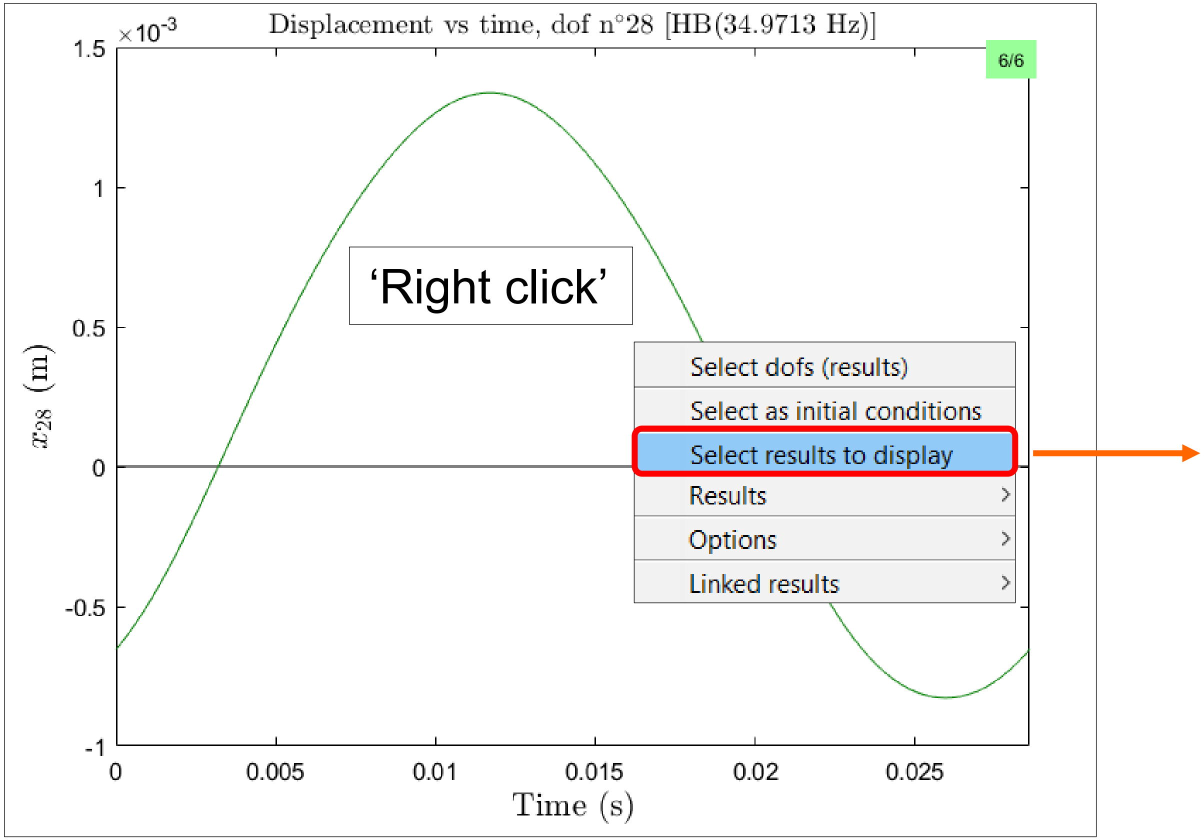
Analysis of the NFRC

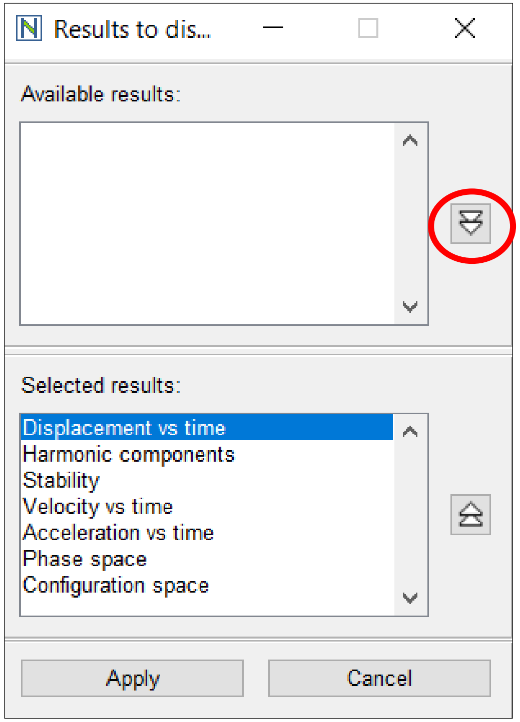
- Use ‘double left click’ on a point to represent its time series reconstructed from the Fourier coefficients, and its Floquet exponent/multipliers.



