Nonlinear Vibrations of Aerospace Structures

University of Liège, Belgium

T07 | Bifurcations

Continuation Parameters Bifurcation Analysis



Nonlinear Identification to Design Software Get familiar with NI2D tools for numerical continuation and bifurcation analysis:

Understand the influence of continuation parameters.

Detect and analyze bifurcations.

Case Study: 2-DOF System With Cubic Nonlinearities

 Create the following 2-DOF system and name it "2DOF_cubic".



NI2D: New model						
Spring/mass system MCK matrices Finite element model LMS model Measured signals						
Nun	nber of masses:	2				
	Mass:	1	Kg			
	Linear damping:	0.002	N.s/m			
l	inear stiffness:	1	N/m			

Select the following parameters.

HB continuation parameters			
Starting point:	0.8	rad/s	d/s
Hz Min:	0	rad/s	d/s N Harmonic Balance parameters
Max:	1.4	rad/s	d/s Number of harmonics: 4 5
Direction:	• • • •		Number of points: 4 512
Fold:	detect localize		Compute stability Reordering
Branch point:	detect localize		Linear mode:
Neimark-Sacker:	detect localize		Amplitude of 1st guess: 1e-005 m
Stepsize:	0.1		Maximum number of iterations: 15
Adaptative Min:	0.01		Relative precision: 1e-006
Мах:	0.05		Scaling factor for displacements: 1
Optimal number of iterations:	3		Scaling factor for time: 1
Maximum number of points:	10000		Apply Cancel
Beta angle:	90	•	
HB parameters App	ly Start	C	Cancel

• Start the continuation.



 Play with the maximum stepsize parameter and analyze its influence on the frequency response.

HB continuation parameters					
Starting point:		0.8	rad/s		
Hz Min:		0	rad/s		
Max:		1.4	rad/s		
Direction:	•-	•			
Fold:	detect	localize			
Branch point:	detect	localize			
Neimark-Sacker:	detect	localize			
Stepsize:		0.1			
Adaptative Min:		0.01			
Max:	<	0.05			
Optimal number of iterations:		3			
Maximum number of points:		10000			
Beta angle:		90	0		
HB parameters App	y (Start	Cancel		

Try 0.05, 0.1, 1, etc.

 Play with the maximum stepsize parameter and analyze its influence on the frequency response.



 Play with the optimal number of iterations parameter and analyze its influence on the frequency response.

HB continuation parameter	S			
Starting point:		0.8	rad/s	
Hz Min:		0	rad/s	
Max:		1.4	rad/s	
Direction:	•-	•		
Fold:	detect	localize		
Branch point:	detect	localize		
Neimark-Sacker:	detect	localize		
Stepsize:		0.1]	
Adaptative Min:		0.01]	Set max. stepsize equal to 1.
Max:		1		
Optimal number of iterations:		3	>	Try 1, 3, 5, etc.
Maximum number of points:		10000]	
Beta angle:		90	 0	
HB parameters App	у	Start	Cancel	

• Play with the optimal number of iterations parameter and analyze its influence on the frequency response.



 Play with the number of harmonics parameter and analyze its influence on the frequency response.

HB continuation paramete	rs		
Starting point:	0.8	rad/s	Try 1 3 5 etc
Hz Min:	0	rad/s	Try 1, 5, 5, 60.
Max:	1.4	rad/s	Harmonic Balance parameters
Direction:	• • • • • • • • • • • • • • • • • • •		Number of harmonics: 💶 🕟 5
Fold:	detect localize		Number of points: 4 512
Branch point:	detect localize		Compute stability Reordering
Neimark-Sacker:	detect localize		Linear mode:
Stepsize:	0.1		Amplitude of 1st guess: 1e-05 m
Adaptative Min:	0.01		Maximum number of iterations: 15
Max:	0.05		Relative precision: 1e-06
Optimal number of iterations:	3		Scaling factor for 1
Maximum number of points:	10000		Scaling factor for time: 1
Beta angle:	90	0	Apply Cancel
HB parameters App	ly Start	Cancel	

 Play with the number of harmonics parameter and analyze its influence on the frequency response.



• Play with the number of time samples parameter and analyze its influence on the frequency response.

HB continuation paramet	ers		
Starting point	. 0.8	rad/s	Trv 32 64 256 etc
🔲 Hz Mir	: 0	rad/s	Try 52, 64, 250, 610.
Max	: 1.4	rad/s	Harmonic Balance parameters
Direction	: • +		Number of harmonics:
Fold	: detect localize		Number of points: 512
Branch point	: 🔲 detect 📃 localize		Compute stability Reordering
Neimark-Sacke	detect localize		Linear mode:
Stepsize	: 0.1		Amplitude of 1st guess: 1e-05 m
Adaptative Mir	: 0.01		Maximum number of iterations: 15
Max			Relative precision: 1e-06
Optimal number of iterations	: 3		Scaling factor for 1
Maximum number of points	: 10000		Scaling factor for time: 1
Beta angle	90	0	Apply Cancel
HB parameters Ap	ply Start	Cancel	

• Play with the number of time samples parameter and analyze its influence on the frequency response.



 Analyze the evolution of Floquet exponents/multipliers in the vicinity of the stable/unstable transitions.



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We observe different types of transition between stable/unstable regions. Are these related to bifurcations?

• Select bifurcation detection and localization in the HB continuation parameters, and recalculate the NFRC.

HB continuation parameters		
Starting point:	0.8	rad/s
Hz Min:	0	rad/s
Max:	1.4	rad/s
Direction:	• - · ·	
Fold:	📝 detect 🛛 📝 localize	
Branch point:	✓ detect ✓ localize	
Neimark-Sacker:	detect 🔽 localize	
Stepsize:	0.1]
Adaptative Min:	0.01]
Max:	0.05]
Optimal number of iterations:	3]
Maximum number of points:	10000]
Beta angle:	90	•
HB parameters Appl	y Start	Cancel



Fold and Neimark-Sacker (NS) bifurcations are detected.



What is the influence of the first NS bifurcation on the system's response to sine excitations?

- Impose sine excitations to the system with frequencies of $\omega = 1.1 \text{ rad/s}$
- Use Newmark's scheme with an integration time of 3000 s.

External force on dof n°1		Newmark parameters [SINE]
Sine Sweep Random User		Final time: 3000 sec
Amplitude: 0.11	N	Time step: 0.01 sec
Frequency: 1.1/2/pi	Hz	Number of time steps: 300000
Phase: 0	•	Number of periods: 100
		Time steps by period: 90
		Saved dofs: (2) all (2) selected
Apply Newmark (F5)	Cancel	Initial cond. Apply Run (F5) Cancel

• Repeat the procedure for $\omega = 1.12$ rad/s.

Before the NS bifurcation, a periodic response is stabilized.



After the NS bifurcation, a quasiperiodic response is stabilized.

Because of the NS bifurcation, periodic solutions lose stability, and a branch of Neimark-Sacker oscillations emanates.