Analysis of the NFRC

- You can choose more results to display.





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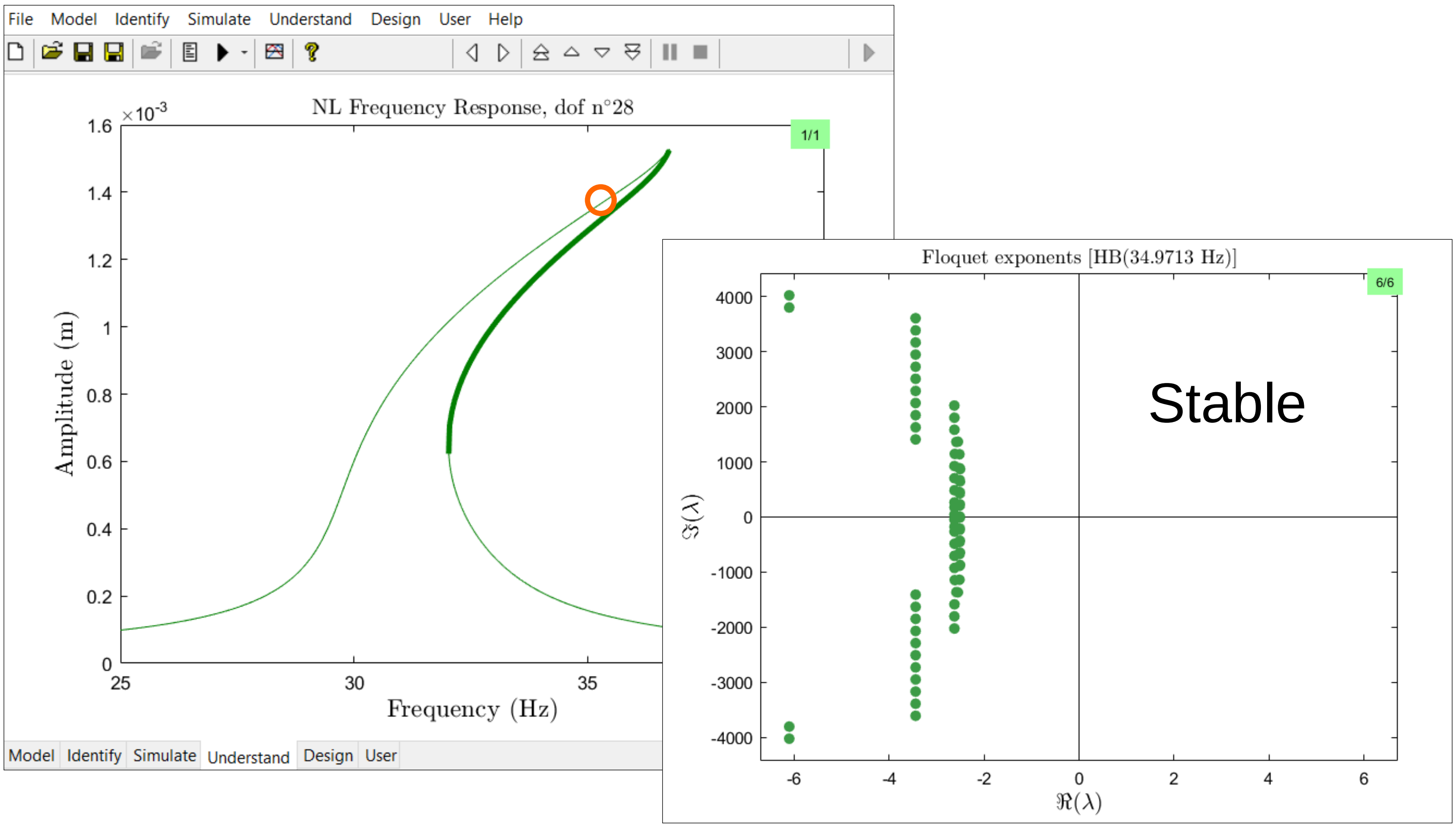
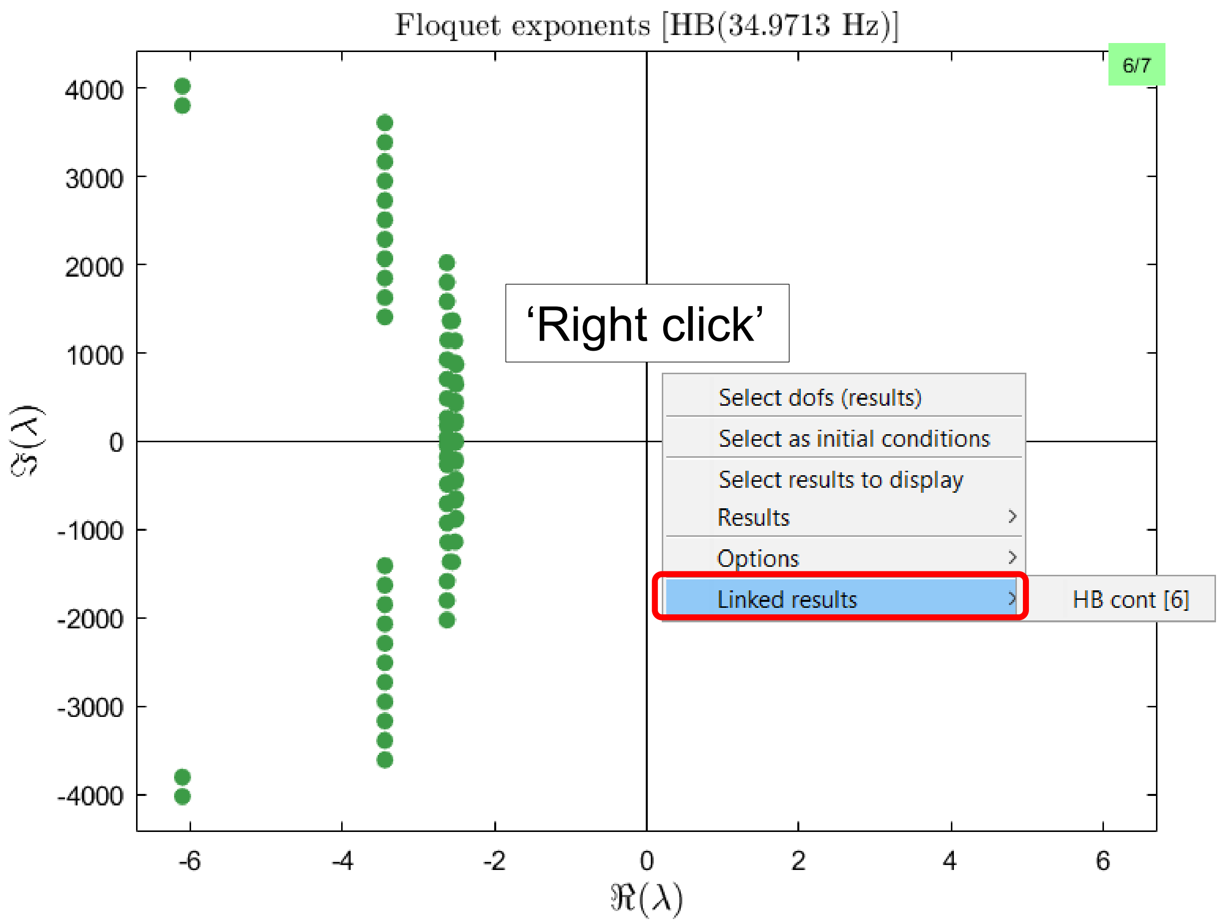


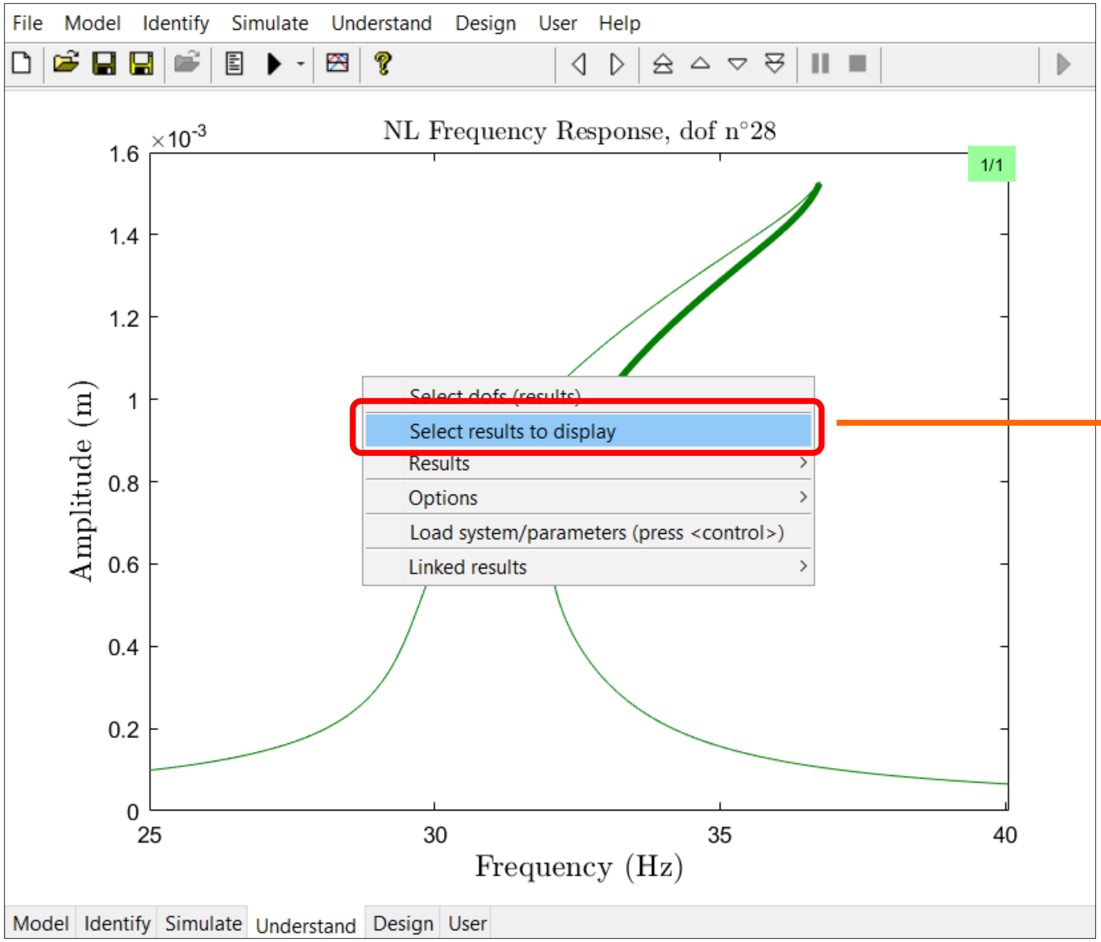
Analysis of the NFRC

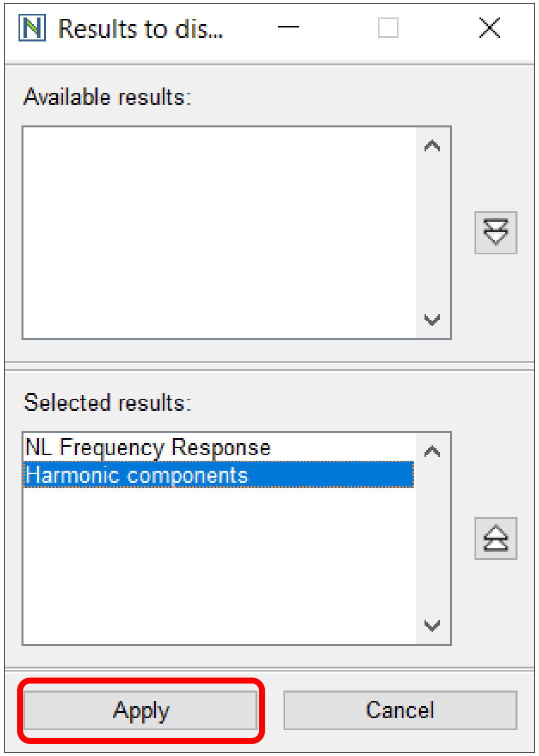
- Go back to the NFRC with a 'right click'.



Analysis of the NFRC

- Among the results to display, select the evolution of the harmonic components along the curve.





Analysis of the NFRC

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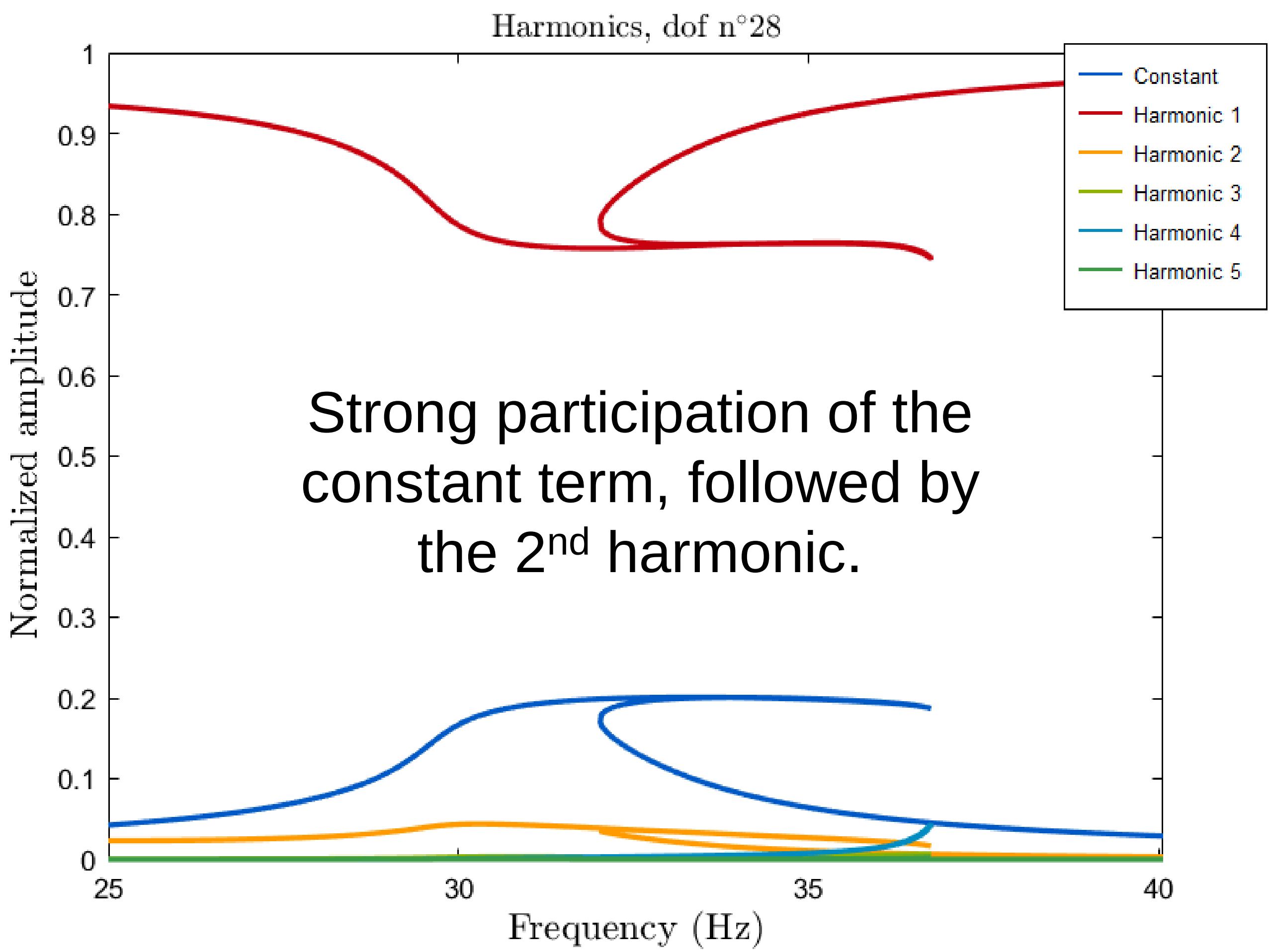
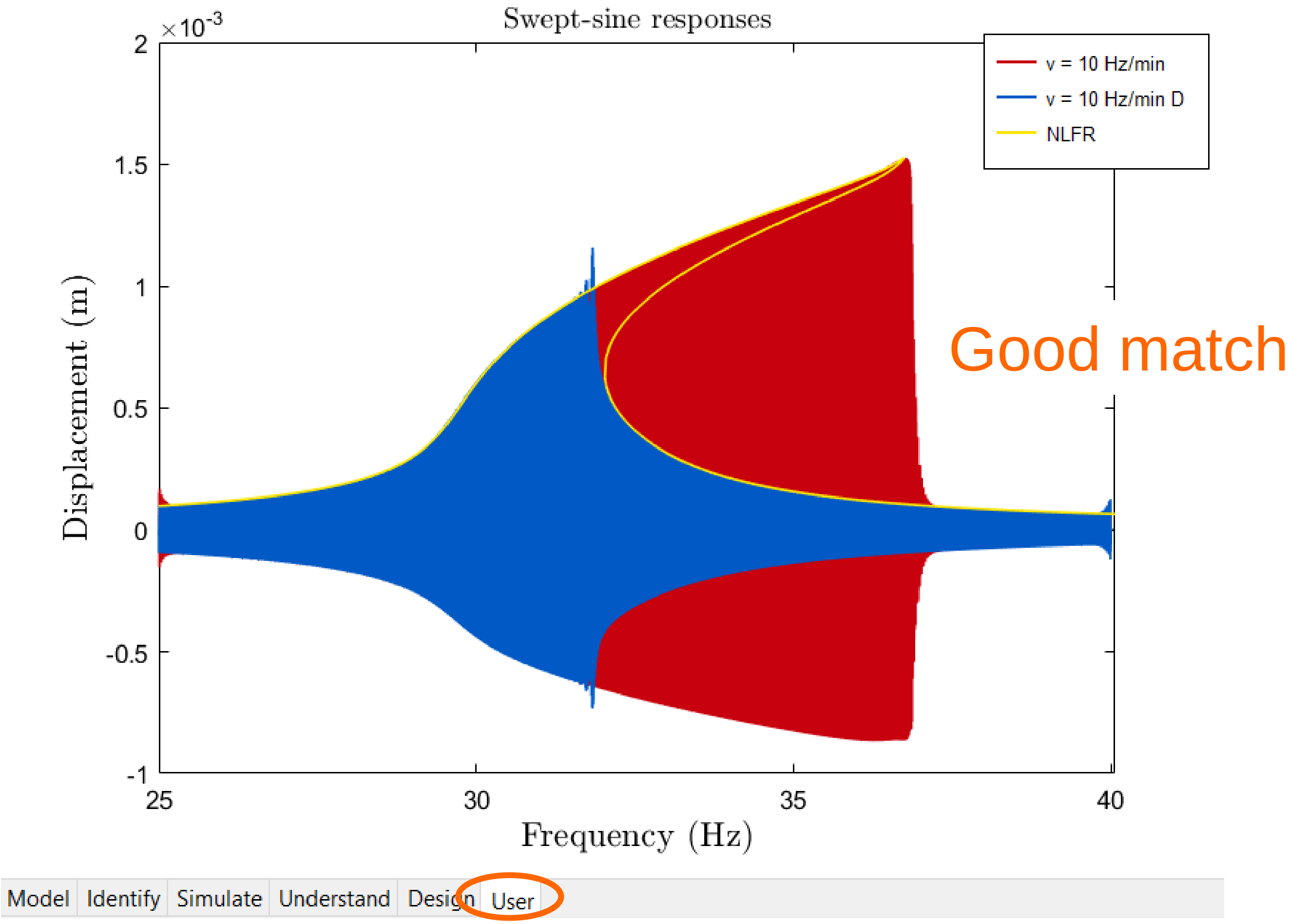


Strong participation of the constant term, followed by the 2nd harmonic.

Analysis of the NFRC

- Add the nonlinear frequency response in the stack curves of the swept-sine responses.



Conclusions

- A small step size is necessary for accurate time integrations (sampling frequency should be approximately 200 times higher than the frequency of interest).
- A small sweep rate is necessary to accurately represent amplitude jumps up and down.
- Sine and swept-sine excitations can reveal coexisting solutions.